Intro Physics Semester I **Name:**

Homework 7: Energy: One or two or no full answer questions will be marked. There will also be a mark for completeness. Homeworks are due usually the day after the chapter they are for is finished. Solutions will be posted soon thereafter. The solutions are intended to be (but not necessarily are) super-perfect and often go beyond a fully correct answer.

Answer Table

Name:

	a	b	c	d	e		a	b	c	d	e
1.	O	O	O	O	O	31.	O	O	O	O	O
2.	O	O	O	O	O	32.	O	O	O	O	O
3.	O	O	O	O	O	33.	O	O	O	O	O
4.	O	O	O	O	O	34.	O	O	O	O	O
5.	O	O	O	O	O	35.	O	O	O	O	O
6.	O	O	O	O	O	36.	O	O	O	O	O
7.	O	O	O	O	O	37.	O	O	O	O	O
8.	O	O	O	O	O	38.	O	O	O	O	O
9.	O	O	O	O	O	39.	O	O	O	O	O
10.	O	O	O	O	O	40.	O	O	O	O	O
11.	O	O	O	O	O	41.	O	O	O	O	O
12.	O	O	O	O	O	42.	O	O	O	O	O
13.	O	O	O	O	O	43.	O	O	O	O	O
14.	O	O	O	O	O	44.	O	O	O	O	O
15.	O	O	O	O	O	45.	O	O	O	O	O
16.	O	O	O	O	O	46.	O	O	O	O	O
17.	O	O	O	O	O	47.	O	O	O	O	O
18.	O	O	O	O	O	48.	O	O	O	O	O
19.	O	O	O	O	O	49.	O	O	O	O	O
20.	O	O	O	O	O	50.	O	O	O	O	O
21.	O	O	O	O	O	51.	O	O	O	O	O
22.	O	O	O	O	O	52.	O	O	O	O	O
23.	O	O	O	O	O	53.	O	O	O	O	O
24.	O	O	O	O	O	54.	O	O	O	O	O
25.	O	O	O	O	O	55.	O	O	O	O	O
26.	O	O	O	O	O	56.	O	O	O	O	O
27.	O	O	O	O	O	57.	O	O	O	O	O
28.	O	O	O	O	O	58.	O	O	O	O	O
29.	O	O	O	O	O	59.	O	O	O	O	O
30.	O	O	O	O	O	60.	O	O	O	O	O

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006 qmult 00100 1 1 1 easy memory: five special forces

1. Five special forces in physics are:

   a) gravitational force, normal force, tension, friction force, and the linear force (the Hooke's law force).
   b) gravitational force, normal force, tension, friction force, and left force (the Captain Hooke's law force).
   c) gravitational force, normal force, tension, friction force, and right force.
   d) gravitational force, normal force, tension, friction force, and air force.
   e) gravitational force, normal force, tension, friction force, and **the Force**.

   SUGGESTED ANSWER: (a)

   Wrong answers:
   b) Avast mateys.
   e) May the Force be with you.

   Redaction: Jeffery, 2001jan01

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006 qmult 00120 1 1 3 easy memory: linear force formula

2. In one dimension, the linear force formula is:

   a) $F = -kx^2$. b) $F = kx$. c) $F = -kx$. d) $F = kx^2$. e) $F = k/x^2$.

   SUGGESTED ANSWER: (c)

   Wrong answers:
   a) A nonsense answer.

   Redaction: Jeffery, 2008jan01

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006 qmult 00250 1 1 4 easy memory: change force contant change period for harmonic ocsillator

3. If you double the force constant of a simple harmonic oscillator, the period changes by a factor of:

   a) 1/2. b) $\sqrt{2}$. c) 2. d) $1/\sqrt{2}$. e) 2/3.

   SUGGESTED ANSWER: (d)

   The period formula for a simple harmonic oscillator is

$$P = 2\pi\sqrt{\frac{m}{k}} \ .$$

   If the initial force constant is k and it is double to get k_{new}, then

$$P_{\text{new}} = 2\pi\sqrt{\frac{m}{k_{\text{new}}}} = 2\pi\sqrt{\frac{m}{2k}} = \frac{P}{\sqrt{2}} \ .$$

   Wrong answers:
   a) A nonsense answer.

   Fortran-95 Code

   Redaction: Jeffery, 2008jan01

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007 qmult 00100 1 4 5 easy deducto-memory: essence energy definition 1

4. "Let's play *Jeopardy*! For \$100, the answer is: Admitting that all short definitions of it are inadequate, one can suggest that it is the conserved essence of structure and transformability. 'Converved' means that the quantity that is neither created nor destroyed. In this case, all forms of the quantity must be summed to have conservation in general though in special cases conservation may hold for a subset of forms. 'Essence of structure' means that the quantity is a general measure of structure, but it does not describe details of structure. 'Structure' is used here in a very general sense to mean the ordering of objects, particles, fields, and motions. 'Essence of transformability' means that the quantity can be transformed into different forms of itself and that all changes in the amounts of the forms result in

changes in structure. If you have some of the quantity, transformations are possible. Knowing which ones occur requires more detailed information."

