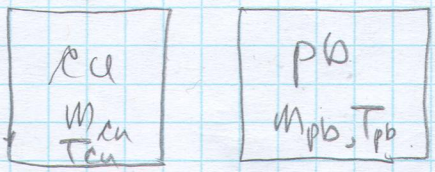
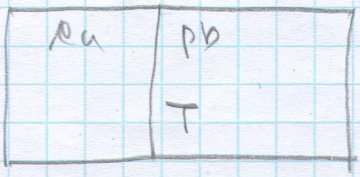
20-7 Let T be the equilibrium temp when the two blocks are in contact.



(a) $m_{Cu} C_{Cu} (T_{Cu} - T) = m_{Pb} C_{Pb} (T - T_{Pb})$



where C_{Cu} – copper specific heat at const pressure

C_{Pb} – lead S.H. at const pres

m – mass

$$m_{Cu} C_{Cu} T_{Cu} - m_{Cu} C_{Cu} T = m_{Pb} C_{Pb} T - m_{Pb} C_{Pb} T_{Pb}$$

$$T = \frac{(m_{Cu} C_{Cu} T_{Cu} + m_{Pb} C_{Pb} T_{Pb})}{(m_{Pb} C_{Pb} + m_{Cu} C_{Cu})}$$

(b) Change of internal energy.

Assuming the expansion of the block is small, work done is ~ 0

Internal energy E_i has to be conserved and will not change in an insulated environment.

(c) the change in entropy ΔS is

$$\Delta S = \int \frac{dQ}{T} \quad \text{and} \quad dQ = mC_p\, dT$$

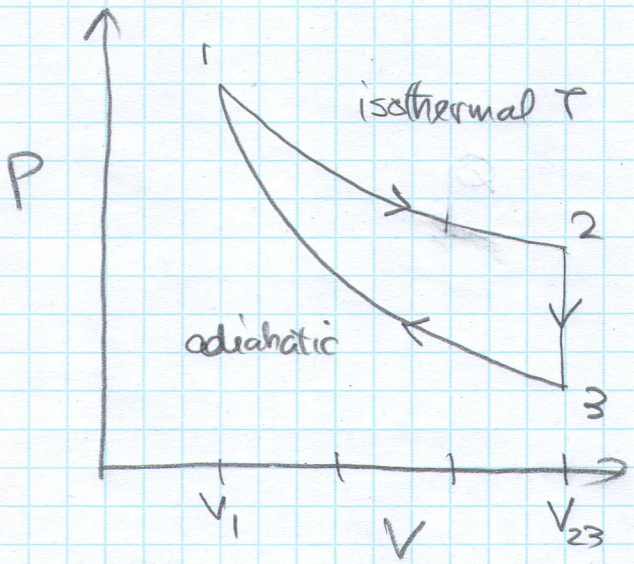
$$\Delta S_{net} = \Delta S_{Cu} + \Delta S_{Pb} = \int_{T_{iCu}}^{T_f} \frac{dQ_{Cu}}{T} + \int_{T_{iPb}}^{T_f} \frac{dQ_{Pb}}{T}$$

$$= m_{Cu} C_{Cu} \int_{T_{iCu}}^{T_f} \frac{dT}{T} + m_{Pb} C_{Pb} \int_{T_{iPb}}^{T_f} \frac{dT}{T}$$

$$= m_{Cu} C_{Cu} \ln\left(\frac{T_f}{T_{iCu}}\right) + m_{Pb} C_{Pb} \ln\left(\frac{T_f}{T_{iPb}}\right)$$

20-17. $V_{23} = 3V_1$

n # of moles of diatomic gas



(a) Find P_2/P_1

For ① → ②, $T = \text{const} = T_1 = T_2$

$P_2V_2 = nRT$

$P_1V_1 = nRT$

$P_2V_2 = P_1V_1 \quad \frac{P_2}{P_1} = \frac{V_1}{V_2} = \frac{1}{3}$

b) Find P_3/P_1

③ → ① adiabatic compression

$PV^\gamma = \text{Const} \Rightarrow P_3V_3^\gamma = P_1V_1^\gamma \quad \frac{P_3}{P_1} = \left(\frac{V_1}{V_3}\right)^\gamma = \left(\frac{1}{3}\right)^\gamma$ with $\gamma = \frac{C_P}{C_V}$

For diatomic gas $C_V = \left(\frac{3}{2} + \frac{2}{2}\right)nR = \frac{5}{2}(nR)$

$C_P = \frac{5}{2}nR + nR = \frac{7}{2}nR$

$\gamma = \frac{C_P}{C_V} = \frac{7}{5} = \underline{1.4}$

$\therefore \frac{P_3}{P_1} = \left(\frac{1}{3}\right)^{1.4}$

(c) Find T_3/T_1 For ③ → ① adiabatic expansion $\frac{PV^\gamma}{nR} = \text{const}$.

