(2025 jan 05) ~~3089~~ 3089

structures
have peculiar velocities ~~sp~~
superimposed on ~~the~~
the recession velocities

Which are NOT
ordinary velocities.
They are NOT motion relative
to inertial frames
but growth of space in GR
— ~~expansion~~ between
definable inertia frames
(e.g., gravitationally bound
celestial frames)

recession velocities (Takes GR to prove this)
do NOT obey $v \leq c$

Ordinary velocities are relative
to an inertial frame — speed
at which some ~~physical~~ thing goes ~~p~~ past you
at one point.

So, in fact, as ~~the~~ Friedmann eqn.
proves (when we get there),
there is no upper limit on how
big a comoving frame can be
— but ~~they~~ must large enough

3090) (2025 jan 05)

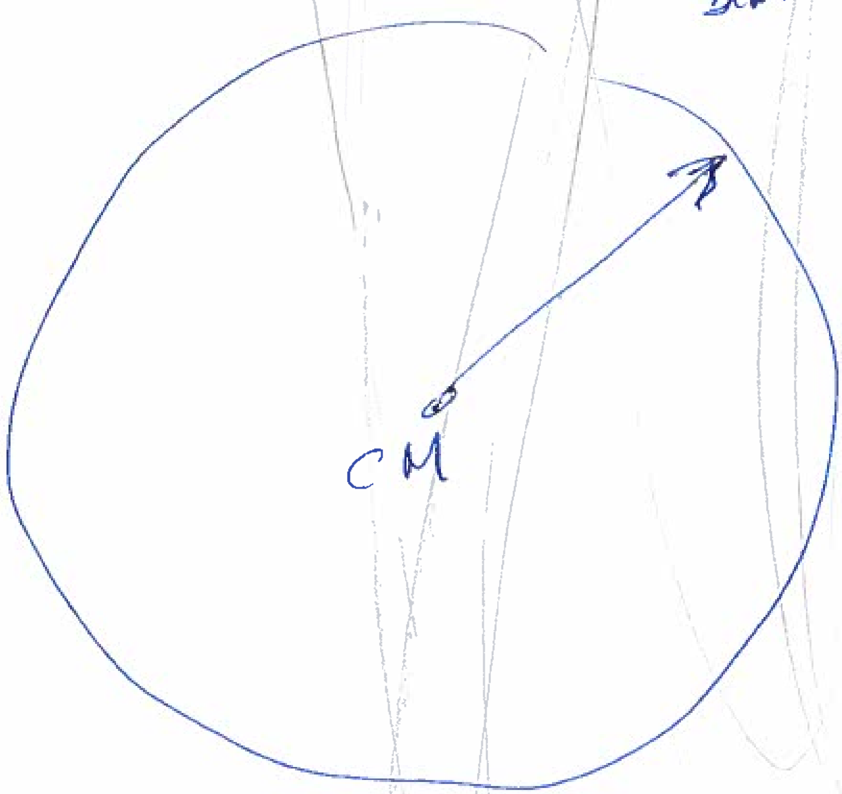
to obey cosmological principle.

How big? Yadav et al (2010) suggested scale size for structure for cos. principle

$$\text{Radial scale} = \frac{260\,\text{Mpc}}{h_{100}} = 370\ \text{Mpc}$$

370 Mpc

cube maybe, but there paper seems elusive on saying anything that definite



Maybe 370 cMpc

comoving Mpc

$$r = a(t)\, r_0$$

$$r_0\,(\text{cMpc}) = \frac{r(\text{Mpc})}{a(t)}$$

Sawala et al (2025) suggests we need to draw a line between structures & patterns

But don't define a structure scale explicitly except it is $<< 1 \text{ Gpc}$ 3091

∴ Maybe Yadav scale is OK

Sawala et al ~~found~~ no upper limit on patterns

arrangements of galaxies, clusters, groups

Just accidental arrangements ~~that~~ have no dynamical or little dynamical significance.

like Great Arc / Great Ring — From one end to the other no tidal forces

found recently by Lopez et al (2012, 2024)

Such patterns turn up in the Sawala et al N-body ~~s~~ simulations everywhere

and are what the ~~the~~ Λ-CDM model gives.

~~Another wa~~ So Yadav scale = 370 Mpc

Sawala scale $\lesssim$ 1 Gpc

Another way is cluster scale

large clusters $\hat{=}$ 3 Mpc virial radius

and virial mass $\cong 10^{15} M_\odot$

(Cimatti, - 165)

3092

$r_{turnaround} = (1.108981...) \left(\frac{M}{M_{\odot}^{12}}\right)^{\frac{1}{3}}$ ($M = 10^{15} M_{\odot}$)

(see p. 3076 2025jun05 notes) $\approx$ [illegible] Mpc

assuming the virial mass which may be a lower limit

Scale cluster >> 10 Mpc

The Yadav scale = 370. Mpc

faute de mieux acceptable

$r_{comoving} \gtrsim$ 370 Mpc (for present)

for cosmological principle to hold inside comoving frame,

recession velocities + orbiting velocities in NR velocities limit can be added

$v = Hr$
$= 70 \cdot 370$
$= 259 \cdot 10^2$
$= 2.5 \times 10^5$ km/s

$\therefore$ Not much smaller than c.

recession velocities

~~local~~ velocities much less

There's no upper limit on size of comoving frame in principle.

But in practice if you want to do a Newtonian large-scale structure calculation (and most are since GR simulation are harder and mostly confirm Newtonian physics simulations are adequate), there is a practicle limit on fan you want to calculate gravity.