What is ____________, Alex?

a) force b) momentum c) angular momentum d) potential e) energy

SUGGESTED ANSWER: (e)

Wrong answers:
a) As Lurch would say AAAARGH.

Redaction: Jeffery, 2008jan01

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007 qmult 00110 1 4 4 easy deducto-memory: change energy definition 2

5. "Let's play *Jeopardy*! For $100, the answer is: A useful, but incomplete definition, is the quantified capacity for change."

What is ____________, Alex?

a) momentum b) force c) potential d) energy e) aspiration

SUGGESTED ANSWER: (d)

Wrong answers:
e) Well no. It's not quantified.

Redaction: Jeffery, 2008jan01

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007 qmult 00120 1 4 5 easy deducto-memory: work energy definition 3
Extra keywords: physci KB-66

6. "Let's play *Jeopardy*! For $100, the answer is: It has been described as that property of something that enables that something to do work."

What is ____________, Alex?

a) inevitability b) noise c) tension d) gravity e) energy

SUGGESTED ANSWER: (e)

Wrong answers:
a) This answer just confounds me: I'm not prepared to accept it.

Redaction: Jeffery, 2001jan01

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007 qmult 00130 1 1 3 easy memory: conservation of energy 1

7. The principle of conservation of energy is that energy is never:

a) adequately defined. b) destroyed, but can be created. c) created or destroyed.
d) created, but can be destroyed. e) destroyed.

SUGGESTED ANSWER: (c)

Wrong answers:
a) It never is adequately defined, but that's not the principle.

Redaction: Jeffery, 2008jan01

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007 qmult 00134 1 1 1 easy memory: energy conservation implication
Extra keywords: physci KB-72

8. The total energy of a closed system (i.e., a system which nothing enters or leaves) is:

a) conserved. b) annihilated. c) created. d) combusted. e) eviscerated.

SUGGESTED ANSWER: (a) This is the principle of conservation of energy. Is the universe as a whole a closed system? Is energy conserved in the universe as a whole?

Wrong answers:
e) Now does this seem likely?

Redaction: Jeffery, 2001jan01

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007 qmult 00140 1 1 1 easy memory: dimensions of energy

9. The physical dimensions of energy are:

   a) ML^2/T^2. b) ML/T^2. c) ML^2. d) ML^2/T. e) M/T^2.

SUGGESTED ANSWER: (a)

Wrong answers:

b) The dimensions of force.

Redaction: Jeffery, 2008jan01

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007 qmult 00142 1 1 1 easy memory: unit of energy, the joule

Extra keywords: physci

10. The standard SI unit of energy and of work is the:

    a) joule (J). b) newton (N). c) kelvin (K). d) bassingthorp (B).
    e) trufflehunter (T).

SUGGESTED ANSWER: (a)

Here's a versicle from the poem016.tex file:

The unit of energy is the joule
and this rhymes with drool,
but it should rhyme with bowel
to be correct for James Joule.

Wrong answers:

e) Trufflehunter was a character in the Narnia stories by C.S. Lewis.

Redaction: Jeffery, 2001jan01

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007 qmult 00180 2 1 1 moderate memory: energy necessity and sufficiency

Extra keywords: physci

11. That one has enough energy for a certain job or transformation that requires energy E is a ________________ condition, but **NOT** a ________________ condition for the job or transformation.

    a) necessary; sufficient b) sufficient; necessary c) inevitable; necessarily so
    d) harmonious; ceremonious e) forbidden; given

SUGGESTED ANSWER: (a) Given the wrong answers, I think this answer must inevitably and necessarily be the best.

Wrong answers:

b) The question says the job needs a certain amount of energy; thus having enough is necessary.
c) You don't have enough energy inevitably: the "not necessarily so" part is right.
e) "Not necessarily so."

Redaction: Jeffery, 2001jan01

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007 qmult 00182 1 4 5 easy deducto-memory: energy and money

Extra keywords: physci

12. "Let's play *Jeopardy*! For \$100, the answer is: Because of its protean nature, energy is very much like this thing in everyday human life which, however, unlike energy is not conserved."

What is/are ____________, Alex?

a) furs b) assignats c) shells d) gold e) money

SUGGESTED ANSWER: (e)

This a rather subjective question, but it's hard to imagine anyone who understands the concepts could not choose answer (e).

Wrong answers:
a) You know in the fur trading days in Canada beaver pelts were sort of like money.
b) During the French Revolution assignats were effectively currency backed up by the value of nationalized church lands.

