Intro Physics Semester I **Name:**

Homework 5: Classical Mechanics I: One or two full answer questions will be marked. There will also be a mark for completeness. Homeworks are due usually the day after the chapter they are for is finished. Solutions will be posted soon thereafter. The solutions are intended to be (but not necessarily are) super-perfect and often go beyond a fully correct answer.

Answer Table Name:

	a	b	c	d	e		a	b	c	d	e
1.	O	O	O	O	O	31.	O	O	O	O	O
2.	O	O	O	O	O	32.	O	O	O	O	O
3.	O	O	O	O	O	33.	O	O	O	O	O
4.	O	O	O	O	O	34.	O	O	O	O	O
5.	O	O	O	O	O	35.	O	O	O	O	O
6.	O	O	O	O	O	36.	O	O	O	O	O
7.	O	O	O	O	O	37.	O	O	O	O	O
8.	O	O	O	O	O	38.	O	O	O	O	O
9.	O	O	O	O	O	39.	O	O	O	O	O
10.	O	O	O	O	O	40.	O	O	O	O	O
11.	O	O	O	O	O	41.	O	O	O	O	O
12.	O	O	O	O	O	42.	O	O	O	O	O
13.	O	O	O	O	O	43.	O	O	O	O	O
14.	O	O	O	O	O	44.	O	O	O	O	O
15.	O	O	O	O	O	45.	O	O	O	O	O
16.	O	O	O	O	O	46.	O	O	O	O	O
17.	O	O	O	O	O	47.	O	O	O	O	O
18.	O	O	O	O	O	48.	O	O	O	O	O
19.	O	O	O	O	O	49.	O	O	O	O	O
20.	O	O	O	O	O	50.	O	O	O	O	O
21.	O	O	O	O	O	51.	O	O	O	O	O
22.	O	O	O	O	O	52.	O	O	O	O	O
23.	O	O	O	O	O	53.	O	O	O	O	O
24.	O	O	O	O	O	54.	O	O	O	O	O
25.	O	O	O	O	O	55.	O	O	O	O	O
26.	O	O	O	O	O	56.	O	O	O	O	O
27.	O	O	O	O	O	57.	O	O	O	O	O
28.	O	O	O	O	O	58.	O	O	O	O	O
29.	O	O	O	O	O	59.	O	O	O	O	O
30.	O	O	O	O	O	60.	O	O	O	O	O

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005 qmult 00100 1 4 2 easy deducto-memory: dynamics defined 1

1. "Let's play *Jeopardy*. For $100, the answer is: The branch of physics that explains motion and acceleration in terms of forces and masses."

   What is ____________, Alex?

   a) kinematics b) dynamics c) statics d) economics e) cinematics

   SUGGESTED ANSWER: (b)

   Wrong answers:

   Redaction: Jeffery, 2001jan01

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005 qmult 00110 1 4 5 easy deducto-memory: dynamics defined 2

2. Dynamics is that branch of physics that:

   a) explains motion and acceleration in terms of the kinematic equations.
   b) explains motion and acceleration in terms of error analysis.
   c) treats dynamos.
   d) treats electricity and magnetism or electromagnetism.
   e) explains motion and acceleration in terms of forces and masses.

   SUGGESTED ANSWER: (e)

   Wrong answers:

   Redaction: Jeffery, 2001jan01

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005 qmult 00130 1 5 1 easy thinking: statics defined

3. The area of physics dealing with **ONLY** cases of balanced forces (or equilibrium) is called:

   a) statics. b) dynamics. c) kinematics. d) kinesiology. e) cinema.

   SUGGESTED ANSWER: (a)

   An easy thinking question. Statics may not have been mentioned explicitly in class. Technically the torques needed to be balanced too, but mentioning that would obscure the question. See French, p. 119. Memory and deduction should help here. But the name alone should be enough.

   Wrong answers:

   e) As Lurch would say, AAAAARGH.

   Redaction: Jeffery, 2008jan01

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005 qmult 00310 1 5 3 easy thinking: what forces do

4. Forces can cause accelerations relative to inertial frames or cancel other forces. Another manifestation (which actually follows from their property of causing acceleration) is that they can cause:

   a) velocity (without causing acceleration).
   b) mass.
   c) bodies to distort: i.e., flex, compress, stretch, etc.
   d) bodies to live
   e) bodies to rule.

   SUGGESTED ANSWER: (c)

   Forces do so much that with suitable qualification almost anything can be a predicate here. But in a definitional general sense "cause acceleration relative to inertial frames" and "cancel other forces" are the main properties. They also distort bodies. This is not really an independent property of force. If accelerations of a body happen relative to other parts of a body, then there will be deformations. Constant velocity deformations can happen too, but an acceleration was needed to create the velocity doing the deforming in the first place.

   If we don't see either an acceleration or a distortion, then how do we know or measure force? Well we often use the 2nd or 3rd law in cases where acceleration zero and distortion is invisible: but distortion is there even if we don't see it. For instance, the normal force of a macroscopically rigid body may not manifest itself either way. But there is a microscopic distortion with the normal force surface nonetheless.

Wrong answers:

a) This is one thing they don't cause. You could twist the meaning of the words to make it true, but it would just be a twisted case.
b) Arguable in some far-out high energy physics way.
d) Again sure, but they don't have to.
e) Nonsense answer.

Redaction: Jeffery, 2001jan01

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005 qmult 00400 1 1 4 easy memory: inertial frames defined

5. Accelerations with respect to ____________—which we will call natural frames for the nonce—though no one calls them that—and only they require forces as causes as prescribed by Newton's 2nd law ($\vec F_{\text{net}} = m\vec a$) in the classical limit. What are natural frames? They have been elusive historically. Newton hypothesized that a primary natural frame defined by the mean position of the fixed stars—absolute space as he called it. But the fixed stars move—as Newton knew himself—and revolve around the center of the Milky Way in complex orbits—as Newton did not know himself. The Milky Way and other galaxies are also in complex orbits in galaxy clusters or otherwise in complex relative motions. In modern cosmological theory, natural frames are frames of reference attached to points in space that participate in the mean expansion of the universe. Space is growing—just accept it. Not all space—not space within bound systems like you, me, and the Milky Way—but the space in between bound systems like galaxy clusters. To every point participating in the mean expansion of the universe attach the origin of a local pimary natural frame. It is called local because sufficiently close to the origin, the frame has the behavior given above. As you move away from the origin, there is a progressive departure from the behavior, but you have to move over distance scales larger than a galaxy cluster for that to become very noticeable. Now any frame in uniform motion (i.e., unaccelerated) with respect to a local primary natural frame is also a local natural frame. Say the pimary frame is unprimed and the non-primary is primed. Then we have

$$\vec r\,' = \vec r - \vec r_{\text{prime}} \ ,$$

where is $\vec r_{\text{prime}}$ is the position of the primed frame in the unprimed frame. Differentiate twice and you get $\vec a\,' = \vec a$. So accelerations in the unprimed frame are exactly those of the primed frame. So Newton's 2nd law ($\vec F_{\text{net}} = m\vec a$) must be obeyed for accelerations relative to non-primary local frames. Forces themselves are frame-independent in classical mechanics. If you need relativistic physics the story changes.

Actually, Newton's 2nd law can be generalized to non-natural frames by introducing what are called inertial forces which are not real forces, but force-like terms that account for using non-natural frames. In fact, using inertial forces is usually the best approach to non-natural frames.

By the by, we can actually identify natural frames in the universe and our own local one very precisely using astronomical measurements. However, for many purposes we can find non-natural frames that are sufficiently close to being natural frames that they can be used as natural frames to some degree of approximation. The local Earth surface (i.e., the ground) is natural enough for many purposes: not long-range gunnery or large-scale weather phenomena. If you need a more natural natural frame, you can use the fixed stars. For highest accuracy, we can use the local primary natural frame using cosmological knowledge.

a) rotating frames. b) accelerated frames. c) non-inertial frames. d) inertial frames.
e) picture frames.

SUGGESTED ANSWER: (d)

Every "natural" should be replaced by "inertial". "Inertial" makes little sense, but by convention we are stuck with it.

Wrong answers:

c) Exactly wrong.

Redaction: Jeffery, 2008jan01

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005 qmult 00510 1 1 3 easy memory: number of Newton's laws

6. How many laws of motion did Newton posit?

   a) 1. b) 2. c) 3. d) 4. e) 5.

   SUGGESTED ANSWER: (c)

   Wrong answers:
   b) Logically he needed only two: his 2nd and 3rd laws. The 1st law is a special case of the 2nd. But for historical and heuristic reasons he must have felt he needed the 1st law.

   Redaction: Jeffery, 2008jan01

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005 qmult 00520 3 5 3 tough thinking: 1st law redundant

7. Newton's 1st law is.

   a) **PHYSICALLY INDEPENDENT** of the other two laws of motion and **CANNOT** be dispensed with as an axiom of Newtonian physics.
   b) **PHYSICALLY INDEPENDENT** of the other two laws of motion, but nonetheless it **CAN** be dispensed with as an axiom of Newtonian physics.
   c) actually a **SPECIAL CASE** of the **2ND LAW**. The case when the net force is zero. Therefore logically we need only two laws of motion. Perhaps for clarity Newton formulated his explicit 1st law and perhaps for the same reason physicists have retained it.
   d) actually a **SPECIAL CASE** of the **3RD LAW**. The case when the net force is zero. Therefore logically we need only two laws of motion. Perhaps for clarity Newton formulated his explicit 1st law and perhaps for the same reason physicists have retained it.
   e) is **INCORRECT**, but is kept in the books for historical reasons.

   SUGGESTED ANSWER: (c)

   A tough thinking question. The students really have to grasp Ockham's razor (which could be painful) and recognize how many basic principles are needed.

   Actually, I'm getting tired of the 1st law. Despite the weight of history, maybe we should just junk it from the textbooks and talk of the two laws of motion with $F = ma$ having the special case of $a = 0$.

   Wrong answers:
   e) Oh, c'mon.

   Redaction: Jeffery, 2008jan01

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005 qmult 00530 1 1 3 easy memory: Newton's 2nd law: 1

8. Newton's 2nd law is:

   a) $m = \vec{F}_{\rm net}\vec{a}$.
   b) $\vec{a} = m\vec{F}_{\rm net}$.
   c) $\vec{F}_{\rm net} = m\vec{a}$.
   d) For every force there is an equal and opposite force.
   e) For every acceleration there is an equal and opposite acceleration.

