Intro Physics Semester I **Name:**

Homework 13: Gravity: One or two full answer questions will be marked. There will also be a mark for completeness. Homeworks are due usually the day after the chapter they are for is finished. Solutions will be posted soon thereafter. The solutions are intended to be (but not necessarily are) super-perfect and often go beyond a fully correct answer.

Answer Table

Name:

	a	b	c	d	e
1.	O	O	O	O	O
2.	O	O	O	O	O
3.	O	O	O	O	O
4.	O	O	O	O	O
5.	O	O	O	O	O
6.	O	O	O	O	O
7.	O	O	O	O	O
8.	O	O	O	O	O
9.	O	O	O	O	O
10.	O	O	O	O	O
11.	O	O	O	O	O
12.	O	O	O	O	O
13.	O	O	O	O	O
14.	O	O	O	O	O
15.	O	O	O	O	O
16.	O	O	O	O	O
17.	O	O	O	O	O
18.	O	O	O	O	O
19.	O	O	O	O	O
20.	O	O	O	O	O
21.	O	O	O	O	O
22.	O	O	O	O	O
23.	O	O	O	O	O
24.	O	O	O	O	O
25.	O	O	O	O	O
26.	O	O	O	O	O
27.	O	O	O	O	O
28.	O	O	O	O	O
29.	O	O	O	O	O
30.	O	O	O	O	O

	a	b	c	d	e
31.	O	O	O	O	O
32.	O	O	O	O	O
33.	O	O	O	O	O
34.	O	O	O	O	O
35.	O	O	O	O	O
36.	O	O	O	O	O
37.	O	O	O	O	O
38.	O	O	O	O	O
39.	O	O	O	O	O
40.	O	O	O	O	O
41.	O	O	O	O	O
42.	O	O	O	O	O
43.	O	O	O	O	O
44.	O	O	O	O	O
45.	O	O	O	O	O
46.	O	O	O	O	O
47.	O	O	O	O	O
48.	O	O	O	O	O
49.	O	O	O	O	O
50.	O	O	O	O	O
51.	O	O	O	O	O
52.	O	O	O	O	O
53.	O	O	O	O	O
54.	O	O	O	O	O
55.	O	O	O	O	O
56.	O	O	O	O	O
57.	O	O	O	O	O
58.	O	O	O	O	O
59.	O	O	O	O	O
60.	O	O	O	O	O

1. "Let's play *Jeopardy*! For \$100, the answer is: He/she discovered the gravitation law (AKA law of universal gravitation) of classical physics. This law shows that the same gravity that holds on Earth also holds throughout the space—insofar as classical physics applies."

   Who is ____________, Alex?

   a) Galileo (1564–1642) b) Isaac Newton (1643–1727) c) James Clark Maxwell (1831–1879) d) Albert Einstein (1879–1955) e) Emmy Noether (1882–1935)

2. William Stukeley (1687–1765) recorded a conversation with Newton at Kensington, 1726 April 15 (less than year before Newton's death at 84):

   "when formerly, the notion of gravitation came into his mind. It was occasioned by the fall of a/an __________, as he sat in contemplative mood. Why should that ____________ always descend perpendicularly to the ground, thought he to himself. Why should it not go sideways or upwards, but constantly to the earth's centre."

   a) peach b) pear c) apple d) orange e) sparrow

3. Gravity is the force between systems with mass and it is:

   a) always attractive. b) always **REPULSIVE**, except perhaps in some cosmological applications. c) either attractive or **REPULSIVE**. d) neither attractive nor **REPULSIVE**. e) neither fish nor fowl.

4. Newton's law of universal gravitation for the force exerted by point mass 1 on point mass 2 (where from 1 to 2 is indicated by subscript 12) is:

   a) $\vec{F}_{12} = -Gm_1m_2r^2\hat{r}_{12}$. b) $\vec{F}_{12} = -\dfrac{Gm_1}{m_2}r^2\hat{r}_{12}$. c) $\vec{F}_{12} = -\dfrac{Gm_1}{m_2}r\hat{r}_{12}$. d) $\vec{F}_{12} = -\dfrac{Gm_1m_2}{r^2}\hat{r}_{12}$. e) $\vec{F}_{12} = -\dfrac{Gm_1m_2}{r^3}\hat{r}_{12}$.

5. Newton's law of gravity is:

   a) inconsistent with Newton's 3rd law of motion. b) consistent with Newton's 3rd law of motion. c) violates Newton's 3rd law of motion. d) Newton's 3rd law of motion. e) Newton's 2nd law of motion.

6. "Let's play *Jeopardy*! For \$100, the answer is: It is the gravitational constant with MKS units $\mathrm{N\,m^2/kg^2}$ It is actually the poorest known of the fundamental constants because gravity is such a weak force between laboratory size objects which are used to measure it."