Redaction: Jeffery, 2001jan01

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007 qmult 00200 1 4 2 easy deducto-memory: work defined

13. "Let's play *Jeopardy*! For \$100, the answer is: In physics, it is a macroscopic process of energy transfer."

What is ____________, Alex?

a) energy b) work c) force d) weight e) sloth

SUGGESTED ANSWER: (b)

Wrong answers:
e) Now is this likely?

Redaction: Jeffery, 2008jan01

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007 qmult 00210 1 1 2 easy memory: differential work formula

14. The differential work formula is:

a) $dW = \vec{F}\,d\vec{s}$. b) $dW = \vec{F}\cdot d\vec{s}$. c) $dW = \vec{F}/d\vec{s}$. d) $dW = F\,ds$. e) $dW = \vec{F}\times d\vec{s}$.

SUGGESTED ANSWER: (b)

Wrong answers:
d) A valid one-dimensional form.

Redaction: Jeffery, 2008jan01

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007 qmult 00222 1 1 3 easy memory: work formula for a constant force 2
Extra keywords: For the vector and dot product literate

15. A constant force $\vec{F}$ acts on a body while that body moves a distance $\Delta\vec{r}$. The work W done on the body by the force is given by:

a) $W = \vec{F}/\Delta\vec{r}$. b) $W = \vec{F}$. c) $W = \vec{F}\cdot\Delta\vec{r}$. d) $W = \vec{F}\cdot\vec{F}\cdot\Delta\vec{r}$. e) $W = \vec{F}\cdot\Delta\vec{r}\cdot\Delta\vec{r}$.

SUGGESTED ANSWER: (c)

a) Vectors cannot be simply divided.
b) Vectors can't equal scalars.
d) A triple vector dot product is not a defined operation.
e) A triple vector dot product is not a defined operation.

Wrong answers:

Redaction: Jeffery, 2001jan01

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007 qmult 00280 1 3 1 easy math: work lifting 100 kg load
Extra keywords: physci KB-95-3

16. How much work is done by a lifter lifting a 100 kg load straight upward 10 m without acceleration?

a) 9800 J. b) 100 J. c) 1000 J. d) 10 J. e) 980 J.

SUGGESTED ANSWER: (a)

The calculation is

$$W = Fd\cos\theta = mgd\cos\theta = 100\times 9.8\times 10\times 1 = 9800\,\mathrm{J}\ ,$$

where F is force (which must be equal in magnitude to gravity), d is distance, m is mass, $\cos\theta = 1$ is the cosine of the angle between the lifting force, and the displacement and $g = 9.8\,\mathrm{m/s^2}$ is the acceleration due to gravity constant.

Wrong answers:

d) Not a good guess.

Redaction: Jeffery, 2001jan01

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007 qmult 00284 1 5 5 easy thinking: no macroscopic work done
Extra keywords: physci KB-95-1

17. A person holds 10 kg grouse at 2.0 m above the ground for 30 s. How much macroscopic net work is done by the person on the grouse?

a) 600 J. b) 20 J. c) 300 J. d) 60 J. e) 0 J.

SUGGESTED ANSWER: (e)

At the macroscopic level there is no change in the system, and so no work is done. Microscopically, in the person's body extra chemical energy must expended to maintain the holding stance, but that is not at the macroscopic level. And there is no easy way to calculate that work anyway and its not done on the grouse who might just as well be resting on a pillar. Actually, 10 kg is really heavy for a grouse I'd guess. Wikipedia hints grouse of a few kilograms are big. But it's just a hypothetical question.

Wrong answers:
a) Nope.

Redaction: Jeffery, 2001jan01

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007 qmult 00300 1 1 3 easy memory: kinetic energy definition
Extra keywords: physci

18. Kinetic energy is:

a) the energy of **POSITION** with formula $KE = mgy$. b) the energy of **MOTION** with formula $KE = mgy$. c) the energy of **MOTION** with formula $KE = (1/2)mv^2$. d) the energy of **POSITION** with formula $KE = (1/2)mv^2$. e) heat energy.

SUGGESTED ANSWER: (c) There are plenty of clues.

Wrong answers:
a) The mgy is the potential energy of gravity near the Earth's surface.
e) Heat energy can be microscopic kinetic energy, but there are other forms of heat energy. When we just say kinetic energy, we are usually thinking of macroscopic kinetic energy.

Redaction: Jeffery, 2001jan01

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007 qmult 00310 1 1 3 easy memory: work-kinetic-energy theorem

19. The work-kinetic-energy theorem is:

a) $KE = \frac{1}{2}mv^2$. b) $\Delta E = W_{\rm non}$. c) $\Delta KE = W$. d) $\Delta KE = \frac{1}{2}W$. e) $\Delta KE = \frac{1}{2}mv^2$.

