

**Intro Physics Semester I****Name:**

**Homework 6: Newtonian Physics: More of the Same:** One or two or no full answer questions will be marked. There will also be a mark for completeness. Homeworks are due usually the day after the chapter they are for is finished. Solutions will be posted soon thereafter. The solutions are intended to be (but not necessarily are) super-perfect and often go beyond a fully correct answer.

**Answer Table****Name:**

	a	b	c	d	e		a	b	c	d	e
1.	O	O	O	O	O	31.	O	O	O	O	O
2.	O	O	O	O	O	32.	O	O	O	O	O
3.	O	O	O	O	O	33.	O	O	O	O	O
4.	O	O	O	O	O	34.	O	O	O	O	O
5.	O	O	O	O	O	35.	O	O	O	O	O
6.	O	O	O	O	O	36.	O	O	O	O	O
7.	O	O	O	O	O	37.	O	O	O	O	O
8.	O	O	O	O	O	38.	O	O	O	O	O
9.	O	O	O	O	O	39.	O	O	O	O	O
10.	O	O	O	O	O	40.	O	O	O	O	O
11.	O	O	O	O	O	41.	O	O	O	O	O
12.	O	O	O	O	O	42.	O	O	O	O	O
13.	O	O	O	O	O	43.	O	O	O	O	O
14.	O	O	O	O	O	44.	O	O	O	O	O
15.	O	O	O	O	O	45.	O	O	O	O	O
16.	O	O	O	O	O	46.	O	O	O	O	O
17.	O	O	O	O	O	47.	O	O	O	O	O
18.	O	O	O	O	O	48.	O	O	O	O	O
19.	O	O	O	O	O	49.	O	O	O	O	O
20.	O	O	O	O	O	50.	O	O	O	O	O
21.	O	O	O	O	O	51.	O	O	O	O	O
22.	O	O	O	O	O	52.	O	O	O	O	O
23.	O	O	O	O	O	53.	O	O	O	O	O
24.	O	O	O	O	O	54.	O	O	O	O	O
25.	O	O	O	O	O	55.	O	O	O	O	O
26.	O	O	O	O	O	56.	O	O	O	O	O
27.	O	O	O	O	O	57.	O	O	O	O	O
28.	O	O	O	O	O	58.	O	O	O	O	O
29.	O	O	O	O	O	59.	O	O	O	O	O
30.	O	O	O	O	O	60.	O	O	O	O	O

1. "Let's play *Jeopardy!* For \$100, the answer is: The macroscopic binding force between smooth surfaces that is parallel to the surfaces."

What is \_\_\_\_\_, Alex?

