Chapter 4 [4001

1) Star Forming Galaxies SFGs (4003)
2) Ci-Table 4.1 (Ci-55) (4004)
3) Stellar Disks (4005)
4) Inclined Disks (4008)
5) Stellar Emissivity (4010)
6) Bulges (4011)
7) Spiral Arms: Just a first word (4013)
8) Star Formation law (4014)
9) Gas Inflows and Outflows (4024)
10) Rotation Curves (4030)
11) Tully-Fisher Relation (4039)
12) Specific ang-mom. (4043)
13) Mass-Metallicity (4045)
14) Star Formation Main Sequence (4047)
14) Size-Mass Relation (4049)

Scaling Relations
empirical

4002

16) Starburst Galaxies
& ULIRGs (~~4~~4050)

9003

1) Star Forming Galaxies

SFGs & ETGs

(which is virtually a synonym for LTGs $\Rightarrow$ late type galaxies)

Spiral galaxies $M_* \gtrsim 10^9 M_\odot$

and dwarf Irregulars dIrr $M_* < 10^9 M_\odot$

by G-54 rule

Irregulars are irregular but may have spiral features

G-54

Recall $M_{DM} \gtrsim 30 M_*$

but much of that mass is rather far out ~ 10 kpc from galaxy center

Starburst Galaxies

↳ few percent of SFG have star formation rates SFRs ~ 10 to 100 larger than most SFGs

G-107

ETGs $\Rightarrow$ Elliptical & Lenticular (S0) galaxies have SFR $\lesssim$ 0.1 SFR_{SFG}

G-54

(So ETGs have some cold gas & dust, but most gas is hot and invisible

except in X-rays where ETGs can be relatively bright but still not very bright

SFG have cold gas T = 10s to 100s K

and they need this to have low pressure support to allow collapse to stars

and to get this cold they

4004

molecules to cool the gas through molecular line cooling.

and to have molecules you need dusty molecular clouds

The dust stops UV and visible light from dissociated the molecules into much less radiating atoms

2) Table 9.1 → lots of data not to memorize, but contemplate while we look at it

Ci-54

Fiducial Total mass discuss table

Quantity	Spirals	dIrrs	MW
Luminosity B ($L_\odot$)	$10^9 - 10^{11}$	$\lesssim 10^9$	3×10^{10}
virial mass ($M_\odot$)	$10^{11} - 10^{13}$	$10^8 - 10^{11}$	$1 - 1.5 \times 10^{12}$
Disk M_* ($M_\odot$)	10^9 – few $\times 10^{11}$	$10^5 - 10^9$	4×10^{10}
Bulge M_* ($M_\odot$) (stars but lower)	$10^8 - 10^{11}$	no bulge	$1 - 1.5 \times 10^{10}$
R_d = Stellar disk scale length (kpc) (Ci-56) e-folding length for surface brightness reduction	1 - 10	< 5	2 - 3
bars	50 - 70%	rare (but contain some smaller)	yes intermediate star → small bar
Molecular Mass ($M_\odot$) → H_2 but traces of other	$10^7 - 10^{10}$ (~~[illegible]~~ third)	uncertain	10^9
Peak Rotating Velocity (km/s) natural unit for most astro / The plateau is ~ 200 km/s → Due to dark matter (lots mostly forgotten)	100 - 900	< 100	220 - 250
SFR ($M_\odot$/yr)	0.1 — few 10s	starburst galaxies $10^{-2} - 1$	1 - 3

3) Stellar Disks

4005

— disks have a mixture of young stars $\lesssim$ 1 Million year to zone

capital Z for metallicity

low metallicity $Z < 0.1 Z_\odot$

— $t_0 \geq 10$ Gyr < Population II or old population

But Dark Matter halos approximate spherical sym. even for disc galaxies

spherically symmetric mass distribution approximation

$$\frac{v^2}{r} = \frac{GM(r)}{r^2}, \quad v^2 = \frac{GM(r)}{r}$$

for circular velocity.

Arms rotate slower than stars — flocculent or spiral density waves or mixture

coherent rotation 10s to 100s km/s

σ = velocity dispersion → random motion

ratio V/σ = { ~10 for large spirals; ~1 for small dIrr

Circular orbits mainly consequence of formation accretion disk

Face-on surface Brightness Profile

Apparently generally works well.

$$I = I_0 e^{-bx}$$

specific intensity = surface brightness

$$= I_0 e^{-\frac{R}{R_d}}$$

$x = \frac{R}{R_e}$

$b(n) = 1.678...$ as we proved on p. 3031 for $n=1$

So $R_d = R_e/b = \frac{R_e}{1.678}$

scale length (1-56)

1006)

And since for a definite orientation

$\underbrace{\text{definite orientation}}_{\text{basic}}$ ~~this~~ is an intrinsic property of disks — on average intrinsic property

In mag/arcsec^2

$$\mu(R) = \mu_0 + 1.0857\left(\frac{R}{R_d}\right)$$

$\lambda_B =$

Freeman law (1970)

$\mu \gtrsim \mu_{0_{\text{Freeman}}} \simeq 21.7\,\text{mag/arcsec}^2$ in B band — Brightness of discs Not bulge.

for large spirals

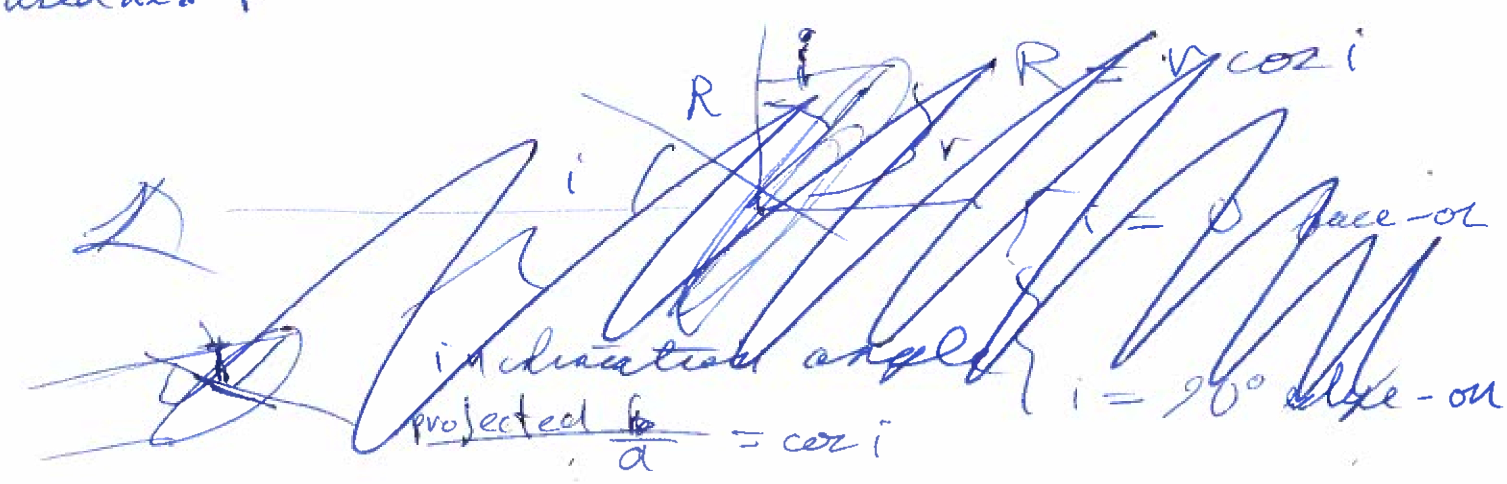
magnitude is wrong-way scale - so these are dimmer

low surface brightness (LSBs) galaxies

have $\mu_0 \gtrsim 23\,\text{mag/arc}^2$.

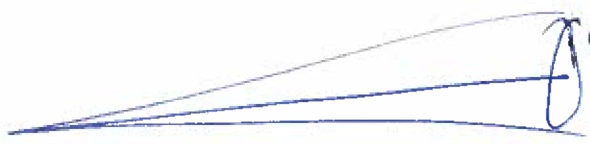
[in magnitude jargon one usually tries to say dimmer/brighter magnitude rather than the confusing equivalent higher/lower magnitude]

but there'd be no confusion if we all used dex



$I_\lambda = I_{\lambda 0}\, e^{-\frac{R}{R_d}}$

can be integrated to get flux times r^2



$I \to \frac{dE}{d\lambda\, dt\, dA\, d\Omega}$

$$r^2 \mathcal{F}_\lambda = \int_0^R I_\lambda 2\pi R\, dR$$

$$= 2\pi I_0 R_d^2 \int_0^x e^{-x} x\, dx$$

$$= 2\pi I_0 R_d^2 \Big[-e^{-x} x \Big|_0^x + \int_0^x e^{-x} dx \Big]$$

integration by parts

$$= 2\pi I_0 R_d^2 \left[1 - (1+x) e^{-x}\right]$$

in general

$$2\pi I_0 R_d^2 \Big[1 - \underbrace{\underbrace{(1+x)(1 - x + \tfrac{1}{2}x^2)}_{x^2 + \frac{1}{2}x^2}}_{\frac{3}{2}x^2}\Big]$$

to lowest order in small x

$$2\pi I_0 R_d^2 \quad \text{for } x \to \infty$$

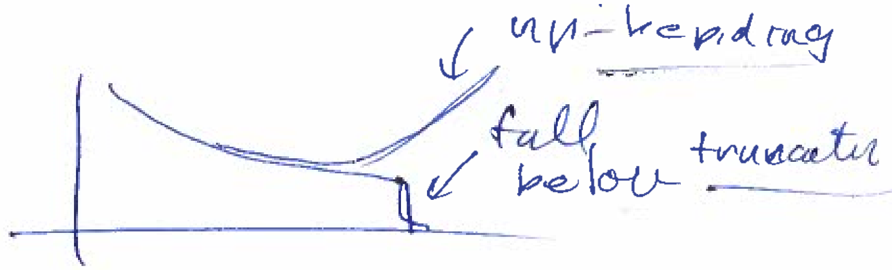
$\frac{R\, dR}{r^2}$ is solid angle, and so $\mathcal{F}_\lambda$ is flux measured at Earth

Many galaxies are exponential as far as R can be seen.

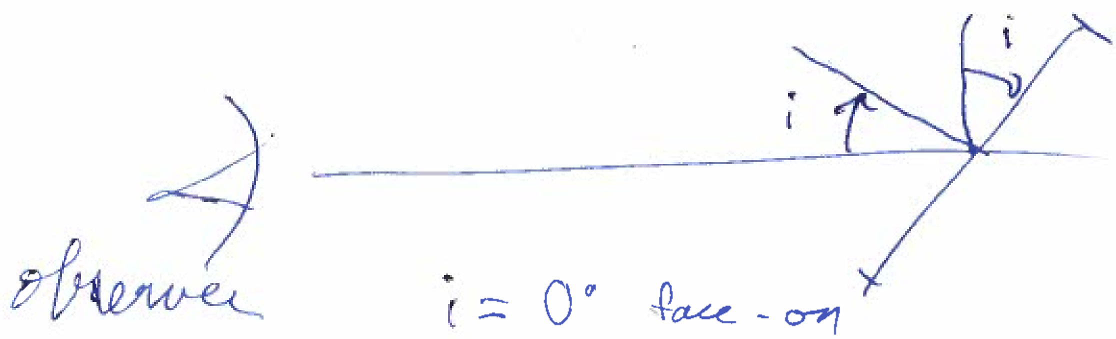
But some (how many debated) ~~?~~ have breaks

4008

truncation or up-bending

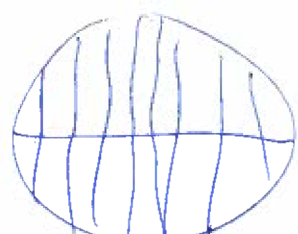


4) Inclined Disks which are most since relatively few are face-on or nearly face-on but similarly relatively few are edge-on or nearly edge-on — most are inclined



$i = 0°$ face-on

$i = 90°$ edge-on



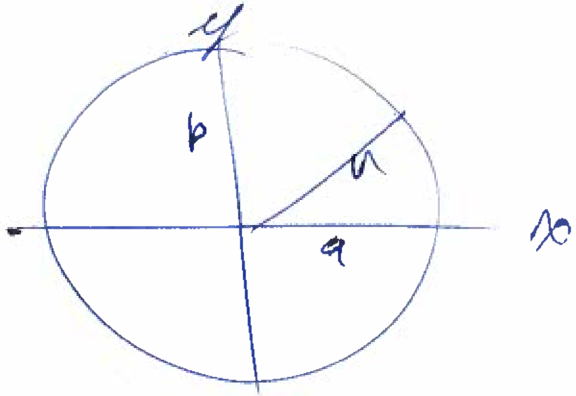
A ~~projected~~ on sky = A cos i

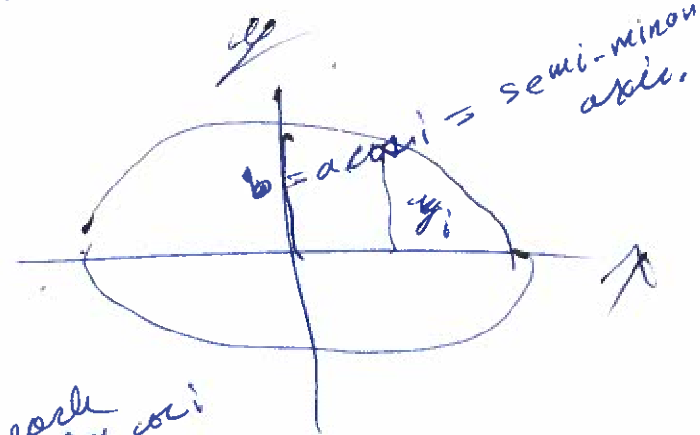
Imagine breaking into stripes

The same is true for all the stripes individually

4009

A inclined circle is an ellipse in projection





$x^2 + y^2 = r^2$

$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$

if $a = b$

$y = \pm a\sqrt{1 - \left(\frac{x}{a}\right)^2}$

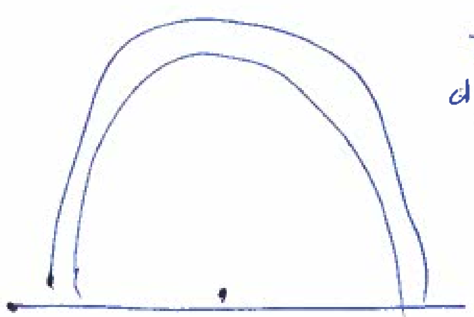
multiply ~~divide~~ each node by cos i

$\rightarrow$ $y_i = y\cos i = \pm a \cos i \sqrt{1 - \left(\frac{x}{a}\right)^2}$, $b = a\cos i$

inclined circle y_i, and so projected is an ellipse.

