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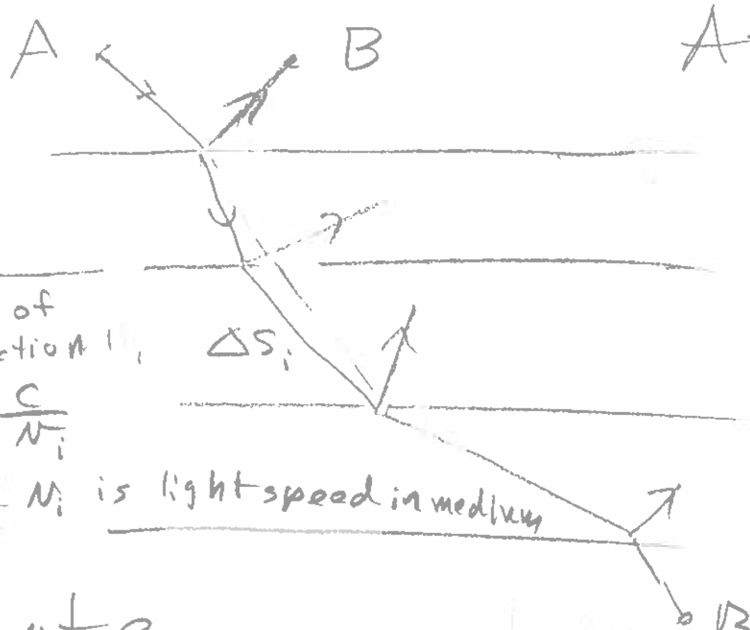
Derivation of Law of Reflection and Snell's Law (AKA Law of Refraction) From modern Fermat's Principle

(Hecht + Zajac -68)

a) Consider a light ray passing through many layers of different media.



index of refraction n_i

$n_i = \frac{c}{N_i}$

where N_i is lightspeed in medium

Note,

Optical Path Length = OPL_{AB}

$\propto \sum_i \frac{\Delta s_i}{\lambda_i} = \sum_i \frac{\Delta s_i}{N_i/f}$ { count of wavelengths; Recall $f\lambda_i = N_i$

$= \frac{f}{c} \sum_i \frac{c}{N_i} \Delta s_i = \frac{f}{c} \sum_i n_i \Delta s_i$

$\propto \sum_i n_i \Delta s_i = OPL_{AB}$

$\propto \sum_i \frac{\Delta s_i}{N_i} =$ Travel time

At each interface there can be a reflected ray and a refracted ray

Both reflected and refracted rays are coherent: i.e., the count of wavelengths is continuous: it is NOT reset or truncated. Just attribute this to the interaction properties at the interfaces. Also frequency f is constant across interfaces

2)

Fermat's principle

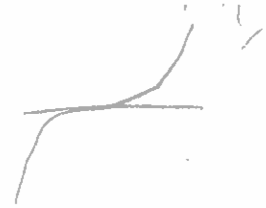
(at least for our purposes here)
is that OPL and travel time
from A to B
are stationary with respect
to variations in
the angles of reflectance
and/or refraction..

What this means that at B,
rays coming along 1st order different paths
arrive with
the same
wavelength
count; i.e.,
they arrive
in phase
(coherently)
and add up
by constructive
interference

At quantum
level individual
travel a range
of paths (but
how large a range?)
and interfere with
themselves only usually