$\frac{T_3}{T_1} = \ln\left(\frac{V_3}{V_1}\right)^{(1-\gamma)} = \ln\left(\frac{V_1}{V_3}\right)^{(\gamma-1)} = \ln\left(\frac{1}{3}\right)^{0.4}$

For path ① → ②

(d) Find $\frac{W}{nRT_1}$; $dQ = dE_i + dW$

For ① → ② $dE_i = 0$ since $T = \text{constant}$

$dQ = dW = pdV$ for same T $\quad p = \frac{nRT}{V}$

$W = \int dW = nRT\int\frac{dV}{V} = nRT_2\ln\left(\frac{V_3}{V_1}\right) = nRT_2 \ln 3$

$\frac{W}{nRT_2} = \frac{W}{nRT_1} = \ln 3$ //

e) Find Q/nRT_1

$$\frac{\int dQ}{nRT_1} = \frac{W}{nRT_1} = \ln 3 \;//$$

f) Find $\frac{\Delta E_{in}}{nRT_1} = 0$ // since $\Delta T = 0$. internal energy is constant

g) $\Delta S/nR$?

$$\Delta S = \int \frac{dQ}{T} = \frac{1}{T}\int dQ = \frac{nRT_1 \ln 3}{T_1}$$

$$\frac{\Delta S}{nR} = \ln 3$$

For path 2→3. Isochoric. = const vol.

e) Find W/nRT_1 $W = \int p\,dV = 0$ since $dV = 0$

$$\frac{W}{nRT_1} = 0$$

i) Find Q/nRT_1 $dQ = dE_i + dW^{\nearrow 0}$

$$\int dE_i = \int_2^3 C_V dT = C_V(T_3 - T_2) = \frac{5}{2}nR(T_3 - T_2)$$

Need to link T_3 to T_1 and therefore T_2 since $T_1 = T_2$

Use path ③→① $\frac{T_3}{T_1} = \left(\frac{V_1}{V_3}\right)^{\gamma-1}$ adiabatic expansion

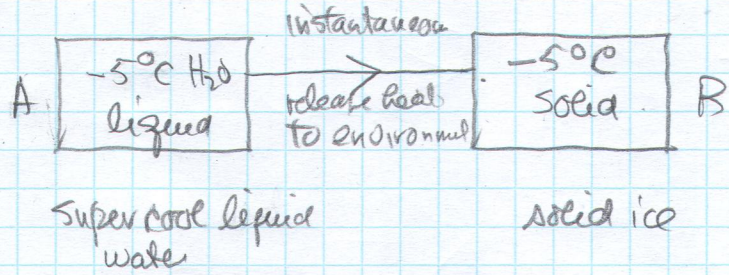
$$\frac{T_3}{T_1} = \left(\frac{V_1}{V_3}\right)^{\gamma-1}$$

$$T_3 = \left(\frac{V_1}{V_3}\right)^{\gamma-1} T_1 \quad \text{sub to} \quad \int dE_i = \frac{5}{2}nR\left(\left(\frac{V_1}{V_3}\right)^{\gamma-1} T_1 - T_2\right)$$

$$= \frac{5}{2}nR\left(\left(\frac{1}{3}\right)^{0.4} - 1\right)T_1$$

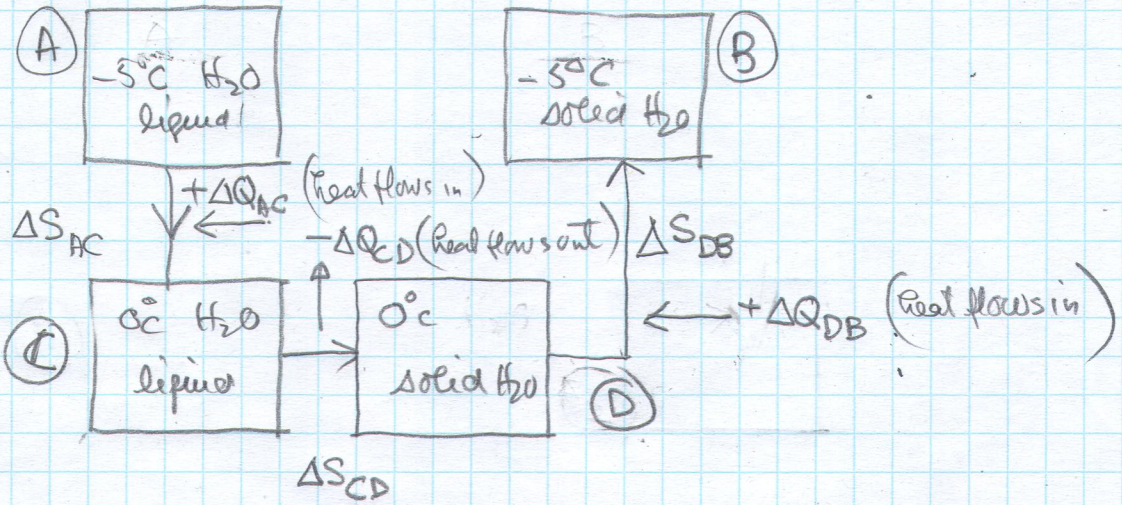
$$\frac{\int dE_i}{nRT_1} = \frac{5}{2}\left(\left(\frac{1}{3}\right)^{0.4} - 1\right) \;//$$