two reasons

① $v << c$ to stay in non-SR limit

② ~~travel~~ travel of gravitational field << t_{motion} of gravity carrier

$\frac{r}{c}$

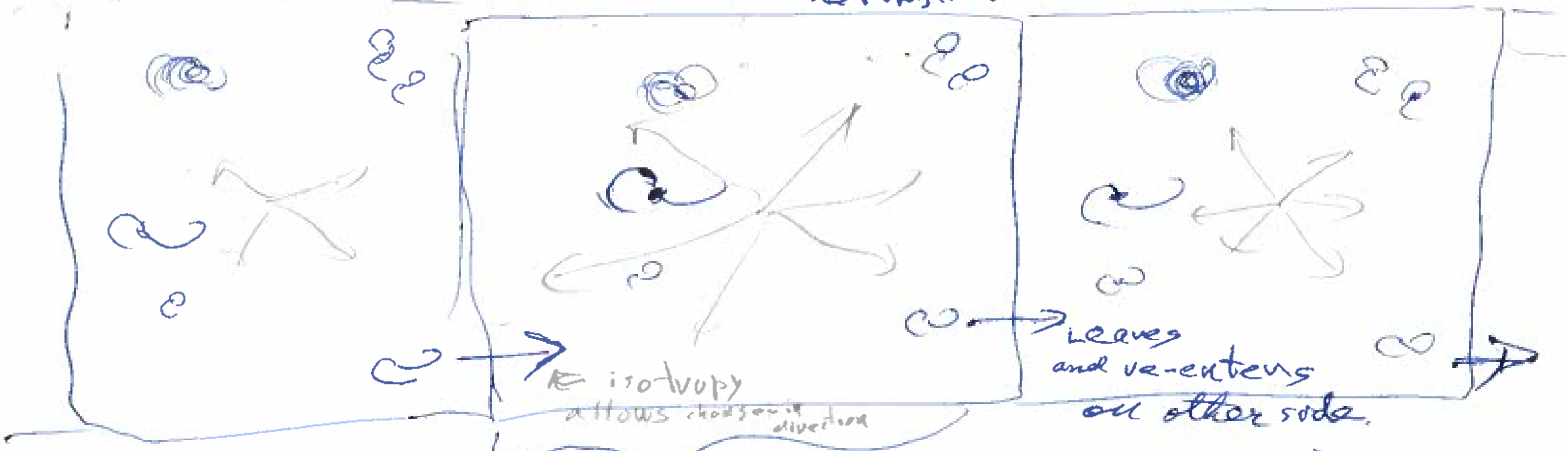
To stay in GR

What I think the large-scale structure simulations do (both N-body and full hydrodynamics) | 2025 Jan 07 | 13093

Illustris, Eagle, etc.

Define a cube computational domain with duplication for periodic BCs.

Just guessing really. Should check with Ken Nagamine



$V_{side} = r_{side_0}\, a(t)$ $\quad V_{side}$

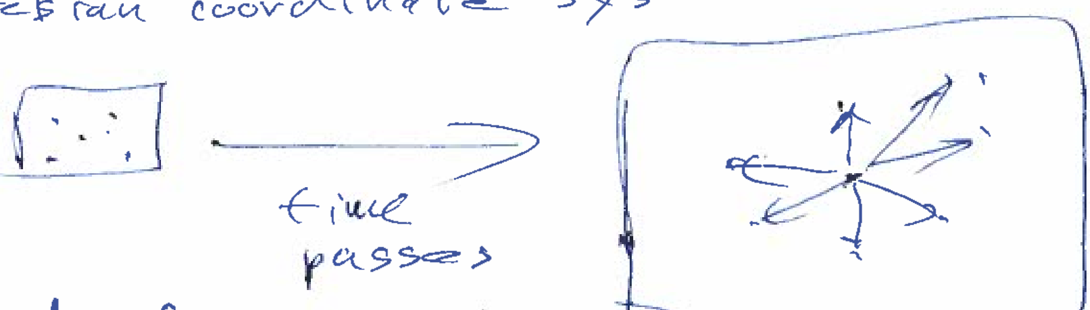
~~cosmic~~

~~$r_{coordinate}$ from somewhere maybe a cube corner~~

Where $a(t)$ is the cosmic scale factor from ~~solution~~ of the Friedmann eqn, so ~~the overall~~ scaling up, t is cosmic time from Big Bang. Includes all density components and cosmological constant term

Let's just think N-body simulation.

Don't need a grid, just cartesian coordinate system



Let $F_\Lambda = \frac{\Lambda}{3} r$

Just push out from the center of cube

Regard as a center of mass.

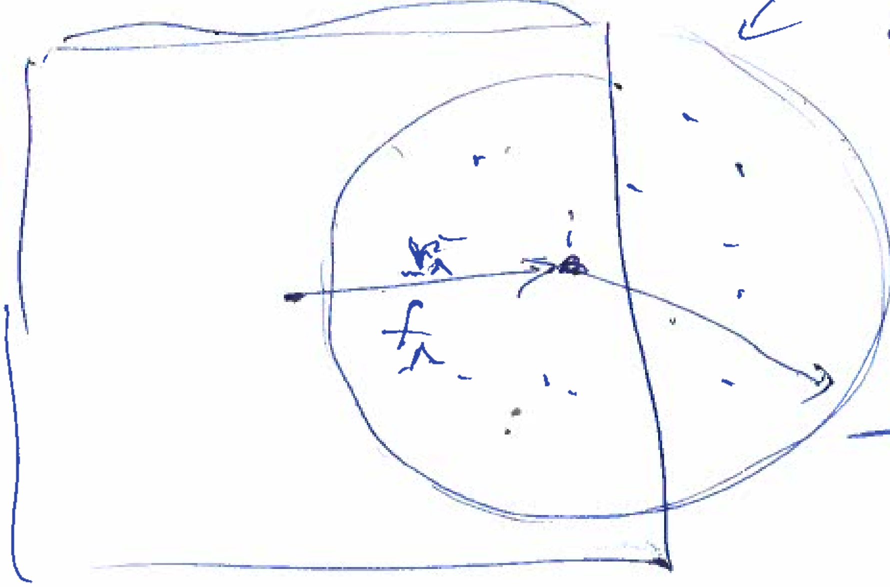
Not sure.

