Intro Physics Semester I **Name:**

Homework 8: Potential Energy and Mechanical Energy: One or two or no full answer questions will be marked. There will also be a mark for completeness. Homeworks are due usually the day after the chapter they are for is finished. Solutions will be posted soon thereafter. The solutions are intended to be (but not necessarily are) super-perfect and often go beyond a fully correct answer.

Answer Table

Name:

	a	b	c	d	e		a	b	c	d	e
1.	O	O	O	O	O	31.	O	O	O	O	O
2.	O	O	O	O	O	32.	O	O	O	O	O
3.	O	O	O	O	O	33.	O	O	O	O	O
4.	O	O	O	O	O	34.	O	O	O	O	O
5.	O	O	O	O	O	35.	O	O	O	O	O
6.	O	O	O	O	O	36.	O	O	O	O	O
7.	O	O	O	O	O	37.	O	O	O	O	O
8.	O	O	O	O	O	38.	O	O	O	O	O
9.	O	O	O	O	O	39.	O	O	O	O	O
10.	O	O	O	O	O	40.	O	O	O	O	O
11.	O	O	O	O	O	41.	O	O	O	O	O
12.	O	O	O	O	O	42.	O	O	O	O	O
13.	O	O	O	O	O	43.	O	O	O	O	O
14.	O	O	O	O	O	44.	O	O	O	O	O
15.	O	O	O	O	O	45.	O	O	O	O	O
16.	O	O	O	O	O	46.	O	O	O	O	O
17.	O	O	O	O	O	47.	O	O	O	O	O
18.	O	O	O	O	O	48.	O	O	O	O	O
19.	O	O	O	O	O	49.	O	O	O	O	O
20.	O	O	O	O	O	50.	O	O	O	O	O
21.	O	O	O	O	O	51.	O	O	O	O	O
22.	O	O	O	O	O	52.	O	O	O	O	O
23.	O	O	O	O	O	53.	O	O	O	O	O
24.	O	O	O	O	O	54.	O	O	O	O	O
25.	O	O	O	O	O	55.	O	O	O	O	O
26.	O	O	O	O	O	56.	O	O	O	O	O
27.	O	O	O	O	O	57.	O	O	O	O	O
28.	O	O	O	O	O	58.	O	O	O	O	O
29.	O	O	O	O	O	59.	O	O	O	O	O
30.	O	O	O	O	O	60.	O	O	O	O	O

1. Potential energy is:

   a) the energy of position: it exists for nonconservative forces.
   b) the energy of position: it exists for conservative forces.
   c) the energy of motion: its formula is $PE = (1/2)mv^2$.
   d) the energy of position: its formula is $PE = (1/2)mv^2$.
   e) heat energy.

2. The work done by a conservative force on an object while the object moves on a path between two endpoints is:

   a) **INDEPENDENT** of the path and endpoints.
   b) **DEPENDENT** on the path.
   c) **INDEPENDENT** of the path between the endpoints.
   d) **DEPENDENT** on the path, but **NOT** on the endpoints.
   e) equal to the path length.

3. "Let's play *Jeopardy*! For \$100, the answer is: $\Delta PE = -W$."

   a) What is the formula relating **POTENTIAL** energy change in a conservative force field to work done by the conservative force (i.e., what is the general potential energy formula), Alex?
   b) What is Faraday's law, Alex?
   c) What are capacitors, Alex?
   d) What is . . . no, no wait . . . what is unicorn circular motion, Alex?
   e) What is the formula relating **KINETIC** energy change in a conservative force field to work done by the conservative force (i.e., what is the work-kinetic-energy theorem), Alex?

