

**Intro Physics Semester I****Name:**

**Homework 2: One-Dimensional Kinematics:** One or two full answer questions will be marked. There will also be a mark for completeness. Homeworks are due usually the day after the chapter they are for is finished. Solutions will be posted soon thereafter. The solutions are intended to be (but not necessarily are) super-perfect and often go beyond a fully correct answer.

**Answer Table****Name:**

	a	b	c	d	e		a	b	c	d	e
1.	O	O	O	O	O	31.	O	O	O	O	O
2.	O	O	O	O	O	32.	O	O	O	O	O
3.	O	O	O	O	O	33.	O	O	O	O	O
4.	O	O	O	O	O	34.	O	O	O	O	O
5.	O	O	O	O	O	35.	O	O	O	O	O
6.	O	O	O	O	O	36.	O	O	O	O	O
7.	O	O	O	O	O	37.	O	O	O	O	O
8.	O	O	O	O	O	38.	O	O	O	O	O
9.	O	O	O	O	O	39.	O	O	O	O	O
10.	O	O	O	O	O	40.	O	O	O	O	O
11.	O	O	O	O	O	41.	O	O	O	O	O
12.	O	O	O	O	O	42.	O	O	O	O	O
13.	O	O	O	O	O	43.	O	O	O	O	O
14.	O	O	O	O	O	44.	O	O	O	O	O
15.	O	O	O	O	O	45.	O	O	O	O	O
16.	O	O	O	O	O	46.	O	O	O	O	O
17.	O	O	O	O	O	47.	O	O	O	O	O
18.	O	O	O	O	O	48.	O	O	O	O	O
19.	O	O	O	O	O	49.	O	O	O	O	O
20.	O	O	O	O	O	50.	O	O	O	O	O
21.	O	O	O	O	O	51.	O	O	O	O	O
22.	O	O	O	O	O	52.	O	O	O	O	O
23.	O	O	O	O	O	53.	O	O	O	O	O
24.	O	O	O	O	O	54.	O	O	O	O	O
25.	O	O	O	O	O	55.	O	O	O	O	O
26.	O	O	O	O	O	56.	O	O	O	O	O
27.	O	O	O	O	O	57.	O	O	O	O	O
28.	O	O	O	O	O	58.	O	O	O	O	O
29.	O	O	O	O	O	59.	O	O	O	O	O
30.	O	O	O	O	O	60.	O	O	O	O	O

1. "Let's play *Jeopardy!* For \$100, the answer is: It is the branch of mathematics that deals with limits, derivatives, differentiation, integrals, and integration."

What is \_\_\_\_\_, Alex?

- a) geometry    b) algebra    c) number theory    d) set theory    e) calculus
2. It is common to specify a change in a quantity (specified by a symbol) by a prefixed capital Greek letter:
- a) Alpha  $\Lambda$ .    b) Delta  $\Delta$ .    c) Lambda  $\Lambda$ .    d) Psi  $\Psi$ .    e) Omega  $\Omega$ .
3. The formula

$$\frac{df}{dx} = \lim_{\Delta x \rightarrow 0} \frac{\Delta f}{\Delta x},$$

where  $f(x)$  is a general function of  $x$  is the not-explicit definition of the:

- a) integration.    b) integral.    c) derivative.    d) differentiation.    e) differential.
4. The quantity  $dx$  in calculus is the:
- a) derivative of  $x$ .    b) differential of  $x$ .    c) integral of  $x$ .    d) integratial of  $x$ .  
e) product of  $d$  and  $x$ .
5. We can approximate derivative  $df/dx$  to some degree of accuracy by:

a)  $\Delta f \Delta x$  .    b)  $\frac{\Delta f}{\Delta x}$  .    c)  $\frac{\Delta x}{\Delta f}$  .    d)  $\frac{d^2 f}{dx^2}$  .    e)  $\frac{df^2}{\Delta x}$  .

6. The derivative of

$$Ax^p,$$

where  $A$  is a general constant and  $p$  is a general power, is:

- a)  $\frac{Ax^{p+2}}{p+2}$  .    b)  $\frac{Ax^p}{p}$  .    c)  $\frac{Ax^{p+1}}{p+1}$  or  $A \ln(x)$  if  $p = -1$ .  
d)  $pAx^{p-1}$  or 0 if  $p$  and  $x$  are both zero.    e)  $Ax^p$  .
7. The expression

$$\int_a^b f(x) dx = F(b) - F(a)$$

is a/an \_\_\_\_\_ and the expression

$$\int f(x) dx = F(x) + C$$

is a/an \_\_\_\_\_.

