Intro Physics Semester I **Name:**

Homework 10: Rotational Kinematics: One or two full answer questions will be marked. There will also be a mark for completeness. Homeworks are due usually the day after the chapter they are for is finished. Solutions will be posted soon thereafter. The solutions are intended to be (but not necessarily are) super-perfect and often go beyond a fully correct answer.

Answer Table

Name:

	a	b	c	d	e		a	b	c	d	e
1.	O	O	O	O	O	31.	O	O	O	O	O
2.	O	O	O	O	O	32.	O	O	O	O	O
3.	O	O	O	O	O	33.	O	O	O	O	O
4.	O	O	O	O	O	34.	O	O	O	O	O
5.	O	O	O	O	O	35.	O	O	O	O	O
6.	O	O	O	O	O	36.	O	O	O	O	O
7.	O	O	O	O	O	37.	O	O	O	O	O
8.	O	O	O	O	O	38.	O	O	O	O	O
9.	O	O	O	O	O	39.	O	O	O	O	O
10.	O	O	O	O	O	40.	O	O	O	O	O
11.	O	O	O	O	O	41.	O	O	O	O	O
12.	O	O	O	O	O	42.	O	O	O	O	O
13.	O	O	O	O	O	43.	O	O	O	O	O
14.	O	O	O	O	O	44.	O	O	O	O	O
15.	O	O	O	O	O	45.	O	O	O	O	O
16.	O	O	O	O	O	46.	O	O	O	O	O
17.	O	O	O	O	O	47.	O	O	O	O	O
18.	O	O	O	O	O	48.	O	O	O	O	O
19.	O	O	O	O	O	49.	O	O	O	O	O
20.	O	O	O	O	O	50.	O	O	O	O	O
21.	O	O	O	O	O	51.	O	O	O	O	O
22.	O	O	O	O	O	52.	O	O	O	O	O
23.	O	O	O	O	O	53.	O	O	O	O	O
24.	O	O	O	O	O	54.	O	O	O	O	O
25.	O	O	O	O	O	55.	O	O	O	O	O
26.	O	O	O	O	O	56.	O	O	O	O	O
27.	O	O	O	O	O	57.	O	O	O	O	O
28.	O	O	O	O	O	58.	O	O	O	O	O
29.	O	O	O	O	O	59.	O	O	O	O	O
30.	O	O	O	O	O	60.	O	O	O	O	O

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004 qmult 00400 1 1 2 easy memory: dividing a circle

1. A circle can be divided into:

   a) 360 divisions only. b) any number of divisions you like. c) 2π divisions only.
   d) π divisions only. e) 360 or 2π divisions only.

   SUGGESTED ANSWER: (b)

   Wrong answers:
   a) A nonsense answer. **Redaction:** Jeffery, 2008jan01

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004 qmult 00410 1 1 2 easy memory: radians in a circle 1

2. How many radians are there in a circle?

   a) π. b) 2π. c) 3π. d) 360°. e) 360.

   SUGGESTED ANSWER: (b)

   Wrong answers:
   e) The trick answer.

   Redaction: Jeffery, 2001jan01

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004 qmult 00420 1 5 1 easy thinking: 24 factors in 360

3. The division of the circle into 360° was an arbitrary choice—and we don't know why. We just know the ancient Mesopotamian mathematicians and astronomers did it this way—you know Mesopotamia—ancient Iraq: "the cradle of civilization". Their choice was just adopted by the ancient Greeks and got passed on to us. In the French Revolutionary epoch, the decimal system was adopted for most measures, but the revolutionaries didn't get around (you might say) to the circle. We can guess that one reasons is that the ancient Mesopotamians had a preference for whole number arithmetic particularly in division and 360 has a lot of whole number factors. How many whole number (i.e., integer) factors does 360 have counting 1 and 360 itself?

   a) 24. b) 360. c) 6. d) 7. e) 12.

   SUGGESTED ANSWER: (a)

   Below are the whole number factors of 360 table format:

count	factor	complement factor
2	1	360
4	2	180
6	3	120
8	4	90
10	5	72
12	6	60
14	8	45
16	9	40
18	10	36
20	12	30
22	15	24
24	18	20

   Wrong answers:
   b) A specious guess.

   Redaction: Jeffery, 2008jan01

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004 qmult 00430 1 3 4 easy math: radian to degree conversion

4. What is the approximate conversion factor from radians to degrees?

   a) 1/60 degrees/radian. b) π degrees/radian. c) 2π degrees/radian.
   d) 60 degrees/radian. e) 360 degrees/radian.

SUGGESTED ANSWER: (d)

Behold

$$180^\circ = \pi \ , \qquad \text{and so} \qquad \frac{180^\circ}{\pi} \approx 57.2958 \approx 60\,\text{degrees/radian} \ .$$

Wrong answers:
a) Wrong conversion factor: this is for degrees to radians.

Redaction: Jeffery, 2008jan01

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004 qmult 00440 1 5 5 easy thinking: the 2 pi unit ti
5. There are 2π radians in a circle. It's rather inconvenient that this means that there are $2\pi = 6.2831853\ldots$ radians in a circle which is an irrational number. For convenience, we could use the revolution (with sympbol Rev: vocalized rev) as a new unit: $1\,\text{Rev} = 2\pi$. One hundredth of an Rv would be a:

a) exaRev. b) megaRev. c) kiloRev. d) deciRev. e) centiRev.