SUGGESTED ANSWER: (c)

Wrong answers:
a) This is the kinetic energy formula.
b) This is the work-energy theorem.

Redaction: Jeffery, 2008jan01

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007 qmult 00370 2 5 4 moderate math: friction killing KE
Extra keywords: physci KB-95-9

20. A moving object has initial $KE = 100$ J and is subjected to a friction force of magnitude 2 N and no other forces. How far does the object go before coming to a stop?

a) 100 m. b) 2 m. c) 1000 m. d) 50 m. e) 0 m.

SUGGESTED ANSWER: (d)

Remember the work-kinetic-energy theorem:

$$\Delta KE = W_{\rm net} \ .$$

The work done by a particular constant force is

$$W = \vec{F} \cdot \Delta\vec{s} = F \Delta s \cos\theta ,$$

where $\vec{F}$ is the force, $\Delta\vec{s}$ is the displacement, and $\cos\theta$ is the cosine of the angle between the force and the displacement vectors. In this case the only force is friction and it is a constant. Friction always opposes the direction of motion, and so $\cos\theta = -1$ (from $\theta = 180^\circ$). So in the present case,

$$\Delta r = -\frac{\Delta KE}{F} = -\left(\frac{0-100}{2}\right) = 50\,\text{m} \ .$$

Wrong answers:

e) Not a good guess.

Redaction: Jeffery, 2001jan01

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007 qfull 0030 1 3 0 easy math: derivation of work-kinetic energy theorem

21. We will now the derive the work-kinetic energy theorem in little-bitty steps. Recall the work-kinetic-energy theorem is

$$W = \Delta KE \ ,$$

where W is the work done by the net external force acting on a body and ΔKE is the change in the body's center-of-mass kinetic energy.

a) Write down the differential definition of work.

b) Write down the integral definition of work with unspecified endpoints.

c) Write down the formula for the work between points $\vec{A}$ and $\vec{B}$ done by net external force on a body and make use of Newton's 2nd law to rewrite this formula in terms of the body's center-of-mass acceleration.

d) Prove that

$$\vec{a} \cdot \vec{v} = \frac{1}{2}\frac{dv^2}{dt} \ .$$

e) Given that $d\vec{s} = \vec{v}\,dt$, now prove the work-kinetic-energy theorem

SUGGESTED ANSWER:

a) Behold:

$$W = \vec{F} \cdot d\vec{s} \ ,$$

where $\vec{F}$ is a force that acts on a body and $d\vec{s}$ is differential vector displacement of the body's center of mass.

b) Behold:

$$W = \int \vec{F} \cdot d\vec{s} \ ,$$

where we have left the endpoints unspecified since we like a clean look unadorned by arbitrary symbols for a general formula.

c) Behold:

$$W = \int_{\vec{A}}^{\vec{B}} \vec{F}_{\rm net} \cdot d\vec{s} = \int_{\vec{A}}^{\vec{B}} m\vec{a} \cdot d\vec{s} \ ,$$

where we have used Newton's 2nd law to rewrite this formula in terms of the body's center-of-mass acceleration.

d) Behold:

$$\vec{a}\cdot\vec{v} = \frac{d\vec{v}}{dt}\cdot\vec{v} = \sum_i \frac{dv_i}{dt}v_i = \sum_i \frac{1}{2}\frac{dv_i^2}{dt} = \frac{1}{2}\frac{dv^2}{dt} \ .$$

e) Behold:

$$W = \int_{\vec{A}}^{\vec{B}} m\vec{a}\cdot\, d\vec{s} = \int_{\vec{A}}^{\vec{B}} m\vec{a}\cdot\vec{v}\,dt = \int_{t_A}^{t_B} \frac{1}{2}m\frac{dv^2}{dt}\,dt = \Delta KE \ ,$$

where kinetic energy is defined by

$$KE = \frac{1}{2}mv^2$$

and ΔKE is the change in kinetic energy between the endpoints of the motion. Thus, we have derived the work-kinetic-energy theorem

$$W = \Delta KE \ .$$

Redaction: Jeffery, 2008jan01

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007 qfull 00340 1 3 0 easy math: work and KE feat

22. You are pushing a 60 kg block across a frictionless surface—which is no mean feat in itself.

a) You exert 100 N of force in the direction of motion and push the block for 30 m. How much work have you done on the block? Show how you got your answer.

b) Assuming you start the block from **REST** and no other forces do work, what is the final kinetic energy and speed of the block? Show how you got your answer.