- a) the linear restoring force    b) the spring force    c) the tension force  
d) the normal force    e) friction
2. The magnitude of the kinetic friction force between a body and a surface equals a coefficient of friction times:
- a) the area of macroscopic contact between the body and the surface.  
b) the magnitude of the normal force acting on the body.  
c) the mass of the body.  
d) the density of the body.  
e) the density of the air surrounding the body.
3. Which is larger: the coefficient of static or kinetic friction?
- a) They are always equal.  
b) Neither. The larger depends on the materials involved and its about a 50-50 split on which is larger.  
c) The kinetic coefficient is always larger.  
d) The kinetic coefficient is usually larger.  
e) The static coefficient is almost always (always?) larger.
4. The friction between sliding surfaces tends to change macroscopic kinetic energy into:
- a) potential energy.    b) rest mass energy.    c) thermal or heat energy.  
d) magnetic energy.    e) nothing.
5. You are pushing a coffee cup across a table (just an ordinary table, not an imaginary frictionless table) at a **CONSTANT** velocity. The magnitudes of the push force and frictional force are  $|F_p|$  and  $|F_f|$ , respectively.
- a)  $|F_f| > |F_p|$  and this is why the cup does not accelerate.  
b) The  $|F_f| < |F_p|$ , but nevertheless the frictional force prevents any acceleration.  
c) There is no frictional force when you push the cup at a constant velocity. Thus,  $|F_f| = 0$  and clearly then  $|F_p| > |F_f|$ .  
d) The  $|F_f|$  must **EQUAL**  $|F_p|$  in order for there to be no acceleration.  
e) The  $|F_f|$  must be **TWICE**  $|F_p|$  in order for there to be no acceleration.
6. Will will now determine the coefficient of static friction  $\mu_{st}$  from an empirical measurement using an adjustable incline. The adjustable angle of incline from the horizontal is  $\theta$ .
- a) Write down Newton's 2nd law for a block sitting at rest on the incline for two directions: perpendicular to the incline and parallel to it. The only forces are the gravity, the normal force, and friction.  
b) The angle of adjustable incline is increased just to the slipping point for the block. Give the parallel 2nd law equation just before slipping occurs.  
c) Solve for the  $\mu_{st}$ .
7. If you double the force constant of a simple harmonic oscillator, the period changes by a factor of:
- a) 1/2.    b)  $\sqrt{2}$ .    c) 2.    d)  $1/\sqrt{2}$ .    e) 2/3.
8. The formula
- $$F_{r,\text{net}}\hat{r} + F_{\theta,\text{net}}\hat{\theta} = m \left[ \left( \frac{d^2r}{dt^2} - r\omega^2 \right) \hat{r} + \left( r\alpha + 2\frac{dr}{dt}\omega \right) \hat{\theta} \right]$$
- is:
- a) Newton's 2nd law in spherical polar coordinates.    b) Newton's 2nd law in polar coordinates.  
c) the centripetal force formula.    d) the centripetal acceleration formula.  
e) the simple harmonic oscillator formula.
9. Uniform circular motion is motion in a/an:

- a) circle at a constant **VELOCITY**.      b) oval at a constant **VELOCITY**.  
 c) oval at a constant **SPEED**.      d) circle at a constant **SPEED**.  
 e) circle at a nonconstant **SPEED**.

10. The formula for the magnitude of centripetal acceleration is:

$$a_{\text{centripetal}} = \frac{v^2}{r},$$

where  $v$  is the speed of a uniform circular motion and  $r$  is the radius of the motion. Say  $v = 10 \text{ m/s}$  and  $r = 5 \text{ m}$ , what is  $a_{\text{centripetal}}$ ?

- a)  $20 \text{ m/s}^2$ .      b)  $10 \text{ m/s}^2$ .      c)  $5 \text{ m/s}^2$ .      d)  $100 \text{ m/s}^2$ .      e)  $15 \text{ m/s}^2$ .

11. The centripetal force is:

- a) a mysterious force that **APPEARS** whenever an object goes into uniform circular motion.  
 b) a mysterious force that tries to throw you **OFF** playground merry-go-rounds.  
 c) in fact  $\vec{F}_{\text{net}}$  of  $\vec{F}_{\text{net}} = m\vec{a}$  when this equation is specialized to the case of uniform circular motion. It is **NOT** a mysterious force that appears whenever you have uniform circular motion. Particular physical forces (e.g., gravity, tension force, and normal force) must act (sometimes in combination) to give a centripetal force which then causes uniform circular motion.  
 d) in fact  $\vec{F}_{\text{net}}$  of  $\vec{F}_{\text{net}} = m\vec{a}$  when this equation is specialized to the case of uniform circular motion. The force itself is **ALWAYS** a field force emanating from the center of motion that pulls on the circling object atom by atom.  
 e) a mysterious force that **DISAPPEARS** whenever an object goes into circular motion.

12. There is a hump on the road with a cylindrical shape. The radius of the hump is  $14.7 \text{ m}$ . In an idealized picture, above about what horizontal speed must a car at the top of the hump lift from the hump?

- a)  $12 \text{ m/s}$ .      b)  $14.7 \text{ m/s}$ .      c)  $144 \text{ m/s}$ .      d)  $10 \text{ m/s}$ .      e)  $10.4 \text{ m/s}$ .