SUGGESTED ANSWER: (e)

I think the idea of revolutions makes sense to me. We could then drop this non-metric degree unit and use centiRevs (3.6°) and milliRevs (0.36°) for most purposes. But no one ever listens to me.

Wrong answers:
a) Eek, 10^{18} ti.

Redaction: Jeffery, 2008jan01

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004 qmult 00450 1 3 1 easy math: hand angular measure
6. Approximately, at arm's length a finger subtends 1°, a fist 10°, and a spread hand 18°. These numbers, of course, vary a bit depending on person and exactly how the operation is done. What are these angles approximately in radians?

a) 1/60, 1/6, and 1/3 radians. b) 60, 600, and 1800 radians.
c) $\pi/12$, $\pi/3$, and $\pi/2$ radians. d) $\pi/12$, $\pi/3$, and π radians. e) $\pi/12$, $\pi/3$, and 2π radians.

SUGGESTED ANSWER: (a)

I've used the conversion factor (π radians)/(180° approximated to (1 radian/60°.

Wrong answers:
b) This looks like a conversion from radians to degrees where one uses the approximate conversion factor 60 degrees/radian.

Redaction: Jeffery, 2008jan01

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004 qmult 00460 1 5 5 easy thinking: covering the Moon
7. Can you cover the Moon with your finger held at arm's length? **HINT:** You could try for yourself if you are not in a a test *mise en scène*.

a) No. The Moon is much larger in angle than a finger. Just think how huge the Moon looks on the horizon sometimes.
b) It depends critically on the size of one's finger and arm. People with huge hands can to it and those without can't.
c) Yes. A finger at arm's length typically subtends about 10° and the Moon subtends 0.01°.
d) No. The Moon's diameter is about 3470 km and a finger is about a centimeter or so in width.
e) Usually yes. A finger at arm's length typically subtends about 1° and the Moon subtends 0.5°.

SUGGESTED ANSWER: (e)

Wrong answers:
d) Yes, this makes sense.

Redaction: Jeffery, 2008jan01

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004 qmult 00470 1 1 5 easy memory: small angle approximations

8. For small angles θ measured in radians and with increasing accuracy as θ goes to zero (where the formulas are in fact exact), one has the small angle approximations:

   a) $\sin\theta \approx \cos\theta \approx 1 - \frac{1}{2}\theta^2$. b) $\cos\theta \approx \tan\theta \approx 1 - \frac{1}{2}\theta^2$. c) $\sin\theta \approx \cos\theta \approx \theta$.
   d) $\cos\theta \approx \tan\theta \approx \theta$. e) $\sin\theta \approx \tan\theta \approx \theta$.

   SUGGESTED ANSWER: (e)

   The proof of these approximations follows from the Taylor expansions of sine and tangent about $\theta = 0$: i.e.,

$$\sin\theta = \sum_{n=0} \frac{1}{(2n+1)!}\theta^{2n+1} = \theta - \frac{1}{6}\theta^3 + \frac{1}{120}\theta^5 - \frac{1}{5040}\theta^7 + \dots ,$$
$$\tan\theta = \theta + \frac{1}{3}\theta^3 + \frac{2}{15}\theta^5 + \frac{17}{315}\theta^7 + \dots ,$$

   where one can get these from

   `http://en.wikipedia.org/wiki/Tangent_function#Series_definitions` .

   Wrong answers:
   a) One has $\cos\theta \approx 1 - (1/2)\theta^2$ for small angles, in fact.
   b) One has $\cos\theta \approx 1 - (1/2)\theta^2$ for small angles, in fact.

   Redaction: Jeffery, 2008jan01

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011 qmult 00100 1 4 4 easy deducto-memory: rotational kinematics

9. "Let's play *Jeopardy*! For $100, the answer is: It is the study of the description of rotational motion or angular motion without reference to forces or torques."

   What is ____________, Alex?

   a) translational dynamics b) rotational dynamics c) translational kinematics
   d) rotational kinematics e) rotational freight

   SUGGESTED ANSWER: (d)

   Wrong answers:
   c) Exactly wrong.
   e) A nonsense answer.

   Redaction: Jeffery, 2008jan01

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011 qmult 00110 1 1 3 easy memory: 1-d rotational kinematics

10. The rotational kinematics analog to 1-dimensional translational (or rectilinear) motion is:

    a) general rotation. b) kernal rotation. c) rotation about a single fixed axis.
    d) rotation about two axes. e) rotation about three axes.

    SUGGESTED ANSWER: (c)

    Wrong answers:
    b) A nonsense answer, but better than colonel rotation.

    Redaction: Jeffery, 2008jan01

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011 qmult 00120 1 1 1 easy memory: angular displacement

11. A change in angular position of a point is a/an:

    a) angular displacement. b) angular velocity. c) angular acceleration.
    d) translational velocity. e) translational acceleration.

    SUGGESTED ANSWER: (a)

    Wrong answers:
    b) Nah.