   SUGGESTED ANSWER: (c) "All I ever learnt in physics was $\vec{F}_{\rm net} = m\vec{a}$."

   Wrong answers:

   Redaction: Jeffery, 2001jan01

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005 qmult 00534 1 5 1 easy memory: Newton's 2nd law class mantra 1

9. From here on in this course, a key thing to remember (to recite to yourself) when faced with any force problem is that Newton's 2nd law ($\vec{F}_{\rm net} = m\vec{a}$) is:

   a) **ALWAYS VALID**. And it is a **VECTOR** equation, and so is always **VALID** component by component. And $\vec{F}_{\rm net}$ is the **VECTOR** sum of all forces acting on the body of mass m. It is not any particular force. If all the forces sum to zero vectorially, $\vec{F}_{\rm net} = m\vec{a} = 0$. If you are given the acceleration, then you can often use $\vec{F}_{\rm net} = m\vec{a}$ to solve for an unknown force.
   b) **ALWAYS VALID**. And it is a **SCALAR** equation. And $\vec{F}_{\rm net}$ is the **SCALAR** sum of all forces acting on the body of mass m. It is not any particular force. If all the forces sum to zero,

$\vec{F}_{\text{net}} = m\vec{a} = 0$. If you are given the acceleration, then you can often use $\vec{F}_{\text{net}} = m\vec{a}$ to solve for an unknown force.

c) **ONLY VALID** when there is a **NON-ZERO** net force. Because the 2nd law is a **VECTOR** equation, it is valid (when it is valid) component by component. And $\vec{F}_{\text{net}}$ is the **VECTOR** sum of all forces acting on the body of mass m. It is not any particular force. If you are given the acceleration, then you can often use $\vec{F}_{\text{net}} = m\vec{a}$ to solve for an unknown force.

d) **ALWAYS INVALID**.

d) **NEVER VALID**.

SUGGESTED ANSWER: (a) "All I ever learnt in physics was $\vec{F}_{\text{net}} = m\vec{a}$."

Wrong answers:

b) Not scalar.

c) It is perfectly valid when $\vec{a} = 0$.

d) Oh, c'mon.

e) Oh, c'mon, again.

Redaction: Jeffery, 2001jan01

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005 qmult 00542 1 5 1 easy thinking: acceleration and third law

Extra keywords: also physci KB-59-15

10. If Newton's 3rd law is true, why then does anything accelerate at all?

a) The equal and opposite forces **DO NOT** have to be on the same body.

b) The equal and opposite forces **DO** have to be on the same body.

c) Nothing moves at all as Parmenides argued in the 5th century BC. Motion is but seeming. Anyway Parmenides seems to have been a pretty smart guy since he's credited with the spherical Earth theory and the discovery that the Moon shines by reflected light.

d) Acceleration has nothing do with forces.

e) Forces have nothing do with acceleration.

SUGGESTED ANSWER: (a) I've provided some leading answers.

Wrong answers:

b) Straight nonsense, since it leads to the opposite conclusion.

c) Parmenides was not really saying that nothing moves at all. He was just arguing from certain premises which he did not necessarily affirm. Actually it is hard to quite know for sure about the big P, since his own words only survive in fragments from his poem in which he lets the unnamed goddess speak for him in oracular manner. Shortly after Parmenides, natural philosophers gave up on poetry and the two have seldom overlapped since. Omar Khayyam (if he really was a poet) and Chaucer (really more of popularizer of science than a practitioner) are possible cases. See D. Furley, "The Greek Cosmologists", p. 36 ff, esp. 41.

Redaction: Jeffery, 2001jan01

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005 qmult 00540 1 4 5 easy deducto-memory: force laws needed

11. "Let's play *Jeopardy*! For \$100, the answer is: Laws that prescribe forces for physical systems. They must exist independent of Newton's 3 laws of motion in order for Newtonian physics to be useful."

What are ____________, Alex?

a) Newton's 3 laws b) accelerations c) velocities d) force inequalities e) force laws

SUGGESTED ANSWER: (e)

Wrong answers:

a) As Lurch would say AAAARGH.

Redaction: Jeffery, 2008jan01

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005 qmult 00570 1 1 5 easy memory: newton defined

12. The base SI unit of force is the:

a) farad (F); $1\,\text{F} = 1\,\text{kg}\,\text{m}/\text{s}^2 \approx 0.22481\,\text{lb} \approx 1/5\,\text{lb}$.

b) henry (H); $1\,\text{H} = 1\,\text{kg}\,\text{m}/\text{s}^2 \approx 0.22481\,\text{lb} \approx 1/5\,\text{lb}$.

c) watt (W); $1\,\mathrm{W} = 1\,\mathrm{kg\,m/s^2} \approx 0.22481\,\mathrm{lb} \approx 1/5\,\mathrm{lb}$.
d) joule (J); $1\,\mathrm{J} = 1\,\mathrm{kg\,m/s^2} \approx 0.22481\,\mathrm{lb} \approx 1/5\,\mathrm{lb}$.
e) newton (N); $1\,\mathrm{N} = 1\,\mathrm{kg\,m/s^2} \approx 0.22481\,\mathrm{lb} \approx 1/5\,\mathrm{lb}$.

SUGGESTED ANSWER: (e) This definition of the newton relies on the exact nature of the 2nd law in the classical limit.

Wrong answers:
a) The unit of capacitance.
b) The unit of inductance.
c) The unit of power.
d) The unit of energy.

Redaction: Jeffery, 2008jan01

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005 qmult 00710 2 3 5 moderate math: stopping a bike
Extra keywords: physci KB-60-21

13. A bicycle-rider system has a mass of 80 kg. The bike is traveling on level and has initial velocity 6 m/s north. What is the constant force needed to stop the bike in 4 s?

a) 80 N south. b) 80 N north. c) 80 N east. d) 100 N south. e) 120 N south.

SUGGESTED ANSWER: (e)

The acceleration to stop the bike is

$$a = \frac{v - v_0}{t} ,$$

where $v = 0$ is the final velocity, $v_0 = 6\,\mathrm{m/s}$ is the initial velocity, $t = 4\,\mathrm{s}$ is the stopping time, and I've chosen north at the positive direction. Thus the constant stopping force is thus

$$F = ma = m\left(\frac{v - v_0}{t}\right) = 80 \times (-1.5) = -120\,\mathrm{N} .$$

South is the negative direction and thus the stopping force is 120 N south.

Wrong answers:
c) East. Are you trying to tip the bike.

Redaction: Jeffery, 2001jan01

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005 qmult 00900 1 1 1 easy memory: force law for gravity

14. The magnitude of the gravitational force on an object of mass m for a uniform gravitational field (such as the gravitational field near the Earth's surface for human-size and somewhat larger objects) is given by the formula:

a) $F = mg$. b) $F = m/g$. c) $F = g/m$. d) $F = ma$. e) $F = m/a$.

SUGGESTED ANSWER: (a)

Wrong answers:
d) Oh, c'mon.

Redaction: Jeffery, 2008jan01

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005 qmult 00910 1 1 4 easy memory: solving mg=ma equation of motion

15. Newton's 2nd law applied to the vertical direction with only the gravity force acting and down defined as positive leads to the scalar equation of motion:

a) $g = m/a$. b) $g = ma$. c) $mg = a$. d) $mg = ma$. e) $m/g = m/a$.

SUGGESTED ANSWER: (d)

Wrong answers:
e) A nonsense answer.

Redaction: Jeffery, 2008jan01

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005 qmult 00920 1 3 3 easy math: person's weight
Extra keywords: physci

16. If you have a mass of 60 kg and $g = 9.8\,\mathrm{m/s^2}$, you weigh about:

    a) 10 N. b) 60 N. c) 600 N. d) 500 N. e) 20 N.

SUGGESTED ANSWER: (c)

Actually you need to remember the gravitational force is the cause of weight. In this case,

$$F = mg = 60 \times 9.8 \approx 600\,\mathrm{N} .$$

Wrong answers:

Redaction: Jeffery, 2001jan01

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005 qmult 00930 2 5 2 easy math: gravity field force and contact forces

17. The force of gravity reaches out across space and pulls on each bit of your body independently of every other bit. We call a force like this a **FIELD FORCE** or a **BODY FORCE**. Why don't you accelerate downward, except when off the ground.

    a) The **GROUND FORCE** reaches out across space and pushes upward on each bit of your body independently of every other bit. The ground force is also a **FIELD FORCE**.
    b) The ground exerts a force on the soles of your feet and the soles of your feet on the next layer of your body and the next layer of your body on the next layer of your body and so on until the top of your head. Each layer pushes up with only enough force to balance the gravity force on the mass above. The ground force and the forces exerted by the layers of our bodies are **CONTACT FORCES**. A **CONTACT FORCE** acts over a very short range: so short that if the distance between the two objects exerting equal and opposite contact forces on each other is more than microscopic there is no contact force at all.
    c) Since you are always off the ground, the question has no answer.
    d) Since you are always off the ground, the question is hypothetical and the answer, speculative.
    e) In orbit, you don't accelerate downward and you are certainly off the ground. So being on the ground may have nothing to do with why you don't accelerate downward.

SUGGESTED ANSWER: (b)

Actually, the students should get the answer easily by deduction, but it takes clear thinking and maybe a bit more physics than has been yet taught to see why answer (b) is right.

Wrong answers:

a) Why should the ground exert a force on the top of your head or elsewhere only when your feet touch the ground. Well whether it could or not, it doesn't.
c) We're not always off the ground and even if we were, other things are on the ground and the question could be answered for those other things.
d) We're not always off the ground and even if we were then other things are on the ground and the question isn't hypothetical and the answer isn't speculative for them.
e) In orbit you are in free fall. You are accelerating downward toward the Earth's center. You just keep missing the Earth. Also this doesn't seem to answer the question.

Redaction: Jeffery, 2001jan01

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005 qmult 00940 1 4 4 easy deducto-memory: mass and weight
Extra keywords: physci KB-13

18. "Let's play *Jeopardy*! For $100, the answer is: they are, respectively, the resistance of a body to acceleration and the magnitude of the force of gravity on a body."

    What are ____________ and ____________, Alex?

    a) acceleration; normal force b) mass; normal force c) force; weight d) mass; weight
    e) gravity; momentum

SUGGESTED ANSWER: (d)

Wrong answers:
b) The normal force can be equal to weight if the normal force cancels gravity as it frequently does.