   What is ____________, Alex?

   a) $1.000\ldots$ b) $2.99792458 \times 10^{-8}$ c) 2.99792458×10^{8} d) $6.67384(80) \times 10^{11}$ e) $6.67384(80) \times 10^{-11}$

7. Newton's classical gravity law is written for point masses. But this presents a paradox since point masses, at least classical point masses (and maybe not even quantum mechanical point masses, but who knows) do not really exist. The paradox is resolved by regarding the law as an ideal limiting form from which actual gravitational forces between masses can be calculated. Four cases of the application of Newton's gravity law come to mind for the situation of two objects.

   1. The general case: Here one makes the classical continuum assumption for matter and integrates up over the matter of the attractor and matter of the attractee in order to find the net force of the attractor on the attractee. As an equation for attractor (object 1) and attractee (object 2), one has

   $$\vec{F}_{12} = -\int\int \frac{G\rho(\vec{r}_1)\rho(\vec{r}_2)}{|\vec{r}_2 - \vec{r}_1|^2} \frac{(\vec{r}_2 - \vec{r}_1)}{|\vec{r}_2 - \vec{r}_1|}\, dV_1\, dV_2 \ ,$$

   where $\vec{F}_{12}$ is the force of object 1 on object 2, $G = 6.67384(80) \times 10^{-11}\,\mathrm{N\,m^2/kg^2}$ (Wikipedia: Gravitational constant 2012mar30) is the gravitational constant, ρ designates density, $\vec{r}_1$ is the position vector for object 1, $\vec{r}_2$ is the position vector for object 1, and the integrals are over the volumes V_1 and V_2 of objects 1 and 2. If one can't make the continuum approximation for some reason, a double summation over point particles can be done.

2. The case for general objects that are relatively far apart: Here the size scales of the two objects are both much smaller than the distance between them. In this case, Newton's gravity law applies directly as an approximation taking some fiducial points in the objects as being their equivalent point locations. The approximation becomes better, the greater the relative distance and the more cleverly the fiducial points are chosen (e.g., their centers of mass, but there may be even better choices), and approaches being exact in the limit that the relative distance goes to infinity. We will not prove this approximation.

3. The case of spherically symmetric objects that are not overlapping: It can be proven that the gravitational field of a spherically symmetric object outside of the object is exactly the field that a point mass would give according to Newton's gravity law if the point mass had the same mass as the object and was located where the object has its center. (The gravitational field is the force per unit mass on a point mass.) The proof is by Gauss's law for gravitation which itself is proven from Newton's law. Then using the logic and Newton's 3rd law, one can prove that non-overlapping spherically symmetric objects attract each other exactly as if they were point masses of the sort specified. Newton himself first proved this result. It was an essential result in understanding the motions of the larger solar system bodies which are nearly spherically symmetric.

4. The case of a relatively large, spherically symmetric object and a relatively small general object outside of the spherically symmetric object. In this case, Newton's gravity law approximately applies as if both objects were point masses: the large object's equivalent point mass is at its center and has its mass and the small object's equivalent point is located at some fiducial point inside of it (its center of mass usually being the best choice) and its mass. For planet and human size objects, the application of gravitation law is virtually exact. This case follows directly from:

   a) Newton's 1st law. b) Newton's 2nd law. c) Newton's 3rd law. d) cases 2 and 3. e) case 3.

8. A fiducial gravitational force is the force between two non-overlapping spherically symmetric objects each of mass 1 kg with the center-to-center distance one meter. The magnitude of this force is:

   a) 1. b) 1/2. c) 6.67428×10^{11}. d) 1.67×10^{-11}. e) 6.67384×10^{-11}.

9. Using Newton's gravitation law

$$\vec{F}_{12} = -\frac{Gm_1m_2}{r^2}\hat{r}_{12}$$

   ($G = 6.67384(80) \times 10^{-11}\,\mathrm{N\,m^2/kg^2}$: e.g., Wikipedia: Gravitational constant, 2012mar30) calculate the magnitude of the force between two 3 kg objects 3 m apart. The answer is:

   a) $60.1 \times 10^{-11} \approx 12 \times 10^{-11}$ lb. b) $20.0 \times 10^{-11} \approx 4 \times 10^{-11}$ lb.
   c) $6.674 \times 10^{-11}\,\mathrm{N} \approx 1.5 \times 10^{-11}$ lb. d) $2.224 \times 10^{-11}\,\mathrm{N} \approx 0.5 \times 10^{-11}$ lb.
   e) $0.741 \times 10^{-11}\,\mathrm{N} \approx 0.15 \times 10^{-11}$ lb.