4. "Let's play *Jeopardy*! For \$100, the answer is: Energy X for a force is an energy type defined, not by its particular intrinsic nature, but because its value for a body is set by the body's location in space. So energy X is a position energy—and probably should have been called that—but it's too late to change centuries of tradition. It is always true that in any real physical case of energy X, the energy is by its nature some kind of field energy: e.g., electric field energy, magnetic field energy, gravitational field energy. A particular field energy may be a potential energy or not depending on the actual system considered. Energy X for some unreal imagined kind of force does not have any more fundamental explanation—unless one imagines one."

   What is ____________, Alex?

   a) polecat b) pole energy c) potentate energy d) potent energy e) potential energy

5. Whether a force is conservative or not depends not only on the fundamental nature of the force, but also on the ____________ which is being considered. For example, the electric force is usually mentioned as a conservative force, but there are ____________ in which it is not: for example, those in which it is generated by the Maxwell-Faraday's law. For another example, the magnetic force is often mentioned as a non-conservative force, but there ____________ in which it is: for example, magnetic dipoles are subject to a conservative magnetic force.

   a) force/forces b) horse/horses c) law/laws d) motion/motions e) system/systems

6. Two forces that are conservative in ordinarily-thought-of systems are:

   a) power and might. b) gravity and kinetic friction. c) gravity and work.
   d) gravity and the linear (or spring) force. e) work and the linear (or spring) force.

7. British American Benjamin Thompson (1753–1814)—ennobled as Count Rumford—while employed as director of the Bavarian arsenal, noticed that in boring cannon—but not causing cannon ennui—that the boring motion and friction seemed to produce unlimited amounts of heat. He concluded:

   a) heat was a substance of which there could only be so much of in any object.
   b) that heat was somehow generated by motion and friction. This conclusion eventually led to the recognition of heat as another form of energy that could be converted from or converted into, e.g., mechanical or chemical energy and to the concept of conservation of energy.
   c) that heat had no relation to motion and friction and was somehow spontaneously generated by cannon.
   d) that cannon could be the plural of cannon.

e) that the biergartens in Munich were much better than the taverns in Boston and that Sam Adams, patriot-founding-father notwithstanding, could have learnt a thing or two about brewing beer.

8. "Let's play *Jeopardy*! For \$100, the answer is: $\Delta E = W_{\text{nonconservative}}$."

   What is the ____________, Alex?

   a) work-energy theorem b) work-kinetic-energy theorem c) potential-energy-work formula
   d) work-potential-energy theorem e) kinetic energy formula

9. Mechanical energy is the sum of kinetic energy and potential energy. It is a conserved quantity:

   a) always.
   b) whenever it has both kinetic and potential energy components.
   c) if all the forces that do net work are **NONCONSERVATIVE**.
   d) if all the forces that do net work are **CONSERVATIVE**.
   e) whenever it is positive.

10. Frequently, in conservation-of-mechanical energy problems, one encounters non-conservative forces that guide the motion and cause accelerations. Mechanical energy is conserved because these ____________ do work because they are always ____________ to the direction of motion. Actually, conservative forces can also be ____________ when they are ____________.

    a) work-doing constraint forces; parallel b) work-doing constraint forces; perpendicular
    c) workless constraint forces; parallel d) workless constraint forces; perpendicular
    e) worthless unconstrained forces; peculiar

11. A brick has mass 1 kg. A dog (from a joke that I'll tell you someday) drops the brick (which it was holding in its mouth or, one might say, with its jowl) 1 m. What is the kinetic energy of the brick just before it hits the ground? **HINT:** The calculator is superfluous.

    a) 9.8 watts. b) 9.8 gems. c) 9.8 newtons. d) 9.8 jowls. e) 9.8 joules.

12. A girl on a swing oscillates between being 2 m off the ground where she is ________ and 1 m off the ground where her speed is a ____________. No nonconservative forces do work. What is her maximum speed?

    a) moving; minimum; 0 m/s. b) at rest; minimum; 0 m/s. c) at rest; maximum; 1 m/s.
    d) at rest; maximum; 2.4 m/s. e) at rest; maximum; 4.4 m/s.