Redaction: Jeffery, 2008jan01

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011 qmult 00130 1 1 2 easy memory: angular variables

12. The usual symbols for the angular kinematic variables angular displacement, angular velocity, and angular acceleration all for rotational motion around a single fixed axis are, respectively:

    a) α, β, γ. b) θ, ω, α. c) θ, α, ω. d) ω, θ, α. e) δ, ϵ, ζ.

    SUGGESTED ANSWER: (b)

    Wrong answers:
    a) Just the first three small Greek letters.

    Redaction: Jeffery, 2008jan01

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011 qmult 00140 1 1 3 easy memory: angular velocity defined

13. About a single fixed axis,

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} \qquad \text{or, if you know calculus,} \qquad \omega = \frac{d\theta}{dt}$$

    is the definition of:

    a) angular displacement. b) angular acceleration. c) angular velocity.
    d) angular momentum. e) torque.

    SUGGESTED ANSWER: (c)

    Wrong answers:
    a) Nope.

    Redaction: Jeffery, 2008jan01

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011 qmult 00150 1 1 2 easy memory: angular acceleration defined

14. About a single fixed axis,

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} \qquad \text{or, if you know calculus,} \qquad \alpha = \frac{d\omega}{dt}$$

    is the definition of

    a) angular displacement. b) angular acceleration. c) angular velocity.
    d) angular momentum. e) torque.

    SUGGESTED ANSWER: (b)

    Wrong answers:
    a) Nope.

    Redaction: Jeffery, 2008jan01

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011 qmult 00180 1 1 3 easy memory: rotational and tangential variables

15. The formulas

$$\Delta s = \Delta\theta r \ , \qquad v = \omega r \ , \qquad a = \alpha r$$

    (where r is the radius of circular motion) relate the rotational variables to the analog:

    a) units. b) transgressional variables. c) tangential variables. d) angular units.
    e) translational units.

    SUGGESTED ANSWER: (c)

    Wrong answers:
    b) Translational variables are arguably correct, but not the best answer in this context, but I thought better not provoke any discussion anyway.

    Redaction: Jeffery, 2008jan01

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011 qmult 00200 1 4 2 easy deducto-memory: rotational kinematic equations

16. The rotational constant-angular-acceleration kinematic equations:

   a) have no resemblance to the linear kinematic equations.
   b) are exactly the same as the linear kinematic equations, except that the angular kinematic equations relate **ANGULAR** rather than linear variables.
   c) are exactly the same as the linear kinematic equations, except that the angular kinematic equations relate **LINEAR** rather than angular variables.
   d) do not allow for angular acceleration.
   e) include torque terms.

   SUGGESTED ANSWER: (b)

   Wrong answers:
   c) Oh, c'mon.

   Redaction: Jeffery, 2001jan01

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011 qmult 00202 1 1 4 easy memory: independent rot. kin. eq.

17. The number of **ALGEBRAICALLY INDEPENDENT** rotational constant-angular-acceleration equations is:

   a) 5. b) 4. c) 3. d) 2. e) 1.

   SUGGESTED ANSWER: (d)

   Wrong answers:
   a) In my formulation there are 5 equations, but only 2 are independent.

   Redaction: Jeffery, 2008jan01

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011 qmult 00232 1 1 5 easy memory: timeless equation use

18. The rotational constant-angular-acceleration equation

$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

   by itself alone does **NEVER** allows you to solve for:

   a) α. b) $\Delta\theta$. c) ω_0. d) ω. e) t.

   SUGGESTED ANSWER: (e)

   The equation is what I call the timeless equation.

   Wrong answers:
   a) Given the other 3 variables that appear in the equation you can solve for this.

   Redaction: Jeffery, 2008jan01

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011 qmult 00310 1 3 3 easy math: find the initial omega 1

19. A wheel spins π radians in 10.0 s with an angular acceleration of 4.00 radians/s^2. What is its final angular velocity?

   a) 80.1 radians/s. b) 203 radians/s. c) 20.3 radians/s. d) 3.14 radians/s. e) 6.28 radians/s.

   SUGGESTED ANSWER: (c)

   Of the 5 standard variables (α, ω_0, ω, $\Delta\theta$, t) of the (constant-angular-acceleration) rotational kinematic equations, we don't know ω nor ω_0. We don't want to know ω_0. This looks like a job for the rarely used 5th rotational constant-acceleration kinematic equation

$$\Delta\theta = -\frac{1}{2}\alpha t^2 + \omega t$$

since it doesn't contain the unwanted variable ω_0, and so allows a solution for the unknown ω from one equation. Behold:

$$\omega = \frac{\Delta\theta + (1/2)\alpha t^2}{t} = \frac{\pi + 200}{10.0} = 20.3\,\text{radians/s} .$$

Wrong answers:
b) Forgot to divide by time.