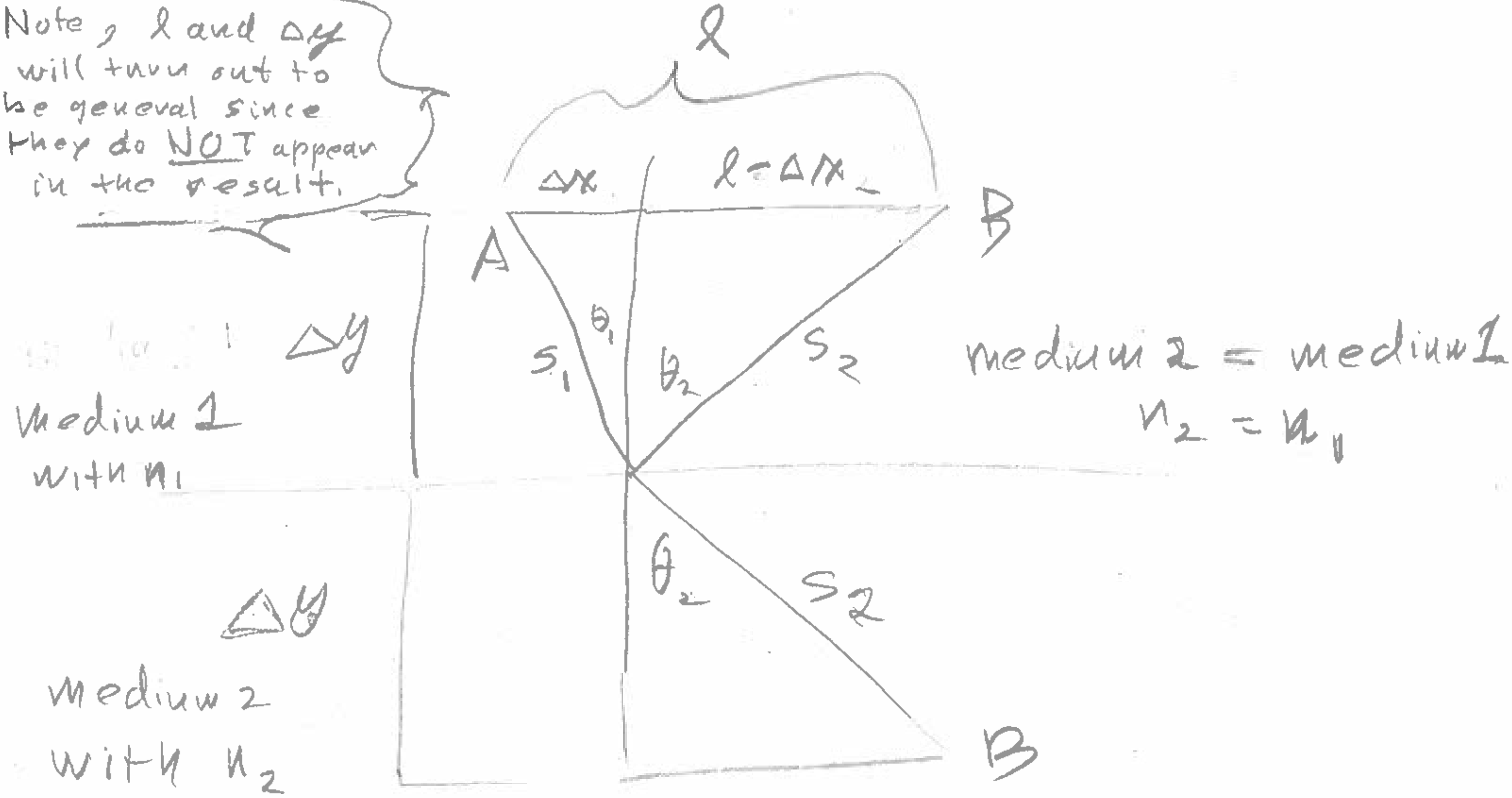
B

Rays at
other points arrive
incoherently and strongly
cancel out by destructive
interference.

The stationary OPL is usually
a minimum  but maxima  and inflection points 
can occur.

b) To find the angles for stationarity, consider

Note, l and Δy will turn out to be general since they do NOT appear in the result.



$$OPL_{AB} = n_1 s_1 + n_2 s_2$$

$$= n_1\sqrt{\Delta x^2+\Delta y^2} + \sqrt{(l-\Delta x)^2+\Delta y^2}$$

Note, nothing forbids us from choosing Δy to be the same in both media.

$$\frac{d\,OPL}{d\Delta x} = n_1 \frac{\Delta x}{\sqrt{\Delta x^2+\Delta y^2}} + n_2 \frac{(l-\Delta x)(-1)}{\sqrt{(l-\Delta x)^2+\Delta y^2}} = 0 \quad \text{for stationarity}$$

$$= n_1 \sin\theta_1 - n_2 \sin\theta_2 = 0$$

Incidence angle — Reflection/Refraction angle

$\therefore \quad n_1 \sin\theta_1 = n_2 \sin\theta_2$

Note, no dependence on l or Δy, and so the result is independent of l and Δy

Law of Reflection $\quad n_1 = n_2$

$\therefore \quad \theta_1 = \theta_2$

Snell's Law $\quad n_1 \neq n_2$

$\therefore \quad n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad$ QED

c) Gaussian Optics and the Paraxial Approximation

In Gaussian Optics, you consider only rays that vary from the optical axis of a system by small angles. This is suitable for, e.g., simple telescope understanding.

The paraxial approximation means that the small angle approximation can be made for angles from the optic axis; i.e.

$$\sin\theta \simeq \theta, \quad \tan\theta \triangleq \theta, \quad \cos\theta \triangleq 1$$

and so, Snell's law reduces to $n_1\theta_1 = n_2\theta_2$ (small angle refraction law).

Kepler in 1611 when he designed the Keplerian telescope knew only the small angle refraction law. Snell's law was known to Kepler's correspond Thomas Harriot, but Harriot kept it a secret and missed his chance for fame. Snell's law was first published in 1637 by Descartes, but he missed out on fame too.