Do they scale up coordinates → seems unfair to bound systems

3094 | 2025 Jan 05

also 3094a →

$V_{side} = V_{side\,0}\, a(t)$



sphere of radius $R_{comoving}$ at some time t

~~Beyond~~

$$\underset{\sim}{g}_i = \sum_j \underset{\sim}{g}_j$$

out to $R_{comoving}$; beyond ~~Ro~~ assume cosmological principle and so by Birkhoff's theorem, no effect inside

the $\underset{\sim}{v}_{i\,new} = \underset{\sim}{v}_{i\,old} + \underset{\sim}{g}_i \underbrace{\Delta t + \frac{\Lambda}{3} r_\Lambda \Delta t}_{\text{time step.}}$

Note particles loses local kinetic energy

local velocity v_1 → v_2

δr gives $\delta v_{recession} = H(t)\,\delta r$
$~~H(t)~~ = \frac{\dot a}{a} v_1 \delta t$

This is the non-relativistic ~~particle~~ particle case. Photons are different. Then there is the cosmological redshift $\lambda \propto a$

$\therefore\ \delta v = v_2 - v_1 = (v_1 - \delta v_{rec}) - v_1$

$\delta v = -\frac{\dot a}{a} v_1 \delta t$

$\frac{1}{v}\frac{dv}{dt} = -\frac{\dot a}{a}$

$\therefore\ \ln v = -\ln a + \text{Const}$

$v(t) = \frac{C}{a(t)}$

LC-16-17

$\therefore\ KE(t) = \frac{1}{2} m v^2 \propto \frac{1}{a^2}$

Note this is loss is only
between points participating in mean
expansion of space

In a bound system ~~the~~ orbital velocities do Not lose local KE.
But the CM velocity can.

Where does this lost KE go?

In a finite
classical system

$v_{relative} = v_1 - v_2$

v_1 v_2

$KE_{local} = \frac{1}{2} m v_{rel}^2$

outside origin of Inertial Fram

$KE = \frac{1}{2} m v_1^2$

So ~~there~~ is lost local KE ~~at the~~ account
~~but~~ you can
for it by referring back ~~to~~
outside origin,

But in an ~~infinite~~ un infinite or finite unbounded
universe there's no determinant origin to refer
to. KE relative to every local comoving
frame is disappearing. And that is the KE
that can do things: — local escapes, orbits

The local KE ~~density~~ in a cube $\propto \left(\frac{1}{a^3}\right)\left(\frac{1}{a^2}\right) = a^{-5}$

For photons it's $\propto \left(\frac{1}{a^3}\right)\left(\frac{1}{a}\right) = a^{-4}$

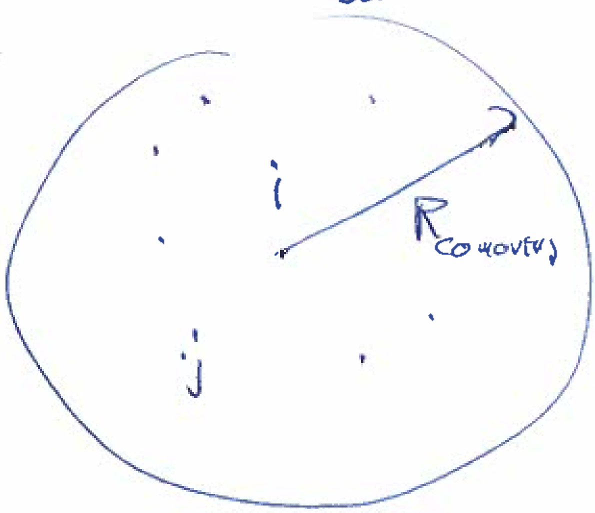
3904b | 2025jan05 (accidentally missed a page)

Now one can
say and some do ~~that~~
~~the~~ E is going into ~~the~~
expansion of the universe,
But for all purposes, it
just seems to be vanishing.

But GR doesn't guarantee
conservation of energy in an
ordinary sense and so we don't
feel too bad.
GR just guarantees itself

Getting back to calculating

$$\underset{\sim}{g}_i = \sum_j \underset{\sim}{g}_{ji}$$ from p. 3094



How many calculations are needed.
The Millenium N-body simulation (2005) then a record used 10^{10} particles.
Therefore every time step need $\sim(10^{10})^2 = 10^{20}$ gravitational calculations

And it is far worse now with hydrodynamics.
But there are faster supercomputers and speed up techniques (e.g. tree codes)

3094d] 2029 Jun 05

How large must $R_{comoving}$ be?
Well you have to capture significant tidal forces, but ~~the~~ gravity field changes have to propagate ~~in~~ ~~fast~~ over a time scale ~~close~~ e~~nough~~ short enough to be approximate as propagating instantaneously as required by Newtonian physics

But to account for scale up fairly you may need to do gravity to CM.

spherical source gravity unchanged by growth of sphere and vanishing and appearing mass

---

Natural unit for Galaxy Motion time scales

$$\frac{1\ \text{kpc}}{1\ \text{km/s}} = 0.977792 2217\ \text{Gyr}$$
$$= 0.98\ \text{Gyr}$$
$$= 1\ \text{Gyr}$$

Why? Galactic and intergalactic distances in kpc or Mpc and velocities in tables in km/s

So $\frac{x\ \text{kpc}}{y\ \text{km/s}} = \frac{x}{y}\ \text{Gyr}$

In GR gravity field changes only propagate at speed of light (relative to local inertial frames)

So relevant time scales in a Natural Time Unit for cosmology [At least for reading off tables of data without endless unit conversions]

t_H Hubble time $= \dfrac{1}{H_0} = \dfrac{1\ \text{Mpc}}{70\ \text{km/s}} = 14.4\,\text{Gyr}\left(\dfrac{67.8}{70}\right)$ (Wik)

fiducial value $70\ \dfrac{\text{km/s}}{\text{Mpc}}$

Somewhat coincidently of same size (Not a general rule of cosmological models)

Age of univers Λ-CDM $= 13.787(20)$ Gyr Planck 2018 (Wik)

$\doteq 13.8$ Gyr

t_s = Orbital time of sun in Milky Way $= \dfrac{2\pi(0.008\ \text{Mpc})}{220\ \text{km/s}} \simeq \dfrac{.05\ \text{Mpc}}{220\ \text{km/s}} = \dfrac{1\ \text{Mpc}}{4400\ \text{km/s}}$

$t_{\text{large galaxy cluster}}$ ~~galcluster~~ $\cong \dfrac{r_{\text{virial}}}{v_{\text{dispersion}}} = \dfrac{3\ \text{Mpc}}{1000\ \text{km/s}} \cong \dfrac{1\ \text{Mpc}}{300\ \text{km/s}} \ll t_H$

$< t_H$ by factor of 4

$t_{\text{gravity field travel time over Yadav scale}} = \alpha\,\dfrac{370\ \text{Mpc}}{3\times 10^5\ \text{km/s}} \cong \alpha\,\dfrac{1\ \text{Mpc}}{10^3\ \text{km/s}}$