10. Two of the basic constituents of ordinary matter are the electron (mass $m = 9.10938215(45) \times 10^{-31}$ kg, charge $q = -1.602176487(40) \times 10^{-19}$ C) and the proton (mass $m = 1.672621636(83) \times 10^{-27}$ kg, charge $q = 1.602176487(40) \times 10^{-19}$ C). The magnitude of the electric force between these particles is given by Coulomb's law

$$F = \frac{kq_1q_2}{r_{12}^2} ,$$

   where q_1 the charge on particle 1, q_2 the charge on particle 2, r_{12} is the distance between the particles, and $k = 8.987551787\ldots \times 10^9$ in MKS units is Coulomb's constant. What is the ratio of the magnitude of the gravitational force between the particles to the Coulomb force between?

   a) $1.000\ldots$. b) 2.3×10^{-39}. c) 2.3×10^{39}. d) 4.4×10^{40}. e) 4.4×10^{-40}.

11. "Let's play *Jeopardy*! For \$100, the answer is: This force is essential in making the large-scale structure of objects (of the kind we know) from middling asteroids to super-clusters of galaxies and perhaps to the universe as whole."

   a) What is ____________, Alex?

   a) a contact force b) friction c) a tension force d) a normal force e) gravity

12. Consider two spherically symmetric objects and the gravitational force between them. What is force for separation r relative to the force for separation r_0 (i.e., $F(r)/F(r_0)$?

a) $(r/r_0)^2$. b) r/r_0. c) r_0/r. d) $(r_0/r)^2$. e) $(r/r_0)^4$.

13. Given two spherically symmetric objects, if their separation is doubled, the force of gravity between them ____________ and, alternatively, if one object's mass is doubled, the force of gravity between them ____________.

    a) decreases by 1/4; is doubled b) decreases by 1/2; is doubled
    c) is doubled; decreases by 1/2 d) decreases by 1/4; increases by 4
    e) stays the same; stays the same

14. An object has mass x and weight y on the Earth's surface. What is its mass and weight at 2 Earth radii above the Earth's surface? Note **ABOVE** the Earth's surface, not **FROM** the Earth's center.

    a) x and $y/2$. b) $x/2$ and $y/2$. c) x and $y/9$. d) x and $y/4$. e) $x/9$ and $y/9$.

15. "Let's play *Jeopardy*! For \$100, the answer is: It can be defined as the gravitational force per unit mass at any point in space. But more fundamentally, it is a vector field that is the cause of the gravitational force: the conventional symbol for this vector field is $\vec{g}$. In modern, physics force fields are real things that are spread through space and cause forces."

    What is ____________, Alex?

    a) the gravitational force b) gravity c) levity d) the gravitational field
    e) Gauss's law

16. The usual symbol for the gravitational field is $\vec{g}$ which has magnitude g. A special case of the gravitational field is the one near the Earth's surface. For this case, the gravitational field has the fiducial magnitude $g = 9.8\,\mathrm{N/kg}$ and is often called little g—big G is the gravitational constant. Context must decide if g means the general gravitational field magnitude or the special case of the gravitational field magnitude near the Earth's surface. The gravitational force on a point mass m in a general gravitational field $\vec{g}$ is given by ____________. The same formula applies to an extended object provided the field is uniform over the extent of the object.

    a) $\vec{F} = \vec{g}/m$ b) $\vec{F} = m/\vec{g}$ c) $\vec{F} = m\vec{g}$ d) $\vec{F} = m^2\vec{g}$ e) $\vec{F} = \vec{g}/m^2$

17. The gravitational field
$$\vec{g} = -\frac{Gm}{r^2}\hat{r}$$
    is that of a:

    a) point mass at the origin. b) cubical mass centered on the origin.
    c) tetrahedral mass centered on the origin. d) dodecahedral mass centered on the origin.
    e) point mass **NOT** at the origin.

18. The gravitational field
$$\vec{g} = -\frac{Gm}{r^2}\hat{r}$$
    is that of spherically symmetric mass distribution centered on the origin where m is all the mass within the sphere of radius ____________. This field result can be easily derived from Gauss's law for gravitation.

    a) $2r$. b) r^2. c) $3r$. d) r. e) $1/r^2$.

19. Given the Earth's mass $M_{\mathrm{Ea}} = 5.9736 \times 10^{24}\,\mathrm{kg}$ and mean radius $R_{\mathrm{Ea}} = 6.37101 \times 10^6\,\mathrm{m}$ (e.g., Wikipedia 2007sep25), compute the magnitude of the Earth's gravitational field (force per unit mass) at the mean Earth's surface assuming spherical symmetry. The formula for the magnitude of gravitational field outside of a spherically-symmetric body of mass M is
$$g = \frac{GM}{r^2}$$
    where r the distance from the body center and the current favored value for the gravitational constant $G = 6.67384(80) \times 10^{-11}\,\mathrm{N\,m^2/kg^2}$ (e.g., Wikipedia: Gravitational constant 2012mar30). By desimplifying the expression for g, one can create a fiducial-value (i.e., reference-value) expression for the magnitude of the gravitational field g in terms of the Earth's values used as fiducial values. Identify this expression.

