Statistical Mechanics Problems

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Introduction

Statistical Mechanics Problems (SMP) is a source book for instructors of statistical mechanics. There is not much here so far and there may never be much.

I would like to thank the Physics & Astronomy Department of University of Nevada, Las Vegas for its support for this work. Thanks also to the students who helped flight-test the problems.

Contents

Appendices

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Chapt. 0 General Questions

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000 qmult 00100 1 4 1 easy deducto-memory: reading and done

1. Did you complete reading for this cosmology lecture before it was lectured/bypassed on in class and the corresponding homework by the day after?

   a) YYYessss! b) Jawohl! c) Da! d) Sí, sí. e) OMG no!

Chapt. 2 History of Cosmology

Multiple-Choice Problems

Full Answer Problems

Chapt. 3 Observational Overview and Miscellaneous Topics

Multiple-Choice Problems

Full-Answer Problems

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002 qfull 01000 1 3 0 easy math: rate equation solutions for statistical systems

2. How does a system approach (i.e., relax to) statistical equilibrium? We will get some insight by solving the rate equations for highly simplified systems.

   NOTE: There are parts a,b,c.

   a) Consider a system consisting of n identical single-particle states and N identical classical particles. States and particles have time-invariant properties. The occupation number for state i is N_i and the transition coefficient from state i to state j is κ_{ij}. The rate equation for state i is

   $$\dot{N}_i = -N_i \sum_{j,j\neq i} \kappa_{ij} + \sum_{j,j\neq i} N_j \kappa_{ji} \ .$$

   Since we have assumed the states and particles are identical, it follows that the κ_{ij} are identical: i.e., the general $\kappa = \kappa_{ij}$. We impose the constraint that particle number is conserved. Thus

   $$N = \sum_i N_i$$

   is a constant. The initial occupation numbers at time zero are N_{i0}. Solve for N_i for the equlilibrium solution and then for the general solution.

   b) The solution to the differential equation in part (a) suggests an approximate interpolation formula for the solution for relaxation for more general systems including those with energy constraints and time-varying transition coefficients (including those that depend on occupation numbers making the system nonlinear). The interpolation formula is

   $$N_i = N_{i\infty} + (N_{i0} - N_{i\infty})e^{-t/\tau_i} + Ate^{-t/\tau_i} \ ,$$

   where N_{i0} is the initial occupation number, $N_{i\infty}$ is the equilibrium or time-infinity occupation number, A is a fitting parameter, and

   $$\frac{1}{\tau_i} = \left(\frac{n}{n-1}\right) \sum_{j,j\neq i} \kappa_{ij}$$

   in order for the interpolation formula to go to the exact solution obtained in part (a) in the limit that all the transition coefficients are identical (and constant in time too). The choice for τ_i is a natural choice, but not a unique one. The κ_{ij} for the interpolation formula of this question part are the time-zero values, but other choices can be made for different versions of the interpolation formula.

As it stands, the interpolation formula is 0th order good at time zero and time infinity. Derive the A value that makes the formula 1st order good at time zero and then show that A goes to zero if all the κ_{ij} become identical.

c) Assume $\kappa_{ij} = \kappa_{ji}$ for all i and j, and derive an equilibrium solution for the rate equations.