$\doteq \alpha * 1$ Gyr

time α factor a fudge factor

speed of light (Wik: grav waves)

3096

2025 jun 09

$t_{\text{sun orbit}} < t_{\text{radar}_x} < t_{\text{galaxy cluster motion}} < t_{\text{Hubble}}$

$\sim \frac{1\text{ Mpc}}{4400\text{ km/s}} < x\,\frac{1\text{ Mpc}}{10^3\text{ km/s}} < \frac{1\text{ Mpc}}{300\text{ km/s}} < \frac{1\text{ Mpc}}{70\text{ km/s}}$

~ 0.25 Gyr $\quad = x * 1$ Gyr $\quad = 3$ Gyr $\quad = 14$ Gyr

probably ~~most~~ most relevant

particle j where evaluating gravi

v_j

remote galaxy cluster

ideally ~~...~~ $t_{\text{radar}} \ll t_{\text{galaxy cluster}}$

and so gravity signal sort of instantaneous.

If $x = 1$, seems not ideal

but but $x = \frac{1}{10}$

then $t_{\text{radar}_x} =$ ~~...~~ 0.1 Gyr $\ll t_{\text{galaxy cluster}}$ by factor of 30.

But is r_{convey} = 37 Mpc enough to eliminate tidal forces? Maybe,

Maybe simulations can use $v_j \hat{=}$ ~~...~~ $\frac{1}{10}\, v_{\text{radar}}$?

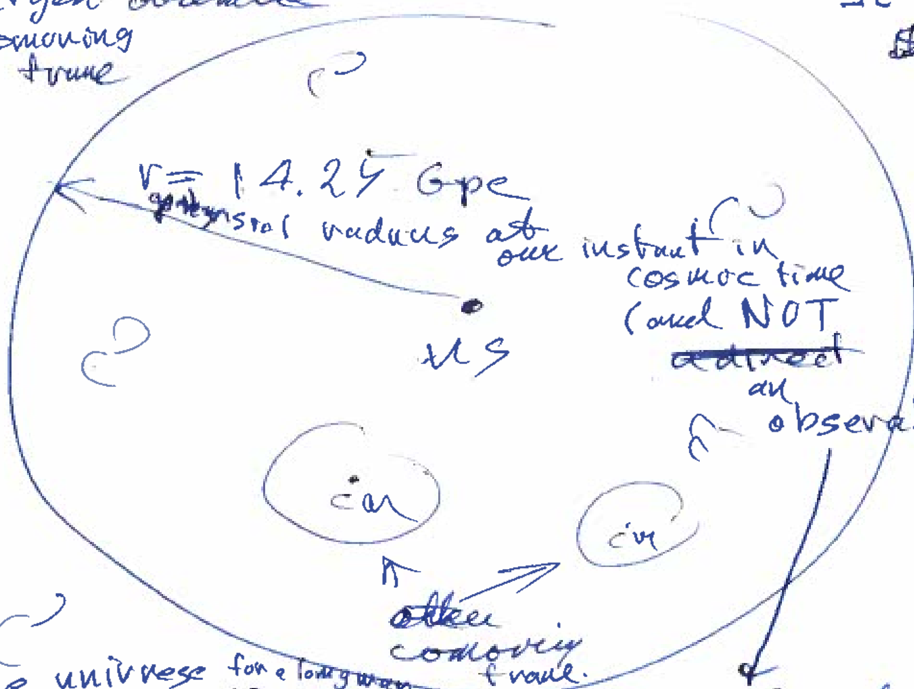
I don't. The practicalities are out of our scope.

3097

What of the observable universe as a whole?

— a sphere centered on us and in the Λ-CDM model Space is nearly flat ~~and so normal~~

— largest ~~observable~~ comoving frame



value from Λ-CDM model

and so it's a normal sphere

— Beyond observable universe for a long way everything is much ~~the~~ same we assume but maybe not to infinite distance.

We can measure very accurately our rotation relative to observable universe using measurement of distant quasars.

↳ Current standard using distant quasars is called International Celestial Reference System (ICRS) (Wik)

These replace fixed stars of traditional astronomy — but they are good enough for most purposes still.

3088

Using CMB with CMB dipole as determined by peculiar velocities

And we can measure our peculiar velocity accurately relative to the comoving frame centered on "us"

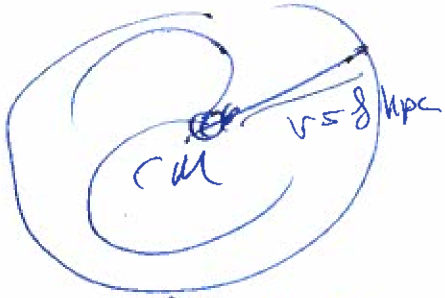
Us being

Solar System CM = 368 km/s in some direction

Milky Way CM = 552 km/s toward center of Hydra

Local Group CM = 627 km/s in some direction.

Can we measure our peculiar acceleration, It seems rather hard.



Milky Way

a_{sun} to CM of MW orbital centripetal acceleration $\doteq \frac{(220\ km/s)^2}{8\ kpc} = \frac{5\times10^4\ km^2/s^2}{8.3\times10^{16}\ km} = 2\times10^{-12}\ km/s$ inward

But the $\ddot r = \ddot a\, r_0$

$= \left(-\frac{0.55}{2}\right)\left(\frac{70}{H_0}\right)^2 \cdot 1\,\mathrm{Mpc}\left(\frac{r_0}{1\,\mathrm{Mpc}}\right)$

$= \left(8.738\times10^{-17}\ \frac{km/s^2}{Mpc}\right)\left(\frac{r_0}{1\,\mathrm{Mpc}}\right)$

If we assume MW CM at rest in its local comoving frame

Of order $\ddot r_{comoving\ sun} \triangleq 72\times10^{-20}\ km/s^2$ outward

This so much smaller than our centripetal acceleration, that accurately measure $\ddot r_{comoving\ sun}$ seems hopeless.

The CMB ~~with~~ is believed to be isotropic in all comoving frames aside from the important fluctuations

$T = 2.7255(57)$ K

and fluctuations ~RMS 100 μK

∴ $\frac{10^{-4}K}{2.7K} \triangleq 4\times10^{-5} = \frac{1}{\frac{1}{4}\times10^{5}} = \frac{1}{2.5\times10^{4}}$

So Not $\frac{1}{10^4}$ or $\frac{1}{10^5}$. It's $\frac{1}{25000}$

Wiki also says $\frac{1}{25000}$

agree (Wiki:CMB: Feature)

The uniformity of the CMB is one of the strongest proofs of the cosmological principle.