Redaction: Jeffery, 2001jan01

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005 qmult 00950 2 5 4 moderate thinking: diving woman and gravity
Extra keywords: physci KB-60-27

19. What is the approximate mass of a woman who weighs 500 N? What is gravitational force that Earth exerts on her. After she jumps **UPWARD** from a diving board, what is her acceleration in the absence of air drag?

   a) About 50 kg, 500 N, and $9.8\,\mathrm{m/s^2}$ downward once she starts moving downward, but **ZERO** before that.
   b) About 50 kg, 50 N, and $9.8\,\mathrm{m/s^2}$ downward once she starts moving downward, but **ZERO** before that.
   c) About 50 kg, 50 N, and $9.8\,\mathrm{m/s^2}$ downward at **ALL** times.
   d) About 50 kg, 500 N, and $9.8\,\mathrm{m/s^2}$ downward at **ALL** times.
   e) None of these questions can be answered with the given information.

SUGGESTED ANSWER: (d)

Remember that weight near the Earth's surface is mg where m is mass and $g = 9.8\,\mathrm{m/s^2}$ is the acceleration due to gravity constant. Now 500 N obviously describes the woman's weight: thus her mass is this value divide by about 10. Her weight is the gravitational force that Earth exerts on her. Once she's left the board the only force on her is gravity and she must accelerate downward at $9.8\,\mathrm{m/s^2}$ no matter what direction she is moving in.

Wrong answers:
e) As Lurch would say: "Aaarh."

Redaction: Jeffery, 2001jan01

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005 qmult 01000 2 1 1 moderate memory: normal force

20. The normal force is:

   a) a repulsive contact force exerted by a surface that points perpendicularly outward from that surface. The force turns out to resist compression. In principle, the force can be calculated from the compressional displacement of the surface from equilibrium, but in elementary problems one usually calculates it from Newton's 2nd or 3rd law assuming the surface to be completely rigid.
   b) $\vec{F}_{\rm net}$ in $\vec{F}_{\rm net} = m\vec{a}$.
   c) the tension force in a rope.
   d) the tension force in a rope that allows you to push on a rope.
   e) an ordinary, run-of-the-mill force, a pedestrian force, a force without pretensions or airs, a downright force, a regular-guy force, just a plain salt-of-the-earth force.

SUGGESTED ANSWER: (a)

I say moderate problem because normal force is not an ordinary day expression and even physicists (like me) can go for long periods of time without remembering the term though we remember the thing.

Wrong answers:
b) No $F_{\rm net}$ is the net some of all forces on a body whatever they may be.
c) The tension force is the tension force and the normal force is the normal force. Of course, at a deeper level both normal and tension forces are manifestations of the interatomic and intermolecular electromagnetic force. They are both contact forces. Friction and pressure forces are in the same boat.
d) Not the least of things that you first learn in physics is that you can't push on rope.
e) Well it is sort of ubiquitous and all-around useful and unspectacular usually, but this is not the best answer in context.

Redaction: Jeffery, 2001jan01

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005 qmult 01040 3 5 2 tough thinking: dynamics and kinematics on slope 1

21. An object of mass m is on a rigid, frictionless slope of angle θ from the horizontal. What is the magnitude of normal force on the object? What is the component of the gravitational force in the positive x direction which is down the slope? What is the expression for the position x of the object as a function of time t when it starts from rest at $t = 0$ and $x = 0$?

    a) $mg\cos\theta$; $mg\cos\theta$; $x = (1/2)(g\cos\theta)t^2$. b) $mg\cos\theta$; $mg\sin\theta$; $x = (1/2)(g\sin\theta)t^2$. c) $mg\sin\theta$; $mg\sin\theta$; $x = (1/2)(g\sin\theta)t^2$. d) $mg\sin\theta$; $mg\sin\theta$; $x = (1/2)(g\cos\theta)t^2$. e) $mg\sin\theta$; $mg\sin\theta$; $x = (g\cos\theta)t$.

SUGGESTED ANSWER: (b)

The normal force must be calculated from the gravitational force from Newton's 2nd law. The y-component of gravitational force is $-mg\cos\theta$ from geometry taking the y-direction as outward normal to the slope. Since there is no acceleration in the y-direction because of the rigidity of the slope, the normal force must be $mg\cos\theta$ outward normal to the slope. From geometry, the x-component of the gravitational force is $mg\sin\theta$ (which is down the slope since it is positive). From the second kinematic equation

$$x = \frac{1}{2}at^2 + v_0 t + x_0 \ ,$$

one finds

$$x = \left(\frac{1}{2}\right) g(\sin\theta)t^2$$

in the present case.

Wrong answers:

e) There has to be a t^2 for constant acceleration.

Redaction: Jeffery, 2008jan01

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005 qmult 01070 1 1 1 mod. thinking: reaction forces of a book

Extra keywords: physci

22. A book sits at rest on a table. The reaction force that follows from Newton's 3rd law to the gravitational force of the Earth on the book is the:

    a) gravitational force of the book on the Earth.
    b) normal (i.e., perpendicular upward) force of the table on the book.
    c) table friction force on the book.
    d) book friction force on the table.
    e) book normal force on the table.

SUGGESTED ANSWER: (a)

This question actually raises an issue that can easily get all snarled up. In order to calculate the acceleration of body, one finds all the forces on the body and does not concern oneself with the forces the body itself exerts which are going to be the reaction forces to the forces applied on the body from the body's perspective.

Thus the table normal force acts on the book and Earth's gravity acts on the book. The respective reaction forces are the normal force of the book on the Earth and the book's gravitational force on the Earth. Note the normal force on the Earth is a contact force that acts directly on the table and causes all kinds of internal table and ground pressure force adjustments.

The book's gravitational force is the vector sum of the book's graviational force on every bit of the Earth. The overall response of the Earth to the book's gravity is pretty minute because of the Earth's huge inertial mass. In fact, it is vastly below detection.

What must the nature of the reaction force be to a force? Though this is not explicit in the short version of the 3rd law, the applied and reaction force must have the same fundamental nature. The reaction force to a gravity force must be a gravity force and the reaction to an electromagnetic force must be an electromagnetic force.

The situation with the electromagnetic force is actually rather complex. For one thing, the electromagnetic force comes in many different manifestations. For another thing, at the macroscopic level, the force and reaction force can be different manifestations: e.g., the elastic force of a solid can be the reaction force to air pressure force. For third thing, the sum of the electric and magnetic force on a particle is inertial frame invariant, but the electric and magnetic forces are individually not inertial frame invariant. For a fourth thing, the 3rd law is not obeyed in all cases by the magnetic force manifestation of the electromagnetic force.

Fortunately, in simple macroscopic cases, the force and reaction forces can usually be identified without difficulty.

What of the those two other forces: the strong nuclear and weak nuclear forces? They are outside of the classical realm, and so intro physics we don't need to worry about them. There is probably some conservation law that stands in place of the 3rd law that applies to them and identifies reaction forces—but yours truly is actually ignorant on this point.

Wrong answers:
b) The reaction force must be force the book exerts.

Redaction: Jeffery, 2001jan01

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005 qmult 01080 2 5 1 moderate thinking: elevator acceleration of woman
Extra keywords: physci KB-61-31

23. A woman who has a mass of 50 kg is in an elevator that is accelerating downward at $2\,\mathrm{m/s^2}$. What is the force the floor exerts on her? What is the force she exerts on the floor?

a) 390 N upward; 390 N downward. b) 390 N downward; 390 N upward.
c) 490 N downward; 490 N upward. d) 490 N upward; 490 N downward.
e) 100 N upward; 100 N downward.

SUGGESTED ANSWER: (a)

The proper way to formulate this problem is to apply Newton's 2nd law to the woman:

$$F_g + F_{\mathrm{normal}} = -mg + F_{\mathrm{normal}} = ma \;,$$

and now solve for F_{normal}:

$$F_{\mathrm{normal}} = m(a+g) = 50 \times (-2 + 9.8) = 390\,\mathrm{N} \;.$$

The normal force is the force the floor exerts on the woman: it is 390 N upward. By the 3rd law, the woman exerts 390 N downward on the floor.

Wrong answers:
e) All things are wrong.

Redaction: Jeffery, 2001jan01

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005 qmult 01100 1 1 1 easy memory: tension defined

24. Tension is the magnitude of the force in an object that resists:

a) extension. b) compression.
c) shearing (i.e., the deformation of the object without change in volume).
d) creaking (i.e., the deformation of the object with noise). e) concession.

SUGGESTED ANSWER: (a)

Wrong answers:
b) The pressure does this.
c) The stress force does this.
d) Nothing resists creaking.

Redaction: Jeffery, 2008jan01

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005 qmult 01110 1 4 4 easy deducto-memory: ideal rope

25. "Let's play *Jeopardy*! For $100, the answer is: It has zero thickness and only resists extension along its length. In fact, resists extension completely. Usually, but not always, it is assumed to have zero mass and be unbreakable"

What is a/an ____________, Alex?

a) ideal monkey b) ideal rigid rod c) ideal surface d) ideal rope e) real rope

SUGGESTED ANSWER: (d)

Wrong answers:

b) An ideal rigid rod is rigid and can't bent unlike an ideal rope.

Redaction: Jeffery, 2008jan01

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005 qmult 01120 1 1 2 easy memory: ideal rope results 1

26. The normal force magnitude per unit length exerted by the curved surface on an ____________ at any general point s is

$$f_{\mathrm{nor}} = \frac{T}{r} ,$$

where s is measured from the start of the ____________, T is the tension at point s r is the radius of curvature at s, and the normal force per unit length points radially outward from the center of curvature. The center of curvature is the center of a circle that approximates the curve at s to first order. The normal force per unit length exerted by the rope on the curved surface is equal in magnitude to f_{nor}, but points radially inward by the 3rd law.

a) ideal rigid rod b) ideal rope c) unreal rigid rod d) uaenrl riigd rod e) ideal door

SUGGESTED ANSWER: (b)

Wrong answers:

d) Asmlot any wrod can be rgceenzoid if you romanldy srcbmale the lteters, epcext for the frist and lsat ltteres.