13. The banking angle formula is \_\_\_\_\_. **HINT:** Use dimensional analysis. Ask yourself what formula has reasonable limiting behavior when input values go to extremes. Or just derive it using the centripetal force formula and the 2nd law.

- a)  $\theta = \tan^{-1}\left(\frac{v}{rg}\right)$       b)  $\theta = \tan^{-1}\left(\frac{rg}{v^2}\right)$       c)  $\theta = \tan^{-1}\left(\frac{v^2 r}{g}\right)$       d)  $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$   
 e)  $\theta = \tan^{-1}\left(\frac{g}{v^2 r}\right)$

14. In what situations, if any, can a body move in a circular path at constant speed without a centripetal force?

- a) None.      b) In certain special non-inertial frames.      c) In all non-inertial frames.  
 d) In all inertial frames.      e) Always.

15. Is-Hilda the ladybug is on a vinyl record spinning at  $78 \text{ rpm}$ . She starts at the center and six-leggedly walks radially outward. Her radial velocity (caused by her walking) is negligible compared to her tangential velocity (caused by the record motion). At  $6 \text{ cm}$  from the center, Is-Hilda suddenly slides off the record. The record counterfactually is a smooth surface—and isn't corrugated—and Is-Hilda is an ideal ladybug without sticky feet. What is the static friction coefficient between her and the record? What is the kinetic friction coefficient if you can determine it?

16. There is a conical pendulum of length  $\ell$  with bob of mass  $m$ . The bob is executing uniform circular motion with velocity  $v$  and radius  $r$ . The pendulum has an ideal mass rope with tension  $T$ . The angle of the rope from the horizontal is  $\theta$ .

- a) Write down Newton's 2nd law for the horizontal directions and vertical directions making use of the centripetal acceleration formula.  
 b) For a person just swinging a conical pendulum by hand, it is probably easiest to set  $\theta$  (at least roughly),  $\ell$  and  $m$ . Find formulae for the variables  $r$ ,  $T$ , and  $v$  as functions of **ONLY**  $\theta$ ,  $\ell$ ,  $m$  and  $g$ . Write the  $T$  formulae with  $\theta$  only appearing in the sine function.  
 c) Give the formula for the period  $P$  of the motion as a function of **ONLY**  $\theta$  and  $\ell$ .

17. Consider an ordinary road corner that is level ground. Such corners are usually not banked very much, in fact.
- What force supplies the centripetal force that allows a vehicle to make the the corner?
  - For a corner turn with radius of curvature  $r$  and a vehicle of mass  $m$ , derive the formula for the maximum turn speed  $v$  before slipping occurs.
  - Say  $r = 10$  m (which seems reasonable for ordinary corners) and  $\mu_{\text{st}} = 1.0$  (as for rubber on concrete in dry conditions approximately), what is the maximum turn speed before slipping? Now say  $\mu_{\text{st}} = 0.30$  (as for rubber on concrete in dry conditions approximately), what is the maximum turn speed before slipping? Convert the answers to miles per hour.
18. An airplane of mass  $m$  is making a turn through a circular bend with a speed of  $v$ . It moves horizontally only: i.e., it's not moving or accelerating downward or upward. In order to make the turn the plane banks at angle  $\theta$  from the horizontal (i.e., its wings are tipped at  $\theta$  to the horizontal with the high side away from the center of the turn). Assume all the force the air can exert on the plane (the aerodynamic lift) acts perpendicularly to the wings. Let  $F_{\text{lift}}$  be the aerodynamic lift magnitude.
- Draw a free body diagram for the airplane in the plane perpendicular to the airplane's instantaneous direction of motion.
  - Apply Newton's 2nd law to airplane in the plane perpendicular to the airplane's instantaneous direction of motion. Remember the course mantra:  