"us" being ~~velocities~~

Solar System CM has $v = \cancel{368}$ km/s in some direction

Milky Way $v = 552$ km/s toward near center of Hydra

Local Group CM $v = 627$ km/s in some direction (~~letter~~ astro frame)

And rotation relative to the observable universe (i.e. absolute rotation) is measured to highest accuracy using cosmologically ~~distant~~ remote quasars and galaxies

(Wiki: International Celestial Reference System)

This supercedes ~~the~~ local fixed stars you see at night for highest accuracy, but the fixed stars were fine most of astronomy history.

Acceleration I think need

CM Milky Way

$\underset{\sim}{a}_{sun} \approx 2 \times 10^{-13}$ km/s²

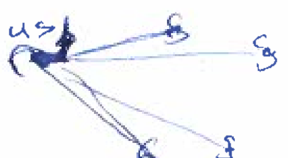
and

$\underset{\sim}{a}_{MW} \approx 10^{-16}$ km/s² relative its local comov frame

So

$\underset{\sim}{a}_{sun-comov} = \underset{\sim}{a}_{sun} - \underset{\sim}{a}_{MW} \approx$ smaller than error in $\underset{\sim}{a}_{sun}$

The acceleration relative ~~of the~~ comoving frame CM is "centered on us"



Acceleration

$a_{\text{solar system about Milky Way center}} = \frac{v^2}{r} = 1.8898\ldots \times 10^{-13}\ \text{km/s}^2 \left(\frac{v}{220\ \text{km/s}}\right)^2 \frac{1}{(r/8.3\ \text{kpc})}$

and $r = a r_0$

$\ddot{r} = \ddot{a} r_0$ $\left(\ddot{a} = (-q) H_0^2 a_0 = 8.7383\ldots \times 10^{-17}\ \frac{\text{km}}{\text{s}^2} \left(\frac{q}{-.55}\right)\left(\frac{H_0}{70}\right)^2\left(\frac{a_0}{1}\right)\right)$ 1 Mpc $(r_0/1\ \text{Mpc})$

Fiducial values

$\ddot{r} \approx 10^{-16}\ \frac{\text{km}}{\text{s}^2}\ r_{Mpc}$

3088 | 2025 jan 05 | N-body can be reduced (N-1)-body similarly taking one body as origin but technique is NOT very useful it seems

Example 2: A 2-body system and its reduction to a 1-body system

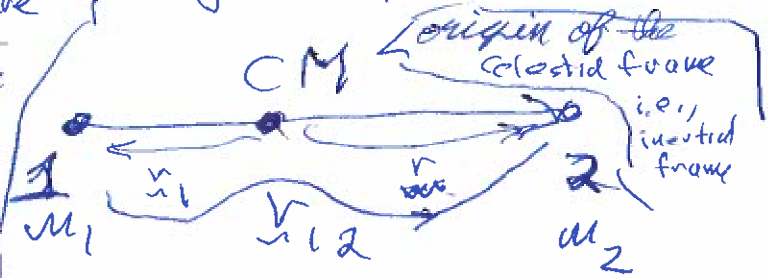
CM inertial frame

The example is for gravity but same reduction procedure happens in other contexts: quantum mechanics, Nuclear rate equations, etc.

The Reduction allows you to solve a one-body problem and get the two-body solution

Amazing it is right for QM study

See p. 3083 equation and we set external forces and Λ-force to negligible and only have gravity as an internal force



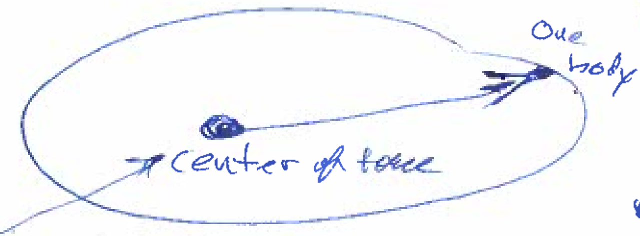
$= -\underset{\sim}{r}_{21}$

Two particles and so two equation

$$-\frac{Gm_1}{r_{12}^2}\hat{r}_{12} = \underset{\sim}{a}_2$$

$$-\frac{Gm_2}{r_{21}^2}\hat{r}_{21} = \underset{\sim}{a}_1$$

An exact analytic solution exists for a one-body problem with a fixed central force that is an inverse-square law force.



thought of as an infinite mass object at resting in an inertial frame

so reduction to a one body problem is a good idea

CM defintn

$$0 = \frac{m_1\underset{\sim}{r}_1 + m_2\underset{\sim}{r}_2}{m_1+m_2}$$

$$\therefore\ \underset{\sim}{r}_1 = -\frac{m_2}{m_1}\underset{\sim}{r}_2$$

$$\underset{\sim}{r}_2 = -\frac{m_1}{m_2}\underset{\sim}{r}_1$$

1 ⟵ 2

$$\underset{\sim}{r}_{21} = \underset{\sim}{r}_1 - \underset{\sim}{r}_2 = \underset{\sim}{r}_1\left(1+\frac{m_1}{m_2}\right)$$

$$\frac{\underset{\sim}{r}_{21}}{1+\frac{m_1}{m_2}} = \underset{\sim}{r}_1$$

1 ⟶ 2

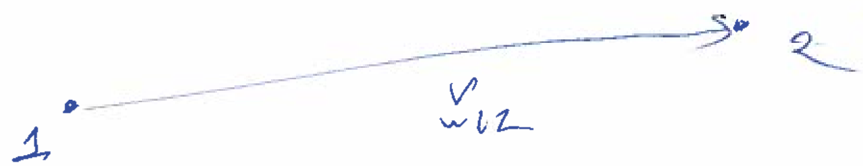
$$\underset{\sim}{r}_{12} = \underset{\sim}{r}_2 - \underset{\sim}{r}_1 = \underset{\sim}{r}_2\left(1+\frac{m_2}{m_1}\right)$$

$$\frac{\underset{\sim}{r}_{12}}{1+\frac{m_2}{m_1}} = \underset{\sim}{r}_2$$

Substituting for $\underset{\sim}{a}_2$ and $\underset{\sim}{a}_1$, we get the dynamic equation {2025 jan05} ~~3089~~ 3101

$$-\frac{G(m_1+m_2)}{r_{12}^2}\hat{r}_{12} = \underset{\sim}{a}_{12} \quad \text{and} \quad -\frac{G(m_1+m_2)}{r_{21}^2}\hat{r}_{21} = \underset{\sim}{a}_{21}$$

which are for the identical solution with the origin interchanged.