Redaction: Jeffery, 2008jan01

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005 qmult 01130 1 1 2 easy memory: ideal rope with constant tension

27. A taut ideal, massless rope should have ______________ tension (i.e., constant magnitude of tension force) between two endpoints provided no external forces parallel to the rope act on it **BETWEEN** the endpoints: there will in general be external applied forces to hold it taut at the endpoints. The rope does not have to be straight. It can be wrapped around constraints as long as their surfaces exert no parallel forces on it.

a) wildly varying b) constant c) complexly varying d) zero e) 9.8 N/kg

SUGGESTED ANSWER: (b)

The tension along the ideal rope can only be changed by external parallel forces. By external we mean not internal to the rope. By parallel we mean the forces act parallel to the rope. Forces that act normal to the rope will change its shape, but not its tension.

What the tension is in this idealized case is is one of those obvious questions that textbooks habitually evade. The analysis of the tension in an ideal rope is given in an appendix in *Newtonian Physics I*, and yes my argument is correct. The case of this problem is a special case that follows from a more general result.

But here perhaps we can give an argument that illustrates that the tension be constant if no parallel forces act.

Wrap and ideal rope around a frictionless wheel once. The rope leaves the surface of the wheel at one point ideal. One branch going straigh off one way and the other branch straight off the other way. The rope is held taut by external applied forces equal in magnitude and opposite in direction, and the whole system is static. By symmetry, there can be no variation in tension going around the wheel. By symmetry, there can be no tension variation in the straight segments. Can there be any variation in tension at the point where the rope leases the wheel? I think the answer can only be no. The internal tension forces just at that point inside the rope must be equal and opposite by the 3rd law and to prevent any acceleration. So the tension must be constant throughout the rope. This argument is for a special case, but there seems no reason why it constant tension in the absence of external parallel forces should not be general. The mathematical analysis in the appendix in *Newtonian Physics I* confirms the conclusions—correctly I think.

Wrong answers:

c) I don't think so.
d) It's true that the tension could be zero if the applied external forces are zero and the rope is just taut by being put that way as an initial condition. But "zero" is not a right answer since is not right for the general case given by the statement. The answer must be true for all cases included in the statement.

Redaction: Jeffery, 2008jan01

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005 qmult 01150 1 3 4 easy math: example tension fore of a rope

28. A **MOTIONLESS** mass of 10 kg is suspended from a rope. What is the tension force that the rope exerts on the mass?

a) 100 N downward. b) 200 N downward. c) 200 N upward. d) 100 N upward.
e) 200 N horizontally.

SUGGESTED ANSWER: (d)

Well one has to remember about tension. And to be motionless gravity has to be balanced by tension.

Wrong answers:

Redaction: Jeffery, 2001jan01

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005 qmult 01170 1 1 2 easy memory: no motion implies net force zero

29. A **MOTIONLESS** mass of 10 kg is suspended from a rope. What is the **NET** force on the mass? It is:

a) about 100 N downward. b) 0 N. c) about 200 N upward. d) about 100 N upward.
e) about 200 N horizontally.

SUGGESTED ANSWER: (b)

Motionless is means no acceleration, which means no net force.

Wrong answers:

Redaction: Jeffery, 2001jan01

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005 qmult 01410 1 1 1 easy memory: elevator acceleration, non-inertial frame

30. An elevator just starts moving upward.

a) You feel slightly heavy for a moment.
b) You feel slightly light for a moment.
c) You feel slightly light and carefree for a moment.
d) You feel totally carefree and ethereal.
e) You come to understand that there are no forces in foxholes.

SUGGESTED ANSWER: (a)

This is actually a profound observation. When you are in an accelerated frame with acceleration $\vec{a}_{\rm frame}$, it is exactly as if you were in a frame with a gravitational field with gravitational force per unit mass or gravitational field $-\vec{a}_{\rm frame}$. We can see how this comes about. Recall (as if you could ever forget)

$$\vec{F}_{\rm net} = m\vec{a} \ .$$

Now say you are in accelerating frame with acceleration $\vec{a}_{\rm frame}$. We can now summandize any acceleration $\vec{a}$ into two parts:

$$\vec{a} = \vec{a}_{\rm relative} + \vec{a}_{\rm frame} \ ,$$

where $\vec{a}_{\rm relative}$ is the acceleration relative to the frame. There is nothing mysterious about $\vec{a}_{\rm relative}$: it is just what you measure if you use the accelerated frame as your frame of reference.

Now we can write

$$\vec{F}_{\text{net}} - m\vec{a}_{\text{frame}} = m\vec{a}_{\text{relative}} \ .$$

For all motions relative to the frame it is exactly as if there were a new mass proportional force given by $-m\vec{a}_{\text{frame}}$. We call this effective force $-m\vec{a}_{\text{frame}}$ an inertial force.

Now you may wonder how a mysterious inertial force caused by being in an accelerated non-inertial frame actually works. Does the frame reach out and push/pull on objects? Well no. Say you are making measurements with respect to an accelerated frame, but you arn't moving with the frame: you are unaccelerated. Well you don't feel a mysterious force, but relative to that frame you are accelerated with acceleration $-\vec{a}_{\text{frame}}$. This is exactly like being in free-fall in a gravitational field. Gravity reaches out an pulls you atom by atom, and so if you don't fight it, you just accelerate atom by atom with gravity and feel no internal stress: i.e., you feel weightless. To fight gravity you must use internal contact forces to support all the parts of your body: every little bit of mass Δm, requires a contact force of magnitude Δmg opposite to gravity to support it against gravity and keep you unaccelerated. In an accelerated frame to keep up with the frame, internal contact forces in your body have to supply a force of magnitude Δma_{frame} in the direction of acceleration. This is exactly the same as in resisting a gravitational force of magnitude Δma_{frame} pointing opposite to the direction of acceleration. Thus the inertial force like gravity is a field force: it only gives rise to internal stresses if you resist it.

One can in fact in accelerated frame define an effective gravitational field:

$$\vec{F}_{g\ \text{eff}} = m(g\hat{g} - \vec{a}_{\text{frame}}) \ ,$$

where $\hat{g}$ is the direction of the real gravitational field. With reference to the accelerated frame, it is exactly as if gravity were $\vec{F}_{g\ \text{eff}}$. Any experiment that you do, mechanical or electromagnetic, cannot distinguish gravity from an inertial force. This observation was one of the things that led Einstein to general relativity. He noted that in an upward accelerating elevator, a light beam should bend down. If the inertial force and gravity were really indistinguishable, gravity must cause light beams to bend too. And indeed gravity does this: e.g., light beams from distant stars that pass near the Sun are noticeably deflected as was first shown during eclipse observations in 1919. In Einstein's general relativity (GR), gravity is like an inertial force: i.e., a force which disappears if you view the system from the correct reference frame: in GR that frame is distorted space-time caused by mass.

As an example of effective gravity consider a rocket launching vertically with acceleration a_{frame}. The effective gravity is

$$F_{g\ \text{eff}} = m(g + a_{\text{frame}}) \ .$$

If the acceleration of the rocket equals g, then the effective g-force is $2g$ per unit mass. If the acceleration is $9g$, then the effective g-force is $10g$ per unit mass. If the acceleration is $-g$, then the effective g-force is 0 per unit mass: i.e., you are in free-fall which is not a good thing on launch.

Wrong answers:

b) That's when the elevator stops moving up.
c) Well maybe, but its not the normal reaction.
d) What are you on man?
e) This is the punchline for the joke I'm still trying to think of.

Redaction: Jeffery, 2001jan01

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005 qfull 00950 2 3 0 moderate math: rocket pod descent on Callisto
Extra keywords: David Bowman and 2001: A Space Odyssey

31. As this is (or was within living memory) 2001, let's say you are David Bowman and you've just arrived at Jupiter. Before going off to investigate that monolith (and go beyond humankind), you decide on a little excursion to Callisto, one of Jupiter's 4 major moons. Assume you are so close to Callisto's surface throughout the maneuvers of this question the gravitational field g_{Cal} can be approximated as a constant.

a) As your landing pod descends straight down to the Callisto surface and when your are relatively close to touchdown, your rocket thrust is 3260 N and your descent velocity is **CONSTANT**. What is the gravitational force on your pod? Take the upward direction as the positive direction.

b) Say you reduce thrust to 2200 N and find that the pod has a downward acceleration of $0.39\,\mathrm{m/s^2}$. What is the mass of your pod including yourself?

c) What's the free-fall acceleration magnitude due to gravity near the Callisto surface (i.e., g_{Cal}, the analog to g for gravity near Earth's surface)? The free-fall acceleration acceleration magnitude is also the gravitational field magnitude.

d) Say you have a mass of 70 kg. What's your **WEIGHT** on Callisto and what is your Callisto weight divided by your Earth weight (i.e., what is the weight **RATIO**)?

e) Now the hard part. After finishing your excursion on the icy surface, you launch and go into uniform circular motion, low-Callisto orbit. The gravitational acceleration is approximately the same as the surface gravitational acceleration and the radius of the orbit is approximately just Callisto's radius of 2400 km. Calculate the orbital speed. Then find the orbital period (i.e., the time to orbit once) in seconds and in hours. **HINT:** Remember centripetal acceleration and $\vec{F}_{\mathrm{net}} = m\vec{a}$.