a) $9.822\,\mathrm{N/kg} \times (m/M_{\mathrm{Ea}})(R_{\mathrm{Ea}}/r)^2$ b) $6.822\,\mathrm{N/kg} \times (m/M_{\mathrm{Ea}})(R_{\mathrm{Ea}}/r)$
c) $6.822\,\mathrm{N/kg} \times (M_{\mathrm{Ea}}/m)(R_{\mathrm{Ea}}/r)$ d) $9.822\,\mathrm{N/kg} \times (M_{\mathrm{Ea}}/m)(r/R_{\mathrm{Ea}})$
e) $9.700\,\mathrm{N/kg} \times (m/M_{\mathrm{Ea}})(r/R_{\mathrm{Ea}})$

20. Given that the Moon's mass $M_{\mathrm{Mo}} = 0.0123 \times M_{\mathrm{Ea}} \approx (1/80)M_{\mathrm{Ea}}$ and mean radius $R_{\mathrm{Ea}} = 0.273 \times R_{\mathrm{Ea}} \approx (1/4)R_{\mathrm{Ea}}$ (e.g., Wikipedia 2007sep25) (where M_{Ea} is the Earth's mass and R_{Ea} is the Earth's mean radius), compute to **3 SIGNIFICANT FIGURES** the gravitational field at the Moon's surface and give it in MKS and in Earth g-force (i.e., in units of $g_{\mathrm{Ea}} = 9.822\,\mathrm{N/kg}$, the Earth's surface mean gravitational field here with a subscript to differentiate it from the more general meaning of g).

a) $2\,\mathrm{N/kg}$; 1/5. b) $9.8225\,\mathrm{N/kg}$; 1.00. c) $1.62\,\mathrm{N/kg}$; 1/3. d) $1.62\,\mathrm{N/kg}$; 0.165.
e) $9.8225\,\mathrm{N/kg}$; 1/5.

21. Calculate the gravitational acceleration g for the Earth's surface given $G = 6.67384(80)\times10^{-11}\,\mathrm{N\,m^2/kg^2}$ (Wikipedia: Gravitational constant 2012mar30), Earth mass $M_{\mathrm{E}} = 5.9736 \times 10^{24}\,\mathrm{kg}$, and mean Earth radius $R_{\mathrm{E}} = 6.37101 \times 10^6\,\mathrm{m}$ (Wikipedia: 2007sep25). What is the percentage difference from the fiducial $g_{\mathrm{fid}} = 9.8\,\mathrm{m/s^2}$ that is commonly used.

Equation Sheet for Introductory Physics Calculus-Based

This equation sheet is intended for students writing tests or reviewing material. Therefore it is neither intended to be complete nor completely explicit. There are fewer symbols than variables, and so some symbols must be used for different things: context must distinguish.

The equations are mnemonic. Students are expected to understand how to interpret and use them.

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1 Constants

$$c = 2.99792458 \times 10^8\,\mathrm{m/s} \approx 2.998 \times 10^8\,\mathrm{m/s} \approx 3 \times 10^8\,\mathrm{m/s} \approx 1\,\mathrm{lyr/yr} \approx 1\,\mathrm{ft/ns} \qquad \text{exact by definition}$$

$$e = 1.602176487(40) \times 10^{-19}\,\mathrm{C}$$

$$G = 6.67384(80) \times 10^{-11}\,\mathrm{N\,m^2/kg^2} \qquad \text{(2012, CODATA)}$$

$$g = 9.8\,\mathrm{m/s^2} \qquad \text{fiducial value}$$

$$k = \frac{1}{4\pi\varepsilon_0} = 8.987551787\ldots \times 10^9 \approx 8.99 \times 10^9 \approx 10^{10}\,\mathrm{N\,m^2/C^2}\text{exact by definition}$$

$$k_{\mathrm{Boltzmann}} = 1.3806504(24) \times 10^{-23}\,\mathrm{J/K} = 0.8617343(15) \times 10^{-4}\,\mathrm{eV/K} \approx 10^{-4}\,\mathrm{eV/K}$$

$$m_e = 9.10938215(45) \times 10^{-31}\,\mathrm{kg} = 0.510998910(13)\,\mathrm{MeV}$$

$$m_p = 1.672621637(83) \times 10^{-27}\,\mathrm{kg} = 938.272013(23), \mathrm{MeV}$$

$$\varepsilon_0 = \frac{1}{\mu_0 c^2} = 8.8541878176\ldots \times 10^{-12}\,\mathrm{C^2/(N\,m^2)} \approx 10^{-11} \qquad \text{vacuum permittivity (exact by definition)}$$

$$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N/A^2} \qquad \text{exact by definition}$$