“Newton's 2nd law is always true and it's always true component by component.”
  - Solve for the  $r$  in terms of the other variables: i.e., in terms of  $m$ ,  $\theta$ ,  $v$ ,  $g$ , and  $F_{\text{lift}}$ .
  - Given  $v = 600$  km/h and  $\theta = 30^\circ$ , what is  $r$ ?
19. An object falling from rest is subject to a drag force of magnitude  $f(v)$ , where  $v$  is the object's speed. The function  $f(v)$  is monotonically increasing with  $v$  and  $f(0) = 0$ , but is otherwise general. The inverse function to  $f(v)$  is  $f^{-1}(x)$ .
- Apply Newton's 2nd law to the object taking the downward direction as positive. **HINT:** You just write down the equation.
  - What is terminal velocity and why should the falling object reach it? What is the formula for the terminal velocity  $v_{\text{ter}}$ ? Make use of  $f^{-1}(x)$ .
  - The evolution to terminal velocity can be crudely divided into two phases: a linear growth phase (when the velocity is growing approximately linearly) and an asymptotic phase (when the velocity is asymptotically approaching terminal velocity). The characteristic time  $t_{\text{ch}}$  border between the two phases is obtained by setting  $f(v) = 0$  in the equation of motion and solving for the time when  $v = v_{\text{ter}}$ . Beyond this time, the pure linear growth must be over. Derive the formula for  $t_{\text{ch}}$ . Then derive the formula for the characteristic length  $\ell_{\text{ch}}$  which is the distance fallen in time  $t_{\text{ch}}$  assuming  $f(v) = 0$ .
  - The formula

$$v = v_{\text{ter}}(1 - e^{-t/t_{\text{ch}}})$$

is a crude approximate solution for velocity in general. If we have linear drag  $f(v) = bv$  (where  $b$  is a constant, then the solution becomes exact. There are cases where linear drag holds: for very low speeds and no turbulence. Sketch a plot of the approximate solution. Verify by substitution into the equation of motion (i.e., our Newton's 2nd law application) that the velocity formula is exact for  $f(v) = bv$ .

# Equation Sheet for Introductory Physics Calculus-Based

This equation sheet is intended for students writing tests or reviewing material. Therefore it is neither intended to be complete nor completely explicit. There are fewer symbols than variables, and so some symbols must be used for different things: context must distinguish.

The equations are mnemonic. Students are expected to understand how to interpret and use them.

## 1 Constants

$$c = 2.99792458 \times 10^8 \text{ m/s} \approx 2.998 \times 10^8 \text{ m/s} \approx 3 \times 10^8 \text{ m/s} \approx 1 \text{ yr/yr} \approx 1 \text{ ft/ns} \quad \text{exact by definition}$$

$$e = 1.602176487(40) \times 10^{-19} \text{ C}$$

$$G = 6.67428(67) \times 10^{-11} \text{ N m}^2/\text{kg}^2 \quad (2006, \text{CODATA})$$

$$g = 9.8 \text{ m/s}^2 \quad \text{fiducial value}$$

$$k = \frac{1}{4\pi\epsilon_0} = 8.987551787 \dots \times 10^9 \approx 8.99 \times 10^9 \approx 10^{10} \text{ N m}^2/\text{C}^2 \text{ exact by definition}$$

$$k_{\text{Boltzmann}} = 1.3806504(24) \times 10^{-23} \text{ J/K} = 0.8617343(15) \times 10^{-4} \text{ eV/K} \approx 10^{-4} \text{ eV/K}$$

$$m_e = 9.10938215(45) \times 10^{-31} \text{ kg} = 0.510998910(13) \text{ MeV}$$

$$m_p = 1.672621637(83) \times 10^{-27} \text{ kg} = 938.272013(23), \text{ MeV}$$

$$\epsilon_0 = \frac{1}{\mu_0 c^2} = 8.8541878176 \dots \times 10^{-12} \text{ C}^2/(\text{N m}^2) \approx 10^{-11} \quad \text{vacuum permittivity (exact by definition)}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2 \quad \text{exact by definition}$$