SUGGESTED ANSWER:

a) Well $\vec{F} = ma$ is always true and its always true component by component. If there is no acceleration, then the force of gravity must cancel the rocket thrust. Thus the gravitational force is

$$F_g = -mg = -F_{\mathrm{th,a}} = -3260\,\mathrm{N}$$

and the direction is downward to the center of Callisto, of course.

b) Well $\vec{F} = ma$ is always ..., and so

$$ma = F_{\mathrm{th,b}} - mg$$

and

$$m = \frac{F_{\mathrm{th,b}} - mg}{a} = \frac{F_{\mathrm{th,b}} - F_{\mathrm{th,a}}}{a} = \frac{-1060}{-0.39} = 2720\,\mathrm{kg}\ .$$

c) Using parts (a) and (b),

$$g_{\mathrm{Cal}} = \frac{|F_g|}{m} = \frac{F_{\mathrm{th,a}}}{m} = 1.2\,\mathrm{m/s^2}\ .$$

The actual equatorial value is $1.235\,\mathrm{m/s^2}$ (Wikipedia: Callisto (moon)).

d) Using part (c),

$$W_{\mathrm{Bow,Cal}} = m_{\mathrm{Bow}} g_{\mathrm{Cal}} = 84\,\mathrm{N}\ .$$

The weight ratio is given by

$$\frac{W_{\mathrm{Bow,Cal}}}{W_{\mathrm{Bow,Earth}}} = \frac{m_{\mathrm{Bow}} g_{\mathrm{Cal}}}{m_{\mathrm{Bow}} g} = \frac{g_{\mathrm{Cal}}}{g} \approx \frac{1.2}{10} = 0.12\ .$$

e) Well the magnitude of centripetal acceleration for uniform circular motion is

$$a_{\mathrm{cen}} = \frac{v^2}{r}\ .$$

The only force causing this acceleration is gravity. Thus

$$F_g = m g_{\mathrm{Cal}} = m a_{\mathrm{cen}} = m\frac{v^2}{r}\ .$$

The orbital speed is then

$$v = \sqrt{g_{\mathrm{Cal}} r} = 1700\ \mathrm{m/s}$$

and the orbital period P is

$$P = \frac{2\pi r}{v} = 2\pi\sqrt{\frac{r}{g_{\mathrm{Cal}}}} = 8900\,\mathrm{s} = 2.5\,\mathrm{h}\ .$$

Fortran Code

```
      print*
      fa=3260.
      fb=2200.
      aa=-0.39
      xmpod=(fb-fa)/aa
      gg=fa/xmpod
      xmbowman=70.*gg
      ratio=gg/9.8
      print*,'xmpod,gg,xmbowman,ratio'
      print*,xmpod,gg,xmbowman,ratio
*            2717.94873  1.19943392  83.9603729  0.122391216
      rcal=2400.e+3
      vv=sqrt(gg*rcal)
      pp=2.*pi*sqrt(rcal/gg)
      print*,'vv,pp,pp/3600.'
      print*,vv,pp,pp/3600.
*            1696.65601  8887.8623  2.46885061
```

Redaction: Jeffery, 2001jan01

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005 qfull 01010 1 3 0 easy math: frictionless incline

32. There is a 2 kg block on a frictionless incline that is at $\theta = 30°$ from the horizontal.

   a) What is the normal force on the block? **HINTS:** Draw a free body diagram and remember the class mantra: "$\vec{F}_{\rm net} = m\vec{a}$ is always true and it's true component by component".

   b) What is the net force down the slope?

   c) What is the acceleration down the slope?

   d) Starting from rest how far does the block slide in 10 s?

SUGGESTED ANSWER:

a) You will have to imagine the diagram. Remember we have no intrinsic formula for the normal force in this course. There can be no such formula for ideal perfectly rigid wall actually. Thus, we must always solve for the normal force from Newton's 2nd or 3rd laws. In this, we are not told a force applied to the incline, and so the 3rd law cannot be used to find the normal force on the block. We must use the 2nd law to find the normal force needed to prevent the block from accelerating perpendicularly to the slope and straight into the incline. Since we assume a perfectly rigid incline (this is implicit in the question), there is no acceleration perpendicular to the incline. We will take the positive y-axis outward perpendicularly to the slope. By geometry this axis is θ to the vertical. Then from the 2nd law applied to the y-axis with $a_y = 0$, we find

$$0 = F_{\rm N} - mg\cos\theta$$

or

$$F_{\rm N} = mg\cos\theta = 17.0\,{\rm N} \ .$$

Since the normal force is not negative, our assumption of zero acceleration is consistent and everything is cool.

b) We take down the slope to be the positive x-direction. The only force component in this direction is due to gravity. Thus from Newton's 2nd law applied in the x-direction, we find

$$F_x = mg\sin\theta = 9.8\,{\rm N} \ .$$

c) Well

$$a_x = \frac{F_x}{m} = 4.9\,{\rm m/s^2} \ .$$

d) Behold:

$$x = \frac{1}{2}at^2 = 245\,\mathrm{m}\ .$$

Fortran Code

```
        print*
        pi=acos(-1.)
        raddeg=180./pi
        gg=9.8
        theta=30.
        xmass=2.
        fn=xmass*gg*cos(theta/raddeg)
        fd=xmass*gg*sin(theta/raddeg)
        aa=gg*sin(theta/raddeg)
        dist=.5*aa*10.**2
        print*,'fn,fd,aa,dist'
        print*,fn,fd,aa,dist
* 16.9740982  9.80000019  4.9000001  245.
```

Redaction: Jeffery, 2001jan01

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005 qfull 01020 1 5 0 easy thinking: incline plane ad infinitum

33. Physics students frequently flub analyzing forces and accelerations on an inclined plane. Let's get it straight.

a) A Cairn terrier named Bit has dog-rolled to a point on an inclined plane. The plane has an angle from the horizontal of θ. Bit's mass is m. What is the component of gravitational force on Bit parallel to the inclined plane? What is the normal force on Bit? **HINT:** Draw a diagram.

b) Fun-loving pig Waldo Pepper (mass m) is sliding down a frictionless incline (with angle θ form the horizontal). What is his acceleration? What is the normal force on Waldo? **HINT:** Draw a diagram.

c) Underdog has just alighted on an inclined plane from which the Wonder Woofer surveys the world with a flint-hard gaze. The inclined plane has an angle of θ from the horizontal. What is the gravitational force component parallel to the inclined plane on the Caring Canine (mass m)? What is the normal force on the Magnificent Mutt? **HINT:** Draw a diagram.

d) A 1992 GM Geo Metro (mass m) is sliding down a frictionless incline (with angle θ form the horizontal) which is sort of like a hill in Moscow, Idaho in January. What is Baby's acceleration? What is the normal force on Baby? **HINT:** Draw a diagram.

SUGGESTED ANSWER:

a) You'll have to imagine the diagram. The component of the gravitational force parallel to the plane is $mg\sin\theta$ down the plane. The normal force is $mg\cos\theta$ normal to the incline plane.

A Cairn terrier is a Toto dog. Ah, they're cute little doggies.

b) Waldo's acceleration is $a = g\sin\theta$ down the incline. The normal force is $mg\cos\theta$ normal to the incline plane.

c) The component of the gravitational force parallel to the plane is $mg\sin\theta$ down the plane. The normal force is $mg\cos\theta$ normal to the incline plane.

d) Baby's acceleration is $a = g\sin\theta$ down the incline. The normal force is $mg\cos\theta$ normal to the incline plane.

Redaction: Jeffery, 2008jan01

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005 qfull 01130 2 5 0 moderate thinking: block and 3rd law

34. You have block just sitting on horizontal flat ground. **HINT:** This is a problem for rumination—or perhaps ruminants.

a) Draw a free body diagram for the block. Indicate all the forces acting on the block. What is the cause of these forces and are they contact or field forces?

b) Now by the 3rd law, what forces does the block exert and on what and where exactly does it exert them?

SUGGESTED ANSWER:

a) The forces on the block are the normal force upward and gravity downward. The normal force is a contact force exerted by the immediate ground surface on the block. Gravity is a field force and it is the vector sum of all the gravitational forces exerted by every bit of the Earth on the block.

b) The block exerts a normal force on the ground surface downward equal and opposite to the normal force the ground surface exerts upward. This force is a contact force. The block exerts a gravitational force on every bit of the Earth: the vector sum of these forces is equal and opposite to the force that gravity exerts on the block. Now the Earth doesn't notice this gravitational force much. It's mass is so big that a force of this size would give only a minute acceleration if acting alone. In fact, there are many other much larger perturbing forces. If the block disappeared, the Earth wouldn't blink.

Redaction: Jeffery, 2001jan01

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005 qfull 01110 2 3 0 moderate math: fuzzy dice at angle

35. You are in a car accelerating at a constant $10\,\mathrm{m/s^2}$ in a constant direction. The car is on level ground. A pair of fuzzy dice is hanging by a cord from the mirror at an angle θ from the vertical. The dice cord is a (massless) ideal rope. Assuming the dice are a point mass, what is this angle? **HINT:** Draw a free body diagram for the dice. Remember the class mantra: "$\vec{F}_{\rm net} = m\vec{a}$ is always true and it's true component by component".

SUGGESTED ANSWER:

I omit the diagram.

Note the fuzzy dice, like the car, must be accelerating in the x-direction and at the car's rate of acceleration. They are constrained to do so.

Since the cord is massless ideal rope only two forces act on it. The dice tension force at one end and the mirror holder tension force at the other end. Gravity can't act on the cord since it is massless. Because the only forces on the cord are at the endpoints the cord must be follow a straight line and tension force it exerts on the dice must be aligned with the cord. The tension must be a constant in the cord since no parallel forces act on it except at the endpoints.

We have the two following equations for the dice from $\vec{F} = m\vec{a}$ using a horizontal-vertical set of coordinate axes:

$$ma_x = T\sin\theta \qquad \text{and} \qquad 0 = T\cos\theta - mg \ ,$$

where m is the dice mass, T is the cord tension, and θ is the angle from the vertical.

Although solving a problem symbolically is best, I usually set to zero immediately quantities that are zero: this saves me from tedious generality.

In this case, we don't know m, T, or θ. So we have 3 unknowns in only 2 equations, and so in general can't solve for all the unknowns. But that is in general. Sometimes in particular cases partial solutions can be extracted. In this case, we can divide

$$ma_x = T\sin\theta \qquad \text{by} \qquad mg = T\cos\theta$$

to get

$$\frac{a_x}{g} = \tan\theta \ ,$$

and thus

$$\theta = \tan^{-1}\left(\frac{a_x}{g}\right) \approx \tan^{-1}\left(\frac{10}{10}\right) = 45^\circ \ .$$

Remarkably, the angle doesn't depend on the mass. Fuzzy dice or an elephant, it's all the same. The cord tension does depend on the mass, of course. In fact, measuring the angle of a hanging object is a way of measuring acceleration:

$$a_x = g\tan\theta \ .$$

We have no way to solve for m and T with the information given.

We can solve the problem in a slightly different way. The tension force added vectorially to the gravitational force must give the net force which is the cause of the acceleration. From the diagram of these forces, it is clear that

$$\tan\theta = \frac{ma_x}{mg} = \frac{a_x}{g},$$

and thus again one has

$$\theta = \tan^{-1}\left(\frac{a_x}{g}\right)$$

again. Since the rope must point in the direction of the tension force, this is the angle that the rope hangs at.