Area $= \pi a b$ (Wiki Ellipse Area)

$= \pi a^2 \cos i$



$dA = \pi a\, da \cos i = \pi a \sqrt{\cos i}\, da \sqrt{\cos i} = \pi R_i dR_i$

QED

$R_{ic} = (ab)^{\frac{1}{2}}$, $dR_i = da\sqrt{\cos i}$

circularized inclined radius $= a\sqrt{\cos i}$

and so $A = \pi R_i^2 = \pi a b$ again

$r^2 dF_{proj} = \underbrace{I\, 2\pi R_i\, dR_i}_{\text{projected}} = I_{\lambda 0}\, e^{-\frac{R_i}{R_{di}}}\, 2\pi R\, dR \cos i$

distance to sun

differential of Flux

$R_i = R\sqrt{\cos i}$

$R_{di} = R_d\sqrt{\cos i}$

and so $x = \frac{R_i}{R_{di}} = \frac{R}{R_d}$

~~$I\, 2\pi\, dR$~~

$= r^2 dF_{face\text{-}on} \cos i$

$\therefore r^2 F_{face\text{-}on} = \frac{r^2 F_{observed}}{\cos i}$

Assuming I, surface brightness is angle-independent and disk is infinitely thin

4001 Of course neither of those assumption is true actually.

5) Stellar ~~luminosity density~~ emissivity ~~for~~ a disk (not a bulge component) parameterized = again all one can say is it works well usually

$$\dot{j}_* = \dot{j}_{*0}\, e^{-\frac{R}{R_d}}\, e^{-\frac{|z|}{h_*}}$$

stars

center of disk component which can be buried in a bulge

$L_\odot$ (E/t) sr Hz pc^3 is the usual unit.

~~$L_\odot$ is usual sr Hz pc^3 unit But~~

$$\frac{dE_\odot}{dt\, d\nu\, d\Omega}$$

R_d disk scale length of order kpc see p. 4004

h_* disk scaleheight $h_* \sim 0.1 R_d$

stellar disk scaleheight Does Not vary strongly with R but $h_{gas}(R)$ does. (Ci-46-47)

But this is thin disk. There is thick disk component to be considered later.

6) Bulges

[40

What of bulge

- 2 kinds (Wiki: Galactic bulge)

— classical bulges which are like elliptical galaxies at center of spirals
→ mostly Population II stars
→ little star formation and so mostly yellow in color enhancement (true color)
— little dust or gas
— random oriented orbits
— formed following mergers like ellipticals mainly streams
→ inside-out quenching
may turn spiral with classical bulges in S0 galaxies?

— disk-like bulges (Pseudo-bulge)

sort of like thicker disks with a prograde rotation.
— complex dust and gas
→ so more star formation
— younger star
~~stop~~
→ more population I
Theories of these
1) mergers not so extreme
2) secular evolution
— Just what happens to spiral galaxies like bars

Milky Way (Wiki: Milky Way)
— a bit undecided
— some same it has a disk bulge but stars there Pop II mainly age $\hat{=}$ 12.8 Gyr
~~Also a small bar~~

Maybe a continuum between two kinds or a superposition of both nature

4012)

When a larger bulge is present then

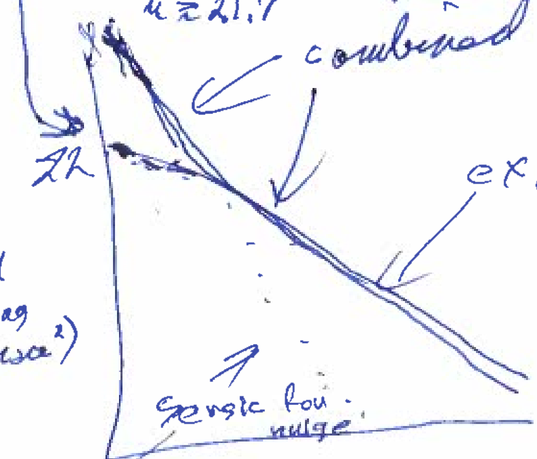
for surface brightness, for Sa Sb SBa SBb

(Milky Way is de Vaucouleurs system SB(rs)bc

One decomposes bulge & disc

I_e or I_0, R_e, $n \gtrsim 2.5$

Parameter I_0, R_d



$n \in [2,10]$

Ei-32

for NGC 3898 Sab (Ci-57)

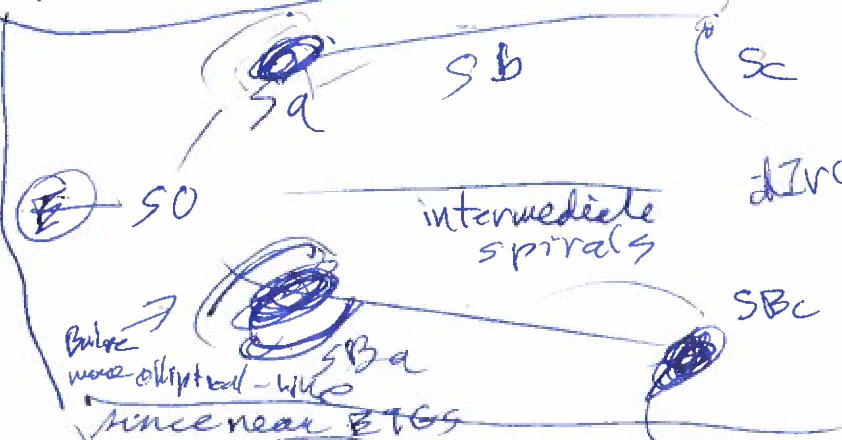
Obliquely seen (Ci-57) but Ci-87 does not give an inclination correction

Bar but a small one

Similar r means ring, s means no ring, so MW in between

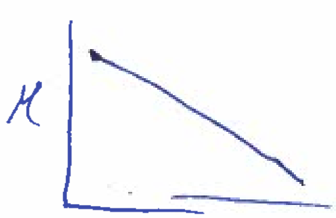
Old rule of galaxy typing if it is in between two types put them both down.

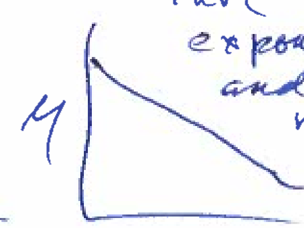
So MW between b and c



Sc Face on

dIrr





Pure exponentials, and so no significant bulge component

7) Spiral galaxies have spiral arms

(But we are going to leave to reading)

Just a brief word here → maybe a later chapter has more and more on bars too (bars = another kind of density wave)

a) grand design spirals
— 2 or 4 arms cover whole disk
(Wiki: grand design spirals only ~10% at [illegible])

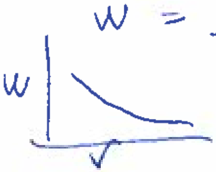
b) Intermediate → the arms lose coherence
Multi-arm spirals (sometimes lumped with flocculent)

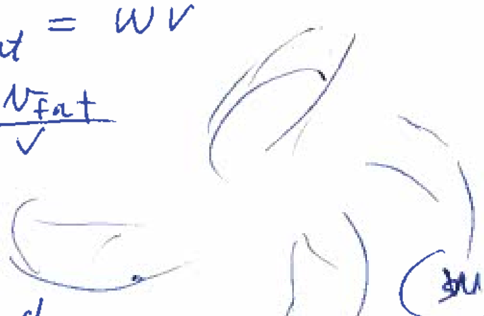
Spiral density waves cause
(Wiki: flocculent)

c) Flocculent → fleecy (Wiki: flocculent) ~30%

$v_{flat} = w r$

$w = \frac{v_{flat}}{r}$





So wind up

— long star formation region that wind up a bit before they disperse and new ones form.
(Wiki self-propagating star formation)

— So not infinitely wound up arms unless there are what lenticular galaxies (S0 galaxies are)

Probably a continuum between extremes

— The arms usually rotate in pinwheel fashion — prograde with the stars
— but slower → traffic jams stars, dust + gas slow down thru them.
→ There are exceptions — retrograde arms. Some counter galaxy-galaxy interaction ...the

4014

8) Star Formation Laws

Jumpover many pages of Ci from Ci-58 to Ci-83.
We can't expand on everything.
Must leave somethings just light background reading.

Ci-55

	Spirals	dIrr	Milky Way
SFG ~~Σ_{SFR} ($M_\odot$/yr)~~	0.1 — few 10s (1-3 for MW; there are starburst)	10^{-4} — 1 (But remember these smaller)	1 - 3
Disk M_* ($M_\odot$)	10^9 — few 10^{11}	10^7 — 10^9	$\sim 4\times10^{10}$

Star Formation Rate Surface Density Σ_{SFR}

SFR per area

Capital Greek Sigma ($M_\odot\,yr^{-1}\,kpc^{-2}$)

Schmidt-Kennicutt Law work well

1959 MW

1998 other galaxies

Like so many galaxy laws - an empirical law that works well usually with a theoretical understanding but not a derivation except a full large-scale structure simulation

surface density of gas (molecular & atomic H usually in units of $M_\odot\,pc^{-2}$)

units

$$\Sigma_{SFR} = B\left(\frac{\Sigma_{gas}}{\cancel{1\,M_\odot\,pc^{-2}}}\right)^{\alpha} \frac{M_\odot}{yr\,kpc^2}$$

$B \approx 10^{-4}$, $\alpha \approx 1.4 \approx \frac{3}{2}$

– the relation holds well for many types of galaxies normal spirals & starburst galaxies

more scatter for metal-poor and low column density galaxies

Σ_{gas} derived from

Column density

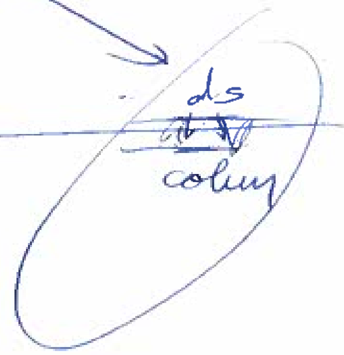
$$\sigma_n = \int_0^s n \, ds$$

Optical depth without opacity

areal number density

number density (per volume)

differential path



Not $\tau_{n\lambda} = \int_0^s \gamma_\lambda n \, ds$

optical depth

cross section.

How is Σ_{gas} ($M_\odot \, pc^{-2}$) determined?

Primordial gas .75 H, .25 He, .001 D most intergalactic gas.

What you ~~both~~ want is H_1 and H_2 gas density.

atomic molecular

this is the bulk material out of which stars form

Metallicity increases with generation of stars but eventually saturated but not forever – see inflow outflow of gas p4024

mass fractions

(0.73 H, .25 He, .02 metals)

Fiducial ~~cosmic~~ galactic present abundances

.01 to .03 maybe

4016

- the metals are a bit uncertain → vary a bit
- but in the context ~~the~~ star formation at cosmic present the effect & ~~amount~~ probably is not too important

He content by mass just a ratio to H

$$\frac{X_{He}}{X_H} = \frac{n_{He} * 4 m_p}{n_H * (1 \text{ or } 2) m_p}$$

for H_1 H_2

$$\therefore \; n_{He} = n_H \left(\frac{1 \text{ or } 2}{4}\right) \frac{X_{He}}{X_H}$$

$$= n_H \left(\frac{1 \text{ or } 2}{4}\right) \left(\frac{1}{3}\right)$$

But H_2 is the key gas for star formation in molecular clouds where dust shields from UV photoionization (UV photon ≥ 11.2 eV can dissociate H_2 Ci-79)