- a) definite integral; indefinite integral    b) indefinite integral; definite integral  
c) antiderivative; derivative    d) differential equation; antidifferential equation  
e) differential; antidifferential
8. Kinematics is:
- a) the description of motion.    b) the techniques of the cinema.    c) dynamics by another name.    d) the study of the causes of motion in terms of physical quantities, most prominently force and mass.    e) the rate of change of acceleration with time.
9. The three quantities that are of most obvious interest in kinematics are displacement, velocity, and:
- a) acceleration.    b) deceleration.    c) mass.    d) force.    e) inertia.
10. The magnitudes of displacement, velocity, and acceleration are usually called distance, speed, and:
- a) acceleration speed.    b) deceleration.    c) acceleration.    d) accelmag.  
e) the unnameable.
11. Displacement, velocity, and acceleration are quantities that have both magnitude and direction, and so are:
- a) scalars.    b) unities.    c) vectors.    d) multiplicities.    e) Templars.

12. “Let’s play *Jeopardy!* For \$100, the answer is: All that is needed and used to indicate direction in one-dimensional kinematics.”

What is \_\_\_\_\_, Alex?

- a) symbol    b) image    c) vision    d) vision quest    e) sign

13. The following equations

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t}, \quad v = \frac{dx}{dt}, \quad a_{\text{avg}} = \frac{\Delta v}{\Delta t}, \quad a = \frac{dv}{dt}$$

are, respectively, the 1-dimensional definitions of average velocity, velocity, \_\_\_\_\_, acceleration.

- a) average deceleration    b) deceleration    c) average acceleration    d) speed  
e) average speed

14. There are different ways of defining average quantities. But one wants definitions that are useful. For example, the conventional definition for average velocity is

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t},$$

where  $\Delta x$  is the displacement that occurred in time  $\Delta t$ . This definition is **USEFUL** since  $v_{\text{avg}}\Delta t$  is:

- a) a random number.    b) the displacement that takes place in time  $\Delta t$ .  
c) a quantity with the dimension of length.    d) a quantity with the dimension of velocity.  
e) a time.

15. Graphs of kinematic variables (i.e., displacement, velocity, acceleration, jerk) versus time can all be called:

- a) emotion diagrams.    b) motion diagrams.    c) velocity diagrams.  
d) acceleration diagrams.    e) graphoi

16. If acceleration is constant, then velocity is linear with time and displacement is \_\_\_\_\_ with time.

- a) constant    b) linear    c) quadratic    d) cubic    e) quartic

17. Cusps, discontinuities, and infinities of functions are all:

- a) anomies.    b) autonomies.    c) anomalies.    d) singularities.    e) multiplicities.

18. Given that one-dimensional acceleration  $a$  is a constant, one obtains the equations

$$v = at + v_0$$

and

$$x = \frac{1}{2}at^2 + v_0t + x_0,$$

where  $v_0$  is an initial velocity and  $x_0$  is an initial displacement. The equations are obtained by:

- a) algebra.    b) distinction.    c) differentiation.    d) antidifferentiation.    e) antialgebra.

19. For 1-dimensional, constant acceleration cases there are \_\_\_\_ **INDEPENDENT** equations. Using these equations, 1-dimensional, constant acceleration problems can be solved. Only \_\_\_\_ unknowns can be solved in general from the \_\_\_\_ **INDEPENDENT** equations. One can derive by **ALGEBRA** extra kinematic equations that speed the solution of problems in some cases. But one can still only solve for \_\_\_\_ unknowns in general.

- a) 5    b) 4    c) 3    d) 2    e) 1

20. The constant-acceleration kinematic equation without the time variable (which can be called the timeless equation for mnemonic reasons) is:

- a)  $v^2 = v_0^2 - 2a\Delta x$ .    b)  $v = v_0 + 2a\Delta x$ .    c)  $v^2 = v_0^2 + 2a/\Delta x$ .    d)  $v^2 = v_0^2 + a\Delta x$   
e)  $v^2 = v_0^2 + 2a\Delta x$ .

