Problems for a Modern Physics Course

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Las Vegas, Nevada

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Introduction

Problems for a Modern Physics Course (PMP) is a problem source book for a modern physics course. The book is available in electronic form to instructors by request to the author. It can be freely used and distributed for educational and non-commercial purposes. Express permission from the author is not required.

The problems are grouped by topics in chapters: see Contents below. The chapters correspond to the chapters of Robert Eisberg and Robert Resnick's *Quantum Physics of Atoms, Molecules, Solids, Nuclei, and Particles*. There are multiple-choice problems and full-answer problems. All the problems have will have complete suggested answers eventually. The answers may be the greatest benefit of PMP. The questions and answers can be posted on the web in pdf format.

At the end of the book is an appendix of answer tables for multiple choice questions.

PMP is currently under construction and whether it will grow to adequate size depends on whether I have any chance to teach the modern physics course again.

Everything is written in plain TEX in my own idiosyncratic style. The problems all have codes and keywords for easy selection electronically or by hand. The keywords will be on the problem code line with additional ones on the extra keyword line which may also have a reference for the problem A fortran program for selecting the problems and outputting them in quiz, assignment, and test formats is also available. Note the quiz, etc. creation procedure is a bit clonky, but it works. User instructors could easily construct their own programs for problem selection.

I would like to thank the Department of Physics & Astronomy of the University of Nevada, Las Vegas for its support for this work. Thanks also to the students who helped flight-test the problems.

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Chapt. 1 Blackbody Radiation

Multiple-Choice Problems

001 qmult 00410 1 1 4 easy memory: blackbody radiation temperature

1. The blackbody radiation spectrum depends only on the:

   a) density of the receiver. b) density of the emitter.
   c) temperature of the receiver. d) temperature of the emitter.
   e) the color of the emitter.

001 qmult 01120 2 1 2 moderate memory: density of states box quantization

2. The density of wavenumber states in wavenumber space (or k-space) per space space volume V in the continuum limit for the box-quantization system (or particle-in-a-box system) is:

   a) linear $1/V$. b) independent of V. c) linear in V. d) quadratic in V.
   e) cubic in V.

001 qmult 02010 1 4 3 easy deducto-memory: Planck idea
Extra keywords: modern physics

3. "Let's play *Jeopardy*! For \$100, the answer is: The person who first proposed that energy states of microscopic systems could form a discrete (or quantized) set instead of a continuum."

   Who is ____________, Alex?

   a) Thomas Young (1773–1829) b) Lord Rayleigh (1842–1919)
   c) Max Planck (1858–1947) d) Wilhelm Wien (1864–1928)
   e) James Jeans (1877–1946)

Full-Answer Problems

001 qfull 01020 1 3 0 easy math: wave equation

4. The standard wave equation in 1 dimension is

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} ,$$

where y is the oscillating quantity, x is the 1 space dimension, t is time, and $|v|$ is the constant phase speed of wave propagation (WA-710). Because this differential equation has more than one independent variable (it has x and t as independent variables), it is a partial differential equation.

   a) Verify that $f(x - vt)$ is a general traveling wave solution of the wave equation where $f(x)$ is any function. What is the initial condition of the solution at time zero? What is the direction of propagation of the solution? Consider the wave system as nonrelativistic and note that v can be positive or negative.

b) In quantum mechanics, it is traditional to write the argument of a 1-dimensional wave as $kx - \omega t$ (rather than $x - vt$), where k is the wavenumber and ω is the angular frequency. The ω is always taken as positive and the sign of k determines the direction of a traveling wave: $k > 0$ gives travel in the positive direction and $k < 0$ gives travel in the negative direction.

Since the wave equation is a linear equation, any two solutions can be added to give another solution. You are given two traveling wave solutions $A\sin(kx - \omega t)$ and $A\sin(-kx - \omega t)$, where A is a constant amplitude, k is a positive wave number, ω is angular frequency, and $\omega/|k| = v$, the phase speed. What is the superposition of the waves (i.e., what is their sum) and what does this superposition amount to physically. **HINT:** The trivial answer is not an answer.

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001 qfull 01310 2 3 0 moderate math: Stirling series

Extra keywords: This is needed to find the Boltzmann distribution

5. Prove the Stirling's approximation version

$$\ln(N!) \approx \left(N + \frac{1}{2}\right)\ln(N) - N + \frac{3}{2} - \frac{3}{2}\ln\left(\frac{3}{2}\right) + \frac{1}{8N} \ ,$$

where N is an integer greater than or equal to 1. For very large N (as in most of statistical mechanics), one usually uses the simpler and more memorable approximation

$$\ln(N!) \approx N\ln(N) - N \ .$$

Actually, there is a more exact Stirling's approximation. This is the real Stirling's series given by Arf-464 and WA-542. Both our Stirling approximation and the Stirling's series become more accurate as N increases. **HINT:** Write $\ln(N!)$ as a sum and approximate the sum by an analytical integral. A sketch comparing the sum in histogram form and the integrand curve helps to get the best simple choices for the integration boundaries. You will also need to a Taylor's series expansion of a form $\ln(1 + x)$ for small x.

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001 qfull 01710 2 3 0 moderate math: Planck spectrum

6. There are several different ways of presenting the Planck or blackbody spectrum. They are all equivalent in a sense, but each is most useful in some special case. The commonest one in astrophysical radiative transfer circles is probably the frequency representation of the Planck specific intensity:

$$B_\nu = \frac{2h\nu^3}{c^2}\frac{1}{\left[e^{h\nu/(kT)} - 1\right]} \ ,$$

where $h = 6.6260693(11) \times 10^{-34}$ J s is Planck's constant, $c = 2.99792458 \times 10^8$ m/s is the vacuum speed of light, ν is frequency (in hertz), $k = 1.3806505(24) \times 10^{-23}$ J/K is Boltzmann's constant, and T is temperature in kelvins. What B_ν is the energy flow per unit time per unit area (perpendicular to the flow direction) per unit frequency per unit solid angle. The energy flow in a particular frequency differential $d\nu$ is $B_\nu\, d\nu$.