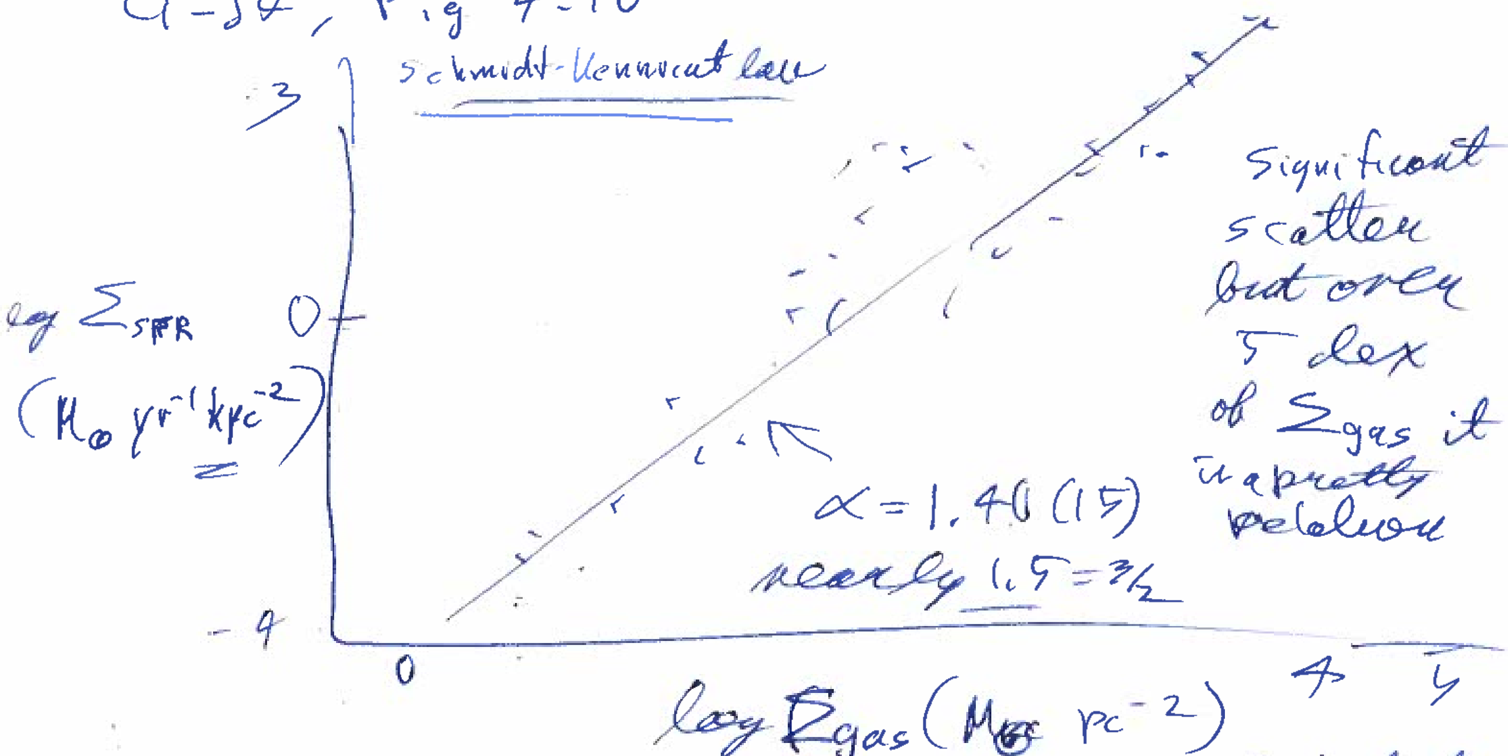
Need molecules to cool the gas and decrease pressure support at which H_2 doesn't do a good job

But H_2 is a very poor emitter in any part of the electromagnetic spectrum including IR and radio at low temperature to which dust is transparent.

→ the main emission channel electric dipole is forbidden by the symmetry of two identical atoms (Ci-79 (Wik: Molecular clouds)

CO carbon monoxide
is a good tracer gas and
a strong emitter
But it is a trace abundance.
So H_2 abundance must
be inferred from CO emission.
→ It is a tricky job.
We won't elaborate here.
See Ci – 76 – 77.

Ci – 84, Fig 4-16



Significant scatter but over 5 dex of Σ_{gas} it is a pretty relation

Why is the $\alpha = 1.40(15)$ understandable. From fairly simple consideration

standard deviation 2 given

4018

SFR density = ρ_{SFR}

Maybe $\dot{\rho}$ would be better idea

$$\frac{\text{Number}}{\text{year kpc}^3}$$

Let say
ρ is gas density

and say $\frac{d\rho}{dt} = -K\rho = \frac{1}{\tau}\rho$

depletion into stars with no inflow of gas

τ is mean lifetime to before depletion.

Solution $\rho = \rho_0(1 - e^{-t/\tau})$

But if there is inflow of new gas

$\frac{d\rho}{dt} = -K\rho + R = 0$ for the steady solution

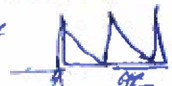
$\rho = \frac{R}{K}$

$\rho_{SFR} = \dot{\rho}_{stars} = K\rho = \frac{\rho}{\tau}$

just the steady rate of stars

Formally τ is the mean lifetime ~~of a random process~~ between events of a random process {Bar—38}

Does not seem correct.
Say



spike flooding by gas and all collapse in tff and then again.
$\rho_{SFR} = \frac{\rho}{\tau}$
But one can have different regions not synchronization and have the same rate.
If refill is at a delay $\tau \approx t_{ff} + \delta t_{delay}$

What is an estimate of τ
the free fall time t_{ff}
for a clump (spherical) of gas of uniform density.

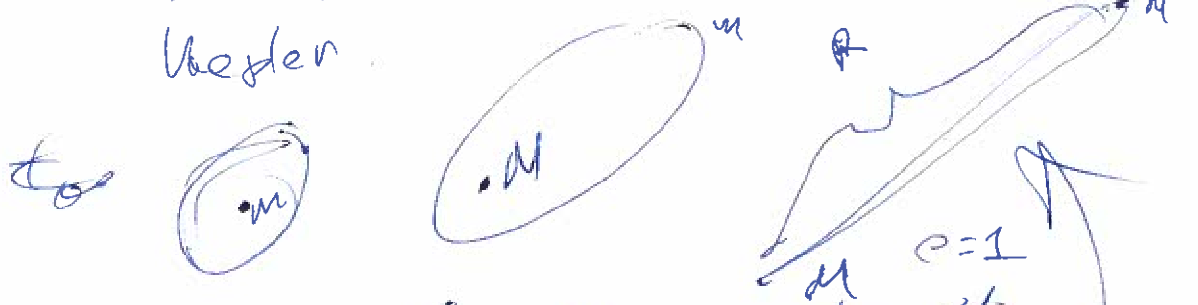
4019

$$t_{ff} = \frac{1}{2} t_{orbit} = \frac{1}{2}\frac{2\pi}{\sqrt{G(M+m)}}\left(\frac{R}{2}\right)^{3/2}$$

Kepler

Wik free fall time

$\frac{R}{2} = a$ semimajor axis

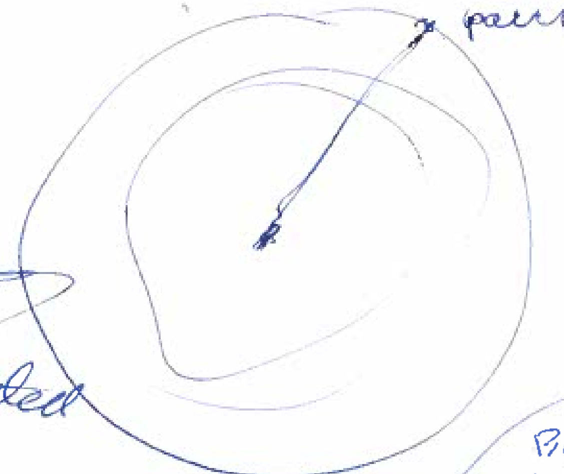


same for all eccentricity $e \in [0, 1]$

$e=1$ is the free fall case.

for $m \ll M$ a test particle

$$t_{ff} = \frac{\pi}{\sqrt{GM}}\left(\frac{R}{2}\right)^{3/2}$$



by shell theorem whole cloud acts as a point source of mass M concentrated at center.

$$M = \frac{4\pi}{3}\rho R^3$$

uniform density

But it is very plausible

$$t_{ff} = \frac{\pi}{\sqrt{G\frac{32}{3}\pi\rho}} = \sqrt{\frac{3\pi}{32G\rho}}$$

a famous result

This result assumes all pressure and ~~rotation~~ angular momentum is zero

same for all shells which don't cross — but there seems no simple obvious proof of this

Really $t_c \geq t_{ff}$

due to having to get rid of pressure support

mostly radiated away by molecules.

angular momentum probably mostly compacts into protostar which speeds up like a figure skater

So $\dot{\rho}_{stars} \simeq \dfrac{\rho}{t_{ff}} \simeq \rho^{3/2}$

$\therefore \rho_{SFR} \propto \rho^{3/2}$

multiply by column height

$\Sigma_{SFR} \sim \Sigma_{gas}^{3/2}$

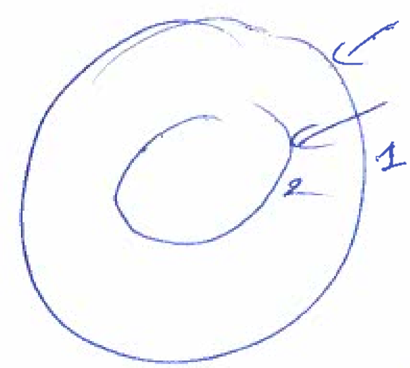
and $\alpha = 1.40(15)$

not so not so far from 3/2

but Σ_{SFR} does grow quite strongly

You probably have to use the Keplerian orbit results to prove this. It can't be proven just by energy conservation, you need evolution in time & think.

But there distributions with infalling shells crossing



Two infinitely thin shells

shell 2 stays at rest while shell 1 falls in by $\vec{g}_{21} = -\dfrac{GM_2}{r^2}\hat{r}$

2

But maybe shell 1 passes thru shell 2 and then coasts at constant velocity while shell 2 accelerates inward do $\vec{g}_{12} = -\dfrac{GM_1}{r^2}\hat{r}$ and then the evolution might get complex.

The Fiducial formula for star formation time scale

free fall

4021

-all sizes so something else decides what very- much

uniform initially a vest pressureless matter

$$t_{ff} = \sqrt{\frac{3\pi}{32 G \rho}} = (1.1\times10^6\,yr)\left(\frac{\mu}{2.3}\right)^{-\frac{1}{2}}\left(\frac{n_{H_2}}{10^3\,cm^{-3}}\right)^{-\frac{1}{2}}$$

order of typical molecular cloud density

μ is the mean molecular/atomic weight

on mass

defined from

mass fraction

$$n = \sum_i \frac{X_i \rho}{A_i m_p} = \frac{\rho}{m_p}\sum_i \frac{X_i}{A_i} = \frac{\rho}{\mu m_p}$$

all species of molecule and ion

Not worrying about electron mass, nuclear binding energy, isotopes (below our interest)

A atomic/molecular weight, but not in units of daltons (atomic mass units) $= \frac{1}{12} M_{C-12}$

but in m_p (mass of protons)

$$\mu^{-1} \equiv \sum \frac{X_i}{A_i}$$

Fiducial value for gas of H_2

$X = .73$ H

$Y = .25$ He

$Z = .02$ metals

the universe is made of hydrogen (which is made of protons) not Daltonium (which is made of Daltons)

rough value for modern universe ~ .1 - .3?

$$\mu^{-1} = \left(\frac{X}{2} + \frac{Y}{4} + \frac{Z}{30}\right) \cong \frac{1}{2.3}$$

just a guess

A very roughly $A_C=12$ $A_O\cong16$ $A_{Si}\cong28$ and $A_{Fe}\cong56$

abundant species

so $\rho = \cancel{M}\, \mu m_p$

and then the formula on p. 4021 follows.

Note the coefficient $\sim 10^{+6}$ yr

so if a molecular cloud ~~fragments~~ into collapsing clumps ~~the~~ the first stars ~~might~~ collapse ~~on~~ time scales of 10^6 y

Star formation region lifetime ~ 30 Myr is a rough number quoted

and probably this limit is $\gtrsim 8\,M_\odot$ blow up a supernova (core collapse SNe)

and $t_{\text{lifetime } 8M_\odot} \simeq 30$ Myr (~~...~~ Ci-86)

Very roughly massive stars do form much faster

so must be after ~~...~~ a freefall stage

~~$6\,M_\odot$~~ on Hayashi tracks With

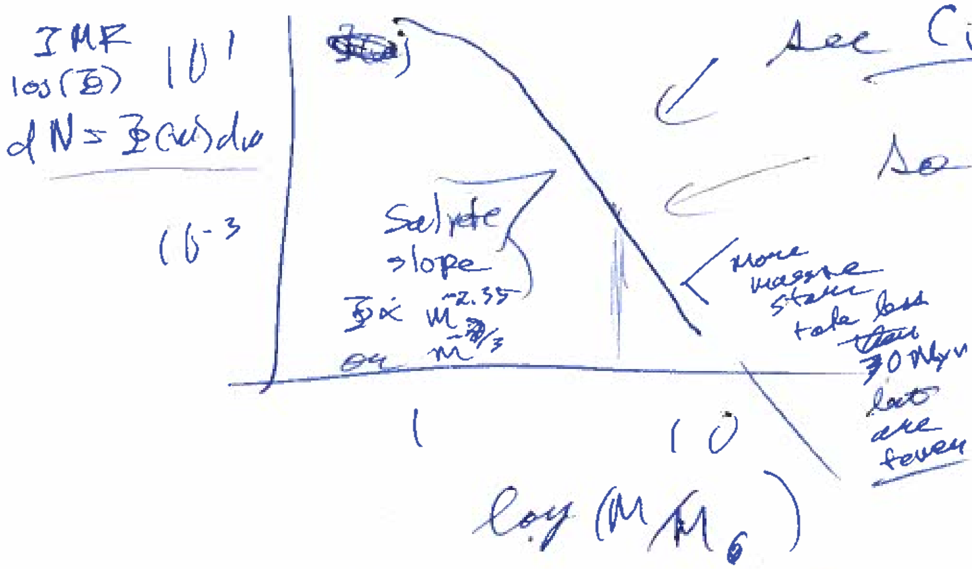
$6\,M_\odot$ take $\sim 10^5$ yr

$0.1\,M_\odot$ takes $\sim 10^8$ yr

more massive stars form even more quickly

$1\,M_\odot$ take $\sim 10^7$ yr.