Fortran-95 Code
```
         print*
         pi_con=acos(-1.d0)
         theta=pi_con
         t=10.d0
         alpha=4.d0
   ! theta=-(1/2)*alpha*t**2+omega*t ! Rarely used 5th kinematic equation.
   ! omega=(theta+(1/2)*alpha*t**2)/t
         omega=(theta+.5d0*alpha*t**2)/t
         print*,'omega'
         print*,omega
   !  20.3141592653590
```

Redaction: Jeffery, 2008jan01

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011 qmult 00440 2 3 3 mod. math: discus centripetal and tangential acceleration

20. A discus thrower (in Greek, a *Diskobolos*) accelerates a discus from rest through π radians accelerating it from angular velocity 0 to angular velocity 15 rad/s. We idealize his motion as circular with radius $r = 0.81$ m. The time of the throwing motion is 0.27 s. What is the net **TRANSLATIONAL** acceleration (not tangential acceleration) of the discus at the end of the throwing motion just before release? Give the direction of acceleration relative to the inward radial direction.

a) 200 m/s^2; 10°. b) 13.9 m/s^2; 188°. c) 188 m/s^2; 13.9°. d) 150 m/s^2; 15°. e) 100 m/s^2; 50°.

SUGGESTED ANSWER: (c)

Behold:

$$\alpha = \frac{\omega - \omega_0}{t} , \qquad a_{\text{tan}} = \alpha r , \qquad a_{\text{cen}} = \omega^2 r ,$$

where a_{tan} is the tangential acceleration and a_{cen} is the centripetal acceleration. We can then find the acceleration magnitude from Pythagorean theorem:

$$a = \sqrt{a_{\text{cen}}^2 + a_{\text{tan}}^2} = 188\,\text{m/s}^2 .$$

The angular direction ϕ is given by

$$\phi = \tan^{-1}\left(\frac{a_{\text{tan}}}{a_{\text{cen}}}\right) = 13.9^\circ .$$

Wrong answers:
a) A nonsense answer.

Fortran-95 Code
```
         print*
         omega0=0.d0
         omega=15.d0
         t=.27d0
         r=.81d0
         alpha=(omega-omega0)/t
         at=alpha*r
```

```
        ac=omega**2*r
        a=sqrt(ac**2+at**2)
        phi=atan(at/ac)*raddeg
        print*,'a,phi'
        print*,a,phi
!  187.72336695254534 13.869686438505038
```

Redaction: Jeffery, 2008jan01

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011 qfull 00330 1 3 0 easy math: Waldo centrifuge, Waldo orbit

Extra keywords: just rotational kinematics.

21. Waldo's back—you know, Waldo Pepper, the Playful Pig—just accept it. This time, the Bold Boar has decided to become an astronaut and is training on NASA's giant centrifuge—the one in the film *The Right Stuff*. Let's guess it has a radius of 10 m. The centrifuge spins in the horizontal: i.e., the centrifuge axis is perpendicular to the ground.

a) Starting from rest the centrifuge goes into a constant angular acceleration phase for 10 s. At this point Waldo—who does indeed have a mass of 150 kg—notes that the vertical weighing scale he is nauseatingly pressed on reads 3000 N. What is the centripetal force on Waldo and what kind of force is the centripetal force?

b) The Sentient Swine now does some math—some correct math. What does Waldo find for his tangential velocity at the 10 s mark? What does he find for his angular velocity at the 10 s mark?

c) Doing a little more correct math, Waldo now finds his angular acceleration from time zero to the 10 s mark. What is this angular acceleration?

d) At this point, the Shaken Bacon does a little Gedanken experiment—just like Einstein—"what if I instantaneously left NASA's big blender and found myself 1000 **KILOMETERS** from the center of Pluto (formerly the 9th planet) and moving perpendicularly to the line to Pluto in free space, but with my current velocity?" What in this case would Waldo's angular velocity be about Pluto—for an instant since after that the Heck-of-a-Hog knows his orbital path will almost certainly not be circular.

SUGGESTED ANSWER:

a) Waldo's pressing on the weighing scale causes it to read 3000 N means that the weighting scale is forcing Waldo into circular motion with a force of 3000 N. The weighing scale force is a normal force. At least it can be called that viewing it from the outside. Internally, the force is probably a spring force of some kind. Note that there must be some normal force by the floor to hold Waldo up against gravity too, but that doesn't come into the problem.

b) Well from

$$F_{\text{centripetal}} = \frac{mv^2}{r} ,$$

Waldo finds his tangential velocity to be

$$v = \sqrt{\frac{rF}{m}} = \sqrt{\frac{10 \times 3000}{150}} = \sqrt{200} \approx 14.142\,\text{m/s}$$

and his angular velocity is

$$\omega = \frac{v}{r} \approx 1.4142\,\text{radians/s} .$$

c) Well we have the angular kinematic equation with constant acceleration

$$\omega = \alpha t + \omega_0 ,$$

where ω is the angular velocity, α is the angular acceleration, t is time, and ω_0 is the initial angular velocity. Rearranging and putting in the numbers, Waldo finds

$$\alpha = \frac{\omega - \omega_0}{t} \approx \frac{1.4142}{10} = 0.14142\,\text{radians/s}^2 .$$

d) Behold:

$$\omega = \frac{v}{r} \approx \frac{14.142}{10^6} = 1.4142 \times 10^{-5}\,\text{radians/s} .$$

Our porcellian friend has actually goofed. At 1000 km from Pluto's center, he is actually embedded in Pluto which has a mean radius of 1195 km (Wikipedia 2007nov07). But it's only a Gedanken experiment.