13. An object is trapped and moving around in some kind of potential well: it's a bound particle we'd say. What are those special points called where the kinetic energy of the particle momentarily goes to zero? Why are they so called?

    a) Turning points—so called because when a particle reaches one, it must come to rest and reverse direction.
    b) Stable static equilibrium points—so called because when a particle reaches one it stops.
    c) Stable static equilibrium points—so called because when a particle reaches one it accelerates.
    d) Unstable static equilibrium points—so called because when a particle reaches one it accelerates.
    e) Turning points—so called because of the film "The Turning Point" starring Shirley Maclaine, Anne Bancroft, and Mikhail Barishnykov.

14. If one makes a sufficiently small displacement from stable static equilibrium of almost any system a with smooth potential energy function and then lets the system evolve in isolation, the system will approximate a/an:

    a) simple harmonic oscillator. b) anharmonic oscillator. c) traveling wave.
    d) unstable equilibrium system. e) a vector field.

15. Why can't a practical balance scale ever be in perfect unstable static equilibrium when balancing?

    a) Nothing is perfect.
    b) Everything is perfect.
    c) There is always some frictional force that makes the balance position, however, slightly metastable or the scale never really balances: the perturbations just grow so slowly that one can make a

measurement before they become obvious. Of course, the less the stabilizing friction and the less the perturbations, the **MORE** exactly a mass can be determined.

d) There is always some frictional force that makes the balance position, however, slightly metastable or the scale never really balances: the perturbations just grow so slowly that one can make a measurement before they become obvious. Of course, the less the stabilizing friction and the less the perturbations, the **LESS** exactly a mass can be determined.

e) No one wants to.

16. Work per unit time or energy transformed per unit time is:

a) power. b) might. c) oomph. d) strength. e) pay.

17. If you could capture it all for useful work, the energy sunlight delivers to a square meter of ground would run one or two ordinary incandescent light bulbs. The power delivered by the Sun to a square meter of ground on average is to order or magnitude:

a) 1 W. b) 10 W. c) 100 W. d) 10^6 W. e) 1 MW.

18. A 50 kg boy runs up a flight of stairs of 5 m in height in 5 s at a constant rate. His power output just to work against gravity is:

a) 50 W. b) 490 W. c) 980 W. d) 10^6 W. e) 1 MW.

19. A 100 kg mountain climber climbs 4000 m in 10 hours. What is his power output going into gravitational potential energy? What is his total power output?

a) 3.92×10^6 W and 3.92×10^6 W.

b) The power going into gravitational potential energy is 109 W. His total power output cannot be exactly calculated since a lot of power must go into waste heat due to frictional forces and into the body heat which is lost to the environment. All one can easily say is that 109 W is a **LOWER BOUND** on the total power output.

c) The power going into gravitational potential energy is 3.92×10^6 W. His total power output cannot be exactly calculated since a lot of power must go into waste heat due to frictional forces and into the body heat which is lost to the environment. All one can easily say is that 3.92×10^6 W is a **LOWER BOUND** on the total power output.

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20. We will now prove the work-kinetic energy theorem. Don't panic.

a) Write down the work-kinetic-energy theorem.

b) Separate the work done W in the work-kinetic-energy theorem in to that done by conservative forces and that done by non-conservative forces. Nothing forbids us from doing this.

c) Use the general formula for the potential energy of conservative forces to eliminate the work done by the conservative forces.

d) Show that the sum of the changes in the kinetic and potential energies equals the work done by the non-conservative forces. This is the work-energy theorem.

e) Another form of the work-energy theorem formula is obtained by defining mechanical energy by

$$E = KE + PE \ .$$

Write it down.

f) If the non-conservative forces do no work—but they may be present as workless constraint forces—what is conserved? What is not necessarily conserved in this case.

