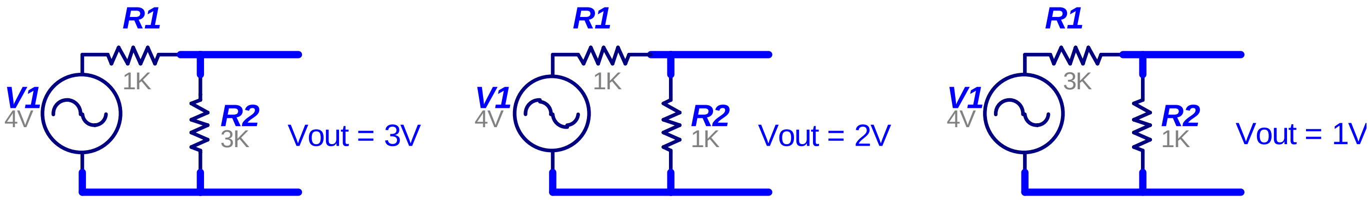
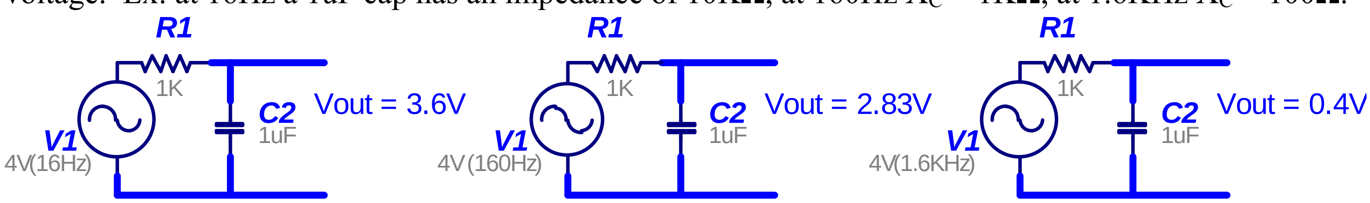
RC Filters

Voltage Divider Review. A voltage divider lowers the output voltage by the ratio of **R2/(R1+R2)**. In the left example the output is ¾ of the input voltage. Similarly the next two have outputs of ½ and ¾ of input. Since the resistors impedance doesn't change with frequency the output voltage will be the same for all frequencies.



Recall that a capacitors reactance (its complex impedance) changes with frequency. A capacitors reactance decreases with increasing frequency, $\mathbf{X_C = 1/(j2\pi fC)}$. Low pass and high pass filters can be made by replacing one of the voltage divider resistors with a capacitor. A low pass filter is shown below. At low frequencies the capacitor is an open circuit and Vout = Vin. As frequency increases the capacitor impedance drops and more voltage is dropped across R1 and less across C2 thus lowering the output voltage. Ex: at 16Hz a 1uF cap has an impedance of 10KΩ, at 160Hz X_C = 1KΩ, at 1.6KHz X_C = 100Ω.



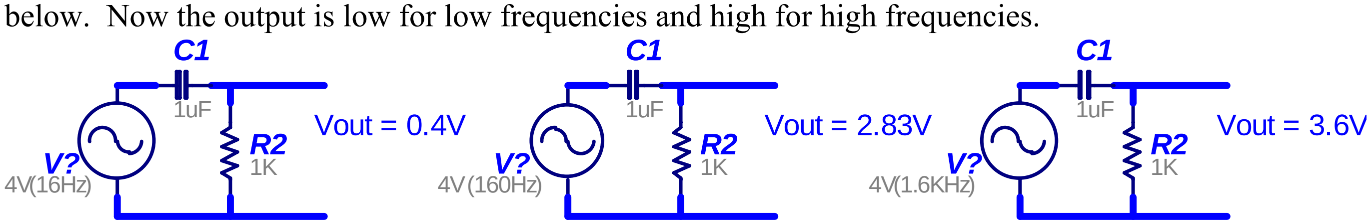
You should be asking yourself; if at 160Hz the capacitor has the same impedance as the resistor why isn't Vout = 2V. It's because the current in the capacitor is not in phase with the voltage across the capacitor. Ex: if you put a sine wave across a resistor; as the voltage increases the current increases at the same time (i.e. in phase). The current in a capacitor is: I = CdV/dt. The current leads the voltage by 90 degrees. Think of charging a capacitor. The current must flow into the capacitor (charging it) before the voltage on the capacitor can increase. If you want to calculate the output voltage and phase you can use the two equations below.

The gain or frequency response of the low pass filter is: (Vout/Vin) = $\dfrac{1}{\sqrt{1+(\omega RC)^2}}$

The phase of the low pass filter (output voltage with respect to the input voltage) is: $\phi = -Tan^{-1}\omega RC$

This link explains Low Pass Filters: http://hyperphysics.phy-astr.gsu.edu/hbase/electric/filcap2.html
Note: You don't need to calculate the exact frequency response of the filter. Just be aware you the gain and phase change with frequency and be able to calculate the corner frequency.

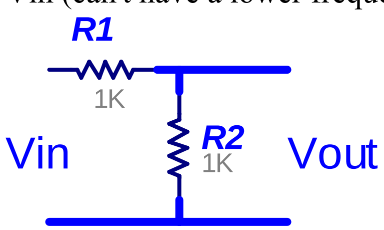
Similarly a high pass filter is made by reversing the placement of the capacitor and resistor as shown below. Now the output is low for low frequencies and high for high frequencies.

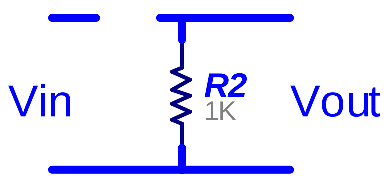


Note that the outputs of the high pass and low pass filters are the same at 160Hz. This is because the capacitor and the resistor have the same impedance at this frequency. This is called the **corner frequency or cutoff frequency** and can be calculated as $\mathbf{f_c = 1/(2\pi RC)}$. For a low pass filter there isn't much attenuation below the corner frequency and above the corner frequency the output drops off linearly (i.e. at 16KHz the output is 10 times smaller than at 1.6KHz). This link explains High Pass Filters: http://hyperphysics.phy-astr.gsu.edu/hbase/electric/filcap.html One can imagine combining high pass and low pass filters to make a band pass or band stop filters.

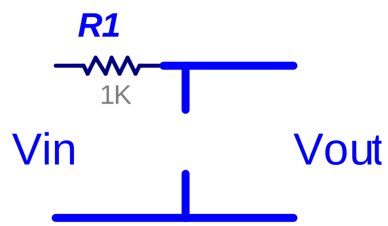
Note: Inductors can be used to make filters but capacitors are smaller and cheaper and therefore are used much more often to construct filters.

Note: If a DC voltage is applied to Vin the capacitor will draw current until fully charged. Once fully charged the capacitor doesn't draw any current (i.e. it's an open circuit). Therefore the filters would essentially appear as shown below. The output of the high pass is zero and the output of the low pass is Vin (can't have a lower frequency than DC).





Highpass at DC



Lowpass at DC

It may help to understand complex impedance. The following link explains complex impedance:
http://hyperphysics.phy-astr.gsu.edu/hbase/electric/impcom.html#c1

The above links were from the following web site:
http://hyperphysics.phy-astr.gsu.edu/hbase/electric/accircon.html#c1

They review many electrical concepts.