The IMF (initial mass function) falls steeply for $M > 1M_\odot$



See Ci-265

So many ~~stars~~ stars going supernova at ~ 30 Myr after star formation started

~~So there are or~~

Star formation is only at 2-3% efficient

The fraction of gas that goes into stars at cosmic present

May disperse the star formation region locally and create fountains eject out of the disk into the CGM but the ejecta has dust formed from the metals in the ejecta which may be compacted elsewhere to create new star forming regions

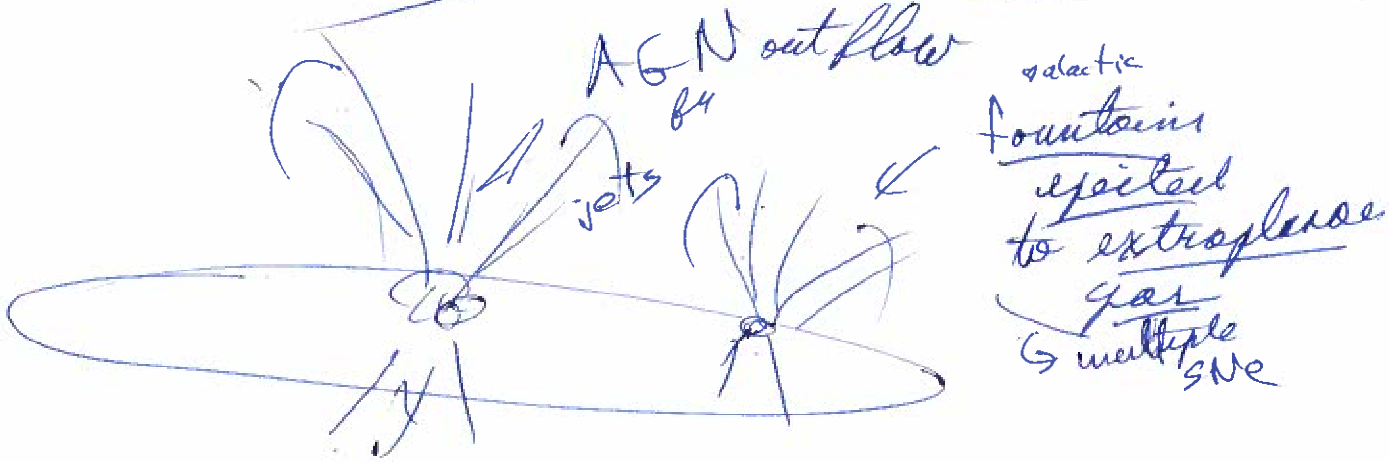
Ci-84-85

deviations from the Schmidt-Kennicutt law do occur and various fixes.

One is not gas column density but gas volume density might be better Ci-85 but that is harder measure.

4024] 9) Gas Inflow and Outflow to Galaxies (i-85-87)

gas accretion

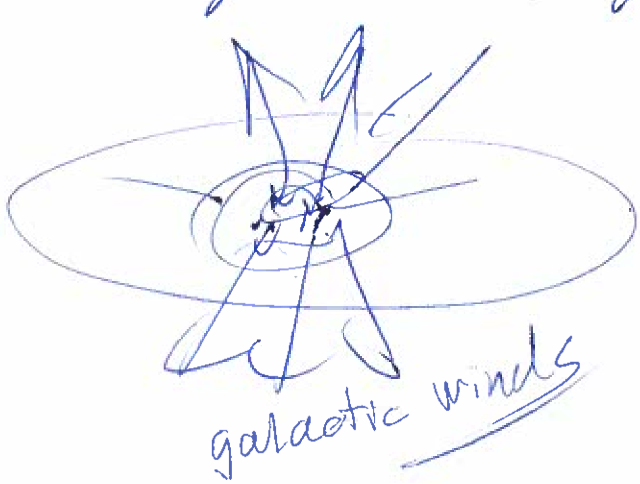


$M_{gas} \cong 10^9 M_\odot$ order of in

SFGs and SFR ~ few $M_\odot$/yr

$\therefore \quad t_{dep} \cong \dfrac{10^9 M_\odot}{1\, M_\odot/yr} \cong 1$ Gyr

but clearly inflows of gas occur to keep SF going in some galaxies for ~13 Gyr.



very strong outflow occurs due to SNe in starbursts galaxies — galactic winds

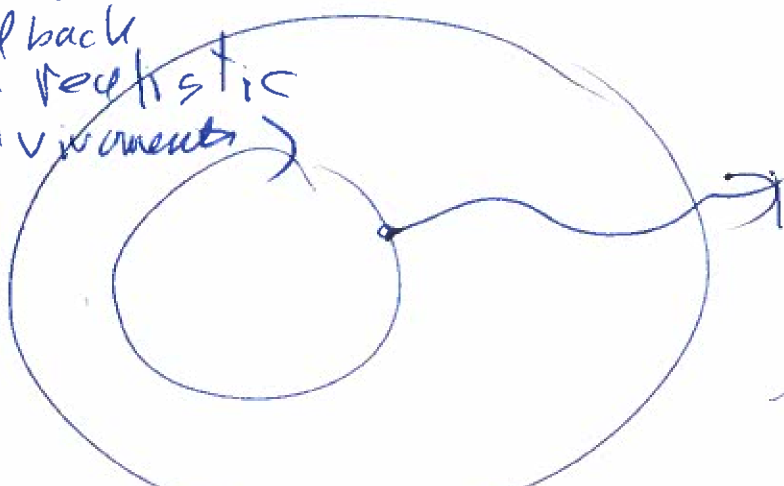
4025

Can gas achieve escape speed from a galaxy

↳ An important question, since Large-Scale Structure simulation require strong feedback to prevent too strong formation at $t_{\text{lookback}} \approx 10-12$ Gyr and prevent star formation from being turned off now → despite gas inflows G1-38?

These simulations have to use a recipe to get the strong feedback (as of 2019?) because they can't do it from detailed physics yet.

FIRE (feedback in realistic environments)



$\Delta KE = W = -\Delta PE + W_{non}$

$\Delta E = \Delta KE + \Delta PE + W_{non}$

If $W_{non} = 0$ ←

$\Delta KE + \Delta PE = 0$

$\Delta E = 0$

$E = KE + PE$ is conserved

No nonconservative force ΔE

Assume $\Delta E = \Delta KE + \Delta PE$, $\Delta E = 0$ mechanical energy conservation.

If $v = 0$ at ∞ and $PE = 0$ by choice

then $\Delta KE = -\Delta PE$

$KE_{init} = -PE_{init}$

$$\frac{1}{2} m v^2 = -m\Phi$$

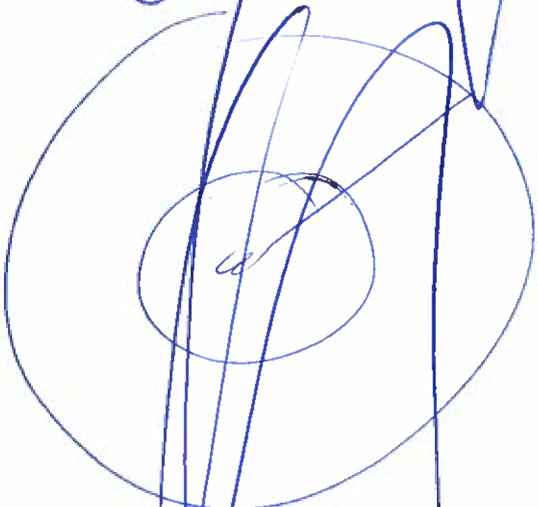
$$\therefore v_{esc} = \sqrt{2|\Phi|} \text{ in general.}$$

4026) Can we estimate v_{esc} from some radius r?

with some heuristic assumptions

educationally useful to see where they lead us

Assume a spherically symmetric mass distribution and the DM halo is often so but also often not.



$$\underset{\sim}{g} = -\frac{GM(r)}{r^2}\hat{r}$$

gravitational field at r.

$$M(r) = \int_0^r 4\pi r^2 \rho\, dr$$

assume $\rho = \rho_0 \left(\frac{r}{r_0}\right)^{-\delta} = \rho_0 x^{-\delta}$

ρ_0 and r_0 some fiducial values to be fitted from data

$$M(r) = 4\pi r_0^3 \rho_0 \frac{x^{-\delta+3}}{3-\delta} = M_0 x^{-\delta+3}$$

~~with~~ Divergences

$\delta = 3$ gives logarithmic divergence, $\delta > 3$ is worse.

a) If $\delta \geq 3$, then Mass diverges at $r \to 0$

b) But if $\delta < 3$ then Mass diverges at $r \to \infty$

~~But~~ we won't worry about this since our estimate of v_{esc} is special limit.

Clearly a ~~a mass dist~~ pure single $\rho = \rho_0 \left(\frac{r}{r_0}\right)^{-\delta}$ density law is not physical for a whole ~~thing~~ galaxy

4027

We can estimate the escape velocity from a Galaxy with some simplifying assumptions,

Let $\rho = \rho_s \left(\frac{r}{r_s}\right)^{-\alpha}$ be a density profile

$$M = \int_0^r \rho \, 4\pi r^2 dr = 4\pi r_s^3 \rho_s \int_0^x x^{-\alpha} x^2 dx$$

$$= 4\pi r_s^3 \rho_s \begin{cases} \frac{x^{3-\alpha}}{3-\alpha} & \text{for } \alpha < 3 \\ \ln(x/x_{in}) + M_{in} & \text{for } \alpha = 3 \\ \left.\frac{x^{3-\alpha}}{3-\alpha}\right|_{x_{in}}^{x} + M_{in} & \text{for } \alpha > 0 \end{cases}$$

for ~~3~~ $\alpha < 3$
- ~~no~~ divergence at $x = 0$
- divergence at $x \to \infty$

for $\alpha = 3$ where we need x_{in} to prevent divergence at $x \to 0$
- divergence as $x \to \infty$
- and as $x_{in} \to 0$

$$\frac{1}{\alpha-3}\left(x_{in}^{3-\alpha} - x^{3-\alpha}\right) + M_{in}$$

divergence as $x_{in} \to 0$

no divergence as $x \to \infty$

The upshot of this result is no single power-law can describe a whole galaxy to ~~The~~ infinity. There is a outer divergence or an inner divergence or both in the logarithmic case.

Let's assume $\alpha < 3$ and there is an outer cut-off radius r_{out}

$$M = M_s x^{3-\alpha}, \quad M_{out} = M_s x_{out}^{3-\alpha}$$

$M_s = 4\pi r_s^3 \rho_s/(3-\alpha)$

Recall the work energy theorem

$$W_{done} = \int \underset{\sim}{F}_{ext} \cdot d\underset{\sim}{s} = \int m \underset{\sim}{a} \cdot d\underset{\sim}{s} = \int m \frac{d\underset{\sim}{v}}{dt} \cdot \underset{\sim}{v}\, dt$$

4028)

$\cancel{\Delta KE}$ Work $= \int m \frac{v^2}{2}\, dt = \Delta\left(\frac{1}{2} m v^2\right)$

$= \Delta KE$

$\therefore \quad \Delta KE = W$

Now $\underset{\text{con}}{F} = -\nabla U$

$\therefore \quad W_{con} = \int -\nabla U \cdot d\underline{s} = -\int dU = \Delta U$

$= \cancel{\int \frac{\partial U}{\partial s}} ds$

$\therefore \quad \Delta KE = -\Delta U + W_{non}$

$\therefore \quad \Delta E = \Delta(KE + U) = W_{non}$

mechanical energy

If $W_{non} = 0$,

$\Delta E = 0$

and $E = KE + U$ is conserved

and $\Delta KE = -\Delta U$

At infinity for at at rest particle

$E = 0 + 0 = 0$

a particle ~~that~~ just escapes to infinity

$\therefore$ has $E = KE_{in} + U_{in} = 0$

$\therefore \quad \Delta KE = -KE_{in}, \quad \Delta U = -U_{in}$

going to infinity

$\therefore \quad KE_{esc} = KE_{in} = -\Delta KE = \Delta U_{in}$

or more clearly $KE_{esc} = -U_{initial}$

$\therefore \quad v_{esc} = \sqrt{2|\Phi_{in}|}$

where $\Phi = \frac{U_{in}}{m}$

is the gravitational potential Not potential energy

But what is ~~the total~~ $\Delta \Phi_i$ | 4028a

$$\Delta \Phi_i = -\int_{r_i}^{\infty}\left(-\frac{GM(r)}{r^2}\hat{r}\right)\cdot dr = \int_{x_1}^{x_2} \frac{GM_s}{r_s^2} x^{1-\alpha}\, dx$$

$$+ \int_{x_2}^{\infty} \frac{GM_s}{r_s^2} x^{-2}\, dx$$

A key point

$\Phi = -\frac{GM(r)}{r}$

is the potential at r assuming nothing above r.

$\therefore U = \int \Phi\, dm$

$= \int_0^r \Phi(r) 4\pi r^2 \rho\, dr$

the contribution of known mass to r with nothing above

and then one can integrate outward

$$= \frac{GM_s}{r_s^2} \frac{x^{2-\alpha}}{2-\alpha}\Bigg|_{x_1}^{x_2} + \frac{GM_s}{r_s^2}(-x^{-1})\Bigg|_{x_2}^{\infty}$$

$$= \underbrace{\frac{GM_s}{r_s^2}\left(\frac{x_2^{2-\alpha} - x_1^{2-\alpha}}{2-\alpha}\right)}_{\pm ve} + \underbrace{\frac{GM_s}{r_s^2}\frac{1}{x_2}}_{+ve}$$

whether $\alpha < 2$

and if $\alpha > 2$

and if $\alpha = 2$ you need a logarithm solution.

Let's assume $\alpha > 2$ and $x_2 >> x_1$

$$\Delta \Phi_i = \frac{GM_s}{r_s^2}\frac{x_1^{2-\alpha}}{\alpha - 2}, \quad \Phi_i = -\Delta\Phi_i$$

$$\therefore v_{esc} = \sqrt{\frac{2GM_s}{r_s}}\sqrt{\frac{x_1}{\alpha-2}} = v_{cir}\sqrt{\frac{2}{\alpha-2}}$$

where we take s as the initial point and so $x_i = 1$.