But what is the dynamic mass,

Infinite mass since fixed in space but with inverse-square law 'charge' (m_1+m_2) for the central force

Well the reduction to be useful must have the same potential energy & kinetic energy.

$$U_{unred} = -\frac{Gm_1m_2}{r_{12}} = U_{red} = -\frac{\cancel{Q}\,\mu G}{r_{12}^2}$$

This interpreted as a charge.

but the potential ~~at~~

$$U_{red} = -\frac{G(m_1+m_2)}{r_{12}}$$

$\therefore \quad Q = m_1+m_2 \quad$ and $\quad \mu = \frac{m_1m_2}{m_1+m_2}$

The reduced mass. dynamic mass of reduced system

Omit

As a interesting aside

$$f(1,2) \stackrel{?}{=} f(2,1)$$

~~order~~ exchange ~~the~~ particles in the function slots and if equal, the function symmetric.

Note $f(1,2) \to f(2,1)$
You exchange all particle values

So $f(1,2) = m_1^2 m_2$
& $f(2,1) = m_2^2 m_1$

for general values

Those two are NOT equal for all m_1 and m_2 values
e.g. $m_1 = 1$, $m_2 = 2$
and so are Not symmetric

But $f = m_1 m_2$
$f = (m_1 + m_2)$
$f = \frac{m_1 m_2}{m_1 + m_2}$
are all symmetric under particle exchange

General considerations
— Potential theory (p. 3015) and the work-kinetic energy theorem (p. 3016) tell us that

$$\frac{1}{2}\mu v_{12}^2 = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2$$

Note symmetric
1 exchange with 2
$\frac{1}{2}\mu v_{21}^2 = \frac{1}{2}m_2 v_2^2 + \frac{1}{2}m_1 v_1^2$
and recall $v_{12} = -v_{21}$
(see p. 3088)
symmetric

~~But~~ as ~~a~~ sanity check we can prove ~~that~~ $\mu = m_1 m_2/(m_1+m_2)$ from this statement directly? ~~directly?~~

$$\frac{1}{2}\mu v_{12}^2 = \frac{1}{2}m_1\left(\frac{v_{21}}{1+\frac{m_1}{m_2}}\right)^2 + \frac{1}{2}m_2\left(\frac{v_{12}}{1+\frac{m_2}{m_1}}\right)^2$$

$$\mu = \frac{m_1}{\left(1+\frac{m_1}{m_2}\right)^2} + \frac{m_2}{\left(1+\frac{m_2}{m_1}\right)^2}$$

$$= \frac{m_1 m_2^2}{(m_1+m_2)^2} + \frac{m_1^2 m_2}{(m_1+m_2)^2}$$

$$= m_1 m_2\left(\frac{m_2+m_1}{(m_1+m_2)^2}\right) = \frac{m_1 m_2}{m_1+m_2}$$

QED.

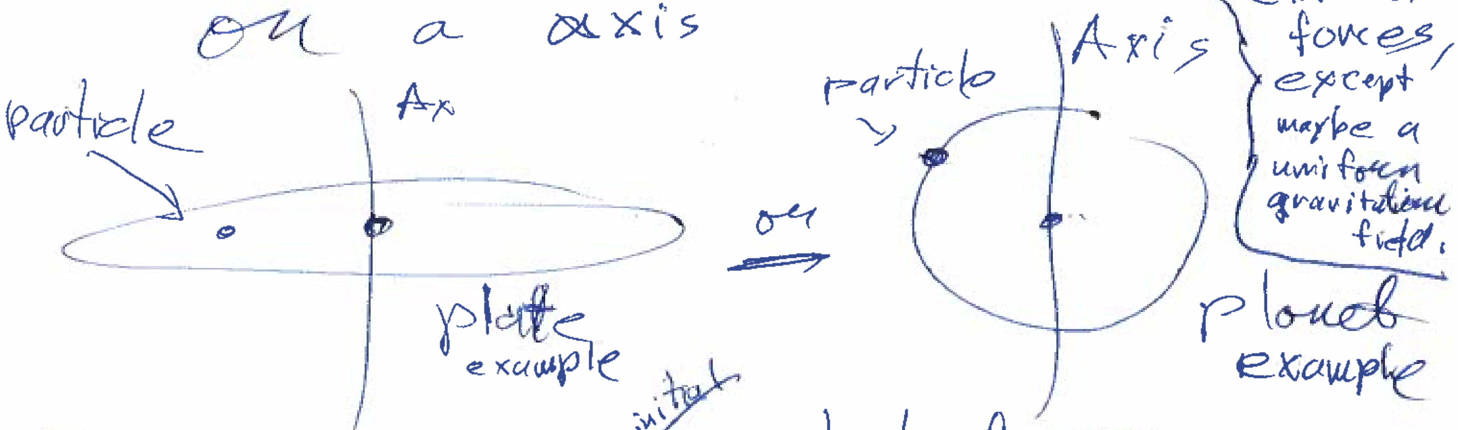
[2025jun05]

~~3091~~
3103

Example 3: Rotating Frame

CM inertial frame / ~~A~~ A system with no significant internal gravity or internal Λ-force.

One puts the origin on a axis and no external forces, except maybe a uniform gravitational field.



There is an initial inertial frame define by the axis unrotating with respect the observable univers

But we impose a rotational frame relative to the observable universe with angular ~~fre~~ velocity w (omega)

the rotational frame ~~for example~~ ~~can~~ moves with a rotating plate or planet for example.

~~2602~~ 3004

2023 Jan 05

In ~~the~~ ~~initial~~ inertial frame

$$\underset{\sim}{F}_{net} = m\,\underset{\sim}{a}$$

net force on particle

particle mass

partical acceleration.