Some students wonder if you can just take the ratio of the "accelerations" meaning a and g to get θ. Well you get the right answer, but g is not an acceleration in this problem: there is no acceleration in the vertical direction. The g just enters as parameter in the gravitational force law. You get the right answer because mass cancels out in this particular problem. As long as that is understood, the solution is valid.

Redaction: Jeffery, 2001jan01

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005 qfull 01140 2 3 0 moderate math: double incline with two blocks

36. You have a frictionless triangular block which gives you two inclines: i.e., double incline. Incline 1 is at θ_1 to the horizontal and incline 2 at θ_2. You have an ideal massless pulley (or altneratively and equivalently, a friction-free bend) at the apex and a taut ideal rope connecting two blocks, one on each slope. The rope is parallel to each incline. The incline 1 block has mass m_1 and the incline 2, mass m_2. The masses of the blocks and the incline angles are the formal knowns of the problem.

a) Write down Newton's 2nd law for each block for the direction along the inclines. Take up as positive for incline 1 and down as positive for incline 2. **HINT:** Draw two free body diagrams on a diagram of the double incline.

b) Derive the formula for acceleration of the two blocks along their respective inclines and the formula for the tension in the rope?

c) Specialize the formulae from the part (b) answer for the case of $\theta_1 = \theta_2 = \pi/2$. This case is Atwood's machine.

d) Specialize the formulae from the part (b) answer for the case of $\theta_1 = 0$ and $\theta_2 = \pi/2$.

e) Specialize the formulae from the part (b) answer for the case of $\theta_2 = \pi/2$.

SUGGESTED ANSWER:

a) You will have to imagine the diagrams. One obtains the equations of motion

$$m_1 a_1 = -m_1 g \sin\theta_1 + T_1 \ ,$$
$$m_2 a_2 = m_2 g \sin\theta_2 - T_2 \ ,$$

where the a's the accelerations and the T's the tension forces. Because the rope and pulley are both ideal, it is clear that there is a common acceleration $a = a_1 = a_2$ and a common tension $T = T_1 = T_2$.

b) Just adding the equations of motion and rearranging gives

$$a = g\left(\frac{m_2\sin\theta_2 - m_1\sin\theta_1}{m_2 + m_1}\right) \ .$$

Dividing each equation by the respective block mass and subtracting and rearranging gives

$$T = g\left(\frac{\sin\theta_1 + \sin\theta_2}{1/m_1 + 1/m_2}\right) = g\left(\frac{m_1 m_2}{m_1 + m_2}\right)(\sin\theta_1 + \sin\theta_2) \ .$$

c) Behold:

$$a = g\left(\frac{m_2 - m_1}{m_2 + m_1}\right)$$
$$T = g\left(\frac{2m_1 m_2}{m_1 + m_2}\right) \ .$$

d) Behold:

$$a = g\left(\frac{m_2}{m_2 + m_1}\right)$$
$$T = g\left(\frac{m_1 m_2}{m_1 + m_2}\right) \ .$$

e) Behold:

$$a = g\left(\frac{m_2 - m_1 \sin\theta_1}{m_2 + m_1}\right) \ ,$$
$$T = g\left(\frac{m_1 m_2}{m_1 + m_2}\right)(\sin\theta_1 + 1) \ .$$

Redaction: Jeffery, 2001jan01

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005 qfull 01150 3 3 0 tough math: monkey on a pulley

37. An ideal rope (i.e., massless, but it has friction so a monkey can grip it) hangs over ideal tree bough (i.e., frictionless). The rope segments on both sides hang vertically. An ideal monkey of mass 5 kg (let's call her object 1) climbs up one side of the rope. A block of 10 kg (let's call it object 2) sits on the ground at least initially and is tied to the end of the opposite side of the rope.

    How the monkey is supported and can be accelerated is a somewhat difficult to understand—but they are everyday effects. She exerts a friction force on a rope segment that is equal and opposite to the friction force of the rope segment on her needed to support her against gravity and accelerate her. The rope segment isn't moving at all, and so the monkey's force on the segment and the tension force on the rope segment must be equal and opposite. So the tension force of the rope on the rope segment is equal to the friction force on the monkey. But that is all a bit complex. Let's just take the monkey and segment of rope she is holding as all part of one monkey system. So tension and gravity are the external forces acting on the monkey system. You may wonder how the tension force can help accelerate the monkey system if the rope segment doesn't move when the monkey accelerates upward or downward. Well it's the center of mass of monkey system that is accelerated. We will get to the center-of-mass concept sometime if not already. For the moment, just regard the monkey system as a particle to be accelerated by tension and gravity. For simplicity, we will just say monkey rather than monkey system.

a) Draw free body diagrams for both monkey and block. Remember the only forces that appear are those that act **ON** the object and they are drawn with their tails at the origin of the diagram.

b) Write down Newton's 2nd law for the monkey (object 1) and the block (object 1). Use symbols **ONLY**. Masses and g are formally knowns. What the unkowns? Can they be solved for without more information.

c) What is the relationship known before any calculation between the block's acceleration and the normal force on the block? If a solution for the normal force yields a negative number (i.e., the normal force is required by the given values to be attractive), what does this mean?

d) If the monkey accelerates as she climbs, what must be her acceleration in order that the normal force on block goes to zero, but the block stays motionless? **NOTE:** The zero normal force and motionless block are conditions we are imposing.

e) Say that the monkey then starts accelerating downward at $3\,\mathrm{m/s^2}$. The block is initially at rest on the ground. What is the tension in the rope? What is the normal force on the block? Does the block stay at rest on the ground?

f) Say that the monkey then starts accelerating upward at at $12\,\mathrm{m/s^2}$—she's really rocketing. The block is initially at rest on the ground. What is the tension in the rope? What is the normal force on the block? Does the block stay at rest on the ground?

SUGGESTED ANSWER:

a) I leave the free body diagrams to your imagination. The monkey is acted on by only two forces—gravity and tension; the block is acted on by three in general—gravity, tension, and normal force.

b) Behold:

$$m_1 a_1 = T - m_1 g \ ,$$
$$m_2 a_2 = F_{\rm N} + T - m_2 g \ ,$$

where $F_{\rm N}$ is the normal force and T is the tension. The tension is the same on both sides of the rope since the frictionless bough exerts no parallel force on the rope as it goes over the bough.

The unknowns are a_1, a_2 $F_{\rm N}$, and T. Since we only have two equations, we can solve only for 2 unknowns in general. So no, we cannot solve for the unknowns without more information.

c) If the block's acceleration is non-zero then the block must be in motion and not sitting on the ground (except for an instant if it just starts accelerating upward). If the block is in motion, the normal force must be zero since the normal force is a contact force and can only exist (for any more than an instant which is not a realistic case), for the block sitting at rest on the ground. If the acceleration is zero, the block can be motionless on the ground or in the air or have a constant velocity in the air.

If the solution for the normal force is negative, then some inconsistent assumption has been made since a real normal force can never become attractive. For example, an example of zero acceleration may be wrong—it is not necessarily wrong.

d) Well our equations of motion special to

$$m_1 a_1 = T - m_1 g \ ,$$
$$0 = T - m_2 g \ ,$$

where we have imposed the given conditions. Now solving for a_1, we get

$$a_1 = \frac{T}{m_1} - g = \frac{m_2}{m_1} g - g = \left(\frac{m_2}{m_1} - 1 \right) g \approx (2-1) \times 10 = 10\,\mathrm{m/s^2}$$

to about 2-digit accuracy.

Now your mind may slightly boggle—mine does—at how the hanging monkey can cause a tension on the rope greater than the monkey's weight and how the tension force can accelerate the monkey without the rope moving. Well just consider a gravity-free case. For the monkey to accelerate along the rope, the monkey exerts a force on the rope and the rope exerts an equal and opposite force on the monkey: this force accelerates the monkey. The tension force in the rope above the topmost monkey's paw part must be equal and opposite to the force the monkey exerts on the rope below its topmost paw—the monkey's paw! Since the rope is massless, the net force on the rope segment below the monkey's paw must be zero no matter what the acceleration of the rope. Actually, the rope isn't accelerating at all in this case.

Our case is different from the gravity-free case in that there must be more tension force to counter the monkey's weight as well as the force to counter the force that accelerates the monkey.

e) Let's make the assumption that the block's acceleration is zero. Now our equations of motion specialize to

$$m_1 a_1 = T - m_1 g \ ,$$
$$0 = F_{\rm N} + T - m_2 g \ .$$

We have two unknowns $F_{\rm N}$ and T. The solution is straightforward:

$$T = m_1(a_1+g) \approx 5\times(-3+10) = 35\,{\rm N}\ ,$$
$$F_{\rm N} = m_2 g - m_1(a_1+g) \approx 10\times 10 - 5\times(-3+10) = 65\,{\rm N}$$

to about 2-digit accuracy.

Since the normal force is positive (i.e., repulsive), our assumption of zero acceleration for the block is consistent and our solution is correct. The block stays on the ground.

f) We can recycle the solution from the part (e) answer again with the assumption of zero acceleration for the block:

$$T = m_1(a_1+g) \approx 5\times(12+10) = 110\,{\rm N}\ ,$$
$$F_{\rm N} = m_2 g - m_1(a_1+g) \approx 10\times 10 - 5\times(12+10) = -10\,{\rm N}$$

to about 2-digit accuracy.

Well our assumption of zero acceleration for the block is wrong since it gives an attractive normal force which can't happen really. So we make the alternative assumption that block is off the ground and accelerating and the normal force is zero. Now our equations of motion specialize to

$$m_1 a_1 = T - m_1 g\ ,$$
$$m_2 a_2 = T - m_2 g$$

and the solutions for the unknowns are

$$T = m_1(a_1+g) \approx 5\times(12+10) = 110\,{\rm N}\ ,$$
$$a_2 = \frac{m_1}{m_2}(a_1+g) - g \approx 11 - 10 = 1\,{\rm N}$$

to about 2-digit accuracy for T and 1-digit accuracy for a_2.

So in this case, the block must accelerate upward and leave the ground after time zero.

Note that rope is now accelerating itself with the same acceleration as the block. But this doesn't affect the monkey's acceleration in this problem since we gave that as a given. The monkey's acceleration relative to the rope is

$$a_{1,{\rm rel}} = a_1 - (-a_2) = 13\,{\rm m/s^2}\ .$$

Note the rope acceleration is upward on the block's side of the bough, but downward on the rope's side and we have taken that into account in calculating the monkey's relative acceleration.