Now if you want the wavelength (i.e., λ) representation, note that

$$\nu = \frac{c}{\lambda} \qquad \text{and} \qquad d\nu = -\frac{c}{\lambda^2}\, d\lambda \ .$$

For an equivalent energy flow to $B_\nu\, d\nu$ in the wavelength representation, one sets

$$B_\lambda\, d\lambda = B_\nu\, d\nu$$

from which it follows that

$$B_\lambda = B_\nu \frac{d\nu}{d\lambda} = B_\nu \frac{c}{\lambda^2} = \frac{2hc^2}{\lambda^5}\frac{1}{\left[e^{hc/(kT\lambda)} - 1\right]} \ ,$$

where we got rid of the minus sign for darn good reasons. One should really write $B_\lambda\,d\lambda = B_\nu\,d\nu$ as $B_\lambda(-d\lambda) = B_\nu\,d\nu$ to account for the fact that a differential increase in ν is a differential decrease in λ and we are trying to equate energy flows in a particular band: but no one ever does this since it looks odd: we just know enough to suppress the minus sign.

What is the energy flux F_ν (energy per unit radiating area per unit time per unit frequency [in the frequency representation]) from a surface radiating like a blackbody? Well imagine a differential patch of surface area dA with outward pointing normal vector: let the angle from normal direction be θ. The amount of area presented by dA perpendicular to a specific intensity beam flowing out at angle θ is $dA\cos\theta$ which a simple diagram will show. Thus, the differential bit of energy flux emerging from dA in differential solid angle $\sin\theta\,d\theta\,d\phi$ (where ϕ is the azimuthal angle) is

$$dF_\nu = B_\nu \cos\theta \sin\theta\,d\theta\,d\phi \ .$$

Since B_ν is angle independent we can integrate for F_ν at once:

$$F_\nu = \int_0^{2\pi}\int_0^{\pi/2} B_\nu \cos\theta\sin\theta\,d\theta\,d\phi = 2\pi\int_0^1 B_\nu\mu\,d\mu = \pi B_\nu = \frac{2\pi hc^2}{\lambda^5}\frac{1}{[e^{hc/(kT\lambda)}-1]} \ ,$$

where we have used the transformation $\mu = \cos\theta$ and $d\mu = -\sin\theta\,d\theta$. So the difference between Planck specific intensity and Planck flux is a pesky little factor of π.

What is the Planck energy density E_ν? Well specific intensity divided by c is the energy density per unit solid angle. The energy density per unit solid angle is $E_\nu/(4\pi)$ since the Planck radiation field is isotropic since it is all a thermodynamic equilibrium radiation field. The division by c can most easily be understood by writing

$$B_\nu = c\frac{E_\nu}{4\pi}$$

and saying (to oneself if no one else) the amount of energy through a bit of area dA perpendicular to the direction of flow in a time dt from a box of volume $dA\,ds$ (where s is the coordinate along the flow direction) is the energy moving in the direction of flow $[E_\nu/(4\pi)]\,dA\,ds$. If one asks for the flow per unit area per unit time (which is just B_ν), one has $E_\nu/(4\pi)]ds/dt$, but photons move at the speed of light and so $ds/dt = c$. So we get the last equation, and so one finds

$$E_\nu = \frac{4\pi}{c}B_\nu = \frac{8\pi h\nu^3}{c^2}\frac{1}{[e^{h\nu/(kT)}-1]} \ .$$

In the wavelength representation one has, of course,

$$E_\lambda = \frac{4\pi}{c}B_\lambda = \frac{8\pi hc^2}{\lambda^5}\frac{1}{[e^{hc/(kT\lambda)}-1]} \ ,$$

which is just ER-19's equation for the energy density.

Wasn't all that edifying. Now on to the problem.

NOTE: There are parts a,b.

a) Integrate

$$F_\nu = \frac{2\pi hc^2}{\lambda^5}\frac{1}{[e^{hc/(kT\lambda)}-1]}$$

over all frequency to find Stefan's Law

$$F = \sigma T^4 \ ,$$

where F is the fequency-integrated flux and $\sigma = 5.670400(40) \times 10^{-8}\,\mathrm{W/m^2/K^4}$. You should be able to find σ in terms of fundamental constants. **HINT:** Change the integration variable to $x = h\nu/(kT)$, remember the geometric series

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \qquad \text{for} \qquad |x| < 1 ,$$

note the factorial function

$$z! = \int_0 t^z e^{-t}\, dt$$

which for z a positive integer n is just $n!$ (Arf-453), and note the Riemann zeta function

$$\zeta(s) = \sum_{n=1}^{\infty} n^{-s} \qquad \text{for} \qquad s > 1$$

(Arf-282) gives $\zeta(4) = \pi^4/90$ (Arf-285).

b) Now prove the Wien displacement law:

$$\lambda_{\rm max} T = \text{constant}$$

where $\lambda_{\rm max}$ is the maximum of B_λ and the Wien constant is $2.8977685(51) \times 10^{-3}\,\mathrm{m\,K}$. Actually the constant cannot be determined exactly analytically. So find a first approximation. **HINT:** Let $x = hc/(kT\lambda)$ and find

$$\frac{dB_\lambda}{d\lambda} = \frac{dB_\lambda}{dx}\frac{dx}{dt} .$$

The maximum of B_λ occurs for $dB_\lambda/dx = 0$. Find the maximizing x value to a good first approximation and then the approximate Wien constant.

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001 qfull 02210 2 5 0 moderate thinking: Earth's effective temperature
Extra keywords: suggested by ER-23-10

7. The solar constant $S = 1366\,\mathrm{W/m^2}$ on average (Wikipedia: Solar radiation, 2008feb18). In fact, it's not exactly constant, due to varying Earth-Sun distance, sunspots, and the 11-year solar cycle. But averaged over those variations, it is really very constant which is good for life on Earth. The solar constant is the power per unit area (or energy flux) on a sphere surrounding the Sun at the Earth's distance from the Sun.

a) What is the **AVERAGE** power per unit area on the Earth? **HINT:** Remember the Earth's a rotating sphere. Think of its cross-sectional area in respect to the solar light flux and its surface area.

b) The Earth's average albedo is $A = 0.30$. The albedo is the fraction of just light **REFLECTED**. What is the average power per unit area **ABSORBED** by the Earth?

c) Assuming the Earth is a perfect blackbody radiator and is in thermal energy steady state (i.e., emits all the energy it **ABSORBS** and maintains a steady state), solve for the Earth's mean temperature. **HINT:** Power per unit area is flux. Blackbody flux is given by the Stefan-Boltzmann law.