Waldo probably was thinking of 1000 km from Pluto's surface: i.e., 2195 km from Pluto's center. In this case, he would be in an orbit—probably a pretty measly orbit with an initial speed of only 10 m/s which sounds pretty small for orbiting any significant celestial body. I bet Waldo would go into a narrow nearly parabolic orbit and crash onto Pluto. For a circular orbit, Waldo would need

$$v = \sqrt{\frac{GM}{r}} \approx \sqrt{\frac{7 \times 10^{-11} \times 1.3 \times 10^{22}}{2.2 \times 10^6}} \approx 6 \times 10^2 = 600\,\text{m/s}$$

(using values from Wikipedia, 2007nov07). Well that's not vastly bigger than Waldo's speed. But I still think there's no chance. He'd crash. Of course, it's all a dream: he's still orbiting in the centrifuge.

Redaction: Jeffery, 2008jan01

Equation Sheet for Introductory Physics Calculus-Based

This equation sheet is intended for students writing tests or reviewing material. Therefore it is neither intended to be complete nor completely explicit. There are fewer symbols than variables, and so some symbols must be used for different things: context must distinguish.

The equations are mnemonic. Students are expected to understand how to interpret and use them.

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1 Constants

$$c = 2.99792458 \times 10^8\,\mathrm{m/s} \approx 2.998 \times 10^8\,\mathrm{m/s} \approx 3 \times 10^8\,\mathrm{m/s} \approx 1\,\mathrm{lyr/yr} \approx 1\,\mathrm{ft/ns} \qquad \text{exact by definition}$$

$$e = 1.602176487(40) \times 10^{-19}\,\mathrm{C}$$

$$G = 6.67428(67) \times 10^{-11}\,\mathrm{N\,m^2/kg^2} \qquad \text{(2006, CODATA)}$$

$$g = 9.8\,\mathrm{m/s^2} \qquad \text{fiducial value}$$

$$k = \frac{1}{4\pi\varepsilon_0} = 8.987551787\ldots \times 10^9 \approx 8.99 \times 10^9 \approx 10^{10}\,\mathrm{N\,m^2/C^2}\text{exact by definition}$$

$$k_{\text{Boltzmann}} = 1.3806504(24) \times 10^{-23}\,\mathrm{J/K} = 0.8617343(15) \times 10^{-4}\,\mathrm{eV/K} \approx 10^{-4}\,\mathrm{eV/K}$$

$$m_e = 9.10938215(45) \times 10^{-31}\,\mathrm{kg} = 0.510998910(13)\,\mathrm{MeV}$$

$$m_p = 1.672621637(83) \times 10^{-27}\,\mathrm{kg} = 938.272013(23), \mathrm{MeV}$$

$$\varepsilon_0 = \frac{1}{\mu_0 c^2} = 8.8541878176\ldots \times 10^{-12}\,\mathrm{C^2/(N\,m^2)} \approx 10^{-11} \qquad \text{vacuum permittivity (exact by definition)}$$

$$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N/A^2} \qquad \text{exact by definition}$$

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2 Geometrical Formulae

$$C_{\text{cir}} = 2\pi r \qquad A_{\text{cir}} = \pi r^2 \qquad A_{\text{sph}} = 4\pi r^2 \qquad V_{\text{sph}} = \frac{4}{3}\pi r^3$$

$$\Omega_{\text{sphere}} = 4\pi \qquad d\Omega = \sin\theta\, d\theta\, d\phi$$

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3 Trigonometry Formulae

$$\frac{x}{r} = \cos\theta \qquad \frac{y}{r} = \sin\theta \qquad \frac{y}{x} = \tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cos^2\theta + \sin^2\theta = 1$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$c^2 = a^2 + b^2 \qquad c = \sqrt{a^2 + b^2 - 2ab\cos\theta_c} \qquad \frac{\sin\theta_a}{a} = \frac{\sin\theta_b}{b} = \frac{\sin\theta_c}{c}$$

$$f(\theta) = f(\theta + 360^\circ)$$

$$\sin(\theta + 180^\circ) = -\sin(\theta) \qquad \cos(\theta + 180^\circ) = -\cos(\theta) \qquad \tan(\theta + 180^\circ) = \tan(\theta)$$

$$\sin(-\theta) = -\sin(\theta) \qquad \cos(-\theta) = \cos(\theta) \qquad \tan(-\theta) = -\tan(\theta)$$

$$\sin(\theta+90^\circ)=\cos(\theta) \qquad \cos(\theta+90^\circ)=-\sin(\theta) \qquad \tan(\theta+90^\circ)=-\tan(\theta)$$

$$\sin(180^\circ-\theta)=\sin(\theta) \qquad \cos(180^\circ-\theta)=-\cos(\theta) \qquad \tan(180^\circ-\theta)=-\tan(\theta)$$

$$\sin(90^\circ-\theta)=\cos(\theta) \qquad \cos(90^\circ-\theta)=\sin(\theta) \qquad \tan(90^\circ-\theta)=\frac{1}{\tan(\theta)}=\cot(\theta)$$