Now a rotating frame is really a continuum of constant acceleration frames. So special treatment is needed to find the inertial forces to turn the rotational frame into an inertial frame.

We will just quote the results

Expand $\underset{\sim}{a} = \underset{\sim}{a}_r + [\underset{\sim}{\omega} \times (\underset{\sim}{\omega} \times \underset{\sim}{r})] + 2(\underset{\sim}{\omega} \times \underset{\sim}{v}_r) + \frac{d\underset{\sim}{\omega}}{dt} \times \underset{\sim}{r}$

cross products

acceleration relative to rotational frame

centrifugal accel.

Euler accel.

velocity relative to rotational frame

$$\underset{\sim}{F}_{net} - m[\underset{\sim}{\omega} \times (\underset{\sim}{\omega} \times \underset{\sim}{r})] - 2m(\underset{\sim}{\omega} \times \underset{\sim}{v}_r) - m\left[\frac{d\underset{\sim}{\omega}}{dt} \times \underset{\sim}{r}\right] = m\underset{\sim}{a}_r$$

centrifugal force

coriolis force

Euler Force

(French 514-524)

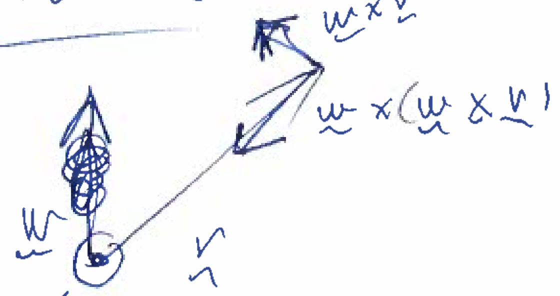
2025 Jun 05

~~3093~~ 3104

Specialize to a plane with $\frac{d\omega}{dt}=0$

a) Centrifugal force

$$\underset{\sim}{\omega}\times(\underset{\sim}{\omega}\times\underset{\sim}{r}) \longrightarrow -\omega^2 r\,\hat{r}$$



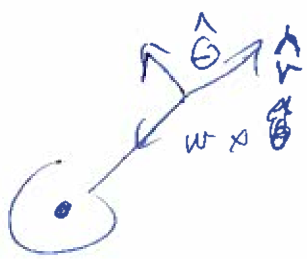
both in plane of page

Points out of page

b) Coriolis force

$$\underset{\sim}{\omega}\times\underset{\sim}{v}_f = \underset{\sim}{\omega}\times\left(v_{fr}\,\hat{r}+v_{f\theta}\,\hat{\theta}\right)$$

$$= \omega\, v_{fr}\,\hat{\theta} - \omega\, v_{f\theta}\,\hat{r}$$



Points out of page

$$\therefore\ \underset{\sim}{F}_{net} + m\omega^2 r\,\hat{r} + m\omega\left(-v_{fr}\,\hat{\theta}+v_{f\theta}\,\hat{r}\right)$$

$$= m\,\underset{\sim}{a}_f$$

centrifugal force points radially outward.

for an object held at rest in the rotating frame it exactly cancels the centripetal force (whatever supplies that) causing circular motion ~~the inertial frame~~

~~3094~~ 3106

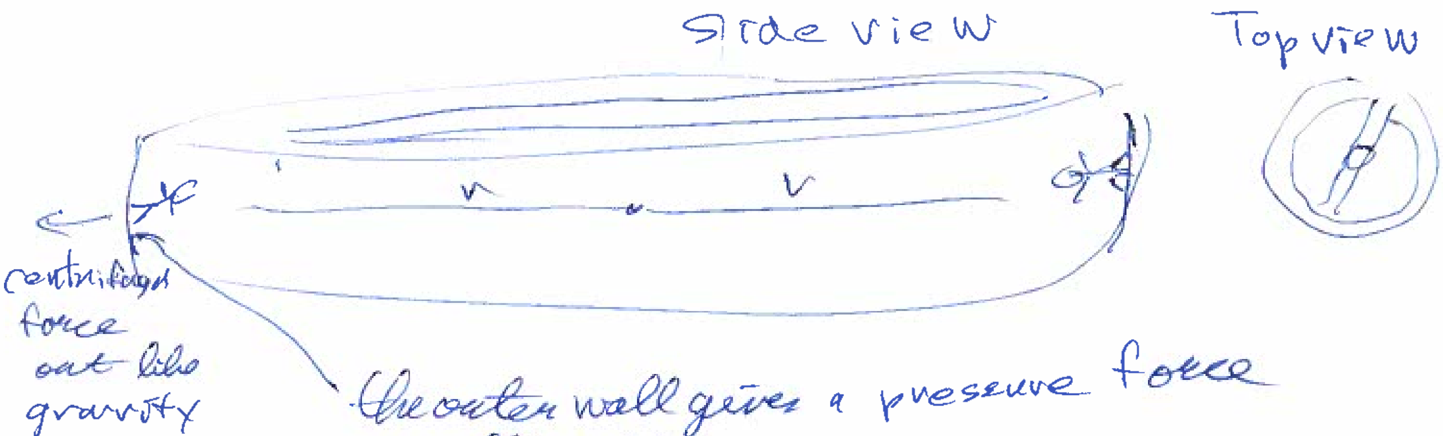
2025 jan 05

c) An interesting aspect of the coriolis force is ~~that~~

it does NOT depend on ~~ω~~ r

~~the~~ magnitude of radius

This is why rotating wheel space station were envisaged like in the film 2001: A Space Odyssey