Chapt. 2 Photons

Multiple-Choice Problems

002 qmult 00210 1 4 5 easy deducto-memory: photoelectric effect

8. "Let's play *Jeopardy*! For \$100, the answer is: It is the emission of electrons from matter caused by the absorption of photons. The effect in some sense includes photoionization as a subcategory since photoionization agrees with the definition, but other cases such as emission of non-localized electrons in materials are also included in the effect and are what one usually thinks of when one says the name of the effect."

   What is the ____________, Alex?

   a) Mössbauer effect b) Hall effect c) quantum Hall effect d) Zeeman effect
   e) photoelectric effect

002 qmult 00320 1 3 1 easy math: work function of gold

9. Given that the work function of gold (Au) is 4.8 eV, what is the maximum wavelength of light that will cause the emission of a photoelectron? **HINT:** $hc = 12398.419\,\text{eV}\,\text{Å}$.

   a) 2600 Å. b) 3000 Å. c) 5000 Å. d) 7000 Å. e) 10000 Å.

002 qmult 00430 1 1 3 easy memory: Compton equation
Extra keywords: ER-37

10. The Compton equation can be derived using the photon picture of electromagnetic radiation and:

    a) the photoelectric effect. b) classical energy and momentum conservation laws.
    c) relativistic energy and momentum conservation laws. d) the Planck spectrum.
    e) the Einstein equation $E = mc^2$.

002 qmult 00440 1 3 5 easy math: proton Compton wavelength

11. The standard Compton equation is

$$\Delta\lambda = \lambda_{\text{scat}} - \lambda_{\text{inc}} = \lambda_{\text{C}}(1 - \cos\theta) ,$$

where λ_{scat} is the wavelength of the scattered photon, λ_{inc} is the wavelength of the incident photon, θ is the scattering angle (i.e., the angle between the incident and scattering directions), and λ_{C} is the Compton wavelength. Note that

$$\lambda_{\text{C}} = \frac{h}{m_e c} = 2.426310238(16) \times 10^{-12}\,\text{m}$$

where h is Planck's constant, m_e is the electron mass, and c is the speed of light. Compton scattering by protons can occur too. What is the proton Compton wavelength?

   a) $2.426310238(16) \times 10^{-15}\,\text{m} \approx 24.3\,\text{fm}$. b) $2.426310238(16) \times 10^{-15}\,\text{m} \approx 2.4\,\text{fm}$.
   c) $2.426310238(16) \times 10^{-12}\,\text{m} \approx 0.024\,\text{Å}$. d) $1.321409855 \times 10^{-14}\,\text{m} \approx 13.2\,\text{fm}$.
   e) $1.321409855 \times 10^{-15}\,\text{m} \approx 1.3\,\text{fm}$.

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002 qmult 00750 1 4 4 easy deducto-memory: positronium

12. "Let's play *Jeopardy*! For \$100, the answer is: A bound state of matter which is usually formed by a positron on its way to annihilation with an electron. It has a mean lifetime of 1.25×10^{-10} s if it forms in the singlet ground state."

    What is ____________, Alex?

    a) pragmatium b) plutonium c) protonium d) positronium e) protesium

Full-Answer Problems

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002 qfull 00390 2 3 0 moderate math: gold foil photoelectric effect
Extra keywords: ER-52

13. X-rays eject photoelectrons from a particular thin gold foil.

    a) The X-rays have wavelength 0.710 Å. What is the energy of an individual photons in units of joules, Kilo-electron-volts (KeV), and $m_e c^2 = 510.998910(13)$ KeV?

    b) The electrons are directed into a region of uniform magnetic field $\vec{B}$ and go into **UNIFORM CIRCULAR MOTION** with radius r as determined by the magnetic force. The observed **MAXIMUM** value of rB (sometimes called the magnetic rigidity [Go-318]) is 1.88×10^{-4} T m where T m are tesla-meters. Find the maximum kinetic energy of the photoelectrons in KeV. **HINT:** Recall the Lorentz force (which includes the magnetic force) is given by

    $$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) ,$$

    where $\vec{E} = 0$ in our case, and the centripetal force magnitude by

    $$F = m\frac{v^2}{r} .$$

    c) What is the minimum work in KeV done by the X-rays in ejecting the photoelectrons? **HINT:** It is not the work function of thick gold sample. That is 4.8 eV (ER-408).

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002 qfull 00830 1 3 0 easy math: photon probability density
Extra keywords: ER-52-8

14. You are given that the probability density for photon removal from a beam along a beam path is

    $$\rho(s) = \frac{e^{-s/\ell}}{\ell} ,$$

    where s is the path coordinate from some initial position and ℓ turns out to be the mean free path.

    NOTE: There are parts a,b,c,d

    a) Derive the probability for removal by point s.

    b) Derive the probability for survival to point s.

    c) Derive the general formula for the moments of the probability distribution. Give special-case results for moments for powers $m = 0, 1, 2$. What is moment for power $m = 1$ called? **HINT:** You will need the factorial function

    $$z! = \int_0^\infty t^z e^{-t}\, dt .$$

    d) Derive the standard deviation formula for the probability density.