Typical disk circular velocity is ~ 200 km/s

$$v_{esc} = 200\sqrt{\frac{2}{\alpha-2}} = \begin{cases} 400 & \text{for } \alpha = 5/2 \\ \to \infty & \text{for } \alpha \to 2 \end{cases}$$

A028b

The upshot is $V_{esc} >> 200$ km/s and Ci-87 says typically of order 800 km/s from Milky way center.

stellar feedback
winds including SNe?
tend to be less than that

Note SN ejecta velocity
near maximum light
are ~ 10^4 km/s
but that is outer matter.
- bulk inner matter is then opaque
and it is moving much
slower ~ 10^3 km/s

So hence the need
for strong feedback recipe
noted on p. 4025.

AGN may supply it
Even probably, but it seems
that ~~developed~~ detailed understanding
is not perfect yet — but
maybe soon: e.g., FIRE project
Feedback in realistic environments.
— Maybe they've solved it already but I haven't noticed.

4030 10) Rotation Curves Ci-9(-93

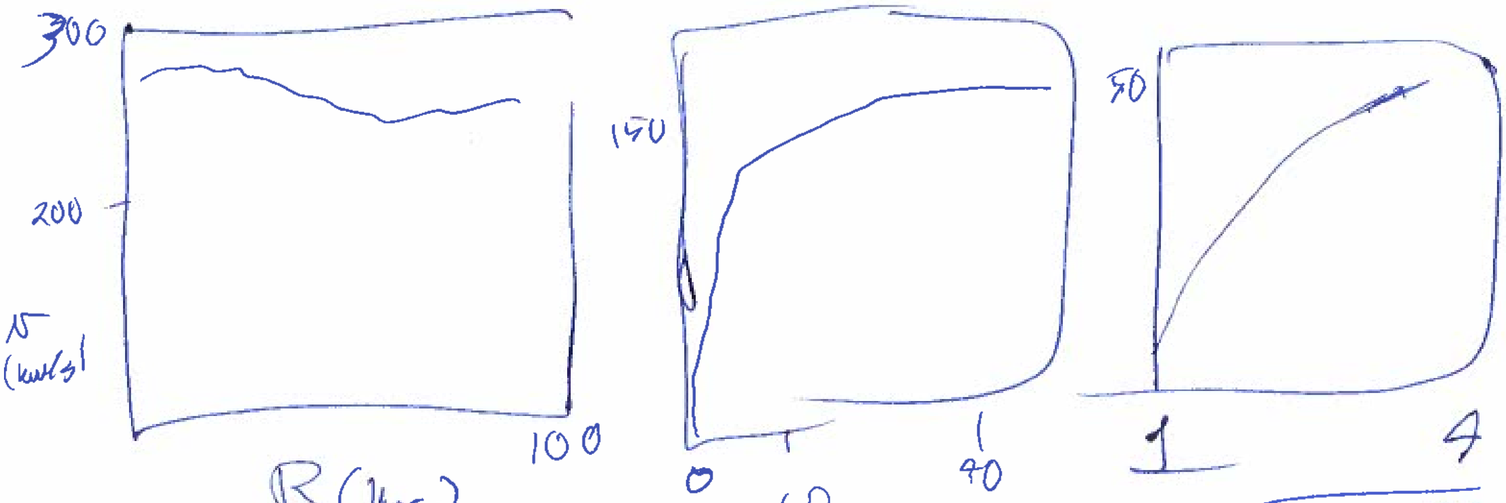
We considered before, so a bit of recapitulation.

3-cases for SFGs

Sa — SOa, SOd — so bulgy

less Bulgy

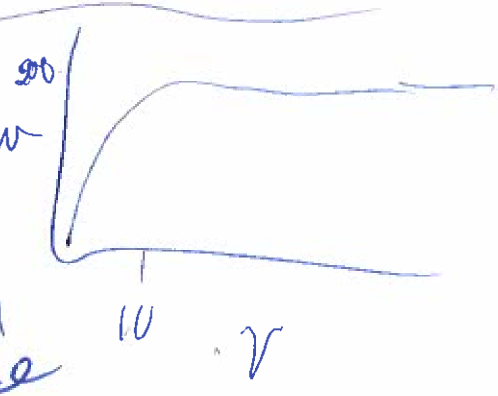
Sa Sc dIrr



see p 4026

No single power law works for whole galaxy model

typical & pustle v but clearly sort a rough average case



$\delta \; 3$ to prevent mass diverg at $r \to 0$

$\delta \geq 3$ to prevent mass divergence at $r \to \infty$

Recall from p. 4027

$$v_c = \sqrt{\frac{GM_0}{r_0} x^{-\alpha+2}}$$

for pure power law density $\rho = \rho_0 \left(\frac{r}{r_0}\right)$

So $\delta = 2$ gives flat circular velocity

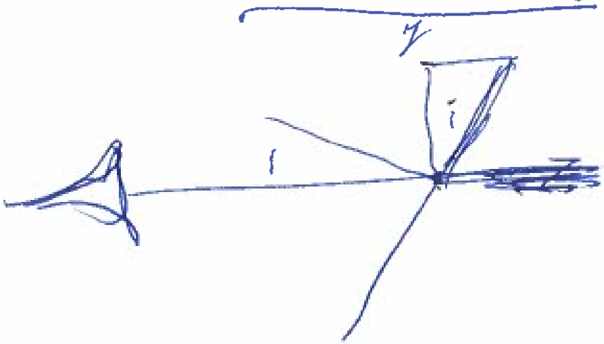
see p. 4026 4027

$$M_{\delta} = 4\pi r_0^3 \rho_0 \frac{x^{-\alpha+3}}{3-\alpha}$$

4031

So how does one determine circular rotation velocity for a disk galaxy?

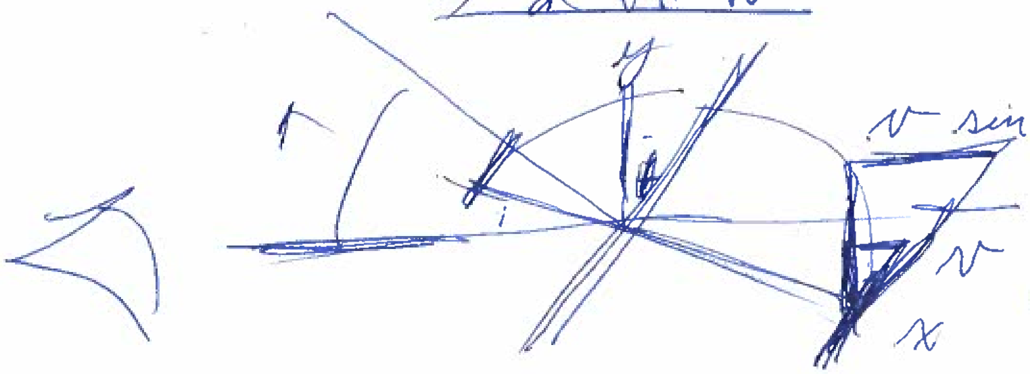
side view



Projection on sky view

assuming ellipticity is constant

3-d view



$y_i = a_i\sqrt{1-\left(\frac{x}{a}\right)^2}$, $a_i = b$

where $y_i = y \cos i$

and $b = a_i = a \cos i$

so $\cos i = \frac{b}{a}$

$i = \cos^{-1}\left(\frac{b}{a}\right)$

along line of sight

$$v(R) = \frac{v_{pro}(R)}{\sin i}$$

indeterminate not infinite since $v_{pro} \to 0$ as $i \to 0$

too small so a ... can't really

$$v(R) = \frac{v_{pro}(R)}{\sqrt{1-\cos^2 i}} = \frac{v_{pro}(R)}{\sqrt{1-\left(\frac{b}{a}\right)^2}}$$

for v

v_{pro} for $b =$

$x = \frac{b}{a}$

$$= v_{pro}(R)\left(1 + \frac{1}{2}\left(\frac{b}{a}\right)^2 + \dots\right)$$

4032

$\tilde{v}_{pro}$ is determined ~~from Doppler shift of the line~~ from the Doppler/Cosmological shift of ven

$v_{pro} = \tilde{v}_{pro} - v_{cm}$

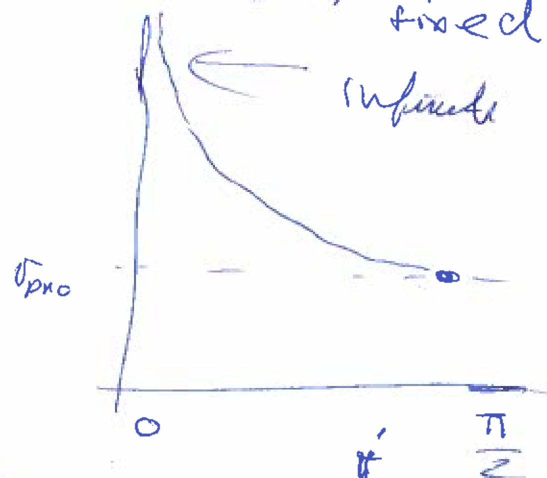
$\tilde{v}_{pr} = c\,\frac{\Delta\lambda_{shift}}{\lambda_{lab}}$

(1st order Doppler formula)

You subtract center of mass velocity of course.

$v_{pro} = \tilde{v}_{pro} - v_{cm}$

Taking v_{pro} fixed



$$v = \frac{v_{pro}}{\sin i}$$

$\sin i = 1 + (i-\frac{\pi}{2})\cos\frac{\pi}{2} - \frac{(i-\frac{\pi}{2})^2}{2}\sin\frac{\pi}{2}$

$\cos i = b/a$ (see p. 4031) so $\sin i = \sqrt{1-(b/a)^2}$

$$= \frac{v_{pro}}{1 - \frac{(i-\frac{\pi}{2})^2}{2}} \quad \text{for } |i-\tfrac{\pi}{2}| \ll 1$$

$$\frac{v_{pro}}{\sqrt{1-(b/a)^2}} \quad \text{also}$$

However, really $i = 0$ is just indeterminate since as $i \to 0$, $v_{pro} \to 0$ too.

You must use some emission/absorption line for precise Doppler shift

Examples: Hα absorption stars, H-21 cm low density gas, Hα emission HII regions, CO molecular clouds.

Detail omit in class

4033

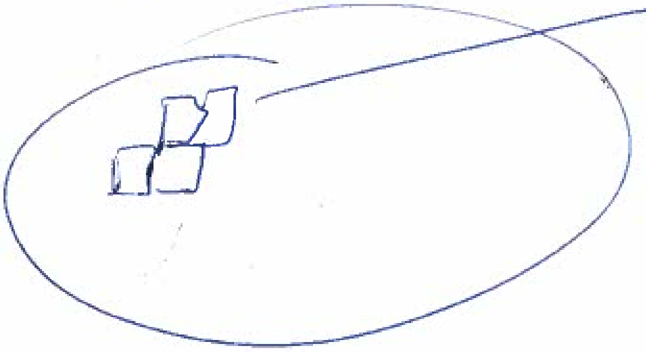
a) Data cube method of velocity determination