SUGGESTED ANSWER:

a) Behold:

$$W = \vec{F}\cdot\Delta\vec{s} = F\Delta s\cos\theta = 3000\,\mathrm{J} \ ,$$

where $\cos\theta = 1$ in this case since the force is in the direction of motion.

b) By the work-kinetic-energy theorem

$$KE_{\rm fi} = W = 3000\,\mathrm{J} \ .$$

The final velocity is

$$v_{\rm fi} = \sqrt{\frac{2KE_{\rm fi}}{m}} = \sqrt{\frac{2W}{m}} = 10\,\mathrm{m/s} \ .$$

Fortran-95 Code

```
          print*
          fff=100.d0
          ss=30.d0
          ww=fff*ss
          xm=60.d0
          vv=sqrt(2.d0*ww/xm)
          print*,'ww,vv'
          print*,ww,vv
!      3000.00000000000          10.0000000000000
```

Redaction: Jeffery, 2001jan01

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007 qfull 00350 1 3 0 easy math: work done on block on incline

Extra keywords: (Ha-157:16e)

23. Power in physics is the rate per unit time of energy transferred or transformed. This is a general definition independent of the where the energy is coming from and where it is going too. If work is the energy transfer process, the differential work formula $dW = \vec{F}\cdot d\vec{s}$ (where dW is differential work done

on a object by external force F and $d\vec{S}$ is differential center-of-mass displacement) leads immediately to the power formula for work:

$$P = \frac{dW}{dt} = \vec{F}\cdot\frac{d\vec{s}}{dt} = \vec{F}\cdot\vec{v}\ ,$$

where $\vec{v}$ is velocity (i.e., center-of-mass velocity) of a displaced object. The MKS unit of work is the joule per second (J/s) which has its own name: the familar watt with symbol (W).

Now that we've preambled, there is a frictionless block of ice of 50 kg on an incline of 30°. A worker pushes the block straight up the incline at a **CONSTANT** velocity of 3.0 m/s.

a) What is the power output (i.e., rate of work done) by the worker?

b) What is the power output by gravity?

c) What is the power output by the normal force? Why?

SUGGESTED ANSWER:

a) If the velocity is constant, the acceleration is zero and the component of the worker's force in the direction of motion $F_{\rm w}$ must exactly balance gravity component in that direction. Thus $F_{\rm w} = mg\sin\theta$ is the component of force in the direction of motion. We don't know the worker's force completely: i.e., $\vec{F}$. But we don't need to: only the component in the direction of motion counts for work. We find

$$P = \frac{dW}{dt} = \vec{F}\cdot\vec{v} = F_{\rm w}v = (mg\sin\theta)v \approx 750\,\mathrm{W}$$

to about 2-digit accuracy. For the algebra-based course,

$$P = \frac{\Delta W}{\Delta t} = (mg\sin\theta)v \approx 750\,\mathrm{W}$$

$$P = \frac{\Delta W}{\Delta t} = \vec{F}\cdot\vec{v} = F_{\rm w}v = (mg\sin\theta)v \approx 750\,\mathrm{W}$$

to about 2-digit accuracy. To better accuracy, the power is 735 W.

b) Clearly the power expended by gravity is

$$P = -mg\hat{y}\cdot\vec{v} = -[mg\cos(180^\circ - \theta_{\rm complement})]v = -[mg\cos(\theta_{\rm complement})]v = -[mg\cos(90^\circ - \theta)]v$$
$$= -(mg\sin\theta)v \approx -750\,\mathrm{W}$$

to about 2-digit accuracy, where y is unit vector pointing the positive vertical direction. To better accuracy, the power is −735 W.

We interpret this negative answer as energy coming out of the block (hence the negative sign) and going into its "position" which is really into the energy of the gravitational field structure of its position. Overall the worker is putting into the block power P and this is being virtually directly transferred by power P going into the block's potential energy. Make sense, yes?

c) The normal force is perpendicular to the direction of motion, and so does zero work and zero work per unit time.

Note the normal force is doing something, not just work as we define it in physics. The normal force and the component of gravity that it cancels are workless constraint forces on the block. They prevent it from rising up from the incline or sinking down through it. You might argue that the since the normal force and the component of gravity cancel, they do nothing. But no. Together they act as a restoring force. If the block is perturbed upward, the normal force turns off and gravity pulls the block back to the incline. Any extra perturbation force pushing downward is canceled by an increase in the normal force which is as strong as it needs to be (when ideal) to prevent any deformation of the incline. So the two forces make the block's equilibrium in the direction perpendicular to the incline a stable equilibrium.

Fortran-95 Code

```
      gg=9.8d0
      xm=50.d0
```

```
      theta=30.d0/raddeg
      sth=sin(theta)
      v=3.d0
      pw=xm*gg*sth*v
      pg=-pw
      print*,'pg,pw'
      print*,pg,pw
!   -735.000000000000        735.000000000000
```

Redaction: Jeffery, 2001jan01

Equation Sheet for Introductory Physics Calculus-Based

This equation sheet is intended for students writing tests or reviewing material. Therefore it is neither intended to be complete nor completely explicit. There are fewer symbols than variables, and so some symbols must be used for different things: context must distinguish.