Chapt. 3 De Broglie's Postulate and Matter Waves

Multiple-Choice Problems

Full-Answer Problems

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003 qfull 00500 3 3 0 tough math: phase velocity, group velocity

15. It's embarrassing thing in elemetary quantum mechanics to admit that the momentum eigenstates or wavenumber eigenstates cannot be normalized. The two eigenstates are the same thing since a momentum eigenvalue p is equal to $\hbar k$ where k is the wavenumber eigenvalue. This means that no particle can every actually be in a wavenumber eigenstate or have a definite wavenumber eigenvalue. A particle can only ever be in superpositions of eigenstates.

    a) The normalization condition for a wave function Ψ is that

    $$\int_{-\infty}^{\infty} \Psi^* \Psi \, dx$$

    be a finite, non-zero number. If this is the case, then one can normalize Ψ by multiplying it by a constant such that one obtains

    $$1 = \int_{-\infty}^{\infty} \Psi^* \Psi \, dx \ .$$

    Since $\Psi^* \Psi$ is a probability density, normnalizability means that the probability of finding a particle somewhere is 1 as logic dictates. The wavenumber eigenstates are given by

    $$\Psi_k(x,t) = \frac{e^{i(kx-\omega t)}}{\sqrt{2\pi}} \ ,$$

    where t is time, $\omega = E/\hbar$ is angular frequency, and the $1/\sqrt{2\pi}$ is a conventional factor. Show that these eigenstates cannot be normalized.

    b) An actual general wave function for a free particle $\Psi(x,t)$ can be expanded in a superposition of wavenumber eigenstates:

    $$\Psi(x,t) = \int_{-\infty}^{\infty} \Phi(k) \Psi_k(x,t) \, dk \ ,$$

    where $\Phi(k)$ is a function in k-space. This $\Psi(x,t)$ is called a wave packet. Now $\Phi(k)$ is actually the Fourier transform of $\Psi(x,0)$. By Plancherel's theorem $\Psi(x,0)$ is the Fourier transform of $\Phi(x,0)$:

    $$\Phi(k) = \int_{-\infty}^{\infty} \Psi(x,0) \frac{e^{-ikx}}{\sqrt{2\pi}} \, dk \ ,$$

Now the phase velocity of any wavenumber eigenstate is

$$v = \frac{\omega}{k} = \frac{E}{p} = \frac{\hbar k^2}{2m} \ .$$

But the classical velocity for a particle with energy E and momentum p is

$$v_{\rm clas} = \frac{2E}{p} = 2v \ .$$

There is a strange paradox here. This can be resolved by considering the concept of group velocity. Assume $\Phi(k)$ is sharply peaked around k_0 for a wave packet. This actually means that $\Psi(x,0)$ is broad about the mean value of x. But this not a limitation since it turns out that the idea of a group isn't well defined for sharply peaked $\Psi(x,0)$ since the wave packet spreads out so quickly. Since $\Phi(k)$ is sharply peaked around k_0 we can Taylor's series expand $\omega(k)$ to first order in k. Do this and write the approximate expression for the wave packet in terms of the function

$$e^{-i(\omega_0-\omega_0' k_0)t} \ .$$

c) Now one of the rules (i.e., micro-postulates) of quantum mechanics is that the physics cannot be changed by a global phase factor in the wave equation: i.e., a factor A of the whole wave function that satisfies

$$A^*A = 1 \ .$$

Use this rule to simplify the expression obtained in the part (b) answer and show that

$$\Psi(x,t) \approx \Psi(x-\omega_0' t, 0) \ .$$

d) What is the "phase velocity" of the wave packet result in part (c) answer. This group "phase velocity" is the group velocity $v_{\rm g}$ for the wave packet. Show that the group velocity is the classical velocity one would expect classically for a particle of momentum $p_0 = \hbar k_0$.

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003 qfull 00510 2 3 0 moderate math: Dopper shift matter wave

16. Something that is never discussed in quantum texts (as far the instructor can tell) is the non-relativistic Doppler shift for matter waves. Perhaps this because one can always just work it for oneself.

a) Given the de Broglie law

$$\lambda = \frac{h}{p}$$

and that one makes frame transformation to a frame with velocity v_0 relative to the initial frame, find the transformation expressions for λ, k, and momentum. The problem is all 1-dimensional. Use prime symbols to indicate quantities in the new frame.

b) Now show that the group velocity transforms consistently: i.e.,

$$v_{\rm g}'|_{k_1'} = v_{\rm g}|_{k_1} - v_0$$

is obtained when one evaluates the group velocity in the primed frame. The subscript "1" denotes the central wavenumber of the wave packet in this case.

Chapt. 4 The Bohr Atom

Multiple-Choice Problems

Full-Answer Problems

Chapt. 5 Schrödinger's Equation and Non-Relativistic Quantum Mechanics

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Multiple-Choice Problems

Full-Answer Problems

Chapt. 6 Applications of One-Dimensional NR Quantum Mechanics

Multiple-Choice Problems

Full-Answer Problems

006 qfull 00720 2 3 0 moderate math: infinie square well features

17. The one-dimensional infinite square well with a symmetric potential and width a is

$$V = \begin{cases} 0 & \text{for } |x| \leq a/2; \\ \infty & \text{for } |x| > a/2. \end{cases}$$

The eigenstates for infinite square well are given by

$$\psi_n(x) = \sqrt{\frac{2}{a}} \times \begin{cases} \cos(kx) & \text{for } n = 1, 3, 5 \ldots; \\ \sin(kx) & \text{for } n = 2, 4, 6 \ldots, \end{cases}$$

where

$$k = \frac{n\pi}{a} \qquad \text{and} \frac{ka}{2} = \frac{n\pi}{2} \ .$$

The n is the quantum number for eigenstates. The eigenstates have been normalized and are guaranteed orthogonal by the mathematics of Hermitian operators of the which the Hamiltonian is one. A quantum number is a dimensionless index (usually integer or half-integer) that specifies the eigenstates and eigenvalues somehow. The eigen-energies are given by

$$E_n = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2}{2m}\left(\frac{\pi}{a}\right)^2 n^2 \ .$$

a) Verify the normalization of eigenstates.

b) Determine $\langle x \rangle$ for the eigenstates.

c) Determine $\langle p_{\text{op}} \rangle$ for the eigenstates. **HINT:** Recall

$$p_{\text{op}} = \frac{\hbar}{i}\frac{\partial}{\partial x} \ .$$

d) Determine $\langle p_{\text{op}}^2 \rangle$ and the momentum standard deviation σ_{p} for the eigenstates.

e) Determine $\langle x^2 \rangle$ and the position standard deviation σ_x in the large n limit. **HINT:** Assume x^2 can be approximated constant over one complete cycle of the probability density $\psi_n^* \psi_n$

f) Now for the boring part. Determine $\langle x^2 \rangle$ and the position standard deviation σ_x exactly now. **HINT:** There probably are several different ways of doing this, but there seem to be no quick tricks to the answer. The indefinite integral

$$\int x^2 \cos(bx)\, dx = \frac{x^2}{b}\sin(bx) + \frac{2}{b^2} x\cos(bx) - \frac{2}{b^3}\sin(bx)$$

might be helpful.

g) Verify that the Heisenberg uncertainty principle

$$\Delta x \Delta p = \sigma_x \sigma_p \geq \frac{\hbar}{2}$$

is satisfied for the infinite square well case.