take small regions and measure H ~~α~~ emission on source

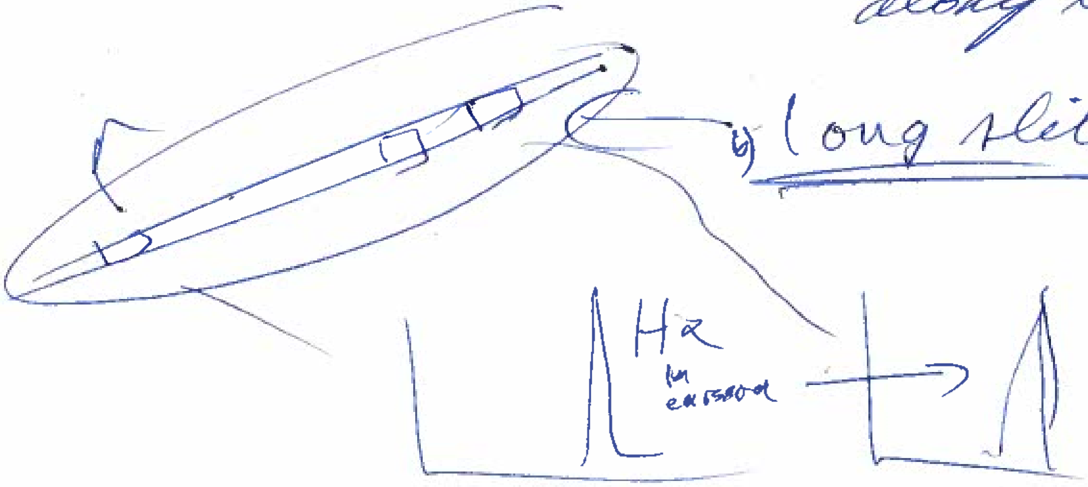
2-d spatial data

1-d ~~in~~ velocity along line of sight

other emission line



b) long slit

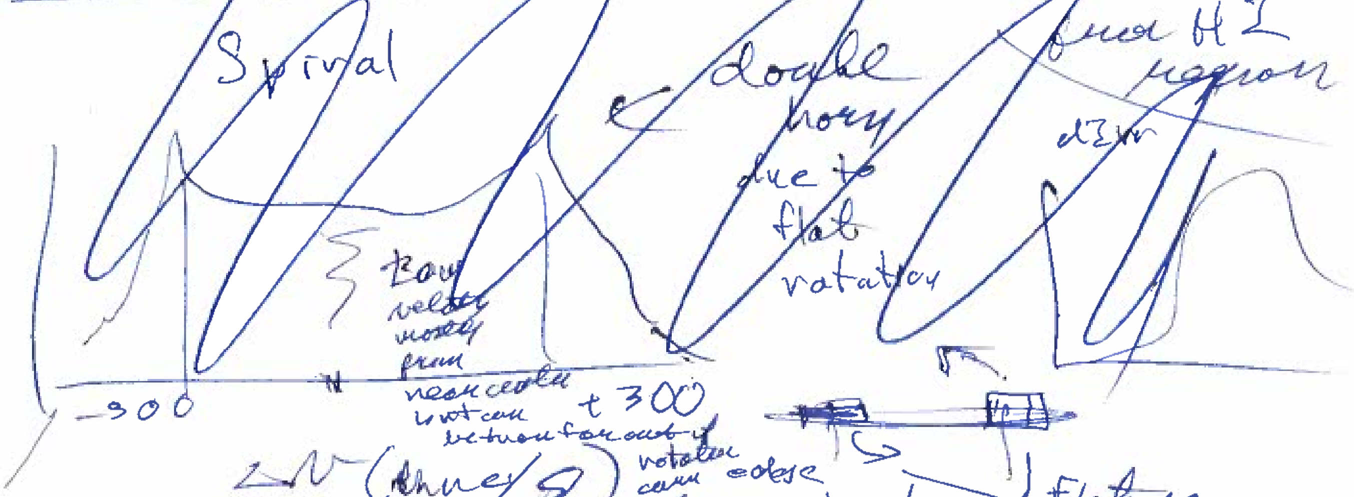


~~Background~~ the star region HI 21cm

Either way can extract a rotation curve

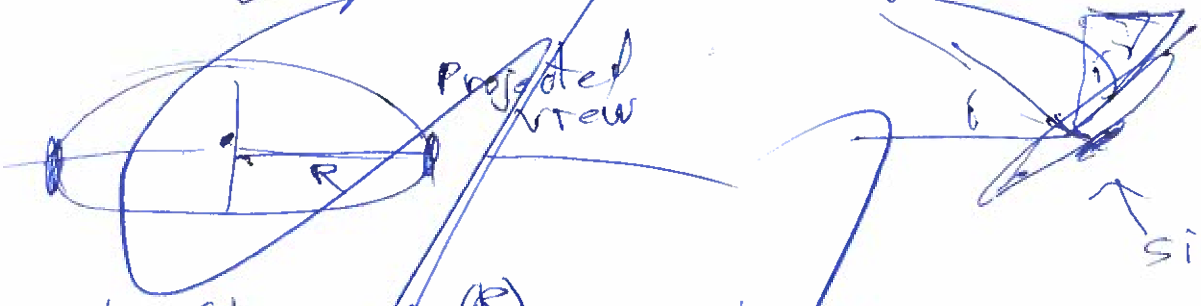
Global emission gives emission at all redshifts

Hα I gives from HI region



4034

Easiest rotation to understand is along the projected long axis of a spiral → a thin spiral ideally



$v_{obs}(R) = v_{rotat}(R) \sin i$ and so $v_{rotat}(R) = v_{obs}/\sin i$

So you could trace out whole rotation curve from a long slit

i = a can't measure

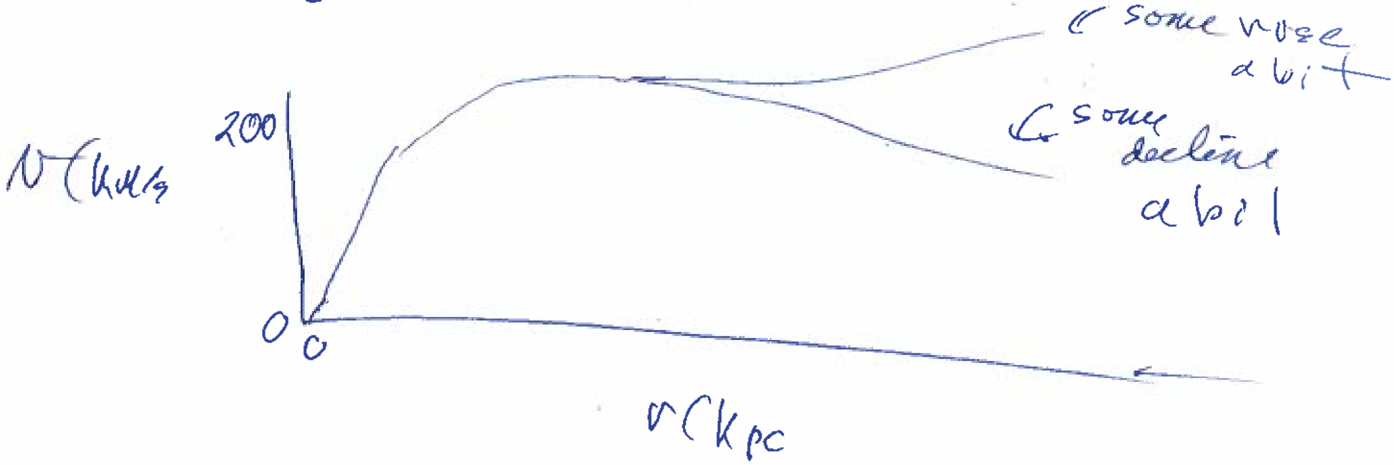
flat saucer

generally strong emission at one velocity shift

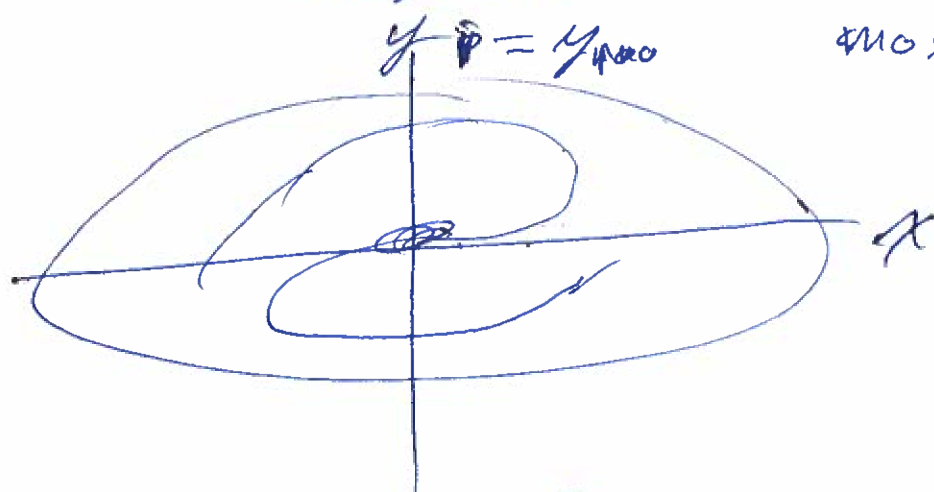
Hence the double horn in global emission

4035

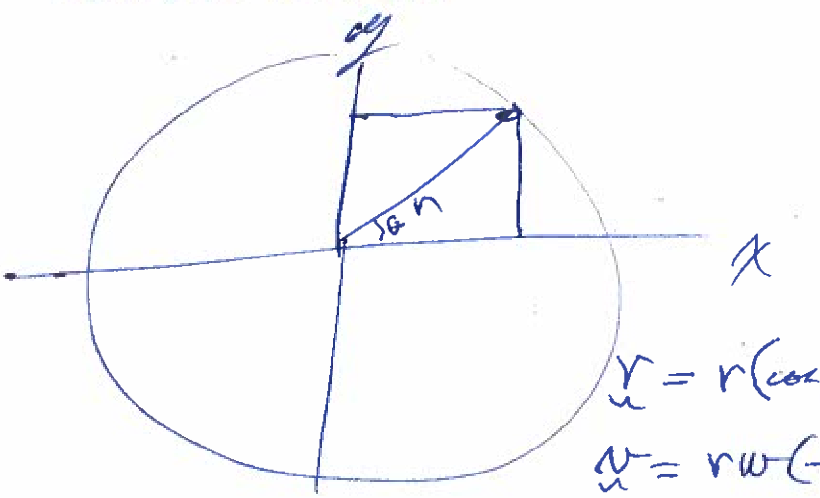
And so one gets a rotation curve



But if one is using ~~H emission~~ $H\alpha$ emission from HII regions, it ~~they~~ will be patchy, and mostly in spiral arms. So one has to measure off the x-axis. What does one do?



Take a face-on view

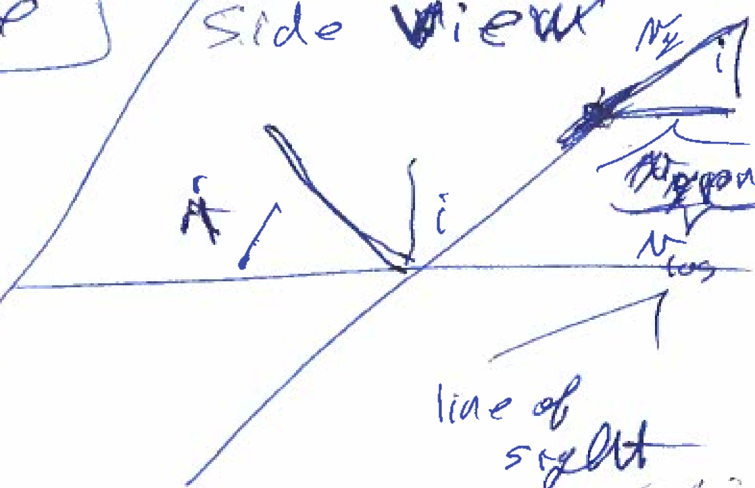


$$\underset{\sim}{r} = r(\cos\omega t, \sin\omega t)$$

$$\underset{\sim}{v} = r\omega(-\sin\omega t, \cos\omega t)$$

z-axis point toward us for right-hand system, but a left-hand system is OK in this context

A036



$v_y \sin i = v_{los}$

∴ $v_{los} = v_{y\,proj}$

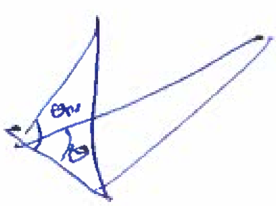
since v_x component in the disk has no los component

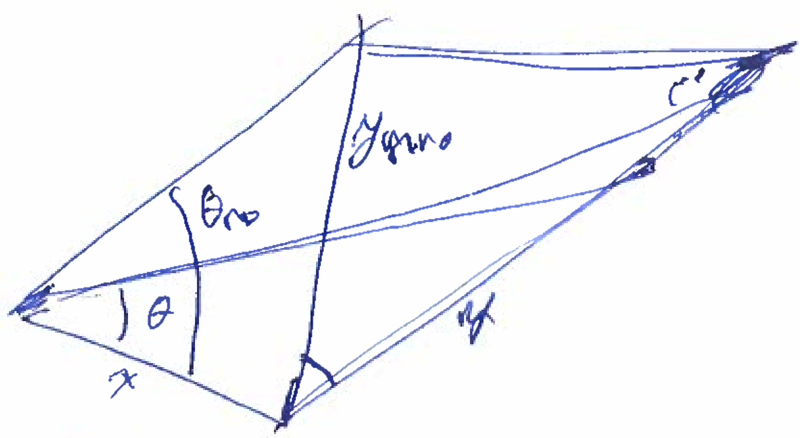
$$v_y = W \cos \omega t$$
$$= v_{rot} \cos\theta$$

angle at time t — it's not changing much in human history

∴ $v_{los} = v_{rot} \cos\theta \sin i$ (Ci — 93)

But what is $\cos\theta$





$$y_{proj} = y \sin i$$

$$\cos\theta = \frac{x}{\sqrt{x^2 + y^2}} = \frac{x}{\sqrt{x^2 + \left(\frac{y_{proj}}{\sin i}\right)^2}}$$

19037

Finally $v_{rot} = \dfrac{v_{los}}{\sin i \sqrt{x^2 + \left(\frac{y v_0}{\sin i}\right)^2}}$

x ~~or~~ $y_{\text{~~}p~~ho}$, $v_{los} = c \dfrac{\Delta\lambda}{\lambda_{H\alpha}}$

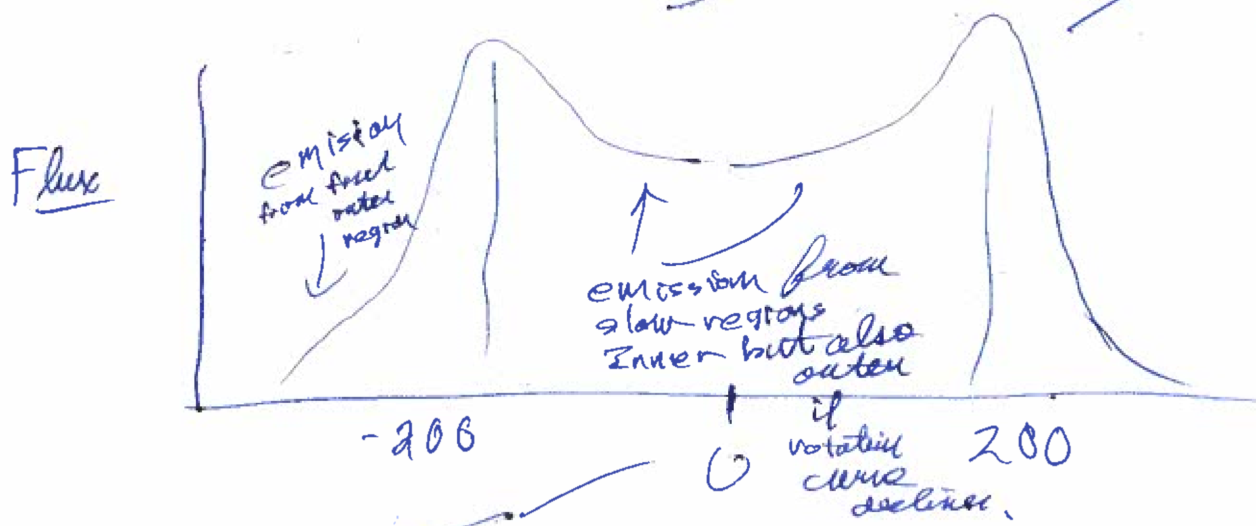
are direct observables ~~though~~ you could parameterize them in other ways as Ci-93 does.