20-21 Consider the irreversible process: supercool liquid water to solid



To calculate the change of entropy ΔS in this irreversible process, one has to connect the initial state A and the final state B with a series of reversible process. the ΔS_{AB} will be the sum of the ΔS_i of individual processes that link state A and B.

Consider the following process that link state A to state B:



And $\Delta S_{AB} = \Delta S_1 + \Delta S_2 + \Delta S_3$

where ΔS_1, ΔS_2 and ΔS_3 are the entropy change between two states in a quasi-static reversible process where the system is in thermodynamic equilibrium at all time

For example: Heating the water from −5°C (268K) to 0°C (273K) slowly is reversible where the system is in thermodynamic equilibrium.

Freezing the water at 0°C (273K) slowly, and

Bring the ice from 0°C (273K) down to −5°C (268K) slowly is also reversible where the change of temperature of the two system $\Delta T \to 0$.

Let's calculate the change of entropy ΔS_{AC}, ΔS_{CD} and ΔS_{DB} between the three state of water/ice.

State A to C

$$\Delta S_{AC} = \int_{T_A}^{T_C} \frac{dQ_{rev}}{T} = mc\int_{T_A}^{T_C} \frac{dT}{T} = mc\ln\left(\frac{T_C}{T_A}\right)$$

with m = mass of H_2O
c = specific heat of liquid H_2O

State C to D

$$\Delta S_{CD} = \int_{T_C}^{T_D} \frac{dQ_{rev}}{T} = \frac{-mL}{T_C}$$

where L = latent heat of fusion

State D to B

$$\Delta S_{DB} = \int_{T_D}^{T_B} \frac{dQ_{rev}}{T} = mc_s\int_{T_D}^{T_B} \frac{dT}{T} = mc_s\ln\left(\frac{T_B}{T_D}\right)$$

c_s = specific heat of ice

$$\Delta S_{AB} = \Delta S_{AC} + \Delta S_{CD} + \Delta S_{DB}$$

$$= mc\ln\left(\frac{T_C}{T_A}\right) + mc_s\ln\left(\frac{T_B}{T_D}\right) - \frac{mL}{T_C} \quad \text{where } T_A - T_B = 268\,K$$

$$= mc\ln\left(\frac{T_C}{T_A}\right) - mc_s\ln\left(\frac{T_D}{T_B}\right) - \frac{mL}{T_C} \quad T_C = T_D = 273\,K$$

$$= m(c - c_s)\ln\left(\frac{T_C}{T_A}\right) - \frac{mL}{T_C}$$

20-28 Consider a two stage Carnot engine.

stage ① Heat input Q_1 @ T_1, work performed W_1 heat Rejected Q_2 @ T_2

stage ② Use heat rejected Q_2 as input, work performed by 2nd stage is W_2 with heat rejected @ Q_3 at T_3

Efficiency of engine $\epsilon = \dfrac{\text{Work extracted from Engine}}{\text{Q input to engine}}$