## 2 Geometrical Formulae

$$C_{\text{cir}} = 2\pi r \quad A_{\text{cir}} = \pi r^2 \quad A_{\text{sph}} = 4\pi r^2 \quad V_{\text{sph}} = \frac{4}{3}\pi r^3$$

$$\Omega_{\text{sphere}} = 4\pi \quad d\Omega = \sin\theta \, d\theta \, d\phi$$

## 3 Trigonometry Formulae

$$\frac{x}{r} = \cos\theta \quad \frac{y}{r} = \sin\theta \quad \frac{y}{x} = \tan\theta = \frac{\sin\theta}{\cos\theta} \quad \cos^2\theta + \sin^2\theta = 1$$

$$\csc\theta = \frac{1}{\sin\theta} \quad \sec\theta = \frac{1}{\cos\theta} \quad \cot\theta = \frac{1}{\tan\theta}$$

$$c^2 = a^2 + b^2 \quad c = \sqrt{a^2 + b^2 - 2ab \cos\theta_c} \quad \frac{\sin\theta_a}{a} = \frac{\sin\theta_b}{b} = \frac{\sin\theta_c}{c}$$

$$f(\theta) = f(\theta + 360^\circ)$$

$$\sin(\theta + 180^\circ) = -\sin(\theta) \quad \cos(\theta + 180^\circ) = -\cos(\theta) \quad \tan(\theta + 180^\circ) = \tan(\theta)$$

$$\sin(-\theta) = -\sin(\theta) \quad \cos(-\theta) = \cos(\theta) \quad \tan(-\theta) = -\tan(\theta)$$

$$\sin(\theta + 90^\circ) = \cos(\theta) \quad \cos(\theta + 90^\circ) = -\sin(\theta) \quad \tan(\theta + 90^\circ) = -\tan(\theta)$$

$$\sin(180^\circ - \theta) = \sin(\theta) \quad \cos(180^\circ - \theta) = -\cos(\theta) \quad \tan(180^\circ - \theta) = -\tan(\theta)$$

$$\sin(90^\circ - \theta) = \cos(\theta) \quad \cos(90^\circ - \theta) = \sin(\theta) \quad \tan(90^\circ - \theta) = \frac{1}{\tan(\theta)} = \cot(\theta)$$

$$\sin(a + b) = \sin(a)\cos(b) + \cos(a)\sin(b) \quad \cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$\sin(2a) = 2\sin(a)\cos(a) \quad \cos(2a) = \cos^2(a) - \sin^2(a)$$

$$\sin(a)\sin(b) = \frac{1}{2}[\cos(a - b) - \cos(a + b)] \quad \cos(a)\cos(b) = \frac{1}{2}[\cos(a - b) + \cos(a + b)]$$

$$\sin(a)\cos(b) = \frac{1}{2}[\sin(a - b) + \sin(a + b)]$$

$$\sin^2\theta = \frac{1}{2}[1 - \cos(2\theta)] \quad \cos^2\theta = \frac{1}{2}[1 + \cos(2\theta)] \quad \sin(a)\cos(a) = \frac{1}{2}\sin(2a)$$

$$\cos(x) - \cos(y) = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos(x) + \cos(y) = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin(x) + \sin(y) = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

#### 4 Approximation Formulae

$$\frac{\Delta f}{\Delta x} \approx \frac{df}{dx} \quad \frac{1}{1-x} \approx 1+x : (x \ll 1)$$

$$\sin\theta \approx \theta \quad \tan\theta \approx \theta \quad \cos\theta \approx 1 - \frac{1}{2}\theta^2 \quad \text{all for } \theta \ll 1$$

#### 5 Quadratic Formula

$$\text{If } 0 = ax^2 + bx + c, \quad \text{then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = -\frac{b}{2a} \pm \sqrt{\left(\frac{b}{2a}\right)^2 - \frac{c}{a}}$$

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## 6 Vector Formulae

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2} \quad \theta = \tan^{-1} \left( \frac{a_y}{a_x} \right) + \pi? \quad \vec{a} + \vec{b} = (a_x + b_x, a_y + b_y)$$