21. You drop a 2.0 kg spanner from rest to a friend standing on the ground which 10 m below the drop height. She will catch the spanner 1.5 m above the ground. Neglect air drag.

    a) What's the work done by gravity in the drop?

    b) What's the change in gravitational potential energy in the drop?

    c) Using an energy conservation calculation find the speed of the spanner as it reaches your friend's hands.

22. This gentleman of fortune Daniel Goodwin in 1981 scaled the Sears Building in Chicago using suction cups and metal clips. The building is 443 m high. Let's guess he had a mass of 70 kg. Let us assume Dan is an **IDEAL** climber: i.e., all the body chemical energy he puts out goes into his own macroscopic kinetic energy or his own gravitational potential energy and into no other forms ever even eventually.

    a) How much body chemical energy did Dan expend getting to the top? He started from rest and ended at rest?

    b) If he'd just climbed the stairs, how much body chemical energy again starting and ending at rest.

    c) Why did he probably use a lot more body chemical energy than an ideal climber?

23. The Steel Dragon in Mie, Japan is one of the world's fastest and tallest roller coasters.

    a) Assuming only gravity does work on a coaster find the formula for its speed v at any height y given that its initial speed and height were, respectively, v_0 and y_0. **NOTE:** We actually have to assume that the coaster is a point mass. Otherwise, we would have to worry about the kinetic energy of its internal parts: i.e., its spinning wheels. Note we are neglecting friction and air drag.

    b) What does the normal force do in the roller coaster system? **NOTE:** We will assume that the tension in the chain or cable that pulls the coaster is negligible, but this might not be the actual case.

    c) Say that $v_0 = 3.0\,\mathrm{m/s}$ and $y_0 = 93.5\,\mathrm{m}$. What is the speed when $y = 0\,\mathrm{m}$?

    d) Why can't we calculate the time it takes for the coaster to go from height y_0 to y in the part (c) case?

    e) What is the **COMPONENT** of the force of gravity along the track direction and what is the **ACCELERATION** if only gravity is acting along the track direction? Take the angle of the track to the horizontal to be θ.

    f) Assuming for the part (c) question that the motion was all downhill and the displacement in the horizontal direction was about the same as in the vertical direction, estimate the travel time between the two locations.

24. A boll weevil of mass m is sitting on top of a hemispherical igloo of radius R. An infinitesimal perturbation starts him sliding down starting from speed **ZERO**. The igloo is frictionless and there is no air drag.

    a) Sketch a diagram of the system with the boll weevil at a general position on the igloo. Indicate angle θ and the forces that act on the boll weevil. Note, forces, not force components.

    b) Find an explicit formula for the boll weevil's speed on the igloo as a function of angle θ on the igloo measured from the vertical using conservation of mechanical energy. **SHOW** how you found the formula.

    c) At some height (measured from the ground) the boll weevil flies off the igloo. Find an explicit formulae for angle and the height at which the boll weevil flies off the igloo. **SHOW** how you found the formulae. **HINT:** This is purely a force analysis problem. Consider the normal force on the boll weevil and note that the radial (component of) acceleration is instantaneously given by the $a = -v^2/r$ just as for uniform circular motion even when v is not constant provided that r is constant. Recall v is just the tangential speed in the radial acceleration formula.

25. Dingo the daredevil dog (who has mass m), starting from **REST**, slides down a frictionless track from an initial height y_0. The track becomes level at $y = 0$, then goes into a circular loop of radius R, and

then goes level at $y = 0$ again. Dingo is actually a particle dog.

a) What is Dingo's speed at any height y assuming he stays on the track? Give the speed as function of y, y_0, g, and, if necessary, m.

b) Find a general formula for the normal force on Dingo when he is on the loop as a function of the angle θ between a general radial vector and the radial vector pointing to the top of the loop. Assume that the direction toward the center is the positive direction. The only variables in the formula should be y_0, R, m, g, and θ. Simplify as much as possible and take radially inward as positive. **HINT:** Note that the magnitude of the radial (component of) acceleration is instantaneously given by $a = v^2/r$ just as for uniform circular motion even when v is not constant provided that r is constant. Recall v is just the tangential speed in this expression and the direction of the radial acceleration is toward the center.