The equations are mnemonic. Students are expected to understand how to interpret and use them.

1 Constants

$$
\begin{aligned}
c &= 2.99792458\times10^{8}\,\mathrm{m/s} \approx 2.998\times10^{8}\,\mathrm{m/s} \approx 3\times10^{8}\,\mathrm{m/s} \approx 1\,\mathrm{lyr/yr} \approx 1\,\mathrm{ft/ns} \qquad \text{exact by definition} \\
e &= 1.602176487(40)\times10^{-19}\,\mathrm{C} \\
G &= 6.67428(67)\times10^{-11}\,\mathrm{N\,m^2/kg^2} \qquad \text{(2006, CODATA)} \\
g &= 9.8\,\mathrm{m/s^2} \qquad \text{fiducial value} \\
k &= \frac{1}{4\pi\varepsilon_0} = 8.987551787\ldots\times10^{9} \approx 8.99\times10^{9} \approx 10^{10}\,\mathrm{N\,m^2/C^2}\text{exact by definition} \\
k_{\mathrm{Boltzmann}} &= 1.3806504(24)\times10^{-23}\,\mathrm{J/K} = 0.8617343(15)\times10^{-4}\,\mathrm{eV/K} \approx 10^{-4}\,\mathrm{eV/K} \\
m_e &= 9.10938215(45)\times10^{-31}\,\mathrm{kg} = 0.510998910(13)\,\mathrm{MeV} \\
m_p &= 1.672621637(83)\times10^{-27}\,\mathrm{kg} = 938.272013(23),\mathrm{MeV} \\
\varepsilon_0 &= \frac{1}{\mu_0 c^2} = 8.8541878176\ldots\times10^{-12}\,\mathrm{C^2/(N\,m^2)} \approx 10^{-11} \qquad \text{vacuum permittivity (exact by definition)} \\
\mu_0 &= 4\pi\times10^{-7}\,\mathrm{N/A^2} \qquad \text{exact by definition}
\end{aligned}
$$

2 Geometrical Formulae

$$C_{\mathrm{cir}} = 2\pi r \qquad A_{\mathrm{cir}} = \pi r^2 \qquad A_{\mathrm{sph}} = 4\pi r^2 \qquad V_{\mathrm{sph}} = \frac{4}{3}\pi r^3$$

$$\Omega_{\mathrm{sphere}} = 4\pi \qquad d\Omega = \sin\theta\,d\theta\,d\phi$$

3 Trigonometry Formulae

$$\frac{x}{r} = \cos\theta \qquad \frac{y}{r} = \sin\theta \qquad \frac{y}{x} = \tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cos^2\theta + \sin^2\theta = 1$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$c^2 = a^2 + b^2 \qquad c = \sqrt{a^2 + b^2 - 2ab\cos\theta_c} \qquad \frac{\sin\theta_a}{a} = \frac{\sin\theta_b}{b} = \frac{\sin\theta_c}{c}$$

$$f(\theta) = f(\theta + 360^\circ)$$

$$\sin(\theta + 180^\circ) = -\sin(\theta) \qquad \cos(\theta + 180^\circ) = -\cos(\theta) \qquad \tan(\theta + 180^\circ) = \tan(\theta)$$

$$\sin(-\theta) = -\sin(\theta) \qquad \cos(-\theta) = \cos(\theta) \qquad \tan(-\theta) = -\tan(\theta)$$

$$\sin(\theta + 90^\circ) = \cos(\theta) \qquad \cos(\theta + 90^\circ) = -\sin(\theta) \qquad \tan(\theta + 90^\circ) = -\tan(\theta)$$

$$\sin(180^\circ - \theta) = \sin(\theta) \qquad \cos(180^\circ - \theta) = -\cos(\theta) \qquad \tan(180^\circ - \theta) = -\tan(\theta)$$

$$\sin(90^\circ - \theta) = \cos(\theta) \qquad \cos(90^\circ - \theta) = \sin(\theta) \qquad \tan(90^\circ - \theta) = \frac{1}{\tan(\theta)} = \cot(\theta)$$

$$\sin(a + b) = \sin(a)\cos(b) + \cos(a)\sin(b) \qquad \cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$\sin(2a) = 2\sin(a)\cos(a) \qquad \cos(2a) = \cos^2(a) - \sin^2(a)$$

$$\sin(a)\sin(b) = \frac{1}{2}\left[\cos(a-b) - \cos(a+b)\right] \qquad \cos(a)\cos(b) = \frac{1}{2}\left[\cos(a-b) + \cos(a+b)\right]$$

$$\sin(a)\cos(b) = \frac{1}{2}\left[\sin(a-b) + \sin(a+b)\right]$$

$$\sin^2\theta = \frac{1}{2}[1 - \cos(2\theta)] \qquad \cos^2\theta = \frac{1}{2}[1 + \cos(2\theta)] \qquad \sin(a)\cos(a) = \frac{1}{2}\sin(2a)$$

$$\cos(x) - \cos(y) = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos(x) + \cos(y) = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin(x) + \sin(y) = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