So by taking measurements over ~~the~~ spiral arms of Hα emission you can work out the inner disk $v_{rot}(\underbrace{r}_{\text{Not projected}})$

To extend beyond ~~the~~ Hα emission regions, one can use H-21 cm line hopefully still in a disk.

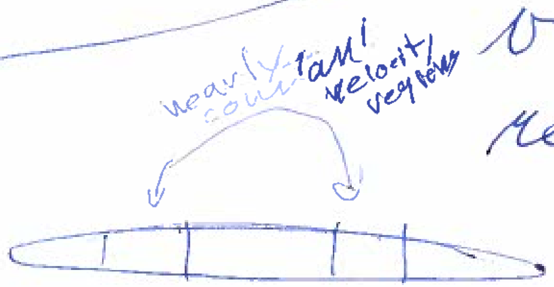
4038

What if you just integrate Hα emission over the whole disk (i~92)



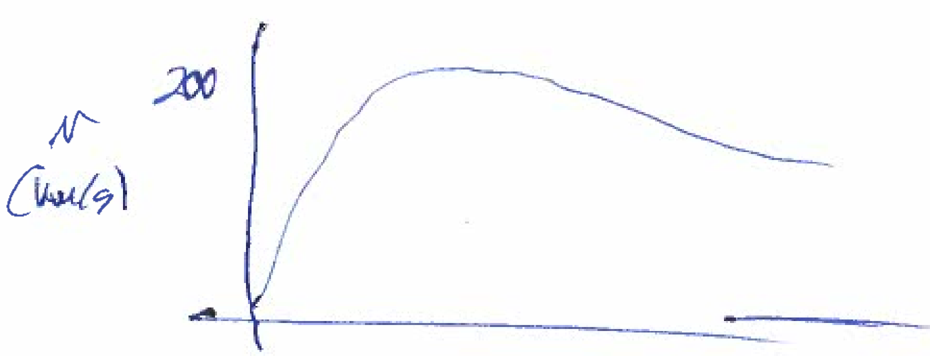
v (km/s) relative to v_{cm}

The double-horn is due to the plateau region — a lot of mass moving at the same velocity



edge-on view

this is particularly clear-cut for an edge-on view



r (kpc)

∴ No Double-horn

Dwarf galaxies have no plateau region

— Just catalog them   Sections 11 — 15   Scaling Relations — Empirical
ci-99 — 108

11) Tully Fisher Relation (TFR)   ci - 99 - 101

4039 Fine test for Large structure formation simulations

a famous scaling law for SFGs → LTCs
spirals

Empirical

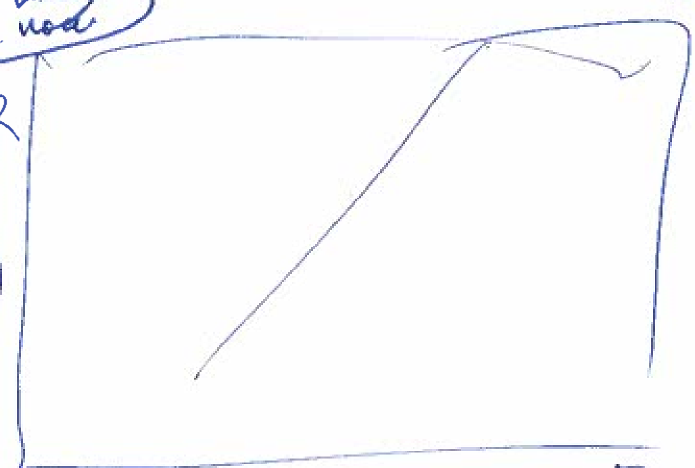
Somehow emerges from cosmic history and probably most have their underlying. But here we just sort of catalog them

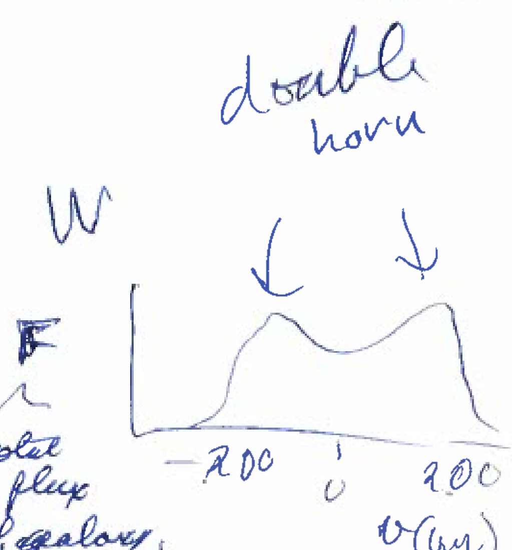
velocity width — Absolute Mag relation

useful up to a point for distance measurement and a fine test of Large-scale structure formation calculations to reproduce

→ established by Brent Tully et al. More than 40 years ago — still active

But I don't know how they are doing now





$W \simeq 2 v_{rot} \sin i$ ← inclination

$W^i \equiv \frac{W}{\sin i}$

$W = \{ W_{20}, W_{90}$ % of peak flux GHW football

— maybe they mean

4040

~~A~~ Absolut mag.

$$M = A - \alpha(\log(W^i) - 2.5)$$

zero point $A = -20.3$ for Ci-100 frame

$\alpha = -9.8$

~~I think~~ $\alpha = 8.8$ must be used

The old wrong way mag scale with $100^{-\frac{1}{5}} = 10^{-2/5}$

5 mag makes 100

$$F = 10^{-\frac{2}{5}[M]} = 10^{(\frac{2}{5})(A + \alpha 2.5 - \alpha\log W^i)}$$

$$= C\,10^{-\frac{2}{5}\alpha\log W^i}$$

$$= C\,W^{+\frac{2}{5}\alpha}$$

$$\beta \equiv \frac{2}{5}\cdot 10 = 4$$

$$F = C W^4$$

but W has to be specified somehow precisely

$1.26 = 10^{0.1}$ fraction σ

Rel err ~ 27%

Another form is

$$\log\left(\frac{L}{10^{10}L_\odot}\right) = B + \beta\log\left(\frac{v_{rotation}}{200\ \mathrm{km/s}}\right)$$

$B \simeq 0.5 \to 1$

$\beta \simeq$ ~~3~~ $\to 5$

$\sigma \sim 0.1$ dex for fit to observing Ci-100 -101

the characteristic $v(R)$ value usually the plateau value defined exactly somehow

~~The~~ The range of B and b depend on light band observed (Not errors)

(9041

TFR has use as a calibrated distance indicator.

Recall $m_{av} - M_{ab} = ?$

~~$M = 10^{-\frac{2}{5}F}$~~

$F = 10^{-\frac{2}{5}M}$
$M = -\frac{5}{2}\log R$

$$\frac{L}{L_{10pc}} = R = \frac{L_{10}/(4\pi r^2)}{L_{10}/4\pi r_{10}^2)}$$

$$\mu = m - M_{av} =$$

$$= -\frac{5}{2}\log(L/L_{10})$$

$$= -\frac{5}{2}\log(r_{10}^2/r^2)$$

$$= 5\log\left(\frac{r}{r_{10}}\right)$$

$$= 5\log(r) - 5\log 10$$

$$= 5\log(r) - 5$$

for r in pc

$r = r_{Mpc} * 10^6$

$$= 5\log(r_{Mpc}) + 25$$ measured

measured

$$m = M + 2.5[\log L \; \cancel{\log(10^{10})}]$$

$$= M + 2.5[\log 10^{10} + B + \beta\log\left(\frac{v_{rotation}}{200\,km/s}\right)]$$

4042

So measure M and v_{rot}

then u use the get distance in Mpc

$$\mu = 5\log\left(\frac{v}{200\ \mathrm{km/s}}\right) + 25$$

$$r_{Mpc} = 10^{\frac{\mu}{5}-5}$$

units of Mpc

However, you have to be carefully to use the good definition for wavelength band and rotation

Also Tully-Fisher method is problematic for cosmological distances since the parameter may evolve over cosmic time

and also its scatter

So the Tully-Fisher relation has NOT resolved ~~the~~ Hubble tension

12) Specific Ang. Mom (4043

— Stellar Mass relation

another empirical scaling relation

surface density

$$j(<R) \equiv \frac{\cancel{2\pi}\int_0^R \Sigma(R')\, v_r(R')\, R'\, 2\pi R' dR'}{\cancel{2\pi}\int_0^R \Sigma(R')\, \cancel{R'}\, 2\pi R' dR'}$$

$v * R$ Recall $L = r \times p$, $L = rp$ for circul orbit

Angular momentum per unit mass

Set $\Sigma(R) \rightarrow \Sigma_{*}(R)$

approximate stars are all ang. mom and the Dark matter has no net rotation

$\Sigma_{DM}(R) \propto v_{rot_{DM}}$

$= 0$ since $M_{rot} \stackrel{?}{=} 0$

But not always

Maybe $v_{rot\,DM}$ couples poorly to Baryonic matter

For a model with $\Sigma \sim e^{-R/R_d}$ and $v_{\phi} = \text{flat} = $ Plateau

α accounts for v_{ϕ} not ~~being~~ applying to all radii and deviations from exponential model

< Factorial function Ara-543

$$j = v_f R_d \frac{\int_0^\infty x^2 e^{-x}\, dx}{\int_0^\infty x e^{-x}\, dx}$$

~~flatten~~

v_{flat} everywhere

$$= v_f R_d \frac{2!}{1!} = 2 v_f R_d$$

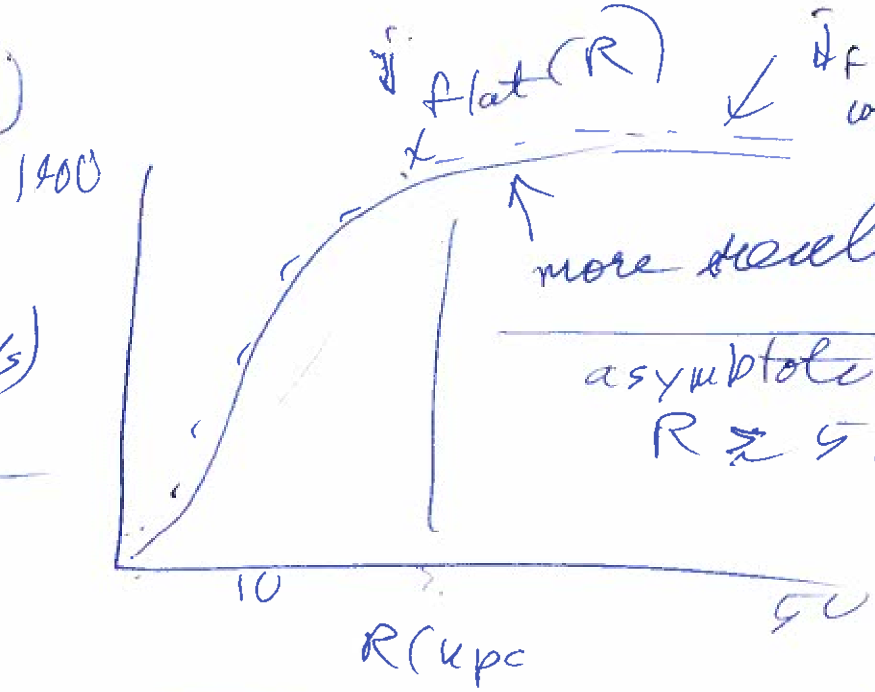
More generally

v_{flat} only on plateau

$j \simeq 2 \alpha R_d v_{\cancel{flat}}$

α a correction factor

4044



j_* ((kpc km/s)

funny unit

j_{flat} at $\alpha=9$ with $v = v_{flat}$ everywhere and $\alpha = 1$

more realistic

asymptotic region $R \gtrsim 5 R_d = 17.5$ kpc

$\alpha \doteq 0.9$ from Fig 4.15 on p.102 with

So most angular momentum is in interior since — ~~high~~ the velocities plateau going outward and the density declines exponentially

Specific Ang-mom — stellar mass relation

Empirical relation and a fine test of structure formation models to ~~introduce~~ reproduce

$$\log\left(\frac{j_*}{10^3 \text{ kpc km/s}}\right) = D + S\log\left(\frac{M_*}{10^{11} M_\odot}\right)$$