c) For what θ is the normal force formula smallest? What is the normal force at this angle?

d) What is the mathematical condition—sufficient, not just necessary—needed so that the normal force formula never specifies an attractive normal force (i.e., a force attracting Dingo to the track) anywhere on the loop? Explain why there is this condition. What happens to Dingo if the formula did specify an attractive normal force?

e) Say $y_0 = 50\,\mathrm{m}$ and the loop radius $R = 10\,\mathrm{m}$. Does Dingo stay on the loop? (A demonstration is needed, not just a yes or no answer. But it's not a long demonstration.)

Equation Sheet for Introductory Physics Calculus-Based

This equation sheet is intended for students writing tests or reviewing material. Therefore it is neither intended to be complete nor completely explicit. There are fewer symbols than variables, and so some symbols must be used for different things: context must distinguish.

The equations are mnemonic. Students are expected to understand how to interpret and use them.

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1 Constants

$$c = 2.99792458 \times 10^8\,\mathrm{m/s} \approx 2.998 \times 10^8\,\mathrm{m/s} \approx 3 \times 10^8\,\mathrm{m/s} \approx 1\,\mathrm{lyr/yr} \approx 1\,\mathrm{ft/ns} \qquad \text{exact by definition}$$

$$e = 1.602176487(40) \times 10^{-19}\,\mathrm{C}$$

$$G = 6.67428(67) \times 10^{-11}\,\mathrm{N\,m^2/kg^2} \qquad \text{(2006, CODATA)}$$

$$g = 9.8\,\mathrm{m/s^2} \qquad \text{fiducial value}$$

$$k = \frac{1}{4\pi\varepsilon_0} = 8.987551787\ldots \times 10^9 \approx 8.99 \times 10^9 \approx 10^{10}\,\mathrm{N\,m^2/C^2}\text{exact by definition}$$

$$k_{\mathrm{Boltzmann}} = 1.3806504(24) \times 10^{-23}\,\mathrm{J/K} = 0.8617343(15) \times 10^{-4}\,\mathrm{eV/K} \approx 10^{-4}\,\mathrm{eV/K}$$

$$m_e = 9.10938215(45) \times 10^{-31}\,\mathrm{kg} = 0.510998910(13)\,\mathrm{MeV}$$

$$m_p = 1.672621637(83) \times 10^{-27}\,\mathrm{kg} = 938.272013(23),\mathrm{MeV}$$

$$\varepsilon_0 = \frac{1}{\mu_0 c^2} = 8.8541878176\ldots \times 10^{-12}\,\mathrm{C^2/(N\,m^2)} \approx 10^{-11} \qquad \text{vacuum permittivity (exact by definition)}$$

$$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N/A^2} \qquad \text{exact by definition}$$

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2 Geometrical Formulae

$$C_{\mathrm{cir}} = 2\pi r \qquad A_{\mathrm{cir}} = \pi r^2 \qquad A_{\mathrm{sph}} = 4\pi r^2 \qquad V_{\mathrm{sph}} = \frac{4}{3}\pi r^3$$

$$\Omega_{\mathrm{sphere}} = 4\pi \qquad d\Omega = \sin\theta\,d\theta\,d\phi$$

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3 Trigonometry Formulae

$$\frac{x}{r} = \cos\theta \qquad \frac{y}{r} = \sin\theta \qquad \frac{y}{x} = \tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cos^2\theta + \sin^2\theta = 1$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$c^2 = a^2 + b^2 \qquad c = \sqrt{a^2 + b^2 - 2ab\cos\theta_c} \qquad \frac{\sin\theta_a}{a} = \frac{\sin\theta_b}{b} = \frac{\sin\theta_c}{c}$$