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2 Geometrical Formulae

$$C_{\mathrm{cir}} = 2\pi r \qquad A_{\mathrm{cir}} = \pi r^2 \qquad A_{\mathrm{sph}} = 4\pi r^2 \qquad V_{\mathrm{sph}} = \frac{4}{3}\pi r^3$$

$$\Omega_{\mathrm{sphere}} = 4\pi \qquad d\Omega = \sin\theta\, d\theta\, d\phi$$

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3 Trigonometry Formulae

$$\frac{x}{r} = \cos\theta \qquad \frac{y}{r} = \sin\theta \qquad \frac{y}{x} = \tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cos^2\theta + \sin^2\theta = 1$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$c^2 = a^2 + b^2 \qquad c = \sqrt{a^2 + b^2 - 2ab\cos\theta_c} \qquad \frac{\sin\theta_a}{a} = \frac{\sin\theta_b}{b} = \frac{\sin\theta_c}{c}$$

$$f(\theta) = f(\theta + 360^\circ)$$

$$\sin(\theta + 180^\circ) = -\sin(\theta) \qquad \cos(\theta + 180^\circ) = -\cos(\theta) \qquad \tan(\theta + 180^\circ) = \tan(\theta)$$

$$\sin(-\theta) = -\sin(\theta) \qquad \cos(-\theta) = \cos(\theta) \qquad \tan(-\theta) = -\tan(\theta)$$

$$\sin(\theta+90^\circ)=\cos(\theta) \qquad \cos(\theta+90^\circ)=-\sin(\theta) \qquad \tan(\theta+90^\circ)=-\tan(\theta)$$

$$\sin(180^\circ-\theta)=\sin(\theta) \qquad \cos(180^\circ-\theta)=-\cos(\theta) \qquad \tan(180^\circ-\theta)=-\tan(\theta)$$

$$\sin(90^\circ-\theta)=\cos(\theta) \qquad \cos(90^\circ-\theta)=\sin(\theta) \qquad \tan(90^\circ-\theta)=\frac{1}{\tan(\theta)}=\cot(\theta)$$

$$\sin(a+b)=\sin(a)\cos(b)+\cos(a)\sin(b) \qquad \cos(a+b)=\cos(a)\cos(b)-\sin(a)\sin(b)$$

$$\sin(2a)=2\sin(a)\cos(a) \qquad \cos(2a)=\cos^2(a)-\sin^2(a)$$

$$\sin(a)\sin(b)=\frac{1}{2}\left[\cos(a-b)-\cos(a+b)\right] \qquad \cos(a)\cos(b)=\frac{1}{2}\left[\cos(a-b)+\cos(a+b)\right]$$

$$\sin(a)\cos(b)=\frac{1}{2}\left[\sin(a-b)+\sin(a+b)\right]$$

$$\sin^2\theta=\frac{1}{2}[1-\cos(2\theta)] \qquad \cos^2\theta=\frac{1}{2}[1+\cos(2\theta)] \qquad \sin(a)\cos(a)=\frac{1}{2}\sin(2a)$$

$$\cos(x)-\cos(y)=-2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos(x)+\cos(y)=2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin(x)+\sin(y)=2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

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4 Approximation Formulae

$$\frac{\Delta f}{\Delta x}\approx\frac{df}{dx} \qquad \frac{1}{1-x}\approx 1+x\ :\ (x<<1)$$

$$\sin\theta\approx\theta \qquad \tan\theta\approx\theta \qquad \cos\theta\approx 1-\frac{1}{2}\theta^2 \qquad \text{all for } \theta<<1$$

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5 Quadratic Formula

If $\quad 0=ax^2+bx+c\ ,\quad$ then $\quad x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}=-\dfrac{b}{2a}\pm\sqrt{\left(\dfrac{b}{2a}\right)^2-\dfrac{c}{a}}$

6 Vector Formulae

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2} \qquad \theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) + \pi? \qquad \vec{a} + \vec{b} = (a_x + b_x, a_y + b_y)$$

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} \qquad \phi = \tan^{-1}\left(\frac{a_y}{a_x}\right) + \pi? \qquad \theta = \cos^{-1}\left(\frac{a_z}{a}\right)$$

$$\vec{a} + \vec{b} = (a_x + b_x, a_y + b_y, a_z + b_z)$$

$$\vec{a} \cdot \vec{b} = ab\cos\theta = a_x b_x + a_y b_y + a_z b_z$$

$$\vec{c} = \vec{a} \times \vec{b} = ab\sin(\theta)\hat{c} = (a_y b_z - b_y a_z, a_z b_x - b_z a_x, a_x b_y - b_x a_y)$$

7 Differentiation and Integration Formulae

$$\frac{d(x^p)}{dx} = px^{p-1} \quad \text{except for } p = 0; \qquad \frac{d(x^0)}{dx} = 0 \qquad \frac{d(\ln|x|)}{dx} = \frac{1}{x}$$