The Total work extracted from the two stage engine W

$$= W_1 + W_2$$

Total heat input to engine is only Q_1

$$\epsilon = \frac{W_1 + W_2}{Q_1}$$

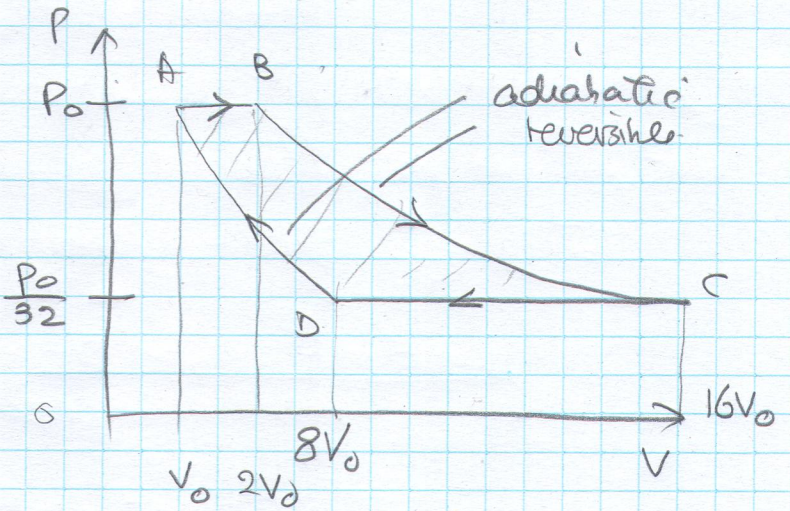
but $W_1 = Q_1 - Q_2$ and $W_2 = Q_2 - Q_3$
sub to ϵ

$$\therefore \epsilon = \frac{Q_1 - Q_2 + Q_2 - Q_3}{Q_1} = \frac{Q_1 - Q_3}{Q_1} = 1 - \frac{Q_3}{Q_1}$$

For carnot engine $\dfrac{Q_3}{Q_1} = \dfrac{T_3}{T_1}$

$$\therefore \epsilon = 1 - \frac{T_3}{T_1}$$

20-34 $n = 1$ mol.



For adiabatic expansion and compression

$$\frac{PV^{\gamma}}{nR} = \text{const.}$$

(B) → (C) adiabatic

$$P_B V_B^{\gamma} = P_C V_C^{\gamma}$$

$$\frac{P_B}{P_C} = \left(\frac{V_C}{V_B}\right)^{\gamma}, \quad \ln\left(\frac{P_B}{P_C}\right) = \gamma \ln\left(\frac{V_C}{V_B}\right)$$

$$\gamma = \frac{\ln\left(\frac{P_B}{P_C}\right)}{\ln\left(\frac{V_C}{V_B}\right)} = \frac{\ln\left(\frac{P_0}{\frac{P_0}{32}}\right)}{\ln\frac{16V_0}{2V_0}} \rightarrow \frac{\ln 32}{\ln 8} = 1.6667 \Rightarrow \text{monoatomic}$$

Efficiency of the engine. $\epsilon = \frac{W}{Q_H} = \frac{Q_H - Q_C}{Q_H} = 1 - \left|\frac{Q_C}{Q_H}\right|$

Need to find Q_C and Q_H

Both processes BC and DA are adiabatic $\Rightarrow dQ = 0$

process A → B will take in heat Q_H while process CD will reject heat by compression

$Q_H = nC_p\Delta T = n\left(\frac{5}{2}R\right)(T_B - T_A)$ and since $pV = nRT$

T_A and T_B are related by. $V = \frac{nRT}{P}$ $\qquad V = \frac{nRT}{P}$

$$\frac{T_A}{T_B}\frac{V_A}{V_B} = \frac{V_0}{2V_0} = \frac{1}{2} \Rightarrow T_B = 2T_A$$

$$\therefore Q_H = \frac{5}{2}nRT_A(2-1) = \frac{5}{2}nRT_A = \frac{5}{2}P_0V_0$$

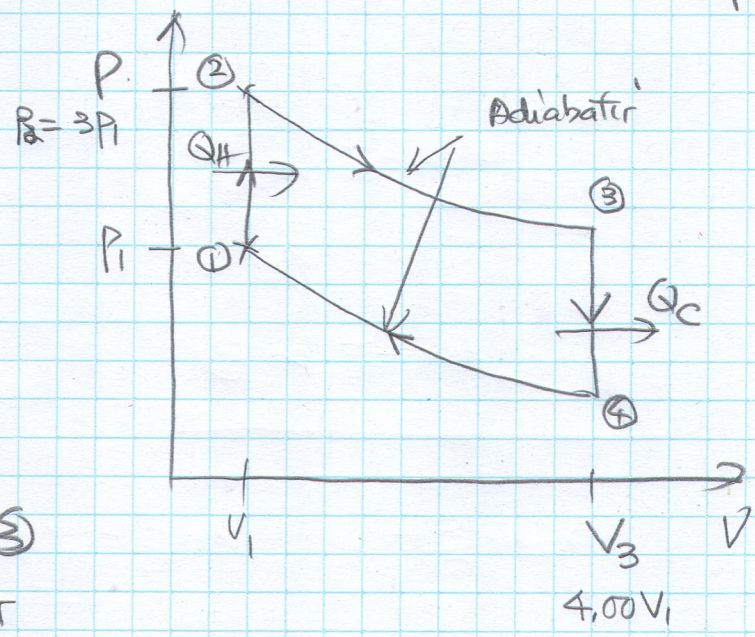
$$Q_C = nC_p\Delta T = nC_p(T_D - T_C) = \frac{5}{2}nR(T_D - T_C)$$