Chapt. 7 The Hydrogenic Atom

Multiple-Choice Problems

Full-Answer Problems

Equation Sheet for Modern Physics

These equation sheets are intended for students writing tests or reviewing material. Therefore they are neither intended to be complete nor completely explicit. There are fewer symbols than variables, and so some symbols must be used for different things: context must distinguish.

The equations are mnemonic. Students are expected to understand how to interpret and use them.

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18 Constants

$$
\begin{aligned}
c &= 2.99792458 \times 10^8\,\text{m/s} \approx 2.998 \times 10^8\,\text{m/s} \approx 3 \times 10^8\,\text{m/s} \approx 1\,\text{lyr/yr} \approx 1\,\text{ft/ns} \\
e &= 1.602176487(40) \times 10^{-19}\,\text{C} \\
E_{\text{Rydberg}} &= 13.60569193(34)\,\text{eV} \\
g_e &= 2.0023193043622 \qquad \text{(electron g-factor)} \\
h &= 6.62606896(33) \times 10^{-34}\,\text{J s} = 4.13566733(10) \times 10^{-15}\,\text{eV s} \\
hc &= 12398.419\,\text{eV Å} \approx 10^4\,\text{eV Å} \\
\hbar &= 1.054571628(53) \times 10^{-34}\,\text{J s} = 6.58211899(16) \times 10^{-16}\,\text{eV s} \\
k &= 1.3806504(24) \times 10^{-23}\,\text{J/K} = 0.8617343(15) \times 10^{-4}\,\text{eV/K} \approx 10^{-4}\,eV/K \\
m_e &= 9.10938215(45) \times 10^{-31}\,\text{kg} = 0.510998910(13)\,\text{MeV} \\
m_p &= 1.672621637(83) \times 10^{-27}\,\text{kg} = 938.272013(23), \text{MeV} \\
\alpha &= e^2/(4\pi\epsilon_0\hbar c) = 7.2973525376(50) \times 10^{-3} = 1/137.035999679(94) \approx 1/137 \\
\lambda_{\text{C}} &= h/(m_e c) = 2.4263102175(33) \times 10^{-12}\,\text{m} = 0.0024263102175(33)\,\text{Å} \\
\mu_{\text{B}} &= 5.7883817555(79) \times 10^{-5}\,\text{eV/T}
\end{aligned}
$$

---

19 Geometrical Formulae

$$
C_{\text{cir}} = 2\pi r \qquad A_{\text{cir}} = \pi r^2 \qquad A_{\text{sph}} = 4\pi r^2 \qquad V_{\text{sph}} = \frac{4}{3}\pi r^3
$$

---

20 Trigonometry

$$
\frac{x}{r} = \cos\theta \qquad \frac{y}{r} = \sin\theta \qquad \frac{y}{x} = \tan\theta \qquad \cos^2\theta + \sin^2\theta = 1
$$

$$
\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b) \qquad \cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)
$$

$$
\cos^2\theta = \frac{1}{2}[1 + \cos(2\theta)] \qquad \sin^2\theta = \frac{1}{2}[1 - \cos(2\theta)] \qquad \sin(2\theta) = 2\sin(\theta)\cos(\theta)
$$

$$
\cos(a)\cos(b) = \frac{1}{2}\left[\cos(a-b) + \cos(a+b)\right] \qquad \sin(a)\sin(b) = \frac{1}{2}\left[\cos(a-b) - \cos(a+b)\right]
$$

$$\sin(a)\cos(b) = \frac{1}{2}\left[\sin(a-b) + \sin(a+b)\right]$$

---

21 **Blackbody Radiation**

$$B_\nu = \frac{2h\nu^3}{c^2}\frac{1}{\left[e^{h\nu/(kT)} - 1\right]} \qquad B_\lambda = \frac{2hc^2}{\lambda^5}\frac{1}{\left[e^{hc/(kT\lambda)} - 1\right]}$$

$$B_\lambda\,d\lambda = B_\nu\,d\nu \qquad \nu\lambda = c \qquad \frac{d\nu}{d\lambda} = -\frac{c}{\lambda^2}$$

$$E = h\nu = \frac{hc}{\lambda} \qquad p = \frac{h}{\lambda}$$

$$F = \sigma T^4 \qquad \sigma = \frac{2\pi^5}{15}\frac{k^4}{c^2h^3} = 5.670400(40)\times 10^{-8}\,\mathrm{W/m^2/K^4}$$

$$\lambda_{\max}T = \text{constant} = \frac{hc}{kx_{\max}} \approx \frac{1.4387751\times 10^{-2}}{x_{\max}}$$

$$B_{\lambda,\mathrm{Wien}} = \frac{2hc^2}{\lambda^5}e^{-hc/(kT\lambda)} \qquad B_{\lambda,\mathrm{Rayleigh-Jeans}} = \frac{2ckT}{\lambda^4}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{c}\nu = \frac{\omega}{c} \qquad k_i = \frac{\pi}{L}n_i \quad \text{standing wave BCs} \qquad k_i = \frac{2\pi}{L}n_i \quad \text{periodic BCs}$$

$$n(k)\,dk = \frac{k^2}{\pi^2}\,dk = \pi\left(\frac{2}{c}\right)\nu^2\,d\nu = n(\nu)\,d\nu$$

$$\ln(z!) \approx \left(z + \frac{1}{2}\right)\ln(z) - z + \frac{1}{2}\ln(2\pi) + \frac{1}{12z} - \frac{1}{360z^3} + \frac{1}{1260z^5} - \ldots$$

$$\ln(N!) \approx N\ln(N) - N$$

$$\rho(E)\,dE = \frac{e^{-E/(kT)}}{kT}\,dE \qquad P(n) = (1 - e^{-\alpha})e^{-n\alpha} \qquad \alpha = \frac{h\nu}{kT}$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \qquad f(x - vt) \qquad f(kx - \omega t)$$