Taylor's series
$$\begin{aligned} f(x) &= \sum_{n=0}^{\infty} \frac{(x-x_0)^n}{n!} f^{(n)}(x_0) \\ &= f(x_0) + (x-x_0)f^{(1)}(x_0) + \frac{(x-x_0)^2}{2!} f^{(2)}(x_0) + \frac{(x-x_0)^3}{3!} f^{(3)}(x_0) + \ldots \end{aligned}$$

$$\int_a^b f(x)\,dx = F(x)|_a^b = F(b) - F(a) \quad \text{where} \quad \frac{dF(x)}{dx} = f(x)$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad \text{except for } n = -1; \qquad \int \frac{1}{x}\,dx = \ln|x|$$

8 One-Dimensional Kinematics

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} \qquad v = \frac{dx}{dt} \qquad a_{\text{avg}} = \frac{\Delta v}{\Delta t} \qquad a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

$$v = at + v_0 \qquad x = \frac{1}{2}at^2 + v_0 t + x_0 \qquad v^2 = v_0^2 + 2a(x - x_0)$$

$$x = \frac{1}{2}(v_0 + v)t + x_0 \qquad x = -\frac{1}{2}at^2 + vt + x_0 \qquad g = 9.8\,\mathrm{m/s^2}$$

$$x_{\text{rel}} = x_2 - x_1 \qquad v_{\text{rel}} = v_2 - v_1 \qquad a_{\text{rel}} = a_2 - a_1$$

$$x' = x - v_{\text{frame}} t \qquad v' = v - v_{\text{frame}} \qquad a' = a$$

9 Two- and Three-Dimensional Kinematics: General

$$\vec{v}_{\text{avg}} = \frac{\Delta \vec{r}}{\Delta t} \qquad \vec{v} = \frac{d\vec{r}}{dt} \qquad \vec{a}_{\text{avg}} = \frac{\Delta \vec{v}}{\Delta t} \qquad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

10 Projectile Motion

$$x = v_{x,0} t \qquad y = -\frac{1}{2} g t^2 + v_{y,0} t + y_0 \qquad v_{x,0} = v_0 \cos\theta \qquad v_{y,0} = v_0 \sin\theta$$

$$t = \frac{x}{v_{x,0}} = \frac{x}{v_0 \cos\theta} \qquad y = y_0 + x \tan\theta - \frac{x^2 g}{2 v_0^2 \cos^2\theta}$$

$$x_{\text{for } y \text{ max}} = \frac{v_0^2 \sin\theta \cos\theta}{g} \qquad y_{\text{max}} = y_0 + \frac{v_0^2 \sin^2\theta}{2g}$$

$$x(y = y_0) = \frac{2 v_0^2 \sin\theta \cos\theta}{g} = \frac{v_0^2 \sin(2\theta)}{g} \qquad \theta_{\text{for max}} = \frac{\pi}{4} \qquad x_{\text{max}}(y = y_0) = \frac{v_0^2}{g}$$

$$x(\theta = 0) = \pm v_0 \sqrt{\frac{2(y_0 - y)}{g}} \qquad t(\theta = 0) = \sqrt{\frac{2(y_0 - y)}{g}}$$

11 Relative Motion

$$\vec{r} = \vec{r}_2 - \vec{r}_1 \qquad \vec{v} = \vec{v}_2 - \vec{v}_1 \qquad \vec{a} = \vec{a}_2 - \vec{a}_1$$

12 Polar Coordinate Motion and Uniform Circular Motion

$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\vec{r} = r\hat{r} \qquad \vec{v} = \frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{r} + r\omega\hat{\theta} \qquad \vec{a} = \frac{d^2\vec{r}}{dt^2} = \left(\frac{d^2 r}{dt^2} - r\omega^2\right)\hat{r} + \left(r\alpha + 2\frac{dr}{dt}\omega\right)\hat{\theta}$$

$$\vec{v} = r\omega\hat{\theta} \qquad v = r\omega \qquad a_{\tan} = r\alpha$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} = -r\omega^2\hat{r} \qquad a_{\text{centripetal}} = \frac{v^2}{r} = r\omega^2 = v\omega$$