$$\frac{T_D}{T_C} = \frac{V_D}{V_C} = \frac{8V_0}{16V_0} = \frac{1}{2} \qquad T_C = 2T_D. \text{ sub to } Q_C = \frac{5}{2}nRT_D(-1)$$

$$\therefore \epsilon = 1 - \left|\frac{Q_C}{Q_H}\right| = 1 - \frac{\frac{5}{2}nRT_D}{\frac{5}{2}nRT_A} = 1 - \frac{T_D}{T_A} = 1 - \left(\frac{V_A}{V_D}\right)^{\gamma-1} = 1 - \left(\frac{V_0}{8V_0}\right)^{0.66} = 1 - 0.25 = 0.75$$

20-35 Internal combustion engine

$\gamma = 1.3$ – polyatomic



(a) Find T_2/T_1

$PV = nRT$

$P_i = \left(\frac{nR}{V}\right)T_i$

$\frac{P_2}{P_1} = \frac{T_2}{T_1} = \frac{3P_1}{P_1} = 3$ //

(b) T_3/T_1 adiabatic ② → ③

$PV^\gamma = \text{Const}$ but $PV = nRT$, $P = \frac{nRT}{V}$

$\therefore nRTV^{\gamma-1} = \text{Const}$

$TV^{\gamma-1} = \text{const}'$

$T_3V_3^{\gamma-1} = T_2V_2^{\gamma-1}$ $\quad V_3 = 4V_1$, $V_2 = V_1$, $T_2 = \frac{P_2}{P_1}T_1$

$\frac{T_3}{T_2} = \left(\frac{V_2}{V_3}\right)^{\gamma-1} = \left(\frac{V_1}{4V_1}\right)^{\gamma-1}$

$\frac{T_3}{T_1} = \left(\frac{1}{4}\right)^{\gamma-1}\left(\frac{P_2}{P_1}\right) = (3)\left(\frac{1}{4}\right)^{0.3}$ //

(c) T_4/T_1 adiabatic ④ → ①

$T_4V_4^{\gamma-1} = T_1V_1^{\gamma-1}$

$\frac{T_4}{T_1} = \left(\frac{V_1}{V_4}\right)^{\gamma-1} = \left(\frac{V_1}{4V_1}\right)^{0.3}$ with $V_4 = V_3$

$= \left(\frac{1}{4}\right)^{0.3}$

(d) P_3/P_1 adiabatic ② → ③

$P_3V_3^\gamma = P_2V_2^\gamma$ with $V_3 = 4V_1$, $V_2 = V_1$, $P_2 = 3P_1$

$\frac{P_3}{P_2} = \frac{P_3}{3P_1} = \left(\frac{V_2}{V_3}\right)^\gamma$ $\quad \therefore \frac{P_3}{P_1} = 3\left(\frac{V_1}{4V_1}\right)^\gamma = 3\left(\frac{1}{4}\right)^{0.3}$ //

$$\text{Efficiency} = \frac{W}{Q_H} = \frac{Q_H - Q_c}{Q_H} = 1 - \left|\frac{Q_c}{Q_H}\right|$$

$$Q_H = \int nC_V\,dT + \int p\,dV \quad (\text{this } \int p\,dV < 0) = nC_V(T_2 - T_1)$$

$$Q_c = nC_V \int dT = nC_V(T_4 - T_3)$$

$$\frac{Q_c}{Q_H} = \frac{T_4 - T_3}{T_2 - T_1}$$

$$\epsilon = 1 - \frac{T_4 - T_3}{T_2 - T_1} \qquad \text{Eq (1)}$$

To find T_4 & T_3, we need to relate T_4 T_3 to P_2 and P_1

Use equation for adiabatic exp. $TV^{\gamma-1} = C'$

$$T_4 V_4^{\gamma-1} = T_1 V_1^{\gamma-1}$$

$$T_4 = T_1\left(\frac{V_1}{V_4}\right)^{\gamma-1} = T_1\left(\frac{1}{4}\right)^{0.3}$$