$$F_{centrifugal} = m\,\omega^2 r$$

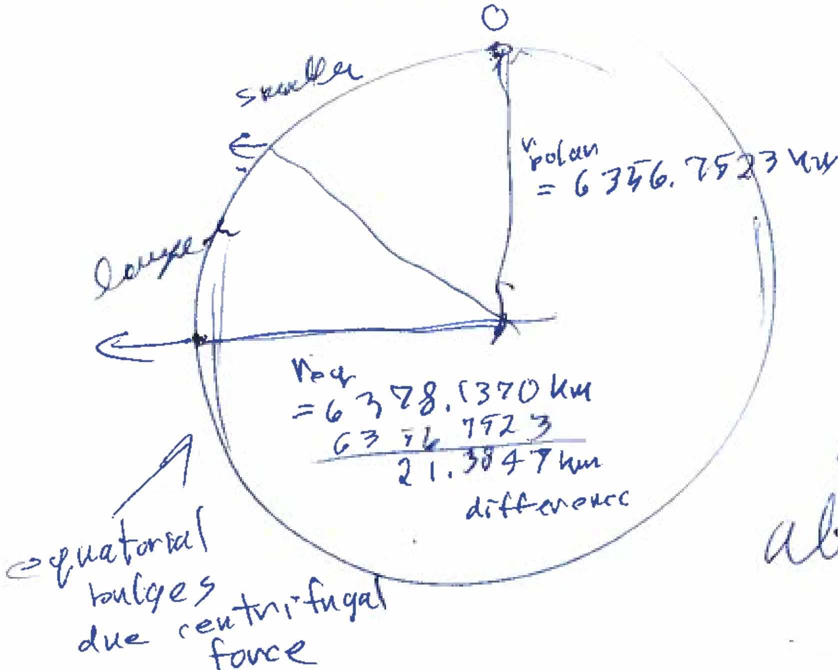
$$g = 9.8\ \mathrm{m/s^2} = \omega^2 r$$

$r \uparrow \quad \omega \downarrow$ which means the Coriolis force $\propto \omega$ (but not r) could be made small enough not to be annoying.

[2025jan05]

~~3095~~ 3107

d) On Earth effective gravity.



$g = 9.8\ m/s^2$ fiducial)

($9.80665\ m/s$ standard)

$g_{pole} \triangleq 9.832\ m/s$

$g_{equator} = 9.780\ m/s^2$

(Wiki: Earth Gravity: Latitude)

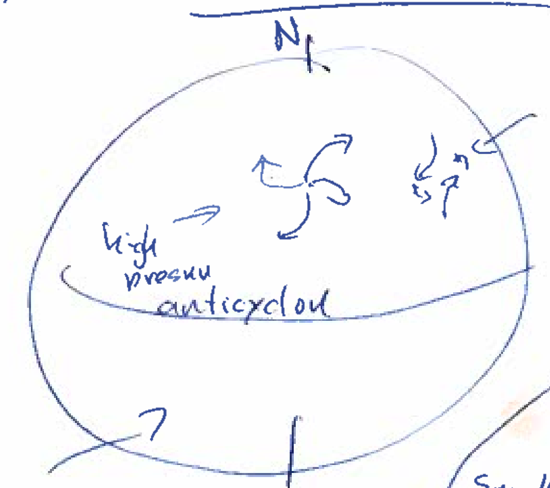
about $\sim 0.5\,\%$ variation.

due to centrifugal force and increased radius at equator due to tidal bulges

Quite easily measured with gravimeters and long ago with pendulums

But below human perception

And Coriolis force noticed for cyclones & anticyclones (big scales)



low ~~high~~ pressure cyclones - strongest be hurricanes

South is mirror image

Small scales you can see Coriolis force only with ~~the~~ sensitive set up

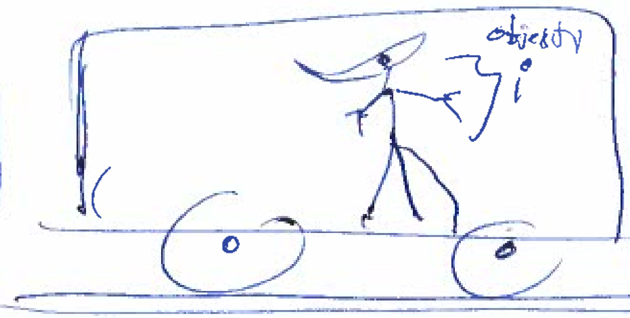
Foucault's pendulum oscillation planes on small scales rotates by coriolis forces. Most easily described at poles.

But the non rotating frame the oscillation is just fixed in space.

3108 2025 jan 05

d) Another CM inertial Frame example

Accelerating rail road car; Just to show generality of approach



Earth centrifugal + coriolis forces negligible

a_{cm} see 3083 for general expression

Where is CM? Exactly it moves around as ~~object~~ moves, but object i

x-direction

no gravity in x-direction

$$\frac{F_i}{m_i} - \frac{\sum_j F_{ex_j}}{M} + 0 + 0 + 0 = \Delta a_i$$

F_{ext_j} is friction force of track on wheels

a_{cm} an inertial force

internal gravity negligible

Λ-force negligible

negligible to CM really. All parts of ~~train~~ can relatively unaccelerate so any point is OK for origin.

y-direction

gravity ~~downward~~

$$\frac{F_i}{m_i} - \frac{\sum_j F_{ex_j}}{M} + (-1)\left(\frac{g_{ext_i}}{m_i} - \frac{\sum_j m_j g_{ext_j}}{M}\right) = \Delta a_i$$

Known to be zero

since g_{ext_i} is constant this term is zero!!!

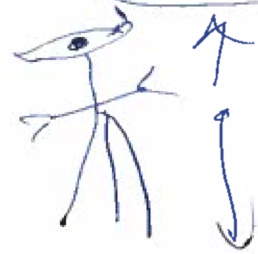
But that seems paradoxical since object i ~~since object~~ i has a gravity force on it.

Resolution: a math cancelation not direct ~~body~~ cancelation of forces on ~~particle~~ i object i

2025 jun 05

3109

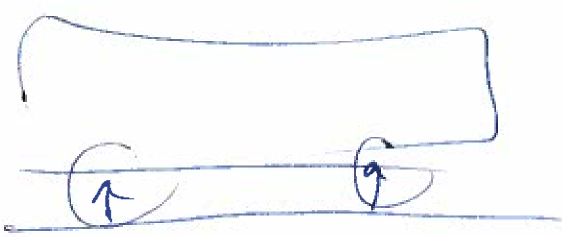
$$\frac{F_i}{m_i} - \frac{g_{ext\,i}}{m_i} = 0$$



gravity on object i

Normal force of car floor on object i

$$\frac{-\sum_j F_{ext\,j}}{m} + \frac{\sum_j m_j g_{ext\,j}}{m} = 0$$



force of gravity on whole train [illegible]

force on wheels by track upward on whole train

Why are ~~these~~ two negative ~~of~~ what you expect?

The equation was for the acceleration on object i

Not whole train.

Their affect on object i acceleration is negative.

3010

2025 jan 05