$$f(\theta) = f(\theta + 360^\circ)$$

$$\sin(\theta + 180^\circ) = -\sin(\theta) \qquad \cos(\theta + 180^\circ) = -\cos(\theta) \qquad \tan(\theta + 180^\circ) = \tan(\theta)$$

$$\sin(-\theta) = -\sin(\theta) \qquad \cos(-\theta) = \cos(\theta) \qquad \tan(-\theta) = -\tan(\theta)$$

$$\sin(\theta+90^\circ)=\cos(\theta) \qquad \cos(\theta+90^\circ)=-\sin(\theta) \qquad \tan(\theta+90^\circ)=-\tan(\theta)$$

$$\sin(180^\circ-\theta)=\sin(\theta) \qquad \cos(180^\circ-\theta)=-\cos(\theta) \qquad \tan(180^\circ-\theta)=-\tan(\theta)$$

$$\sin(90^\circ-\theta)=\cos(\theta) \qquad \cos(90^\circ-\theta)=\sin(\theta) \qquad \tan(90^\circ-\theta)=\frac{1}{\tan(\theta)}=\cot(\theta)$$

$$\sin(a+b)=\sin(a)\cos(b)+\cos(a)\sin(b) \qquad \cos(a+b)=\cos(a)\cos(b)-\sin(a)\sin(b)$$

$$\sin(2a)=2\sin(a)\cos(a) \qquad \cos(2a)=\cos^2(a)-\sin^2(a)$$

$$\sin(a)\sin(b)=\frac{1}{2}\left[\cos(a-b)-\cos(a+b)\right] \qquad \cos(a)\cos(b)=\frac{1}{2}\left[\cos(a-b)+\cos(a+b)\right]$$

$$\sin(a)\cos(b)=\frac{1}{2}\left[\sin(a-b)+\sin(a+b)\right]$$

$$\sin^2\theta=\frac{1}{2}[1-\cos(2\theta)] \qquad \cos^2\theta=\frac{1}{2}[1+\cos(2\theta)] \qquad \sin(a)\cos(a)=\frac{1}{2}\sin(2a)$$

$$\cos(x)-\cos(y)=-2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos(x)+\cos(y)=2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin(x)+\sin(y)=2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

4 Approximation Formulae

$$\frac{\Delta f}{\Delta x}\approx\frac{df}{dx} \qquad \frac{1}{1-x}\approx 1+x\ :\ (x<<1)$$

$$\sin\theta\approx\theta \qquad \tan\theta\approx\theta \qquad \cos\theta\approx 1-\frac{1}{2}\theta^2 \qquad \text{all for } \theta<<1$$

5 Quadratic Formula

$$\text{If} \qquad 0=ax^2+bx+c\ , \qquad \text{then} \qquad x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=-\frac{b}{2a}\pm\sqrt{\left(\frac{b}{2a}\right)^2-\frac{c}{a}}$$

6 Vector Formulae

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2} \qquad \theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) + \pi? \qquad \vec{a} + \vec{b} = (a_x + b_x, a_y + b_y)$$

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} \qquad \phi = \tan^{-1}\left(\frac{a_y}{a_x}\right) + \pi? \qquad \theta = \cos^{-1}\left(\frac{a_z}{a}\right)$$

$$\vec{a} + \vec{b} = (a_x + b_x, a_y + b_y, a_z + b_z)$$

$$\vec{a} \cdot \vec{b} = ab\cos\theta = a_x b_x + a_y b_y + a_z b_z$$

$$\vec{c} = \vec{a} \times \vec{b} = ab\sin(\theta)\hat{c} = (a_y b_z - b_y a_z, a_z b_x - b_z a_x, a_x b_y - b_x a_y)$$

7 Differentiation and Integration Formulae

$$\frac{d(x^p)}{dx} = px^{p-1} \quad \text{except for } p = 0; \qquad \frac{d(x^0)}{dx} = 0 \qquad \frac{d(\ln|x|)}{dx} = \frac{1}{x}$$