22 Photons

$$KE = h\nu - w \qquad \Delta\lambda = \lambda_{\rm scat} - \lambda_{\rm inc} = \lambda_{\rm C}(1-\cos\theta)$$

$$\ell = \frac{1}{n\sigma} \qquad \rho = \frac{e^{-s/\ell}}{\ell} \qquad \langle s^m \rangle = \ell^m m!$$

23 Matter Waves

$$\lambda = \frac{h}{p} \qquad p = \hbar k \qquad \Delta x \Delta p \geq \frac{\hbar}{2} \qquad \Delta E \Delta t \geq \frac{\hbar}{2}$$

$$\Psi(x,t) = \int_{-\infty}^{\infty} \phi(k)\Psi_k(x,t)\,dk \qquad \phi(k) = \int_{-\infty}^{\infty} \Psi(x,0)\frac{e^{-ikx}}{\sqrt{2\pi}}\,dk$$

$$v_{\rm g} = \left.\frac{d\omega}{dk}\right|_{k_0} = \frac{\hbar k_0}{m} = \frac{p_0}{m} = v_{\rm clas,0}$$

24 Non-Relativistic Quantum Mechanics

$$H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V \qquad T = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} \qquad H\Psi = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V\Psi = i\hbar\frac{\partial\Psi}{\partial t}$$

$$\rho = \Psi^*\Psi \qquad \rho\,dx = \Psi^*\Psi\,dx$$

$$A\phi_i = a_i\phi_i \qquad f(x) = \sum_i c_i\phi_i \qquad \int_a^b \phi_i^*\phi_j\,dx = \delta_{ij} \qquad c_j = \int_a^b \phi_j^* f(x)\,dx$$

$$[A,B] = AB - BA$$

$$P_i = |c_i|^2 \qquad \langle A \rangle = \int_{-\infty}^{\infty} \Psi^* A \Psi\,dx = \sum_i |c_i|^2 a_i \qquad H\psi = E\psi \qquad \Psi(x,t) = \psi(x)e^{-i\omega t}$$

$$p_{\rm op}\phi = \frac{\hbar}{i}\frac{\partial\phi}{\partial x} = p\phi \qquad \phi = \frac{e^{ikx}}{\sqrt{2\pi}} \qquad \frac{\partial^2\psi}{\partial x^2} = \frac{2m}{\hbar^2}(V-E)\psi$$

$$|\Psi\rangle \qquad \langle\Psi| \qquad \langle x|\Psi\rangle = \Psi(x) \qquad \langle\vec{r}|\Psi\rangle = \Psi(\vec{r}) \qquad \langle k|\Psi\rangle = \Psi(k) \qquad \langle\Psi_i|\Psi_j\rangle = \langle\Psi_j|\Psi_i\rangle^*$$

$$\langle\phi_i|\Psi\rangle = c_i \qquad 1_{\text{op}} = \sum_i |\phi_i\rangle\langle\phi_i| \qquad |\Psi\rangle = \sum_i |\phi_i\rangle\langle\phi_i|\Psi\rangle = \sum_i c_i|\phi_i\rangle$$

$$1_{\text{op}} = \int_{-\infty}^{\infty} dx\,|x\rangle\langle x| \qquad \langle\Psi_i|\Psi_j\rangle = \int_{-\infty}^{\infty} dx\,\langle\Psi_i|x\rangle\langle x|\Psi_j\rangle \qquad A_{ij} = \langle\phi_i|A|\phi_j\rangle$$

$$Pf(x) = f(-x) \qquad P\frac{df(x)}{dx} = \frac{df(-x)}{d(-x)} = -\frac{df(-x)}{dx} \qquad Pf_{\text{e/o}}(x) = \pm f_{\text{e/o}}(x)$$

$$P\frac{df_{\text{e/o}}(x)}{dx} = \mp\frac{df_{\text{e/o}}(x)}{dx}$$

---

25 **Spherical Harmonics**

$$Y_{0,0} = \frac{1}{\sqrt{4\pi}} \qquad Y_{1,0} = \left(\frac{3}{4\pi}\right)^{1/2}\cos(\theta) \qquad Y_{1,\pm1} = \mp\left(\frac{3}{8\pi}\right)^{1/2}\sin(\theta)e^{\pm i\phi}$$

$$L^2 Y_{\ell m} = \ell(\ell+1)\hbar^2 Y_{\ell m} \qquad L_z Y_{\ell m} = m\hbar Y_{\ell m} \qquad |m| \le \ell \qquad m = -\ell, -\ell+1, \ldots, \ell-1, \ell$$

0	1	2	3	4	5	6	...
s	p	d	f	g	h	i	...

---

26 **Hydrogenic Atom**

$$\psi_{n\ell m} = R_{n\ell}(r)Y_{\ell m}(\theta,\phi) \qquad \ell \le n-1 \qquad \ell = 0, 1, 2, \ldots, n-1$$

$$a_z = \frac{a_0}{Z}\left(\frac{m_e}{m_{\text{reduced}}}\right) \qquad a_0 = \frac{\hbar}{m_e c\alpha} = \frac{\lambda_{\text{C}}}{2\pi\alpha} \qquad m_{\text{reduced}} = \frac{m_1 m_2}{m_1 + m_2}$$

$$R_{10} = 2a_Z^{-3/2}e^{-r/a_Z} \qquad R_{20} = \frac{1}{\sqrt{2}}a_Z^{-3/2}\left(1 - \frac{1}{2}\frac{r}{a_Z}\right)e^{-r/(2a_Z)}$$

$$R_{21} = \frac{1}{\sqrt{24}}a_Z^{-3/2}\frac{r}{a_Z}e^{-r/(2a_Z)}$$

$$R_{n\ell} = -\left\{ \left(\frac{2}{na_Z} \right)^3 \frac{(n-\ell-1)!}{2n[(n+\ell)!]^3} \right\}^{1/2} e^{-\rho/2} \rho^\ell L_{n+\ell}^{2\ell+1}(\rho) \qquad \rho = \frac{2r}{na_Z}$$

$$L_q(x) = e^x \left(\frac{d}{dx} \right)^q \left(e^{-x} x^q \right) \quad \text{Rodrigues's formula for the Laguerre polynomials}$$

$$L_q^j(x) = \left(\frac{d}{dx} \right)^j L_q(x) \qquad \text{Associated Laguerre polynomials}$$

$$\langle r \rangle_{n\ell m} = \frac{a_Z}{2} \left[3n^2 - \ell(\ell+1) \right]$$

$$\text{Nodes} = (n-1) - \ell \qquad \text{not counting zero or infinity}$$

$$E_n = -\frac{1}{2} m_e c^2 \alpha^2 \frac{Z^2}{n^2} \frac{m_{\text{reduced}}}{m_e} = -E_{\text{Ryd}} \frac{Z^2}{n^2} \frac{m_{\text{reduced}}}{m_e} \approx -13.606 \times \frac{Z^2}{n^2} \frac{m_{\text{reduced}}}{m_e} \text{ eV}$$