21. You have just traveled the back roads from Knoxville to Nashville. Your average speed was 60 mi/h, but you occasionally hit an instantaneous speed of 130 mi/h. (Could be you’re hauling white lightning.) Your odometer travel distance is 250 miles. How long have you been on the road?

- a) 1/4 hours.    b) 10 hours.    c) 4.17 hours.    d) 6 hours.    e) about 2 hours.
22. You have just traveled 400 km on a trip to Knoxville and back. Knoxville is due east of your starting point. It took 8 hours. Your average **VELOCITY** (with velocity definitely meaning a vector here) was:
- a) 0 km/h with an indeterminate direction.    b) 50 km/h west.    c) 100 km/h east.  
d) 200 km/h west.    e) 400 km/h north.
23. You move 3 meters due west and then, **WITHOUT** a discontinuous change in direction, go onto a circular path (circle **RADIUS** 2 meters) bending to the left until you are headed due east. This has taken you 10 seconds. Your average speed is approximately \_\_\_\_\_. **HINT:** Draw a diagram.
- a) 3 m/s.    b) 1.55 m/s.    c) 0.3 m/s.    d) 15.5 m/s.    e) 0.9 m/s.
24. The speed of sound in air (at 1 atm pressure and 20°C) is 343 m/s. A lightning flash occurs 1.5 km away. How long until you hear thunder?
- a) 228.7 s.    b) 0.0044 s.    c) 4.4 s.    d) 515 s.    e) 1.5 s.
25. If an object's speed changes, the object:
- a) stops.    b) accelerates.    c) starts.    d) goes forward.    e) hesitates.
26. Say you are moving in the positive  $x$ -direction, but your speed is decreasing.
- a) Your acceleration points in the **POSITIVE**  $x$ -direction.  
b) Your acceleration points in the **NEGATIVE**  $x$ -direction.  
c) You are **NOT** accelerating at all. You are decelerating.  
d) Your acceleration points **PERPENDICULAR** to the  $x$ -axis.  
e) Your acceleration points in the **POSITIVE**  $x$ -direction. Acceleration always points in the same direction as velocity.
27. At time zero you are moving at 10 m/s in the positive  $y$ -direction. At time 10 s, you are moving at 15 m/s in the positive  $y$ -direction. What is your average acceleration over the 10 s of travel? What is your instantaneous acceleration at time 5 s?
- a) The average acceleration and the 5 s instantaneous acceleration are both 0.5 m/s<sup>2</sup> in the **POSITIVE**  $y$ -direction.  
b) The average acceleration and the 5 s instantaneous acceleration are both 0.5 m/s<sup>2</sup> in the **NEGATIVE**  $y$ -direction.  
c) The average acceleration is 0.5 m/s<sup>2</sup> in the **NEGATIVE**  $y$ -direction. There is **NOT** enough information to determine the 5 s instantaneous acceleration.  
d) The average acceleration and the 5 s instantaneous accelerations are both 5 m/s<sup>2</sup> in the **NEGATIVE**  $y$ -direction.  
e) The average acceleration is 0.5 m/s<sup>2</sup> in the **POSITIVE**  $y$ -direction. There is **NOT** enough information to determine the 5 s instantaneous acceleration.
28. The acceleration due to gravity near the Earth's surface (whose magnitude is denoted by  $g$  and whose value is nearly constant and is near fiducial value 9.8 m/s<sup>2</sup>):
- a) points east.    b) points down toward the Earth's center more or less.    c) points straight up more or less.  
d) has no direction at all.    e) points toward the Moon.
29. As an object falls freely under gravity. Neglecting air drag, its speed \_\_\_\_\_ and its acceleration is \_\_\_\_\_.
- a) increases; constant.    b) decreases; increases.    c) increases; increases.  
d) decreases; decreases.    e) is constant; constant.
30. It is reported that Galileo (circa 1590) dropped balls of different mass at the same time from the top of the Leaning Tower of Pisa in order to demonstrate that:
- a) the heavier ball hit the ground first by a large margin.  
b) the lighter ball hit the ground first by a large margin.  
c) both balls hit the ground at more or less the same time.  
d) the balls would levitate toward the Moon.