Taylor's series
$$\begin{aligned} f(x) &= \sum_{n=0}^{\infty} \frac{(x-x_0)^n}{n!} f^{(n)}(x_0) \\ &= f(x_0) + (x-x_0)f^{(1)}(x_0) + \frac{(x-x_0)^2}{2!} f^{(2)}(x_0) + \frac{(x-x_0)^3}{3!} f^{(3)}(x_0) + \ldots \end{aligned}$$

$$\int_a^b f(x)\,dx = F(x)|_a^b = F(b) - F(a) \quad \text{where} \quad \frac{dF(x)}{dx} = f(x)$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad \text{except for } n = -1; \qquad \int \frac{1}{x}\,dx = \ln|x|$$

8 One-Dimensional Kinematics

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} \qquad v = \frac{dx}{dt} \qquad a_{\text{avg}} = \frac{\Delta v}{\Delta t} \qquad a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

$$v = at + v_0 \qquad x = \frac{1}{2}at^2 + v_0 t + x_0 \qquad v^2 = v_0^2 + 2a(x - x_0)$$

$$x = \frac{1}{2}(v_0 + v)t + x_0 \qquad x = -\frac{1}{2}at^2 + vt + x_0 \qquad g = 9.8\,\text{m/s}^2$$

$$x_{\text{rel}} = x_2 - x_1 \qquad v_{\text{rel}} = v_2 - v_1 \qquad a_{\text{rel}} = a_2 - a_1$$

$$x' = x - v_{\text{frame}} t \qquad v' = v - v_{\text{frame}} \qquad a' = a$$

9 Two- and Three-Dimensional Kinematics: General

$$\vec{v}_{\text{avg}} = \frac{\Delta \vec{r}}{\Delta t} \qquad \vec{v} = \frac{d\vec{r}}{dt} \qquad \vec{a}_{\text{avg}} = \frac{\Delta \vec{v}}{\Delta t} \qquad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

10 Projectile Motion

$$x = v_{x,0} t \qquad y = -\frac{1}{2} g t^2 + v_{y,0} t + y_0 \qquad v_{x,0} = v_0 \cos\theta \qquad v_{y,0} = v_0 \sin\theta$$

$$t = \frac{x}{v_{x,0}} = \frac{x}{v_0 \cos\theta} \qquad y = y_0 + x \tan\theta - \frac{x^2 g}{2 v_0^2 \cos^2\theta}$$

$$x_{\text{for } y \text{ max}} = \frac{v_0^2 \sin\theta \cos\theta}{g} \qquad y_{\max} = y_0 + \frac{v_0^2 \sin^2\theta}{2g}$$

$$x(y = y_0) = \frac{2 v_0^2 \sin\theta \cos\theta}{g} = \frac{v_0^2 \sin(2\theta)}{g} \qquad \theta_{\text{for max}} = \frac{\pi}{4} \qquad x_{\max}(y = y_0) = \frac{v_0^2}{g}$$

$$x(\theta = 0) = \pm v_0 \sqrt{\frac{2(y_0 - y)}{g}} \qquad t(\theta = 0) = \sqrt{\frac{2(y_0 - y)}{g}}$$

11 Relative Motion

$$\vec{r} = \vec{r}_2 - \vec{r}_1 \qquad \vec{v} = \vec{v}_2 - \vec{v}_1 \qquad \vec{a} = \vec{a}_2 - \vec{a}_1$$

12 Polar Coordinate Motion and Uniform Circular Motion

$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\vec{r} = r\hat{r} \qquad \vec{v} = \frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{r} + r\omega\hat{\theta} \qquad \vec{a} = \frac{d^2\vec{r}}{dt^2} = \left(\frac{d^2 r}{dt^2} - r\omega^2\right)\hat{r} + \left(r\alpha + 2\frac{dr}{dt}\omega\right)\hat{\theta}$$

$$\vec{v} = r\omega\hat{\theta} \qquad v = r\omega \qquad a_{\tan} = r\alpha$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} = -r\omega^2\hat{r} \qquad a_{\text{centripetal}} = \frac{v^2}{r} = r\omega^2 = v\omega$$