Similarly $T_3 = T_2\left(\frac{V_2}{V_3}\right)^{\gamma-1} = T_2\left(\frac{V_1}{4V_1}\right)^{0.3} = T_2\left(\frac{1}{4}\right)^{0.3}$

$$\therefore T_4 - T_3 = (T_1 - T_2)\left(\frac{1}{4}\right)^{0.3} = -(T_2 - T_1)\left(\frac{1}{4}\right)^{0.3}$$

$$\left|\frac{Q_c}{Q_H}\right| = \left|\frac{T_4 - T_3}{T_2 - T_1}\right| = \left|\frac{-(T_2 - T_1)\left(\frac{1}{4}\right)^{0.3}}{(T_2 - T_1)}\right| =$$

$$\epsilon = 1 - \left(\frac{1}{4}\right)^{0.3} = 1 - 0.66 = 0.34 \quad \approx 34\%$$

20-39 Consider a carnot air conditioner. – Reversible cycle. 10

Max Coefficient of performance $COP = \frac{T_C}{\Delta T}$ with T_C in Kelvin, ΔT in Kelvin

$$T_C = \left(\frac{70-32}{9}\right)(5) + 273\ K$$

$$T_H = \left(\frac{96-32}{9}\right)(5) + 273\ K$$

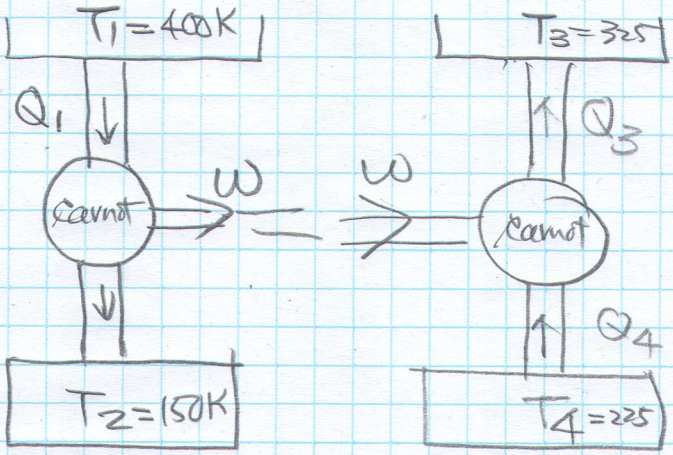
$$COP = \frac{\left(\frac{70-32}{9}\right)\left(\frac{5}{9}\right) + 273}{\left(\frac{90-32}{9}\right)\frac{5}{9} - \left(\frac{70-32}{9}\right)\frac{5}{9}}$$

$$= \frac{21.1}{11.11}$$

$$= 1.9$$

∴ for 1 joule of electric energy, 1.9 joules are removed from the room

20-43



Heat Engine Refrigerator

For Carnot engine

$$\epsilon = \frac{W_H}{Q_1}$$

$$= \left(1 - \frac{T_C}{T_H}\right)$$

$$W_H = \left(1 - \frac{T_2}{T_1}\right) Q_1 \quad \text{—①}$$

For Carnot refrigerator

$$\text{COP} = K = \frac{Q_4}{W_R}$$ for refrigerator in general

$$= \frac{T_C}{\Delta T}$$ efficiency for carnot refrigerat

$$K = \frac{T_4}{T_3 - T_4}$$ (where $T_C = T_4$ $\Delta T = T_3 - T_4$)

$$\therefore Q_4 = K W_R \quad \text{—②}$$

the heat engine and refrigerator are linked by same W i.e $W_H = W_R = W$

Furthermore $W + Q_4 = Q_3$ —③ conservation of energy. internal energy does not change.