Redaction: Jeffery, 2001jan01

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005 qfull 01160 3 5 0 tough thinking: massive rope and block system

38. You have a uniform-density rectangular block of mass $m_{\rm b}$ on a frictionless, horizontal surface. You are pulling it along with a uniform-density rope of mass $m_{\rm r}$. the total mass of the block-rope system is $m = m_{\rm b} + m_{\rm r}$. Aside from its mass and weight the rope is ideal. The two ends of the rope are at the same height which is well above the ground. The motion of the system is entirely 1-dimensional. This is an entirely symbolical and thinking question until part (e).

a) Must the rope sag at least a little? Why or why not?

b) Assume any sag of the rope is negligible for the rest of this question and that the block-rope system can be regarded as rigid and non-rotating. Given that there is force F in the positive x-direction pulling on the rope and holding it taut and that no other external forces in the x-direction on the block-rope system, what is the x-direction center-of-mass acceleration a of the block-rope system? What is the acceleration of every bit of the block-rope system?

c) Say the positive x direction is to the right. Let m_x be the total mass of the system to the left of point x along the length of the block-rope system. What is the internal tension force at x accelerating the mass m_x as a function **ONLY** of F and the relevant masses?

d) What are forces F_x at middle of the rope, the point of block-rope contact, and at the middle of the block as functions **ONLY** of F and the relevant masses?

e) Given the masses $m_{\rm b} = 5.0\,{\rm kg}$ and $m_{\rm r} = 0.1\,{\rm kg}$, and the acceleration of the system $a = 2.0\,{\rm m/s^2}$, what is F?

f) What is the block's speed at 10 m from the starting point if it starts from rest?

SUGGESTED ANSWER:

a) Yes. The rope has mass and weight, and so there is a gravitational force on every bit of it downward. A straight ideal rope can only exert a tension force parallel to the rope itself. Thus, it cannot cancel its own weight, and so must be accelerating downward. In our case, the rope must sag a little so that the tension forces on any bit of rope are not exactly parallel and thus sum to have a component in the upward direction that cancels the weight of the bit.

b) Using Newton's 2nd law we find the center-of-mass acceleration to be

$$a = \frac{F}{m} \ .$$

Since the block-rope system moves as a rigid, non-rotating body, this acceleration a is also the acceleration of every bit of block-rope system.

c) Since every bit of the block-rope system is acceleratin a, so is the mass m_x, and thus we have

$$F_x = m_x a \ .$$

Therefore

$$F_x = \frac{m_x}{m} F \ .$$

d) Making use of the part (c) answer, the forces F_x at middle of the rope, the point of block-rope contact, and at the middle of the block are, respectively,

$$F_x = \left[\frac{m_{\rm b} + (1/2)m_r}{m}\right] F \ , \qquad F_x = \left(\frac{m_{\rm b}}{m}\right) F \ , \qquad F_x = \left[\frac{(1/2)m_{\rm b}}{m}\right] F \ .$$

e) Using Newton's 2nd law, we find

$$F = (m_{\rm b} + m_{\rm r})a = 10.2\,{\rm N} \ .$$

g) Using the appropriate kinematic equation (the timeless equation, in fact), we find

$$v = \sqrt{v_0^2 + 2a(x - x_0)} = \sqrt{2.0 \times 2.0 \times 10.} \approx 6.3\,{\rm m/s} \ .$$

Redaction: Jeffery, 2001jan01

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006 qfull 01210 2 3 0 moderate math: pig on slide: with and sans friction

Extra keywords: HRW-113-14p

39. A free-spirited pig (let's call him Waldo Pepper) loves sliding down chutes (pronounced shoots).

a) Say there is a frictionless chute and Waldo accelerates down at $5.0\,{\rm m/s^2}$. What is the inclination angle of the chute? **HINT:** Remember that free body diagram.

b) Now the evil magician of physics turns on kinetic friction and Waldo only accelerates down at $3.0\,{\rm m/s^2}$? What is the coefficient of kinetic friction? **HINT:** Update that free body diagram.

c) What's Waldo's mass? Explain your answer. **HINT:** The evil magician of physics is being spiteful.

SUGGESTED ANSWER:

Long ago in the 1970s, there was this Robert Redford film *The Great Waldo Pepper* in which Redford played daredevil pilot Waldo Pepper. Our pig Waldo thinks of himself as a daredevil flier: hence his name.

a) Well in the incline direction $\vec{F} = m\vec{a}$ reduces to

$$ma = mg\sin\theta \ ,$$

and so

$$\theta = \sin^{-1}\left(\frac{a}{g}\right) \approx 31^\circ \ ,$$

to about 2-digit accuracy.

b) First we need the normal force. Remember that for a perfectly rigid surface, there is no intrinsic formula for the normal force and we must find it by applying Newton's 2nd or 3rd law. In this case, we have to apply the 2nd law. We know that Waldo doesn't accelerate in the normal direction to the chute, and that leads to

$$F_{\rm N} = mg\cos\theta \ .$$

Applying the 2nd law to the direction down the incline gives

$$ma = mg\sin\theta - \mu_{\rm ki} mg\cos\theta \ ,$$

and so solving for $\mu_{\rm ki}$ gives

$$\mu_{\rm ki} = \frac{g\sin\theta - a}{g\cos\theta} = \tan\theta - \frac{a}{g\cos\theta} \approx 0.24$$

to about 2-digit accuracy. Although the coefficient of kinetic friction can actually vary over a large range ($\sim$ 0.01–1.5: WP, p. 103), 0.3 is a typical value for many substances, and so our result is typical.

c) Can't tell: all the force formulae are mass proportional and mass cancels out of all solvable 2nd law expressions at our disposal. The solvable equations that one can deduce from $F = ma$ are all mass independant. A pig of any mass—say Waldo's big brother Weirdo—would yield the same inclination angle and kinetic friction coefficient as Waldo. I sort of think of Waldo as a mature 150 kg: I'd stay away from chutes if I were him.

Fortran Code

```
      print*
      pi=acos(-1.)
      raddeg=180./pi
      a1=5.0
      a2=3.0
      gg=9.8
      thetar=asin(a1/gg)
      theta=thetar*raddeg
      xmu=tan(thetar)-(a2/gg)/cos(thetar)
      print*,'theta,xmu'
      print*,theta,xmu
*             30.677423  0.237289493
```

Redaction: Jeffery, 2001jan01

Equation Sheet for Introductory Physics Calculus-Based

This equation sheet is intended for students writing tests or reviewing material. Therefore it is neither intended to be complete nor completely explicit. There are fewer symbols than variables, and so some symbols must be used for different things: context must distinguish.

The equations are mnemonic. Students are expected to understand how to interpret and use them.

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1 Constants

$$c = 2.99792458 \times 10^8\,\mathrm{m/s} \approx 2.998 \times 10^8\,\mathrm{m/s} \approx 3 \times 10^8\,\mathrm{m/s} \approx 1\,\mathrm{lyr/yr} \approx 1\,\mathrm{ft/ns} \qquad \text{exact by definition}$$

$$e = 1.602176487(40) \times 10^{-19}\,\mathrm{C}$$

$$G = 6.67428(67) \times 10^{-11}\,\mathrm{N\,m^2/kg^2} \qquad \text{(2006, CODATA)}$$

$$g = 9.8\,\mathrm{m/s^2} \qquad \text{fiducial value}$$

$$k = \frac{1}{4\pi\varepsilon_0} = 8.987551787\ldots \times 10^9 \approx 8.99 \times 10^9 \approx 10^{10}\,\mathrm{N\,m^2/C^2}\text{exact by definition}$$

$$k_{\text{Boltzmann}} = 1.3806504(24) \times 10^{-23}\,\mathrm{J/K} = 0.8617343(15) \times 10^{-4}\,\mathrm{eV/K} \approx 10^{-4}\,\mathrm{eV/K}$$

$$m_e = 9.10938215(45) \times 10^{-31}\,\mathrm{kg} = 0.510998910(13)\,\mathrm{MeV}$$

$$m_p = 1.672621637(83) \times 10^{-27}\,\mathrm{kg} = 938.272013(23), \mathrm{MeV}$$

$$\varepsilon_0 = \frac{1}{\mu_0 c^2} = 8.8541878176\ldots \times 10^{-12}\,\mathrm{C^2/(N\,m^2)} \approx 10^{-11} \qquad \text{vacuum permittivity (exact by definition)}$$

$$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N/A^2} \qquad \text{exact by definition}$$

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2 Geometrical Formulae

$$C_{\text{cir}} = 2\pi r \qquad A_{\text{cir}} = \pi r^2 \qquad A_{\text{sph}} = 4\pi r^2 \qquad V_{\text{sph}} = \frac{4}{3}\pi r^3$$

$$\Omega_{\text{sphere}} = 4\pi \qquad d\Omega = \sin\theta\, d\theta\, d\phi$$

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3 Trigonometry Formulae

$$\frac{x}{r} = \cos\theta \qquad \frac{y}{r} = \sin\theta \qquad \frac{y}{x} = \tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cos^2\theta + \sin^2\theta = 1$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$c^2 = a^2 + b^2 \qquad c = \sqrt{a^2 + b^2 - 2ab\cos\theta_c} \qquad \frac{\sin\theta_a}{a} = \frac{\sin\theta_b}{b} = \frac{\sin\theta_c}{c}$$

$$f(\theta) = f(\theta + 360^\circ)$$

$$\sin(\theta + 180^\circ) = -\sin(\theta) \qquad \cos(\theta + 180^\circ) = -\cos(\theta) \qquad \tan(\theta + 180^\circ) = \tan(\theta)$$

$$\sin(-\theta) = -\sin(\theta) \qquad \cos(-\theta) = \cos(\theta) \qquad \tan(-\theta) = -\tan(\theta)$$

$$\sin(\theta + 90^\circ) = \cos(\theta) \qquad \cos(\theta + 90^\circ) = -\sin(\theta) \qquad \tan(\theta + 90^\circ) = -\tan(\theta)$$

$$\sin(180^\circ - \theta) = \sin(\theta) \qquad \cos(180^\circ - \theta) = -\cos(\theta) \qquad \tan(180^\circ - \theta) = -\tan(\theta)$$

$$\sin(90^\circ - \theta) = \cos(\theta) \qquad \cos(90^\circ - \theta) = \sin(\theta) \qquad \tan(90^\circ - \theta) = \frac{1}{\tan(\theta)} = \cot(\theta)$$

$$\sin(a + b) = \sin(a)\cos(b) + \cos(a)\sin(b) \qquad \cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$\sin(2a) = 2\sin(a)\cos(a) \qquad \cos(2a) = \cos^2(a) - \sin^2(a)$$

$$\sin(a)\sin(b) = \frac{1}{2}\left[\cos(a - b) - \cos(a + b)\right] \qquad \cos(a)\cos(b) = \frac{1}{2}\left[\cos(a - b) + \cos(a + b)\right]$$

$$\sin(a)\cos(b) = \frac{1}{2}\left[\sin(a - b) + \sin(a + b)\right]$$

$$\sin^2\theta = \frac{1}{2}[1 - \cos(2\theta)] \qquad \cos^2\theta = \frac{1}{2}[1 + \cos(2\theta)] \qquad \sin(a)\cos(a) = \frac{1}{2}\sin(2a)$$

$$\cos(x) - \cos(y) = -2\sin\left(\frac{x + y}{2}\right)\sin\left(\frac{x - y}{2}\right)$$

$$\cos(x) + \cos(y) = 2\cos\left(\frac{x + y}{2}\right)\cos\left(\frac{x - y}{2}\right)$$

$$\sin(x) + \sin(y) = 2\sin\left(\frac{x + y}{2}\right)\cos\left(\frac{x - y}{2}\right)$$