- e) the Leaning Tower was leaning.
31. A ball is thrown downward at 12 m/s. About what is its **SPEED** 2.0 s later assuming no air drag?  
 a) 20 m/s.    b) 32 m/s.    c) 22 m/s.    d)  $-2$  m/s.    e) 12 m/s.
32. A tall archer with her longbow shoots an arrow straight up at 100 m/s. The arrow rises, slows, holds for an instant, and then descends picking up speed. The rise time, neglecting air drag, is:  
 a) 100 s.    b) 100.2 s.    c) 9.8 s.    d) 10.2 s.    e) 980 s.
33. How fast is a person falling after 3 s starting from rest? Recall the acceleration due to gravity is  $g = 9.8 \text{ m/s}^2$  (which is the fiducial value). Neglect air drag.  
 a) 29.4 m/s.    b) 44.1 m/s.    c) 9.8 m/s.    d) 88.2 m/s.    e) At the speed of light.
- 34.\* A human falls off some high scaffolding. About how far does he/she fall in 3 seconds? (Neglect air drag.)  
 a) 44 m.    b) 88 m.    c) 22 m.    d) 9.8 m.    e) 4.9 m.
35. "Let's play *Jeopardy!* For \$100, the answer is: It occurs to a dense falling object falling near the Earth's surface when the force of gravity and the force of air drag (AKA air resistance) cancel to give no net force on an object."

What is \_\_\_\_\_, Alex?

- a) acceleration upward    b) acceleration downward    c) terminal velocity  
 d) initial velocity    e) parabolic motion
36. The terminal velocity of a human in air is about 120 mi/h. At this speed how long does it take to fall 2 miles.  
 a) 2 minutes.    b) 1 minute.    c) 1 hour.    d) 2 hours.    e) 1 second.
37. "Let's play *Jeopardy!* For \$100, the answer is: These features allow cat victims of the high-rise syndrome (i.e., the propensity to taking flying leaps into oblivion—cats being so darn smart you know) to survive falls of more than  $\sim 20$  m without major injuries—sometimes that is."

What are \_\_\_\_\_, Alex?

- a) feline insouciance, savoir-faire, panache, et je-ne-sais-quoi.  
 b) the cat **WRONGING** reflex and relatively **LOW** terminal velocity when spread-eagled  
 c) the cat **WRONGING** reflex and relatively **HIGH** terminal velocity when spread-eagled  
 d) the cat **RIGHTING** reflex and relatively **HIGH** terminal velocity when spread-eagled  
 e) the cat **RIGHTING** reflex and relatively **LOW** terminal velocity when spread-eagled
38. Differentiation is an extremely important mathematical operation in physics.

- a) Differentiate the power-law formula

$$y = Ax^p ,$$

where  $A$  is a constant and  $p$  is a general power.

- b) Find the derivatives of

$$y = A , \quad y = Ax , \quad y = Ax^2 .$$

39. Jerk is derivative of acceleration. The term jerk is not used much by a lot of people. Say jerk for a one-dimensional case is a constant  $b$ . This means that  $b = da/dt$ , where  $a$  is acceleration. Using antidifferentiation (i.e., integration), write down the expressions for acceleration, velocity, and position remembering that a constant of integration arises at every integration.
40. A car starts from **REST** with acceleration  $1 \text{ m/s}^2$ .
- a) What is its velocity after 10 s from its **START POSITION**?  
 b) What is its displacement traveled after 10 s from its **START POSITION**?  
 c) What is its velocity when it has traveled 100 m from its **START POSITION**?