---

27 Spin, Magnetic Dipole Moment, Spin-Orbit Interaction

$$S_{\text{op}}^2 = \frac{3}{4} \hbar \begin{pmatrix} 1 & 0 \\ 01 & \end{pmatrix} \qquad s = \frac{1}{2} \qquad s(s+1) = \frac{3}{4} \qquad S = \sqrt{s(s+1)}\hbar = \frac{\sqrt{3}}{2}\hbar$$

$$S_{z,\text{op}} = \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad m_s = \pm\frac{1}{2} \qquad \chi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \chi_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\mu_{\text{b}} = \frac{e\hbar}{2m_e} = 9.27400915(26) \times 10^{-24}\,\text{J/T} = 5.7883817555(79) \times 10^{-5}\,\text{eV/T}$$

$$\mu_{\text{nuclear}} = \frac{e\hbar}{2m_p} = 5.05078324(13) \times 10^{-27}\,\text{J/T} = 3.1524512326(45) \times 10^{-8}\,\text{eV/T}$$

$$\vec{\mu}_\ell = -g_\ell \mu_{\text{b}} \frac{\vec{L}}{\hbar} \qquad \mu_\ell = g_\ell \mu_{\text{b}} \ell(\ell+1) \qquad \mu_{\ell,z} = -g_\ell \mu_{\text{b}} \frac{L_z}{\hbar} \qquad \mu_{\ell,z} = -g_\ell \mu_{\text{b}} m_\ell$$

$$\vec{\tau} = \vec{\mu} \times \vec{B} \qquad PE = -\vec{\mu} \cdot \vec{B} \qquad \vec{F} = \Delta(\vec{\mu} \cdot \vec{B}) \qquad F_z = \sum_j \mu_j \frac{\partial B_j}{\partial z} \qquad \vec{\omega} = \frac{g_\ell \mu_{\text{b}}}{\hbar} \vec{B}$$

$$\vec{J} = \vec{L} + \vec{S} \qquad J = \sqrt{j(j+1)}\hbar \qquad j = |\ell - s|, |\ell - s + 1|, \ldots, \ell + s \quad \text{triangle rule}$$

$$J_z = m_j\hbar \qquad m_j = -j, -j+1, \ldots, j-1, j$$

$$E(n,\ell,\pm 1/2, j) = -\frac{E_{\rm Ryd}}{n^2}\frac{m}{m_e}\left[1 + \frac{\alpha^2}{n^2}\left(\frac{n}{j+1/2} - \frac{3}{4}\right)\right]$$

28 **Special Relativity**

$$c = 2.99792458 \times 10^8\,\mathrm{m/s} \approx 2.998 \times 10^8\,\mathrm{m/s} \approx 3 \times 10^8\,\mathrm{m/s} \approx 1\,\mathrm{lyr/yr} \approx 1\,\mathrm{ft/ns}$$

$$\beta = \frac{v}{c} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}} \qquad \gamma(\beta << 1) = 1 + \frac{1}{2}\beta^2 \qquad \tau = ct$$

Galilean Transformations	Lorentz Transformations
$x' = x - \beta\tau$	$x' = \gamma(x - \beta\tau)$
$y' = y$	$y' = y$
$z' = z$	$z' = z$
$\tau' = \tau$	$\tau' = \gamma(\tau - \beta x)$
$\beta'_{\rm obj} = \beta_{\rm obj} - \beta$	$\beta'_{\rm obj} = \dfrac{\beta_{\rm obj} - \beta}{1 - \beta\beta_{\rm obj}}$

$$\ell = \ell_{\rm proper}\sqrt{1-\beta^2} \qquad \Delta\tau_{\rm proper} = \Delta\tau\sqrt{1-\beta^2}$$

$$m = \gamma m_0 \qquad p = mv = \gamma m_0 c\beta \qquad E_0 = m_0c^2 \qquad E = \gamma E_0 = \gamma m_0 c^2 = mc^2$$

$$E = mc^2 \qquad E = \sqrt{(pc)^2 + (m_0c^2)^2}$$

$$KE = E - E_0 = \sqrt{(pc)^2 + (m_0c^2)^2} - m_0c^2 = (\gamma - 1)m_0c^2$$

$$f = f_{\rm proper}\sqrt{\frac{1-\beta}{1+\beta}} \quad \text{for source and detector separating}$$

$$f(\beta << 1) = f_{\rm proper}\left(1 - \beta + \frac{1}{2}\beta^2\right)$$

$$f_{\rm trans} = f_{\rm proper}\sqrt{1-\beta^2} \qquad f_{\rm trans}(\beta << 1) = f_{\rm proper}\left(1 - \frac{1}{2}\beta^2\right)$$

$$\tau = \beta x + \gamma^{-1}\tau' \quad \text{for lines of constant } \tau'$$

$$\tau = \frac{x - \gamma^{-1}x'}{\beta} \quad \text{for lines of constant } x'$$

$$x' = \frac{x_{\text{intersection}}}{\gamma} = x'_{x\ \text{scale}}\sqrt{\frac{1-\beta^2}{1+\beta^2}} \qquad \tau' = \frac{\tau_{\text{intersection}}}{\gamma} = \tau'_{\tau\ \text{scale}}\sqrt{\frac{1-\beta^2}{1+\beta^2}}$$

$$\theta_{\text{Mink}} = \tan^{-1}(\beta)$$

Appendix 8 Multiple-Choice Problem Answer Tables

Note: For those who find scantrons frequently inaccurate and prefer to have their own table and marking template, the following are provided. I got the template trick from Neil Huffacker at University of Oklahoma. One just punches out the right answer places on an answer table and overlays it on student answer tables and quickly identifies and marks the wrong answers