13 Very Basic Newtonian Physics

$$\vec{r}_{\mathrm{cm}} = \frac{\sum_i m_i \vec{r}_i}{m_{\mathrm{total}}} = \frac{\sum_{\mathrm{sub}} m_{\mathrm{sub}} \vec{r}_{\mathrm{cm\ sub}}}{m_{\mathrm{total}}} \qquad \vec{v}_{\mathrm{cm}} = \frac{\sum_i m_i \vec{v}_i}{m_{\mathrm{total}}} \qquad \vec{a}_{\mathrm{cm}} = \frac{\sum_i m_i \vec{a}_i}{m_{\mathrm{total}}}$$

$$\vec{r}_{\mathrm{cm}} = \frac{\int_V \rho(\vec{r})\vec{r}\,dV}{m_{\mathrm{total}}}$$

$$\vec{F}_{\mathrm{net}} = m\vec{a} \qquad \vec{F}_{21} = -\vec{F}_{12} \qquad F_g = mg \qquad g = 9.8\,\mathrm{m/s^2}$$

$$\vec{F}_{\mathrm{normal}} = -\vec{F}_{\mathrm{applied}} \qquad F_{\mathrm{linear}} = -kx$$

$$f_{\mathrm{normal}} = \frac{T}{r} \qquad T = T_0 - F_{\mathrm{parallel}}(s) \qquad T = T_0$$

$$F_{\mathrm{f\ static}} = \min(F_{\mathrm{applied}}, F_{\mathrm{f\ static\ max}}) \qquad F_{\mathrm{f\ static\ max}} = \mu_{\mathrm{static}} F_{\mathrm{N}} \qquad F_{\mathrm{f\ kinetic}} = \mu_{\mathrm{kinetic}} F_{\mathrm{N}}$$

$$v_{\mathrm{tangential}} = r\omega = r\frac{d\theta}{dt} \qquad a_{\mathrm{tangential}} = r\alpha = r\frac{d\omega}{dt} = r\frac{d^2\theta}{dt^2}$$

$$\vec{a}_{\mathrm{centripetal}} = -\frac{v^2}{r}\hat{r} \qquad \vec{F}_{\mathrm{centripetal}} = -m\frac{v^2}{r}\hat{r}$$

$$F_{\mathrm{drag,lin}} = bv \qquad v_{\mathrm{T}} = \frac{mg}{b} \qquad \tau = \frac{v_{\mathrm{T}}}{g} = \frac{m}{b} \qquad v = v_{\mathrm{T}}(1 - e^{-t/\tau})$$

$$F_{\mathrm{drag,quad}} = bv^2 = \frac{1}{2}C\rho A v^2 \qquad v_{\mathrm{T}} = \sqrt{\frac{mg}{b}}$$

14 Energy and Work

$$dW = \vec{F}\cdot d\vec{s} \qquad W = \int \vec{F}\cdot d\vec{s} \qquad KE = \frac{1}{2}mv^2 \qquad E_{\mathrm{mechanical}} = KE + PE$$

$$P_{\mathrm{avg}} = \frac{\Delta W}{\Delta t} \qquad P = \frac{dW}{dt} \qquad P = \vec{F}\cdot\vec{v}$$

$$\Delta KE = W_{\mathrm{net}} \quad \Delta PE_{\text{of a conservative force}} = -W_{\text{by a conservative force}} \quad \Delta E = W_{\mathrm{nonconservative}}$$

$$F = -\frac{dPE}{dx} \qquad \vec{F} = -\nabla PE \qquad PE = \frac{1}{2}kx^2 \qquad PE = mgy$$