41. At time zero, there is a car stopped at a light and a truck moving at a constant velocity of 30.0 m/s is 200 m behind the car. Also at time zero, the car starts accelerating forward at 1.00 m/s<sup>2</sup>.
- Draw a **QUALITATIVE** plot of position  $x$  versus time  $t$  showing the trajectory curves of both vehicles. What are the three qualitatively different ways the curves can intersect?
  - At what time or times do the car and truck pass? Is it possible for there to be only one pass with the given conditions?
42. At time zero, there is a car (a Jaguar) stopped at a red light and a Mercedes (that has just run the red) is moving at a constant 50 m/s and is 20 m ahead of the Jag. Also at time zero, the stoplight turns green and the Jag (driven by an atavistic categorical imperative) starts accelerating forward at 5.0 m/s<sup>2</sup>.
- What is 50 m/s in miles per hour? Note there are 1609 meters to the mile.
  - Given the conditions, must the Jag pass the Mercedes? Why? (If your answer is no, you can skip the rest of the question.)
  - Calculate the time to the pass?
  - Calculate the distance from the stoplight to the pass?
  - How fast is the Jag going at the pass in meters per second **AND** miles per hour?
43. Say that raindrops fall 1800 m to the ground starting from rest.
- Assuming there is no air drag (AKA air resistance) what is touch-down velocity?
  - Would it be safe to walk in the rain without air drag? Compare the drop speed to bullet speed? You will find some useful information about raindrop speeds at  
<http://hypertextbook.com/facts/2007/EvanKaplan.shtml>  
 and about bullet speeds at  
<http://hypertextbook.com/facts/1999/MariaPereyra.shtml> .  
 Of course, this is no help on a test.
44. Super-Armadillo is on the **MOON** where the free-fall acceleration  $g = 1.6 \text{ m/s}^2$  downward and there is really no air drag (AKA air resistance).
- He flies **STRAIGHT UP** from the surface starting from **REST** with an **ACCELERATION** of 5 m/s<sup>2</sup> upward for 20 s. Then his superpower stalls and he can no longer accelerate, but he still has the velocity he had at the end of his self-powered acceleration phase for an instant. What is his **HEIGHT** and his **VELOCITY** when his superpower failure occurs?
  - After stalling, his acceleration is due to gravity. How high above the **LUNAR GROUND** does he rise before beginning to fall?
  - Eventually Super-Armadillo must fall back to the lunar surface. What is his crash **SPEED**? (Fortunately, his invulnerability is still operative.)
45. The 5 standard kinematic equations of constant-acceleration, 1-dimensional motion were derived assuming motion with respect to some fixed reference frame. But what if you had say a two-object constant-acceleration, 1-dimensional problem that you wished to reduce to a one-object problem by changing reference frames: i.e., changing to the frame of one of the objects or in other words to a relative frame.

Note the relative position (or displacement), velocity, and acceleration for object 2 in the frame of object 1 are defined by, respectively,

$$x_{\text{rel}} = x_2 - x_1 ,$$

$$v_{\text{rel}} = v_2 - v_1 ,$$

$$a_{\text{rel}} = a_2 - a_1 .$$

These formulae just express what one means by relative position, velocity, and expression in one dimension.

- a) Show that the first two kinematic equations apply to relative motion if the objects involved both have constant acceleration (relative to some standard frame of reference like the ground). Recall these equations are:

$$v = at + v_0 \quad \text{and} \quad x = \frac{1}{2}at^2 + v_0t + x_0 ,$$

where the subscript “0” indicates initial value.

- b) Show that the other 3 kinematic equations apply to relative motion. **HINT:** This isn't very hard if you know the trick.

# Equation Sheet for Introductory Physics Calculus-Based

This equation sheet is intended for students writing tests or reviewing material. Therefore it is neither intended to be complete nor completely explicit. There are fewer symbols than variables, and so some symbols must be used for different things: context must distinguish.

The equations are mnemonic. Students are expected to understand how to interpret and use them.

## 1 Constants

$$c = 2.99792458 \times 10^8 \text{ m/s} \approx 2.998 \times 10^8 \text{ m/s} \approx 3 \times 10^8 \text{ m/s} \approx 1 \text{ yr/yr} \approx 1 \text{ ft/ns} \quad \text{exact by definition}$$

$$e = 1.602176487(40) \times 10^{-19} \text{ C}$$

$$G = 6.67428(67) \times 10^{-11} \text{ N m}^2/\text{kg}^2 \quad (2006, \text{CODATA})$$

$$g = 9.8 \text{ m/s}^2 \quad \text{fiducial value}$$

$$k = \frac{1}{4\pi\epsilon_0} = 8.987551787 \dots \times 10^9 \approx 8.99 \times 10^9 \approx 10^{10} \text{ N m}^2/\text{C}^2 \text{ exact by definition}$$

$$k_{\text{Boltzmann}} = 1.3806504(24) \times 10^{-23} \text{ J/K} = 0.8617343(15) \times 10^{-4} \text{ eV/K} \approx 10^{-4} \text{ eV/K}$$

$$m_e = 9.10938215(45) \times 10^{-31} \text{ kg} = 0.510998910(13) \text{ MeV}$$

$$m_p = 1.672621637(83) \times 10^{-27} \text{ kg} = 938.272013(23), \text{ MeV}$$

$$\epsilon_0 = \frac{1}{\mu_0 c^2} = 8.8541878176 \dots \times 10^{-12} \text{ C}^2/(\text{N m}^2) \approx 10^{-11} \quad \text{vacuum permittivity (exact by definition)}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2 \quad \text{exact by definition}$$