$$\sin(a+b)=\sin(a)\cos(b)+\cos(a)\sin(b) \qquad \cos(a+b)=\cos(a)\cos(b)-\sin(a)\sin(b)$$

$$\sin(2a)=2\sin(a)\cos(a) \qquad \cos(2a)=\cos^2(a)-\sin^2(a)$$

$$\sin(a)\sin(b)=\frac{1}{2}\left[\cos(a-b)-\cos(a+b)\right] \qquad \cos(a)\cos(b)=\frac{1}{2}\left[\cos(a-b)+\cos(a+b)\right]$$

$$\sin(a)\cos(b)=\frac{1}{2}\left[\sin(a-b)+\sin(a+b)\right]$$

$$\sin^2\theta=\frac{1}{2}[1-\cos(2\theta)] \qquad \cos^2\theta=\frac{1}{2}[1+\cos(2\theta)] \qquad \sin(a)\cos(a)=\frac{1}{2}\sin(2a)$$

$$\cos(x)-\cos(y)=-2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos(x)+\cos(y)=2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin(x)+\sin(y)=2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

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4 Approximation Formulae

$$\frac{\Delta f}{\Delta x}\approx\frac{df}{dx} \qquad \frac{1}{1-x}\approx 1+x\ :\ (x<<1)$$

$$\sin\theta\approx\theta \qquad \tan\theta\approx\theta \qquad \cos\theta\approx 1-\frac{1}{2}\theta^2 \qquad \text{all for } \theta<<1$$

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5 Quadratic Formula

If $\quad 0=ax^2+bx+c\ ,\quad$ then $\quad x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}=-\dfrac{b}{2a}\pm\sqrt{\left(\dfrac{b}{2a}\right)^2-\dfrac{c}{a}}$

6 Vector Formulae

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2} \qquad \theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) + \pi? \qquad \vec{a} + \vec{b} = (a_x + b_x, a_y + b_y)$$

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} \qquad \phi = \tan^{-1}\left(\frac{a_y}{a_x}\right) + \pi? \qquad \theta = \cos^{-1}\left(\frac{a_z}{a}\right)$$

$$\vec{a} + \vec{b} = (a_x + b_x, a_y + b_y, a_z + b_z)$$

$$\vec{a} \cdot \vec{b} = ab\cos\theta = a_x b_x + a_y b_y + a_z b_z$$

$$\vec{c} = \vec{a} \times \vec{b} = ab\sin(\theta)\hat{c} = (a_y b_z - b_y a_z, a_z b_x - b_z a_x, a_x b_y - b_x a_y)$$

7 Differentiation and Integration Formulae

$$\frac{d(x^p)}{dx} = px^{p-1} \quad \text{except for } p = 0; \qquad \frac{d(x^0)}{dx} = 0 \qquad \frac{d(\ln|x|)}{dx} = \frac{1}{x}$$

Taylor's series
$$\begin{aligned} f(x) &= \sum_{n=0}^{\infty} \frac{(x-x_0)^n}{n!} f^{(n)}(x_0) \\ &= f(x_0) + (x-x_0) f^{(1)}(x_0) + \frac{(x-x_0)^2}{2!} f^{(2)}(x_0) + \frac{(x-x_0)^3}{3!} f^{(3)}(x_0) + \ldots \end{aligned}$$

$$\int_a^b f(x)\,dx = F(x)|_a^b = F(b) - F(a) \quad \text{where} \quad \frac{dF(x)}{dx} = f(x)$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad \text{except for } n = -1; \qquad \int \frac{1}{x}\,dx = \ln|x|$$

8 One-Dimensional Kinematics

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} \qquad v = \frac{dx}{dt} \qquad a_{\text{avg}} = \frac{\Delta v}{\Delta t} \qquad a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

$$v = at + v_0 \qquad x = \frac{1}{2}at^2 + v_0 t + x_0 \qquad v^2 = v_0^2 + 2a(x - x_0)$$

$$x = \frac{1}{2}(v_0 + v)t + x_0 \qquad x = -\frac{1}{2}at^2 + vt + x_0 \qquad g = 9.8\,\mathrm{m/s^2}$$

$$x_{\text{rel}} = x_2 - x_1 \qquad v_{\text{rel}} = v_2 - v_1 \qquad a_{\text{rel}} = a_2 - a_1$$

$$x' = x - v_{\text{frame}}t \qquad v' = v - v_{\text{frame}} \qquad a' = a$$

9 Two- and Three-Dimensional Kinematics: General

$$\vec{v}_{\text{avg}} = \frac{\Delta\vec{r}}{\Delta t} \qquad \vec{v} = \frac{d\vec{r}}{dt} \qquad \vec{a}_{\text{avg}} = \frac{\Delta\vec{v}}{\Delta t} \qquad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