13 **Very Basic Newtonian Physics**

$$\vec{r}_{\rm cm} = \frac{\sum_i m_i \vec{r}_i}{m_{\rm total}} = \frac{\sum_{\rm sub} m_{\rm sub} \vec{r}_{\rm cm\ sub}}{m_{\rm total}} \qquad \vec{v}_{\rm cm} = \frac{\sum_i m_i \vec{v}_i}{m_{\rm total}} \qquad \vec{a}_{\rm cm} = \frac{\sum_i m_i \vec{a}_i}{m_{\rm total}}$$

$$\vec{r}_{\rm cm} = \frac{\int_V \rho(\vec{r})\vec{r}\, dV}{m_{\rm total}}$$

$$\vec{F}_{\rm net} = m\vec{a} \qquad \vec{F}_{21} = -\vec{F}_{12} \qquad F_g = mg \qquad g = 9.8\,{\rm m/s^2}$$

$$\vec{F}_{\rm normal} = -\vec{F}_{\rm applied} \qquad F_{\rm linear} = -kx$$

$$f_{\rm normal} = \frac{T}{r} \qquad T = T_0 - F_{\rm parallel}(s) \qquad T = T_0$$

$$F_{\rm f\ static} = \min(F_{\rm applied}, F_{\rm f\ static\ max}) \qquad F_{\rm f\ static\ max} = \mu_{\rm static} F_{\rm N} \qquad F_{\rm f\ kinetic} = \mu_{\rm kinetic} F_{\rm N}$$

$$v_{\rm tangential} = r\omega = r\frac{d\theta}{dt} \qquad a_{\rm tangential} = r\alpha = r\frac{d\omega}{dt} = r\frac{d^2\theta}{dt^2}$$

$$\vec{a}_{\rm centripetal} = -\frac{v^2}{r}\hat{r} \qquad \vec{F}_{\rm centripetal} = -m\frac{v^2}{r}\hat{r}$$

$$F_{\rm drag,lin} = bv \qquad v_{\rm T} = \frac{mg}{b} \qquad \tau = \frac{v_{\rm T}}{g} = \frac{m}{b} \qquad v = v_{\rm T}(1 - e^{-t/\tau})$$

$$F_{\rm drag,quad} = bv^2 = \frac{1}{2}C\rho A v^2 \qquad v_{\rm T} = \sqrt{\frac{mg}{b}}$$

14 **Energy and Work**

$$dW = \vec{F}\cdot d\vec{s} \qquad W = \int \vec{F}\cdot d\vec{s} \qquad KE = \frac{1}{2}mv^2 \qquad E_{\rm mechanical} = KE + PE$$

$$P_{\rm avg} = \frac{\Delta W}{\Delta t} \qquad P = \frac{dW}{dt} \qquad P = \vec{F}\cdot\vec{v}$$

$$\Delta KE = W_{\rm net} \quad \Delta PE_{\rm of\ a\ conservative\ force} = -W_{\rm by\ a\ conservative\ force} \quad \Delta E = W_{\rm nonconservative}$$

$$F = -\frac{dPE}{dx} \qquad \vec{F} = -\nabla PE \qquad PE = \frac{1}{2}kx^2 \qquad PE = mgy$$

15 **Momentum**

$$\vec{F}_{\text{net}} = m\vec{a}_{\text{cm}} \qquad \Delta KE_{\text{cm}} = W_{\text{net,external}} \qquad \Delta E_{\text{cm}} = W_{\text{not}}$$

$$\vec{p} = m\vec{v} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}_{\text{total}}}{dt}$$

$$m\vec{a}_{\text{cm}} = \vec{F}_{\text{net non-flux}} + (\vec{v}_{\text{flux}} - \vec{v}_{\text{cm}})\frac{dm}{dt} = \vec{F}_{\text{net non-flux}} + \vec{v}_{\text{rel}}\frac{dm}{dt}$$

$$v = v_0 + v_{\text{ex}} \ln\left(\frac{m_0}{m}\right) \qquad \text{rocket in free space}$$

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16 **Collisions**

$$\vec{I} = \int_{\Delta t} \vec{F}(t)\,dt \qquad \vec{F}_{\text{avg}} = \frac{\vec{I}}{\Delta t} \qquad \Delta p = \vec{I}_{\text{net}}$$

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f} \qquad \vec{v}_{\text{cm}} = \frac{\vec{p}_1 + \vec{p}_2}{m_{\text{total}}}$$

$$KE_{\text{total }f} = KE_{\text{total }i} \qquad \text{1-d Elastic Collision Expression}$$

$$v_{1'} = \frac{(m_1 - m_2)v_1 + 2m_2v_2}{m_1 + m_2} \qquad \text{1-d Elastic Collision Expression}$$

$$v_{2'} - v_{1'} = -(v_2 - v_1) \qquad v_{\text{rel}'} = -v_{\text{rel}} \qquad \text{1-d Elastic Collision Expressions}$$