4 Approximation Formulae

$$\frac{\Delta f}{\Delta x} \approx \frac{df}{dx} \qquad \frac{1}{1-x} \approx 1 + x \ : \ (x << 1)$$

$$\sin\theta \approx \theta \qquad \tan\theta \approx \theta \qquad \cos\theta \approx 1 - \frac{1}{2}\theta^2 \qquad \text{all for } \theta << 1$$

5 Quadratic Formula

$$\text{If} \qquad 0 = ax^2 + bx + c\,, \qquad \text{then} \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = -\frac{b}{2a} \pm \sqrt{\left(\frac{b}{2a}\right)^2 - \frac{c}{a}}$$

6 Vector Formulae

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2} \qquad \theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) + \pi? \qquad \vec{a} + \vec{b} = (a_x + b_x, a_y + b_y)$$

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} \qquad \phi = \tan^{-1}\left(\frac{a_y}{a_x}\right) + \pi? \qquad \theta = \cos^{-1}\left(\frac{a_z}{a}\right)$$

$$\vec{a} + \vec{b} = (a_x + b_x, a_y + b_y, a_z + b_z)$$

$$\vec{a} \cdot \vec{b} = ab\cos\theta = a_x b_x + a_y b_y + a_z b_z$$

$$\vec{c} = \vec{a} \times \vec{b} = ab\sin(\theta)\hat{c} = (a_y b_z - b_y a_z, a_z b_x - b_z a_x, a_x b_y - b_x a_y)$$

7 Differentiation and Integration Formulae

$$\frac{d(x^p)}{dx} = px^{p-1} \quad \text{except for } p = 0; \qquad \frac{d(x^0)}{dx} = 0 \qquad \frac{d(\ln|x|)}{dx} = \frac{1}{x}$$

Taylor's series

$$\begin{aligned} f(x) &= \sum_{n=0}^{\infty} \frac{(x-x_0)^n}{n!} f^{(n)}(x_0) \\ &= f(x_0) + (x-x_0)f^{(1)}(x_0) + \frac{(x-x_0)^2}{2!} f^{(2)}(x_0) + \frac{(x-x_0)^3}{3!} f^{(3)}(x_0) + \ldots \end{aligned}$$

$$\int_a^b f(x)\,dx = F(x)|_a^b = F(b) - F(a) \quad \text{where} \quad \frac{dF(x)}{dx} = f(x)$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad \text{except for } n = -1; \qquad \int \frac{1}{x}\,dx = \ln|x|$$

8 One-Dimensional Kinematics

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} \qquad v = \frac{dx}{dt} \qquad a_{\text{avg}} = \frac{\Delta v}{\Delta t} \qquad a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

$$v = at + v_0 \qquad x = \frac{1}{2}at^2 + v_0 t + x_0 \qquad v^2 = v_0^2 + 2a(x - x_0)$$

$$x = \frac{1}{2}(v_0 + v)t + x_0 \qquad x = -\frac{1}{2}at^2 + vt + x_0 \qquad g = 9.8\,\text{m/s}^2$$

$$x_{\text{rel}} = x_2 - x_1 \qquad v_{\text{rel}} = v_2 - v_1 \qquad a_{\text{rel}} = a_2 - a_1$$

$$x' = x - v_{\text{frame}} t \qquad v' = v - v_{\text{frame}} \qquad a' = a$$

9 Two- and Three-Dimensional Kinematics: General

$$\vec{v}_{\text{avg}} = \frac{\Delta \vec{r}}{\Delta t} \qquad \vec{v} = \frac{d\vec{r}}{dt} \qquad \vec{a}_{\text{avg}} = \frac{\Delta \vec{v}}{\Delta t} \qquad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