10 Projectile Motion

$$x = v_{x,0}t \qquad y = -\frac{1}{2}gt^2 + v_{y,0}t + y_0 \qquad v_{x,0} = v_0\cos\theta \qquad v_{y,0} = v_0\sin\theta$$

$$t = \frac{x}{v_{x,0}} = \frac{x}{v_0\cos\theta} \qquad y = y_0 + x\tan\theta - \frac{x^2 g}{2v_0^2\cos^2\theta}$$

$$x_{\text{for } y \text{ max}} = \frac{v_0^2\sin\theta\cos\theta}{g} \qquad y_{\text{max}} = y_0 + \frac{v_0^2\sin^2\theta}{2g}$$

$$x(y = y_0) = \frac{2v_0^2\sin\theta\cos\theta}{g} = \frac{v_0^2\sin(2\theta)}{g} \qquad \theta_{\text{for max}} = \frac{\pi}{4} \qquad x_{\text{max}}(y = y_0) = \frac{v_0^2}{g}$$

$$x(\theta = 0) = \pm v_0\sqrt{\frac{2(y_0 - y)}{g}} \qquad t(\theta = 0) = \sqrt{\frac{2(y_0 - y)}{g}}$$

11 Relative Motion

$$\vec{r} = \vec{r}_2 - \vec{r}_1 \qquad \vec{v} = \vec{v}_2 - \vec{v}_1 \qquad \vec{a} = \vec{a}_2 - \vec{a}_1$$

12 Polar Coordinate Motion and Uniform Circular Motion

$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\vec{r} = r\hat{r} \qquad \vec{v} = \frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{r} + r\omega\hat{\theta} \qquad \vec{a} = \frac{d^2\vec{r}}{dt^2} = \left(\frac{d^2r}{dt^2} - r\omega^2\right)\hat{r} + \left(r\alpha + 2\frac{dr}{dt}\omega\right)\hat{\theta}$$

$$\vec{v} = r\omega\hat{\theta} \qquad v = r\omega \qquad a_{\tan} = r\alpha$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} = -r\omega^2\hat{r} \qquad a_{\text{centripetal}} = \frac{v^2}{r} = r\omega^2 = v\omega$$

13 Very Basic Newtonian Physics

$$\vec{r}_{\text{cm}} = \frac{\sum_i m_i \vec{r}_i}{m_{\text{total}}} = \frac{\sum_{\text{sub}} m_{\text{sub}} \vec{r}_{\text{cm sub}}}{m_{\text{total}}} \qquad \vec{v}_{\text{cm}} = \frac{\sum_i m_i \vec{v}_i}{m_{\text{total}}} \qquad \vec{a}_{\text{cm}} = \frac{\sum_i m_i \vec{a}_i}{m_{\text{total}}}$$

$$\vec{r}_{\text{cm}} = \frac{\int_V \rho(\vec{r})\vec{r}\,dV}{m_{\text{total}}}$$

$$\vec{F}_{\text{net}} = m\vec{a} \qquad \vec{F}_{21} = -\vec{F}_{12} \qquad F_g = mg \qquad g = 9.8\,\text{m/s}^2$$

$$\vec{F}_{\text{normal}} = -\vec{F}_{\text{applied}} \qquad F_{\text{linear}} = -kx$$

$$f_{\text{normal}} = \frac{T}{r} \qquad T = T_0 - F_{\text{parallel}}(s) \qquad T = T_0$$

$$F_{\text{f static}} = \min(F_{\text{applied}}, F_{\text{f static max}}) \qquad F_{\text{f static max}} = \mu_{\text{static}} F_{\text{N}} \qquad F_{\text{f kinetic}} = \mu_{\text{kinetic}} F_{\text{N}}$$

$$v_{\text{tangential}} = r\omega = r\frac{d\theta}{dt} \qquad a_{\text{tangential}} = r\alpha = r\frac{d\omega}{dt} = r\frac{d^2\theta}{dt^2}$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} \qquad \vec{F}_{\text{centripetal}} = -m\frac{v^2}{r}\hat{r}$$

$$F_{\text{drag,lin}} = bv \qquad v_{\text{T}} = \frac{mg}{b} \qquad \tau = \frac{v_{\text{T}}}{g} = \frac{m}{b} \qquad v = v_{\text{T}}(1 - e^{-t/\tau})$$

$$F_{\text{drag,quad}} = bv^2 = \frac{1}{2}C\rho A v^2 \qquad v_{\text{T}} = \sqrt{\frac{mg}{b}}$$

14 Energy and Work

$$dW = \vec{F} \cdot d\vec{s} \qquad W = \int \vec{F} \cdot d\vec{s} \qquad KE = \frac{1}{2}mv^2 \qquad E_{\text{mechanical}} = KE + PE$$

$$P_{\text{avg}} = \frac{\Delta W}{\Delta t} \qquad P = \frac{dW}{dt} \qquad P = \vec{F} \cdot \vec{v}$$

$$\Delta KE = W_{\text{net}} \quad \Delta PE_{\text{of a conservative force}} = -W_{\text{by a conservative force}} \quad \Delta E = W_{\text{nonconservative}}$$

$$F = -\frac{dPE}{dx} \qquad \vec{F} = -\nabla PE \qquad PE = \frac{1}{2}kx^2 \qquad PE = mgy$$