D is normalization

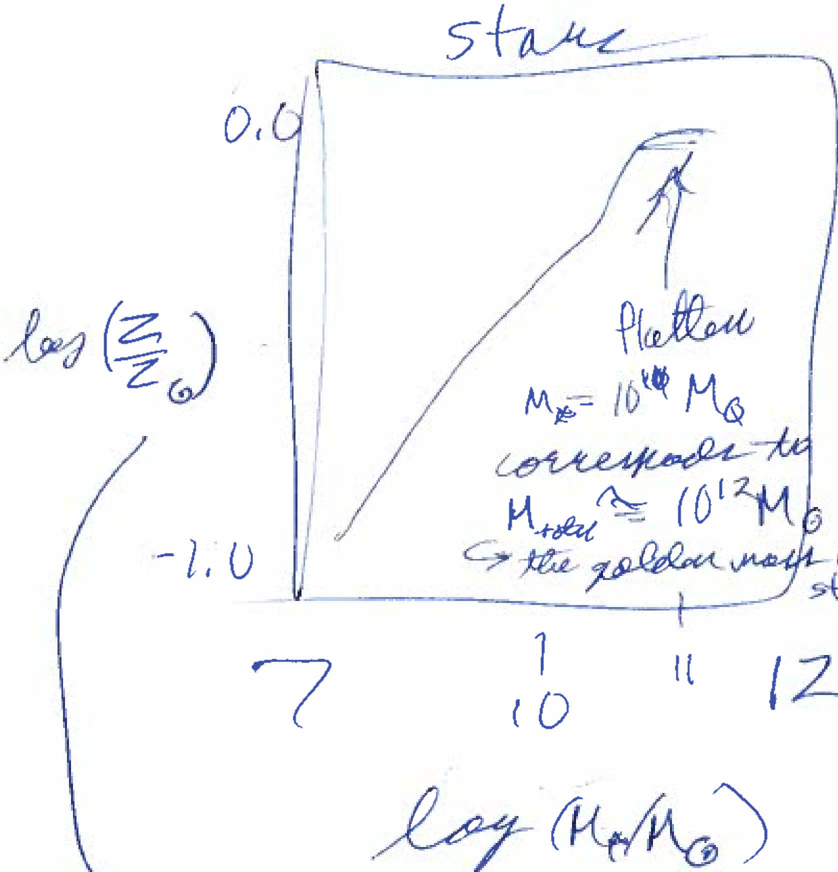
$S \simeq 0.55 \cong \frac{1}{2}$

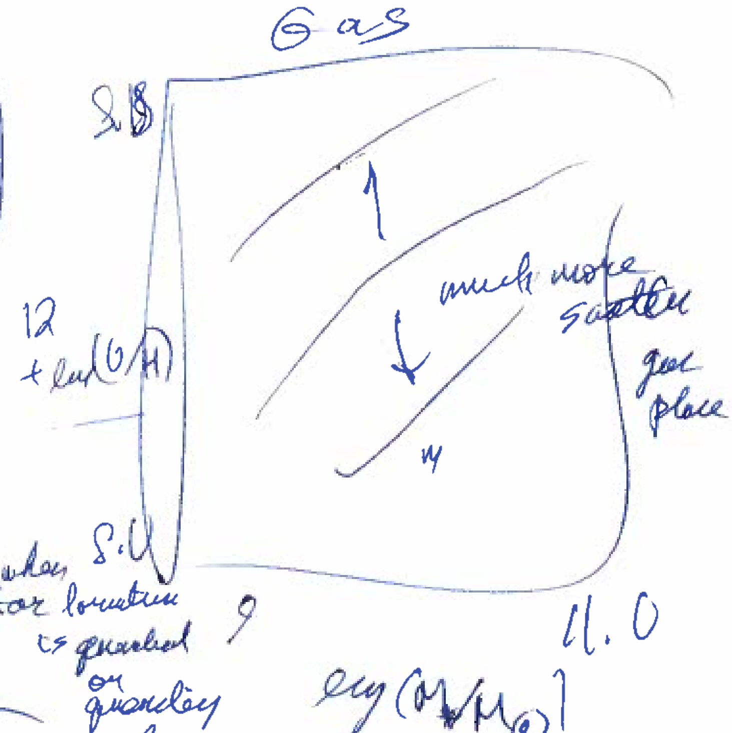
Dark matter seems to have little specific Ang mom or to be decoupled from stellar mass

13) Mass-Metallicity Relation

4045

Here we present not explain, but again a fine test of large-scale structure simulations





$Z_\odot = 0.02$ fiducially and traditionally but it is absolute size is debated.

Asplund et al 2009 gave $Z_\odot = 0.0134$ very precise
But accuracy still debated

I would hazard the gas phase is more current and reflects complex inflow/outflow from variously enriched ISM/CGM

– stars have a long time since they reflect past too

4046) — On the older hand stars of all ages represent integrated metallicity over ~~[illegible]~~ years?

~~d Irr can~~ have low $Z = 0.01\,Z_\odot$

they're star forming still — not quenched

→ enriched gas flows have easier escape and ?

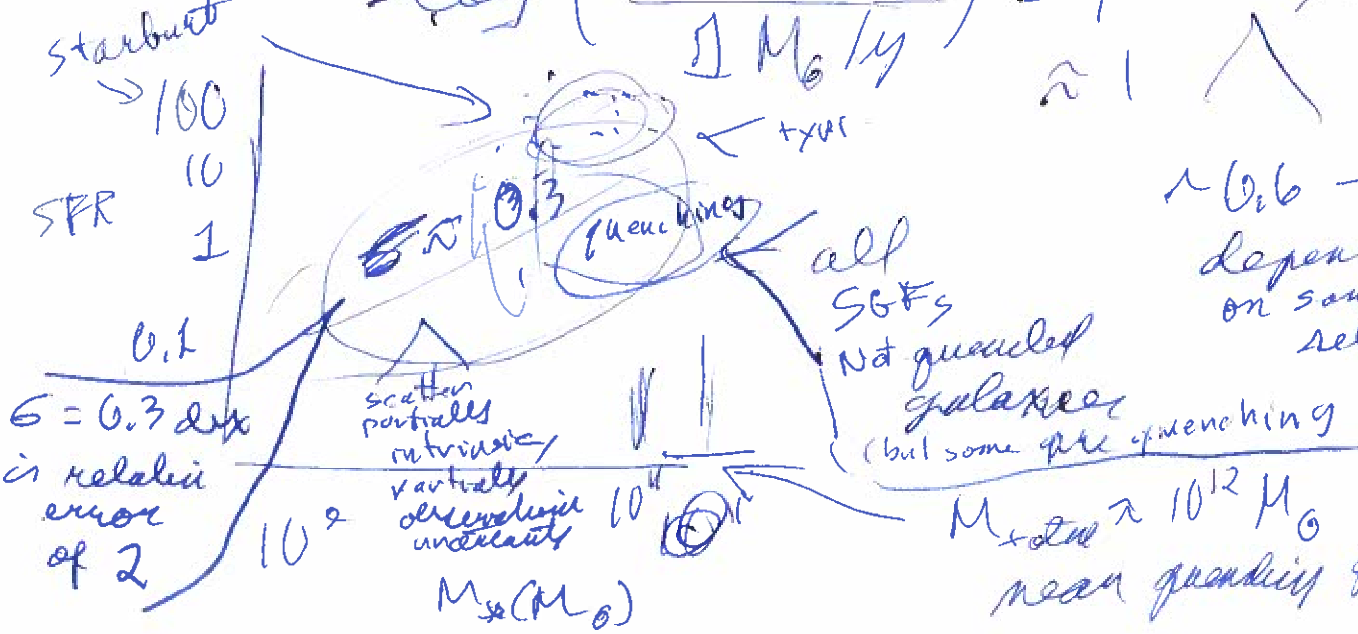
so ISM and stars formed from ~~more~~ gas closer to primordial.

14) Star formation Main sequence

$$\log\left(\frac{SFR}{1\,M_\odot/yr}\right) = \gamma + K \log\left(\frac{M_*}{10^{10} M_\odot}\right)$$

$\gamma \approx 1$

$K \sim 0.6 \to 1.0$ depending on sample selection



Digression on error in dex

$$y = 10^{x} = e^{x \ln 10} \quad \underbrace{}_{x \text{ is in dex}}$$

4047

$$\cancel{\frac{\Delta y}{y} = \Delta\left(e^{x\ln(10)}\right)}$$

$$\frac{dy}{dx} = \frac{d\,e^{x\ln 10}}{dx} = \ln 10\, e^{x \ln 10}$$

$$\frac{dy}{y} = \frac{\ln 10\, e^{x\ln 10}\, dx}{y}$$

$$= \ln 10\, dx \quad \text{relative error}$$

$$\cong 2.30\, dx$$

$$\text{Relerr} = \frac{\Delta y}{y} = \Delta x \cancel{\ln 10} \,/\, 2.30\, \underbrace{\Delta x}_{\text{in dex}}$$ to 1st order in small Δx

so small enough	0.23	0.1	but not $10^{0.1} = 1.26$ factor
first order begins to fail	0.46	0.2	but $10^{0.2} = 1.585$
first order fails	0.69	0.3	but $10^{0.3} = 2$

4048

specific star formation rate $\quad sSFR = \frac{SFR}{M_*} \propto \frac{M_*^k}{M_*} \propto M_*^{k-1}$

units of $\frac{1}{\text{years}}$

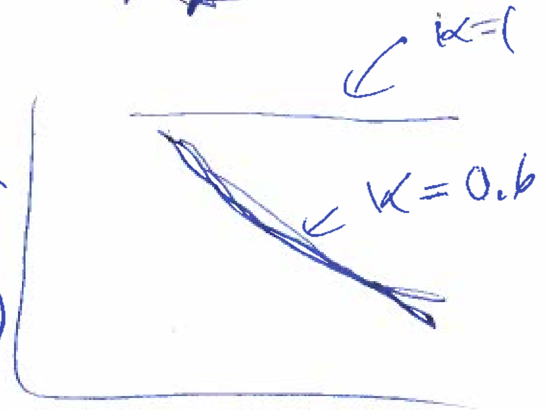
$\in [0.6, 1]$ depending on selection

$t_{\text{building characteristic time}} = \frac{1}{sSFR}$

$= \begin{cases} 10\ \text{Gyr in MS galaxy} \ (1-10\text{G}) \\ \frac{1\ M_\odot/\text{yr}}{10^{11}\ M_\odot} \ \ 10^2\ \text{Gyr} \\ \sim \frac{100\ M_\odot/\text{yr}}{10^{11}\ M_\odot} = 10^{-9} = 1\ \text{Gyr in starburst galaxie} \end{cases}$

from plot

log sSFR



log ~~sSFR~~ M_*

Scalo birthrate parameter

$b \equiv \frac{SFR}{\langle SFR \rangle}$ ← time average

$\frac{I}{\Delta t} = \frac{\int_0^\infty SFR\, dt}{\Delta t}$

$M_* = \int SFR\, dt - \int \text{mass loss}\, dt$

from stars
- SNe, strong winds
- lock up in compact remnants

Mass dt ejected by ~~stars~~ SNe and other ejection to ISM process

strong wind

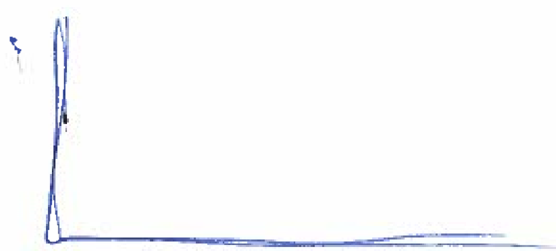
- and mass locked up in compact remnant.

15) Size - Mass Relation (4049)

another Factoid for Large-Scale structure simulation to get right

atomic H gas

$$\log\left(\frac{R_H}{10\,\mathrm{kpc}}\right) = K + \chi \log\left(\frac{M_{HI}}{10^{10} M_\odot}\right)$$



$M_{HI} = M_{HI0}$

$$\Sigma_{HI}(R) = \Sigma_{HI0}\, e^{-R/R_H}$$

surface density

$$M_{HI} = \int_0^R \Sigma_{HI}\, 2\pi R\, dR$$

$$= 2\pi \Sigma_{HI0} R_d^2 \int_0^{x\to\infty} x e^{-x}\, dx$$

$$M_{HI} = 2\pi \Sigma_{HI0} R_d^2$$

$z!$ for $x \to \infty$

$= 1! = 1$

Ar-543

if this were constant $M_{HI} \propto R_d^2$ or $R_d \propto M_H^{1/2}$ and the exponential model exact.

$$R_d = \sqrt{\frac{M_{HI}}{2\pi \Sigma_{HI0}}}$$

$$\log R_d = -\tfrac{1}{2}\log(2\pi \Sigma_{HI0}) + \tfrac{1}{2}\log M_{HI}$$

However the exponential model is not exact and ~~[illegible]~~ similarly ... from g ... any disc ...

$$\log\left(\frac{R_H}{10\,\mathrm{kpc}}\right) = \underbrace{K}_{0.5} + \underbrace{\chi \log\left(\frac{M_{HI}}{10^{10} M_\odot}\right)}_{\sim 0.5 - 0.55}$$

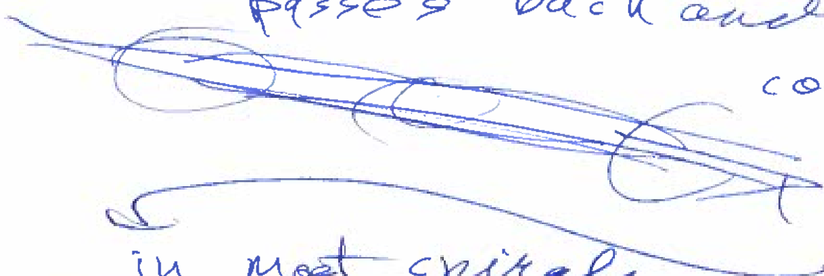
but intrinsic scatter is ~~[illegible]~~ ~ 0.1 dex

4040) 16) Star Burst Galaxies & ULIRGs — the short version

SFRs in SFGs that are 10 to 100 higher than normal SFGs are starburst galaxies.

- ~~[illegible]~~ collisions or mergers are very likely often the trigger

passes back and forth



concentrating gas & dust in the center region

in most spirals the bulges have low SFR and are mostly old stars — the ~~[illegible]~~ yellow bulges contrast with the blue/pink/brown mix in the arms.

the strong dust concentration can make for very high IR emission ($8 - 10^3\ \mu m$ integrated) in some starbursts

L4051

luminous IR galaxies
$\log(L_{IR}/L_\odot) > 11$

Ultra luminous IR galaxies ULIRGs $\log(L_{IR}/L_\odot) > 12$

But after a star burst
of order $t \lesssim 10^8$ yr $= 0.1$ Gyr

of a merger exceed
$M_{total} \gtrsim 10^{12} M_\odot$ = Golden mass

turns into an EGT

quenching follows
it seems
on 1 - 2 Gyr time scale
→ so not instantaneous

Conspiracy of nature
- combination of halo size and shape and SMBH — leads to heating of gas and so much too hot to cool
— dust evaporates
→ seen in X-ray emission

EGTs do have some cool gas, dust, SF but usually ~~often~~ at much lower rate than LGTs
spirals & dIrr