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17 **Rotational Kinematics**

$$2\pi = 6.2831853\ldots \qquad \frac{1}{2\pi} = 0.15915494\ldots$$

$$\frac{180^\circ}{\pi} = 57.295779\ldots \approx 60^\circ \qquad \frac{\pi}{180^\circ} = 0.017453292\ldots \approx \frac{1}{60^\circ}$$

$$\theta = \frac{s}{r} \qquad \omega = \frac{d\theta}{dt} = \frac{v}{r} \qquad \alpha = \frac{d^2\theta}{dt^2} = \frac{d\omega}{dt} = \frac{a}{r} \qquad f = \frac{\omega}{2\pi} \qquad P = \frac{1}{f} = \frac{2\pi}{\omega}$$

$$\omega = \alpha t + \omega_0 \qquad \Delta\theta = \frac{1}{2}\alpha t^2 + \omega_0 t \qquad \omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

$$\Delta\theta = \frac{1}{2}(\omega_0 + \omega)t \qquad \Delta\theta = -\frac{1}{2}\alpha t^2 + \omega t$$

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18 **Rotational Dynamics**

$$\vec{L} = \vec{r} \times \vec{p} \qquad \vec{\tau} = \vec{r} \times \vec{F} \qquad \vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt}$$

$$L_z = RP_{xy} \sin \gamma_L \qquad \tau_z = RF_{xy} \sin \gamma_\tau \qquad L_z = I\omega \qquad \tau_{z,\text{net}} = I\alpha$$

$$I = \sum_i m_i R_i^2 \qquad I = \int R^2 \rho \, dV \qquad I_{\text{parallel axis}} = I_{\text{cm}} + mR_{\text{cm}}^2 \qquad I_z = I_x + I_y$$

$$I_{\text{cyl,shell,thin}} = MR^2 \qquad I_{\text{cyl}} = \frac{1}{2}MR^2 \qquad I_{\text{cyl,shell,thick}} = \frac{1}{2}M(R_1^2 + R_2^2)$$

$$I_{\text{rod,thin,cm}} = \frac{1}{12}ML^2 \qquad I_{\text{sph,solid}} = \frac{2}{5}MR^2 \qquad I_{\text{sph,shell,thin}} = \frac{2}{3}MR^2$$

$$a = \frac{g \sin \theta}{1 + I/(mr^2)}$$

$$KE_{\text{rot}} = \frac{1}{2}I\omega^2 \qquad dW = \tau_z \, d\theta \qquad P = \frac{dW}{dt} = \tau_z \omega$$

$$\Delta KE_{\text{rot}} = W_{\text{net}} = \int \tau_{z,\text{net}} \, d\theta \qquad \Delta PE_{\text{rot}} = -W = -\int \tau_{z,\text{con}} \, d\theta$$

$$\Delta E_{\text{rot}} = KE_{\text{rot}} + \Delta PE_{\text{rot}} = W_{\text{non,rot}} \qquad \Delta E = \Delta KE + KE_{\text{rot}} + \Delta PE = W_{\text{non}} + W_{\text{rot}}$$

19 Static Equilibrium

$$\vec{F}_{\text{ext,net}} = 0 \qquad \vec{\tau}_{\text{ext,net}} = 0 \qquad \vec{\tau}_{\text{ext,net}} = \tau'_{\text{ext,net}} \quad \text{if } F_{\text{ext,net}} = 0$$

$$0 = F_{\text{net } x} = \sum F_x \qquad 0 = F_{\text{net } y} = \sum F_y \qquad 0 = \tau_{\text{net}} = \sum \tau$$

20 Gravity

$$\vec{F}_{1 \text{ on } 2} = -\frac{Gm_1 m_2}{r_{12}^2}\hat{r}_{12} \qquad \vec{g} = -\frac{GM}{r^2}\hat{r} \qquad \oint \vec{g} \cdot d\vec{A} = -4\pi GM$$

$$PE = -\frac{Gm_1 m_2}{r_{12}} \qquad V = -\frac{GM}{r} \qquad v_{\text{escape}} = \sqrt{\frac{2GM}{r}} \qquad v_{\text{orbit}} = \sqrt{\frac{GM}{r}}$$

$$P^2 = \left(\frac{4\pi^2}{GM}\right) r^3 \qquad P = \left(\frac{2\pi}{\sqrt{GM}}\right) r^{3/2} \qquad \frac{dA}{dt} = \frac{1}{2}r^2\omega = \frac{L}{2m} = \text{Constant}$$

$$R_{\text{Earth,mean}} = 6371.0\,\text{km} \qquad R_{\text{Earth,equatorial}} = 6378.1\,\text{km} \qquad M_{\text{Earth}} = 5.9736 \times 10^{24}\,\text{kg}$$

$$R_{\text{Earth mean orbital radius}} = 1.495978875 \times 10^{11}\,\text{m} = 1.0000001124\,\text{AU} \approx 1.5 \times 10^{11}\,\text{m} \approx 1\,\text{AU}$$

$$R_{\text{Sun,equatorial}} = 6.955 \times 10^{8} \approx 109 \times R_{\text{Earth,equatorial}} \qquad M_{\text{Sun}} = 1.9891 \times 10^{30}\,\text{kg}$$