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} \quad \phi = \tan^{-1} \left( \frac{a_y}{a_x} \right) + \pi? \quad \theta = \cos^{-1} \left( \frac{a_z}{a} \right)$$

$$\vec{a} + \vec{b} = (a_x + b_x, a_y + b_y, a_z + b_z)$$

$$\vec{a} \cdot \vec{b} = ab \cos \theta = a_x b_x + a_y b_y + a_z b_z$$

$$\vec{c} = \vec{a} \times \vec{b} = ab \sin(\theta) \hat{c} = (a_y b_z - b_y a_z, a_z b_x - b_z a_x, a_x b_y - b_x a_y)$$

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## 7 Differentiation and Integration Formulae

$$\frac{d(x^p)}{dx} = px^{p-1} \quad \text{except for } p = 0; \quad \frac{d(x^0)}{dx} = 0 \quad \frac{d(\ln|x|)}{dx} = \frac{1}{x}$$

Taylor's series

$$f(x) = \sum_{n=0}^{\infty} \frac{(x-x_0)^n}{n!} f^{(n)}(x_0)$$

$$= f(x_0) + (x-x_0)f^{(1)}(x_0) + \frac{(x-x_0)^2}{2!}f^{(2)}(x_0) + \frac{(x-x_0)^3}{3!}f^{(3)}(x_0) + \dots$$

$$\int_a^b f(x) dx = F(x)|_a^b = F(b) - F(a) \quad \text{where} \quad \frac{dF(x)}{dx} = f(x)$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} \quad \text{except for } n = -1; \quad \int \frac{1}{x} dx = \ln|x|$$

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## 8 One-Dimensional Kinematics

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} \quad v = \frac{dx}{dt} \quad a_{\text{avg}} = \frac{\Delta v}{\Delta t} \quad a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

$$v = at + v_0 \quad x = \frac{1}{2}at^2 + v_0t + x_0 \quad v^2 = v_0^2 + 2a(x - x_0)$$

$$x = \frac{1}{2}(v_0 + v)t + x_0 \quad x = -\frac{1}{2}at^2 + vt + x_0 \quad g = 9.8 \text{ m/s}^2$$

$$x_{\text{rel}} = x_2 - x_1 \quad v_{\text{rel}} = v_2 - v_1 \quad a_{\text{rel}} = a_2 - a_1$$

$$x' = x - v_{\text{frame}}t \quad v' = v - v_{\text{frame}} \quad a' = a$$

## 9 Two- and Three-Dimensional Kinematics: General

$$\vec{v}_{\text{avg}} = \frac{\Delta \vec{r}}{\Delta t} \quad \vec{v} = \frac{d\vec{r}}{dt} \quad \vec{a}_{\text{avg}} = \frac{\Delta \vec{v}}{\Delta t} \quad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

## 10 Projectile Motion

$$x = v_{x,0}t \quad y = -\frac{1}{2}gt^2 + v_{y,0}t + y_0 \quad v_{x,0} = v_0 \cos \theta \quad v_{y,0} = v_0 \sin \theta$$

$$t = \frac{x}{v_{x,0}} = \frac{x}{v_0 \cos \theta} \quad y = y_0 + x \tan \theta - \frac{x^2 g}{2v_0^2 \cos^2 \theta}$$

$$x_{\text{for } y \text{ max}} = \frac{v_0^2 \sin \theta \cos \theta}{g} \quad y_{\text{max}} = y_0 + \frac{v_0^2 \sin^2 \theta}{2g}$$

$$x(y = y_0) = \frac{2v_0^2 \sin \theta \cos \theta}{g} = \frac{v_0^2 \sin(2\theta)}{g} \quad \theta_{\text{for max}} = \frac{\pi}{4} \quad x_{\text{max}}(y = y_0) = \frac{v_0^2}{g}$$

$$x(\theta = 0) = \pm v_0 \sqrt{\frac{2(y_0 - y)}{g}} \quad t(\theta = 0) = \sqrt{\frac{2(y_0 - y)}{g}}$$

## 11 Relative Motion

$$\vec{r} = \vec{r}_2 - \vec{r}_1 \quad \vec{v} = \vec{v}_2 - \vec{v}_1 \quad \vec{a} = \vec{a}_2 - \vec{a}_1$$