4 Approximation Formulae

$$\frac{\Delta f}{\Delta x} \approx \frac{df}{dx} \qquad \frac{1}{1 - x} \approx 1 + x \ : \ (x << 1)$$

$$\sin\theta \approx \theta \qquad \tan\theta \approx \theta \qquad \cos\theta \approx 1 - \frac{1}{2}\theta^2 \qquad \text{all for } \theta << 1$$

5 Quadratic Formula

If $\quad 0 = ax^2 + bx + c\ ,\quad$ then $\quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = -\dfrac{b}{2a} \pm \sqrt{\left(\dfrac{b}{2a}\right)^2 - \dfrac{c}{a}}$

6 Vector Formulae

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2} \qquad \theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) + \pi? \qquad \vec{a} + \vec{b} = (a_x + b_x, a_y + b_y)$$

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} \qquad \phi = \tan^{-1}\left(\frac{a_y}{a_x}\right) + \pi? \qquad \theta = \cos^{-1}\left(\frac{a_z}{a}\right)$$

$$\vec{a} + \vec{b} = (a_x + b_x, a_y + b_y, a_z + b_z)$$

$$\vec{a} \cdot \vec{b} = ab\cos\theta = a_x b_x + a_y b_y + a_z b_z$$

$$\vec{c} = \vec{a} \times \vec{b} = ab\sin(\theta)\hat{c} = (a_y b_z - b_y a_z, a_z b_x - b_z a_x, a_x b_y - b_x a_y)$$

7 Differentiation and Integration Formulae

$$\frac{d(x^p)}{dx} = px^{p-1} \quad \text{except for } p = 0; \qquad \frac{d(x^0)}{dx} = 0 \qquad \frac{d(\ln|x|)}{dx} = \frac{1}{x}$$

Taylor's series

$$\begin{aligned} f(x) &= \sum_{n=0}^{\infty} \frac{(x-x_0)^n}{n!} f^{(n)}(x_0) \\ &= f(x_0) + (x-x_0)f^{(1)}(x_0) + \frac{(x-x_0)^2}{2!} f^{(2)}(x_0) + \frac{(x-x_0)^3}{3!} f^{(3)}(x_0) + \ldots \end{aligned}$$

$$\int_a^b f(x)\,dx = F(x)|_a^b = F(b) - F(a) \quad \text{where} \quad \frac{dF(x)}{dx} = f(x)$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad \text{except for } n = -1; \qquad \int \frac{1}{x}\,dx = \ln|x|$$

8 One-Dimensional Kinematics

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} \qquad v = \frac{dx}{dt} \qquad a_{\text{avg}} = \frac{\Delta v}{\Delta t} \qquad a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

$$v = at + v_0 \qquad x = \frac{1}{2}at^2 + v_0 t + x_0 \qquad v^2 = v_0^2 + 2a(x - x_0)$$

$$x = \frac{1}{2}(v_0 + v)t + x_0 \qquad x = -\frac{1}{2}at^2 + vt + x_0 \qquad g = 9.8\,\text{m/s}^2$$

$$x_{\text{rel}} = x_2 - x_1 \qquad v_{\text{rel}} = v_2 - v_1 \qquad a_{\text{rel}} = a_2 - a_1$$

$$x' = x - v_{\text{frame}} t \qquad v' = v - v_{\text{frame}} \qquad a' = a$$

9 Two- and Three-Dimensional Kinematics: General

$$\vec{v}_{\text{avg}} = \frac{\Delta \vec{r}}{\Delta t} \qquad \vec{v} = \frac{d\vec{r}}{dt} \qquad \vec{a}_{\text{avg}} = \frac{\Delta \vec{v}}{\Delta t} \qquad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

10 Projectile Motion

$$x = v_{x,0} t \qquad y = -\frac{1}{2} g t^2 + v_{y,0} t + y_0 \qquad v_{x,0} = v_0 \cos\theta \qquad v_{y,0} = v_0 \sin\theta$$

$$t = \frac{x}{v_{x,0}} = \frac{x}{v_0 \cos\theta} \qquad y = y_0 + x \tan\theta - \frac{x^2 g}{2 v_0^2 \cos^2\theta}$$

$$x_{\text{for } y \text{ max}} = \frac{v_0^2 \sin\theta \cos\theta}{g} \qquad y_{\text{max}} = y_0 + \frac{v_0^2 \sin^2\theta}{2g}$$

$$x(y = y_0) = \frac{2 v_0^2 \sin\theta \cos\theta}{g} = \frac{v_0^2 \sin(2\theta)}{g} \qquad \theta_{\text{for max}} = \frac{\pi}{4} \qquad x_{\text{max}}(y = y_0) = \frac{v_0^2}{g}$$

$$x(\theta = 0) = \pm v_0 \sqrt{\frac{2(y_0 - y)}{g}} \qquad t(\theta = 0) = \sqrt{\frac{2(y_0 - y)}{g}}$$

11 Relative Motion

$$\vec{r} = \vec{r}_2 - \vec{r}_1 \qquad \vec{v} = \vec{v}_2 - \vec{v}_1 \qquad \vec{a} = \vec{a}_2 - \vec{a}_1$$

12 Polar Coordinate Motion and Uniform Circular Motion

$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\vec{r} = r\hat{r} \qquad \vec{v} = \frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{r} + r\omega\hat{\theta} \qquad \vec{a} = \frac{d^2\vec{r}}{dt^2} = \left(\frac{d^2 r}{dt^2} - r\omega^2\right)\hat{r} + \left(r\alpha + 2\frac{dr}{dt}\omega\right)\hat{\theta}$$

$$\vec{v} = r\omega\hat{\theta} \qquad v = r\omega \qquad a_{\tan} = r\alpha$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} = -r\omega^2\hat{r} \qquad a_{\text{centripetal}} = \frac{v^2}{r} = r\omega^2 = v\omega$$

13 Very Basic Newtonian Physics

$$\vec{r}_{\rm cm} = \frac{\sum_i m_i \vec{r}_i}{m_{\rm total}} = \frac{\sum_{\rm sub} m_{\rm sub} \vec{r}_{\rm cm\ sub}}{m_{\rm total}} \qquad \vec{v}_{\rm cm} = \frac{\sum_i m_i \vec{v}_i}{m_{\rm total}} \qquad \vec{a}_{\rm cm} = \frac{\sum_i m_i \vec{a}_i}{m_{\rm total}}$$

$$\vec{r}_{\rm cm} = \frac{\int_V \rho(\vec{r})\vec{r}\,dV}{m_{\rm total}}$$

$$\vec{F}_{\rm net} = m\vec{a} \qquad \vec{F}_{21} = -\vec{F}_{12} \qquad F_g = mg \qquad g = 9.8\,{\rm m/s^2}$$

$$\vec{F}_{\rm normal} = -\vec{F}_{\rm applied} \qquad F_{\rm linear} = -kx$$

$$f_{\rm normal} = \frac{T}{r} \qquad T = T_0 - F_{\rm parallel}(s) \qquad T = T_0$$

$$F_{\rm f\ static} = \min(F_{\rm applied}, F_{\rm f\ static\ max}) \qquad F_{\rm f\ static\ max} = \mu_{\rm static} F_{\rm N} \qquad F_{\rm f\ kinetic} = \mu_{\rm kinetic} F_{\rm N}$$

$$v_{\rm tangential} = r\omega = r\frac{d\theta}{dt} \qquad a_{\rm tangential} = r\alpha = r\frac{d\omega}{dt} = r\frac{d^2\theta}{dt^2}$$

$$\vec{a}_{\rm centripetal} = -\frac{v^2}{r}\hat{r} \qquad \vec{F}_{\rm centripetal} = -m\frac{v^2}{r}\hat{r}$$

$$F_{\rm drag,lin} = bv \qquad v_{\rm T} = \frac{mg}{b} \qquad \tau = \frac{v_{\rm T}}{g} = \frac{m}{b} \qquad v = v_{\rm T}(1 - e^{-t/\tau})$$

$$F_{\rm drag,quad} = bv^2 = \frac{1}{2}C\rho A v^2 \qquad v_{\rm T} = \sqrt{\frac{mg}{b}}$$

14 Energy and Work

$$dW = \vec{F}\cdot d\vec{s} \qquad W = \int \vec{F}\cdot d\vec{s} \qquad KE = \frac{1}{2}mv^2 \qquad E_{\rm mechanical} = KE + PE$$

$$P_{\rm avg} = \frac{\Delta W}{\Delta t} \qquad P = \frac{dW}{dt} \qquad P = \vec{F}\cdot\vec{v}$$

$$\Delta KE = W_{\rm net} \quad \Delta PE_{\rm of\ a\ conservative\ force} = -W_{\rm by\ a\ conservative\ force} \quad \Delta E = W_{\rm nonconservative}$$

$$F = -\frac{dPE}{dx} \qquad \vec{F} = -\nabla PE \qquad PE = \frac{1}{2}kx^2 \qquad PE = mgy$$