10 Projectile Motion

$$x = v_{x,0} t \qquad y = -\frac{1}{2} g t^2 + v_{y,0} t + y_0 \qquad v_{x,0} = v_0 \cos\theta \qquad v_{y,0} = v_0 \sin\theta$$

$$t = \frac{x}{v_{x,0}} = \frac{x}{v_0 \cos\theta} \qquad y = y_0 + x \tan\theta - \frac{x^2 g}{2 v_0^2 \cos^2\theta}$$

$$x_{\text{for } y \text{ max}} = \frac{v_0^2 \sin\theta \cos\theta}{g} \qquad y_{\text{max}} = y_0 + \frac{v_0^2 \sin^2\theta}{2g}$$

$$x(y = y_0) = \frac{2 v_0^2 \sin\theta \cos\theta}{g} = \frac{v_0^2 \sin(2\theta)}{g} \qquad \theta_{\text{for max}} = \frac{\pi}{4} \qquad x_{\text{max}}(y = y_0) = \frac{v_0^2}{g}$$

$$x(\theta = 0) = \pm v_0 \sqrt{\frac{2(y_0 - y)}{g}} \qquad t(\theta = 0) = \sqrt{\frac{2(y_0 - y)}{g}}$$

11 Relative Motion

$$\vec{r} = \vec{r}_2 - \vec{r}_1 \qquad \vec{v} = \vec{v}_2 - \vec{v}_1 \qquad \vec{a} = \vec{a}_2 - \vec{a}_1$$

12 Polar Coordinate Motion and Uniform Circular Motion

$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\vec{r} = r\hat{r} \qquad \vec{v} = \frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{r} + r\omega\hat{\theta} \qquad \vec{a} = \frac{d^2\vec{r}}{dt^2} = \left(\frac{d^2 r}{dt^2} - r\omega^2\right)\hat{r} + \left(r\alpha + 2\frac{dr}{dt}\omega\right)\hat{\theta}$$

$$\vec{v} = r\omega\hat{\theta} \qquad v = r\omega \qquad a_{\tan} = r\alpha$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} = -r\omega^2\hat{r} \qquad a_{\text{centripetal}} = \frac{v^2}{r} = r\omega^2 = v\omega$$

13 Very Basic Newtonian Physics

$$\vec{r}_{\text{cm}} = \frac{\sum_i m_i \vec{r}_i}{m_{\text{total}}} = \frac{\sum_{\text{sub}} m_{\text{sub}} \vec{r}_{\text{cm sub}}}{m_{\text{total}}} \qquad \vec{v}_{\text{cm}} = \frac{\sum_i m_i \vec{v}_i}{m_{\text{total}}} \qquad \vec{a}_{\text{cm}} = \frac{\sum_i m_i \vec{a}_i}{m_{\text{total}}}$$

$$\vec{r}_{\text{cm}} = \frac{\int_V \rho(\vec{r})\vec{r}\, dV}{m_{\text{total}}}$$

$$\vec{F}_{\text{net}} = m\vec{a} \qquad \vec{F}_{21} = -\vec{F}_{12} \qquad F_g = mg \qquad g = 9.8\,\text{m/s}^2$$

$$\vec{F}_{\text{normal}} = -\vec{F}_{\text{applied}} \qquad F_{\text{linear}} = -kx$$

$$f_{\text{normal}} = \frac{T}{r} \qquad T = T_0 - F_{\text{parallel}}(s) \qquad T = T_0$$

$$F_{\text{f static}} = \min(F_{\text{applied}}, F_{\text{f static max}}) \qquad F_{\text{f static max}} = \mu_{\text{static}} F_{\text{N}} \qquad F_{\text{f kinetic}} = \mu_{\text{kinetic}} F_{\text{N}}$$

$$v_{\text{tangential}} = r\omega = r\frac{d\theta}{dt} \qquad a_{\text{tangential}} = r\alpha = r\frac{d\omega}{dt} = r\frac{d^2\theta}{dt^2}$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} \qquad \vec{F}_{\text{centripetal}} = -m\frac{v^2}{r}\hat{r}$$

$$F_{\text{drag,lin}} = bv \qquad v_{\text{T}} = \frac{mg}{b} \qquad \tau = \frac{v_{\text{T}}}{g} = \frac{m}{b} \qquad v = v_{\text{T}}(1 - e^{-t/\tau})$$

$$F_{\text{drag,quad}} = bv^2 = \frac{1}{2}C\rho A v^2 \qquad v_{\text{T}} = \sqrt{\frac{mg}{b}}$$

14 Energy and Work

$$dW = \vec{F} \cdot d\vec{s} \qquad W = \int \vec{F} \cdot d\vec{s} \qquad KE = \frac{1}{2}mv^2 \qquad E_{\text{mechanical}} = KE + PE$$

$$P_{\text{avg}} = \frac{\Delta W}{\Delta t} \qquad P = \frac{dW}{dt} \qquad P = \vec{F} \cdot \vec{v}$$

$$\Delta KE = W_{\text{net}} \quad \Delta PE_{\text{of a conservative force}} = -W_{\text{by a conservative force}} \quad \Delta E = W_{\text{nonconservative}}$$

$$F = -\frac{dPE}{dx} \qquad \vec{F} = -\nabla PE \qquad PE = \frac{1}{2}kx^2 \qquad PE = mgy$$