## 2 Geometrical Formulae

$$C_{\text{cir}} = 2\pi r \quad A_{\text{cir}} = \pi r^2 \quad A_{\text{sph}} = 4\pi r^2 \quad V_{\text{sph}} = \frac{4}{3}\pi r^3$$

$$\Omega_{\text{sphere}} = 4\pi \quad d\Omega = \sin\theta \, d\theta \, d\phi$$

## 3 Trigonometry Formulae

$$\frac{x}{r} = \cos\theta \quad \frac{y}{r} = \sin\theta \quad \frac{y}{x} = \tan\theta = \frac{\sin\theta}{\cos\theta} \quad \cos^2\theta + \sin^2\theta = 1$$

$$\csc\theta = \frac{1}{\sin\theta} \quad \sec\theta = \frac{1}{\cos\theta} \quad \cot\theta = \frac{1}{\tan\theta}$$

$$c^2 = a^2 + b^2 \quad c = \sqrt{a^2 + b^2 - 2ab \cos\theta_c} \quad \frac{\sin\theta_a}{a} = \frac{\sin\theta_b}{b} = \frac{\sin\theta_c}{c}$$

$$f(\theta) = f(\theta + 360^\circ)$$

$$\sin(\theta + 180^\circ) = -\sin(\theta) \quad \cos(\theta + 180^\circ) = -\cos(\theta) \quad \tan(\theta + 180^\circ) = \tan(\theta)$$

$$\sin(-\theta) = -\sin(\theta) \quad \cos(-\theta) = \cos(\theta) \quad \tan(-\theta) = -\tan(\theta)$$



$$\sin(\theta + 90^\circ) = \cos(\theta) \quad \cos(\theta + 90^\circ) = -\sin(\theta) \quad \tan(\theta + 90^\circ) = -\tan(\theta)$$

$$\sin(180^\circ - \theta) = \sin(\theta) \quad \cos(180^\circ - \theta) = -\cos(\theta) \quad \tan(180^\circ - \theta) = -\tan(\theta)$$

$$\sin(90^\circ - \theta) = \cos(\theta) \quad \cos(90^\circ - \theta) = \sin(\theta) \quad \tan(90^\circ - \theta) = \frac{1}{\tan(\theta)} = \cot(\theta)$$

$$\sin(a + b) = \sin(a)\cos(b) + \cos(a)\sin(b) \quad \cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$\sin(2a) = 2\sin(a)\cos(a) \quad \cos(2a) = \cos^2(a) - \sin^2(a)$$

$$\sin(a)\sin(b) = \frac{1}{2}[\cos(a - b) - \cos(a + b)] \quad \cos(a)\cos(b) = \frac{1}{2}[\cos(a - b) + \cos(a + b)]$$

$$\sin(a)\cos(b) = \frac{1}{2}[\sin(a - b) + \sin(a + b)]$$

$$\sin^2\theta = \frac{1}{2}[1 - \cos(2\theta)] \quad \cos^2\theta = \frac{1}{2}[1 + \cos(2\theta)] \quad \sin(a)\cos(a) = \frac{1}{2}\sin(2a)$$

$$\cos(x) - \cos(y) = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos(x) + \cos(y) = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin(x) + \sin(y) = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

#### 4 Approximation Formulae

$$\frac{\Delta f}{\Delta x} \approx \frac{df}{dx} \quad \frac{1}{1-x} \approx 1+x : (x \ll 1)$$

$$\sin\theta \approx \theta \quad \tan\theta \approx \theta \quad \cos\theta \approx 1 - \frac{1}{2}\theta^2 \quad \text{all for } \theta \ll 1$$

#### 5 Quadratic Formula

$$\text{If } 0 = ax^2 + bx + c, \quad \text{then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = -\frac{b}{2a} \pm \sqrt{\left(\frac{b}{2a}\right)^2 - \frac{c}{a}}$$