Sub ③ to ② to link Q_3 and Q_4

$$Q_4 = K(Q_3 - Q_4) = KQ_3 - KQ_4$$

$$Q_4(1+K) = KQ_3$$

$$Q_4 = \frac{KQ_3}{1+K}$$

sub to ② to replace Q_4

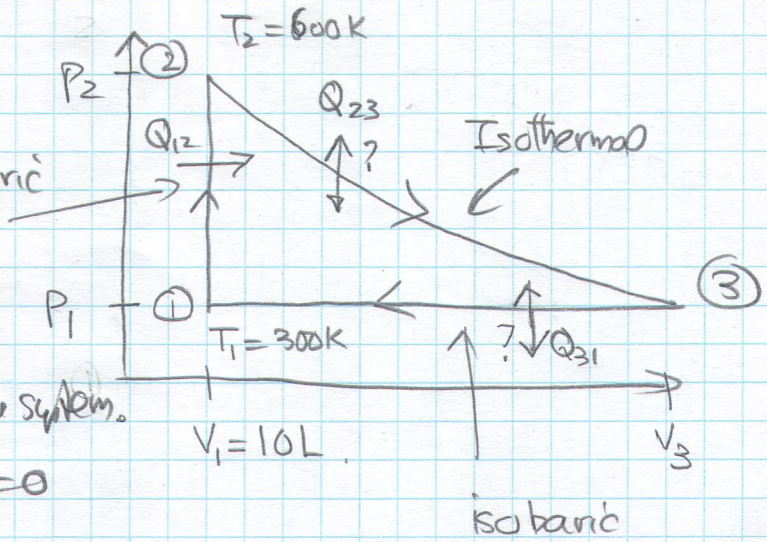
$$\left(\frac{K}{1+K}\right) Q_3 = KW = K\left(1 - \frac{T_2}{T_1}\right) Q_1$$

$$\frac{Q_3}{Q_1} = \frac{K\left(1 - \frac{T_2}{T_1}\right)}{\left(\frac{K}{1+K}\right)} = \left(1 - \frac{T_2}{T_1}\right)(1+K)$$

20-52. monoatomic gas

given $n = 1$ mole,

find Q_H, W, and ϵ.



To find Q_H in one cycle

Q_H is heat energy flows into the system.

→ Consider ① → ② with $dV = 0$

$$dQ_{12} = nC_V dT + p\,dV \quad (p\,dV \to 0)$$

$$Q_{12} = nC_V \int dT = nC_V(T_2 - T_1) \longrightarrow \text{Eq 1}$$

$T_2 = 600K$, $T_1 = 300K$

$C_V = \frac{3}{2} N_A R$

with R = Boltzmann constant

$N_A R = R$, ideal gas const.

→ Consider ② → ③ isothermal $dT = 0$

$$dQ_{23} = dE_i + p\,dV \quad \text{with } dE_i = 0 \text{ since } T \text{ is const.}$$

$$Q_{23} = \int p\,dV$$

$$= nRT_2 \int_{V_2}^{V_3} \frac{dV}{V} \quad \text{with } T_2 = T_3$$

$$= nRT_2 \ln\left(\frac{V_3}{V_2}\right)$$

$$\frac{V_3}{V_2} = \frac{P_2}{P_3}$$

use ideal gas law $PV = nRT$

and $P_3 = P_1$ since ③ → ① is isobaric.

$$\frac{V_3}{V_2} = \frac{P_2}{P_1} = \frac{T_2}{T_1}$$

using $PV = nRT$ ① → ②.

∴ $Q_{23} = nRT_2 \ln\left(\frac{T_2}{T_1}\right)$ ⟶ Eq 2

→ Now consider isobaric process ③ → ① with $dp = 0$

$$dQ_{31} = nC_p\,dT + p\,dV$$

$$Q_{31} = nC_p \int_{T_3}^{T_1} dT + P_1 \int_{V_3}^{V_1} dV = nC_p(T_1 - T_3) + P_1(V_1 - V_3)$$

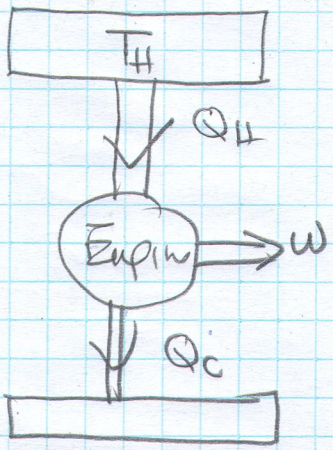
Need to exp. T_3 and V_3 interms of T_1 and V_1.

Again use ideal gas laws. $PV = nRT$

For ③→① P = const $\quad V_1 = \frac{nR}{P} T_C$

$\frac{V_3}{V_1} = \frac{T_3}{T_1} = \frac{T_2}{T_1} \Rightarrow V_3 = \left(\frac{T_2}{T_1}\right) V_1$

$T_3 = T_2$. isothermal from ②→③



$$Q_{31} = nC_p(T_1 - T_2) + P_1\left(V_1 - V_1\left(\frac{T_2}{T_1}\right)\right)$$

$$= nC_p(T_1 - T_2) + P_1V_1\left(1 - \frac{T_2}{T_1}\right)$$

Q_{31} is negative since $T_2 > T_1$ and $\frac{T_2}{T_1} > 1$ ∴ Q_{31} heat flows out of the syst

$Q_{31} = Q_C$ and $Q_{12} + Q_{23} = Q_H$.