Answer Table for the Multiple-Choice Questions

	a	b	c	d	e		a	b	c	d	e
29.	O	O	O	O	O	6.	O	O	O	O	O
30.	O	O	O	O	O	7.	O	O	O	O	O
31.	O	O	O	O	O	8.	O	O	O	O	O
32.	O	O	O	O	O	9.	O	O	O	O	O
33.	O	O	O	O	O	10.	O	O	O	O	O

Answer Table for the Multiple-Choice Questions

	a	b	c	d	e
34.	O	O	O	O	O
35.	O	O	O	O	O
36.	O	O	O	O	O
37.	O	O	O	O	O
38.	O	O	O	O	O
39.	O	O	O	O	O
40.	O	O	O	O	O
41.	O	O	O	O	O
42.	O	O	O	O	O
43.	O	O	O	O	O

	a	b	c	d	e
11.	O	O	O	O	O
12.	O	O	O	O	O
13.	O	O	O	O	O
14.	O	O	O	O	O
15.	O	O	O	O	O
16.	O	O	O	O	O
17.	O	O	O	O	O
18.	O	O	O	O	O
19.	O	O	O	O	O
20.	O	O	O	O	O

Answer Table for the Multiple-Choice Questions

	a	b	c	d	e		a	b	c	d	e
44.	O	O	O	O	O	16.	O	O	O	O	O
45.	O	O	O	O	O	17.	O	O	O	O	O
46.	O	O	O	O	O	18.	O	O	O	O	O
47.	O	O	O	O	O	19.	O	O	O	O	O
48.	O	O	O	O	O	20.	O	O	O	O	O
49.	O	O	O	O	O	21.	O	O	O	O	O
50.	O	O	O	O	O	22.	O	O	O	O	O
51.	O	O	O	O	O	23.	O	O	O	O	O
52.	O	O	O	O	O	24.	O	O	O	O	O
53.	O	O	O	O	O	25.	O	O	O	O	O
54.	O	O	O	O	O	26.	O	O	O	O	O
55.	O	O	O	O	O	27.	O	O	O	O	O
56.	O	O	O	O	O	28.	O	O	O	O	O
57.	O	O	O	O	O	29.	O	O	O	O	O
58.	O	O	O	O	O	30.	O	O	O	O	O

NAME:

Answer Table for the Multiple-Choice Questions

	a	b	c	d	e		a	b	c	d	e
59.	O	O	O	O	O	26.	O	O	O	O	O
60.	O	O	O	O	O	27.	O	O	O	O	O
61.	O	O	O	O	O	28.	O	O	O	O	O
62.	O	O	O	O	O	29.	O	O	O	O	O
63.	O	O	O	O	O	30.	O	O	O	O	O
64.	O	O	O	O	O	31.	O	O	O	O	O
65.	O	O	O	O	O	32.	O	O	O	O	O
66.	O	O	O	O	O	33.	O	O	O	O	O
67.	O	O	O	O	O	34.	O	O	O	O	O
68.	O	O	O	O	O	35.	O	O	O	O	O
69.	O	O	O	O	O	36.	O	O	O	O	O
70.	O	O	O	O	O	37.	O	O	O	O	O
71.	O	O	O	O	O	38.	O	O	O	O	O
72.	O	O	O	O	O	39.	O	O	O	O	O
73.	O	O	O	O	O	40.	O	O	O	O	O
74.	O	O	O	O	O	41.	O	O	O	O	O
75.	O	O	O	O	O	42.	O	O	O	O	O
76.	O	O	O	O	O	43.	O	O	O	O	O
77.	O	O	O	O	O	44.	O	O	O	O	O
78.	O	O	O	O	O	45.	O	O	O	O	O
79.	O	O	O	O	O	46.	O	O	O	O	O
80.	O	O	O	O	O	47.	O	O	O	O	O
81.	O	O	O	O	O	48.	O	O	O	O	O
82.	O	O	O	O	O	49.	O	O	O	O	O
83.	O	O	O	O	O	50.	O	O	O	O	O

Answer Table

	a	b	c	d	e
84.	O	O	O	O	O
85.	O	O	O	O	O
86.	O	O	O	O	O
87.	O	O	O	O	O
88.	O	O	O	O	O
89.	O	O	O	O	O
90.	O	O	O	O	O
91.	O	O	O	O	O
92.	O	O	O	O	O
93.	O	O	O	O	O
94.	O	O	O	O	O
95.	O	O	O	O	O
96.	O	O	O	O	O
97.	O	O	O	O	O
98.	O	O	O	O	O
99.	O	O	O	O	O
100.	O	O	O	O	O
101.	O	O	O	O	O
102.	O	O	O	O	O
103.	O	O	O	O	O
104.	O	O	O	O	O
105.	O	O	O	O	O
106.	O	O	O	O	O
107.	O	O	O	O	O
108.	O	O	O	O	O
109.	O	O	O	O	O
110.	O	O	O	O	O
111.	O	O	O	O	O
112.	O	O	O	O	O
113.	O	O	O	O	O

Name:

	a	b	c	d	e
31.	O	O	O	O	O
32.	O	O	O	O	O
33.	O	O	O	O	O
34.	O	O	O	O	O
35.	O	O	O	O	O
36.	O	O	O	O	O
37.	O	O	O	O	O
38.	O	O	O	O	O
39.	O	O	O	O	O
40.	O	O	O	O	O
41.	O	O	O	O	O
42.	O	O	O	O	O
43.	O	O	O	O	O
44.	O	O	O	O	O
45.	O	O	O	O	O
46.	O	O	O	O	O
47.	O	O	O	O	O
48.	O	O	O	O	O
49.	O	O	O	O	O
50.	O	O	O	O	O
51.	O	O	O	O	O
52.	O	O	O	O	O
53.	O	O	O	O	O
54.	O	O	O	O	O
55.	O	O	O	O	O
56.	O	O	O	O	O
57.	O	O	O	O	O
58.	O	O	O	O	O
59.	O	O	O	O	O
60.	O	O	O	O	O