## 12 Polar Coordinate Motion and Uniform Circular Motion

$$\omega = \frac{d\theta}{dt} \quad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\vec{r} = r\hat{r} \quad \vec{v} = \frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{r} + r\omega\hat{\theta} \quad \vec{a} = \frac{d^2\vec{r}}{dt^2} = \left(\frac{d^2r}{dt^2} - r\omega^2\right)\hat{r} + \left(r\alpha + 2\frac{dr}{dt}\omega\right)\hat{\theta}$$

$$\vec{v} = r\omega\hat{\theta} \quad v = r\omega \quad a_{\text{tan}} = r\alpha$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} = -r\omega^2\hat{r} \quad a_{\text{centripetal}} = \frac{v^2}{r} = r\omega^2 = v\omega$$

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### 13 Very Basic Newtonian Physics

$$\vec{r}_{\text{cm}} = \frac{\sum_i m_i \vec{r}_i}{m_{\text{total}}} = \frac{\sum_{\text{sub}} m_{\text{sub}} \vec{r}_{\text{cm sub}}}{m_{\text{total}}} \quad \vec{v}_{\text{cm}} = \frac{\sum_i m_i \vec{v}_i}{m_{\text{total}}} \quad \vec{a}_{\text{cm}} = \frac{\sum_i m_i \vec{a}_i}{m_{\text{total}}}$$

$$\vec{r}_{\text{cm}} = \frac{\int_V \rho(\vec{r}) \vec{r} dV}{m_{\text{total}}}$$

$$\vec{F}_{\text{net}} = m\vec{a} \quad \vec{F}_{21} = -\vec{F}_{12} \quad F_g = mg \quad g = 9.8 \text{ m/s}^2$$

$$\vec{F}_{\text{normal}} = -\vec{F}_{\text{applied}} \quad F_{\text{linear}} = -kx$$

$$f_{\text{normal}} = \frac{T}{r} \quad T = T_0 - F_{\text{parallel}}(s) \quad T = T_0$$

$$F_{\text{f static}} = \min(F_{\text{applied}}, F_{\text{f static max}}) \quad F_{\text{f static max}} = \mu_{\text{static}} F_{\text{N}} \quad F_{\text{f kinetic}} = \mu_{\text{kinetic}} F_{\text{N}}$$

$$v_{\text{tangential}} = r\omega = r \frac{d\theta}{dt} \quad a_{\text{tangential}} = r\alpha = r \frac{d\omega}{dt} = r \frac{d^2\theta}{dt^2}$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r} \hat{r} \quad \vec{F}_{\text{centripetal}} = -m \frac{v^2}{r} \hat{r}$$

$$F_{\text{drag, lin}} = bv \quad v_{\text{T}} = \frac{mg}{b} \quad \tau = \frac{v_{\text{T}}}{g} = \frac{m}{b} \quad v = v_{\text{T}}(1 - e^{-t/\tau})$$

$$F_{\text{drag, quad}} = bv^2 = \frac{1}{2} C \rho A v^2 \quad v_{\text{T}} = \sqrt{\frac{mg}{b}}$$

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### 14 Energy and Work

$$dW = \vec{F} \cdot d\vec{s} \quad W = \int \vec{F} \cdot d\vec{s} \quad KE = \frac{1}{2}mv^2 \quad E_{\text{mechanical}} = KE + PE$$

$$P_{\text{avg}} = \frac{\Delta W}{\Delta t} \quad P = \frac{dW}{dt} \quad P = \vec{F} \cdot \vec{v}$$

$$\Delta KE = W_{\text{net}} \quad \Delta PE_{\text{of a conservative force}} = -W_{\text{by a conservative force}} \quad \Delta E = W_{\text{nonconservative}}$$

$$F = -\frac{dPE}{dx} \quad \vec{F} = -\nabla PE \quad PE = \frac{1}{2}kx^2 \quad PE = mgy$$