$$Q_H = nC_V(T_2 - T_1) + nRT_2 \ln\left(\frac{T_2}{T_1}\right) //$$

To calculate net work in one recycle

$\Delta W = W_{23} + W_{31}$

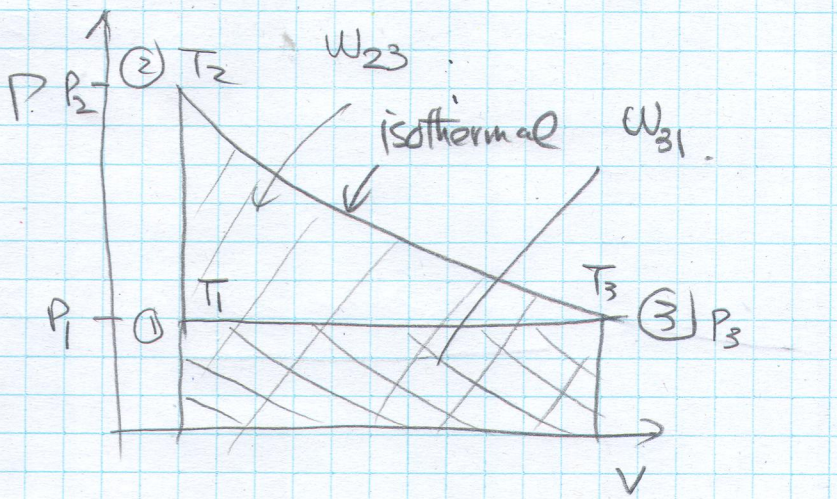
Consider W_{23}

$$W_{23} = \int P\,dV \quad \text{and} \quad P = \frac{nRT}{V}$$

$$= nRT\int_{V_2}^{V_3} \frac{dV}{V} \qquad T = T_2 = T_3 \text{ isothermal}$$

$$= nRT_2 \ln\left(\frac{V_3}{V_2}\right)$$

$$= nRT_2 \ln\left(\frac{T_2}{T_1}\right) \quad \text{Eq ⑤ from ②→③ process}$$



$$W_{31} = \int_{V_3}^{V_1} P\,dV = P(V_1 - V_3) = P\left(V_1 - \frac{T_2}{T_1}V_1\right) = P_1V_1\left(1 - \frac{T_2}{T_1}\right) \quad \text{since } P_1 = P_3 = P$$

$$= nRT_1\left(1 - \frac{T_2}{T_1}\right) \quad \text{Eq ⑥}$$

Note. $\frac{T_2}{T_1} > 1$. W_{31} has a -ve value.

$$\Delta W = nRT_2 \ln\left(\frac{T_2}{T_1}\right) + nRT_1\left(1 - \frac{T_2}{T_1}\right) \quad //$$

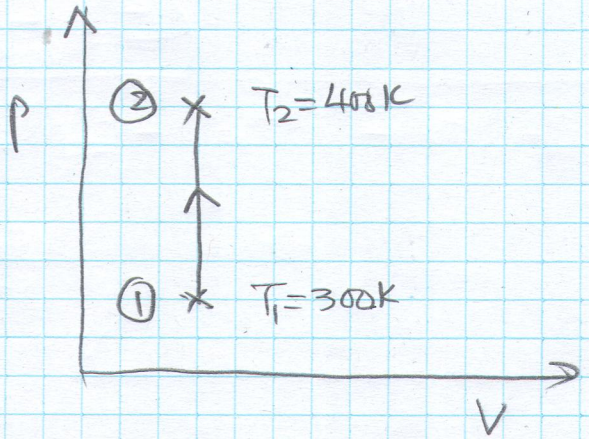
$$\epsilon = \frac{\Delta W}{Q_H} = \frac{nRT_2 \ln\left(\frac{T_2}{T_1}\right) + nRT_1\left(1 - \frac{T_2}{T_1}\right)}{nC_V(T_2 - T_1) + nRT_2 \ln\left(\frac{T_2}{T_1}\right)} \quad //$$

with $T_1 = 300\,K$, $T_2 = 600\,K$, $n = 1\,mole$,

$C_V = \frac{3}{2}nR$ $\quad C_P = \frac{5}{2}nR$

20-5) $n = 1$ mole monoatomic gas

Find ΔS.



$$\Delta S = \int \frac{dQ}{T}$$

$$dQ = dE_{int} + dW$$

$$= nC_V dT + dW \quad \text{(} dW \to 0 \text{ since } dV = 0\text{)}$$

$$\int \frac{dQ}{T} = nC_V \int_{T_1}^{T_2} \frac{dT}{T}$$

$$= nC_V \ln\left(\frac{T_2}{T_1}\right)$$

and $C_V = \frac{3}{2}nR$ or $= \frac{3}{2}nN_A k$.