15 Momentum

$$\vec{F}_{\rm net} = m\vec{a}_{\rm cm} \qquad \Delta KE_{\rm cm} = W_{\rm net,external} \qquad \Delta E_{\rm cm} = W_{\rm not}$$

$$\vec{p} = m\vec{v} \qquad \vec{F}_{\rm net} = \frac{d\vec{p}}{dt} \qquad \vec{F}_{\rm net} = \frac{d\vec{p}_{\rm total}}{dt}$$

$$m\vec{a}_{\rm cm} = \vec{F}_{\text{net non-flux}} + (\vec{v}_{\rm flux} - \vec{v}_{\rm cm})\frac{dm}{dt} = \vec{F}_{\text{net non-flux}} + \vec{v}_{\rm rel}\frac{dm}{dt}$$

$$v = v_0 + v_{\rm ex}\ln\left(\frac{m_0}{m}\right) \qquad \text{rocket in free space}$$

16 Collisions

$$\vec{I} = \int_{\Delta t} \vec{F}(t)\,dt \qquad \vec{F}_{\rm avg} = \frac{\vec{I}}{\Delta t} \qquad \Delta p = \vec{I}_{\rm net}$$

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f} \qquad \vec{v}_{\rm cm} = \frac{\vec{p}_1 + \vec{p}_2}{m_{\rm total}}$$

$$KE_{{\rm total}\ f} = KE_{{\rm total}\ i} \qquad \text{1-d Elastic Collision Expression}$$

$$v_{1'} = \frac{(m_1 - m_2)v_1 + 2m_2v_2}{m_1 + m_2} \qquad \text{1-d Elastic Collision Expression}$$

$$v_{2'} - v_{1'} = -(v_2 - v_1) \qquad v_{\rm rel'} = -v_{\rm rel} \qquad \text{1-d Elastic Collision Expressions}$$

17 Rotational Kinematics

$$2\pi = 6.2831853\ldots \qquad \frac{1}{2\pi} = 0.15915494\ldots$$

$$\frac{180^\circ}{\pi} = 57.295779\ldots \approx 60^\circ \qquad \frac{\pi}{180^\circ} = 0.017453292\ldots \approx \frac{1}{60^\circ}$$

$$\theta = \frac{s}{r} \qquad \omega = \frac{d\theta}{dt} = \frac{v}{r} \qquad \alpha = \frac{d^2\theta}{dt^2} = \frac{d\omega}{dt} = \frac{a}{r} \qquad f = \frac{\omega}{2\pi} \qquad P = \frac{1}{f} = \frac{2\pi}{\omega}$$

$$\omega = \alpha t + \omega_0 \qquad \Delta\theta = \frac{1}{2}\alpha t^2 + \omega_0 t \qquad \omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

$$\Delta\theta = \frac{1}{2}(\omega_0 + \omega)t \qquad \Delta\theta = -\frac{1}{2}\alpha t^2 + \omega t$$

18 Rotational Dynamics

$$\vec{L} = \vec{r} \times \vec{p} \qquad \vec{\tau} = \vec{r} \times \vec{F} \qquad \vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt}$$

$$L_z = RP_{xy} \sin \gamma_L \qquad \tau_z = RF_{xy} \sin \gamma_\tau \qquad L_z = I\omega \qquad \tau_{z,\text{net}} = I\alpha$$

$$I = \sum_i m_i R_i^2 \qquad I = \int R^2 \rho \, dV \qquad I_{\text{parallel axis}} = I_{\text{cm}} + mR_{\text{cm}}^2 \qquad I_z = I_x + I_y$$

$$I_{\text{cyl,shell,thin}} = MR^2 \qquad I_{\text{cyl}} = \frac{1}{2}MR^2 \qquad I_{\text{cyl,shell,thick}} = \frac{1}{2}M(R_1^2 + R_2^2)$$

$$I_{\text{rod,thin,cm}} = \frac{1}{12}ML^2 \qquad I_{\text{sph,solid}} = \frac{2}{5}MR^2 \qquad I_{\text{sph,shell,thin}} = \frac{2}{3}MR^2$$

$$a = \frac{g \sin \theta}{1 + I/(mr^2)}$$

$$KE_{\text{rot}} = \frac{1}{2}I\omega^2 \qquad dW = \tau_z \, d\theta \qquad P = \frac{dW}{dt} = \tau_z \omega$$

$$\Delta KE_{\text{rot}} = W_{\text{net}} = \int \tau_{z,\text{net}} \, d\theta \qquad \Delta PE_{\text{rot}} = -W = -\int \tau_{z,\text{con}} \, d\theta$$

$$\Delta E_{\text{rot}} = KE_{\text{rot}} + \Delta PE_{\text{rot}} = W_{\text{non,rot}} \qquad \Delta E = \Delta KE + KE_{\text{rot}} + \Delta PE = W_{\text{non}} + W_{\text{rot}}$$

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19 Static Equilibrium

$$\vec{F}_{\text{ext,net}} = 0 \qquad \vec{\tau}_{\text{ext,net}} = 0 \qquad \vec{\tau}_{\text{ext,net}} = \tau'_{\text{ext,net}} \qquad \text{if } F_{\text{ext,net}} = 0$$

$$0 = F_{\text{net } x} = \sum F_x \qquad 0 = F_{\text{net } y} = \sum F_y \qquad 0 = \tau_{\text{net}} = \sum \tau$$