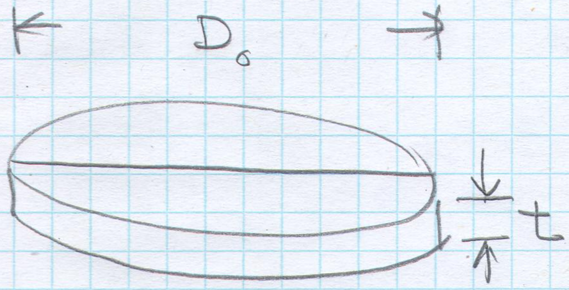
D1

Ch18 – 14.



Temp change = 100°C.

D_0 increases by 0.18% when ΔT is 100°C.

$$\frac{\Delta D}{D_0} = \frac{0.18}{100}$$

$$\frac{D_f - D_0}{D_0} = \frac{0.18}{100}$$

$$\frac{D_f}{D_0} = \frac{0.18}{100} + 1 = \frac{100.18}{100} = 1.0018$$

a) Let A be the area of the face: $A = \frac{\pi D^2}{4}$, A_f = final area after temp inc. A_0 = initial area before temp increases

$$\frac{A_f}{A_0} = \frac{\cancel{\pi} D_f^2}{\cancel{4}} \frac{\cancel{4}}{\cancel{\pi} D_0^2} = \left(\frac{D_f}{D_0}\right)^2$$

Fractional increase in area

$$\frac{A_f - A_0}{A_0} = \frac{A_f}{A_0} - 1 = \left(\frac{D_f}{D_0}\right)^2 - 1$$

$$\% \text{ increase in area} = \left(\frac{A_f - A_0}{A_0}\right) 100\% = \left[\left(\frac{D_f}{D_0}\right)^2 - 1\right] 100\%$$

$$= 0.36\%$$

b) Let t be the thickness. $t_0 = D_0$ and $t_f = D_f$

$$\frac{t_f}{t_0} = \frac{D_f}{D_0}$$

$$\% \text{ increase}: \left(\frac{t_f - t_0}{t_0}\right) 100\% = \left(\frac{t_f}{t_0} - 1\right) 100\% = \left(\frac{D_f}{D_0} - 1\right) 100\%$$

$$= 0.18\%$$

(f) Since $D_f - D_0 = \Delta D = D_0 \alpha \Delta T$

where α is the coeff of linear expansion and $\Delta T = 100°C$

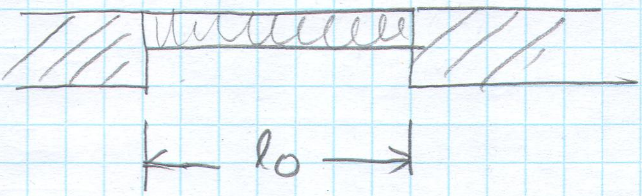
$$\frac{\Delta D}{D_0} = \alpha \Delta T \rightarrow \alpha = \frac{\Delta D}{D_0} \frac{1}{\Delta T} = \frac{0.18\%}{100} = 0.0018\%/°C$$

since β = volume exp coeff $= 3\alpha$

$$\frac{\Delta V}{V_0} = \beta \Delta T = 3\alpha \Delta T = (3)(0.0018)(100)$$

$$= \underline{\underline{0.54\ \%}}$$

18-21 Let l_0 be the length of the bar before expansion takes place. Lets assume that the spacing does not change when the temp is elevated.

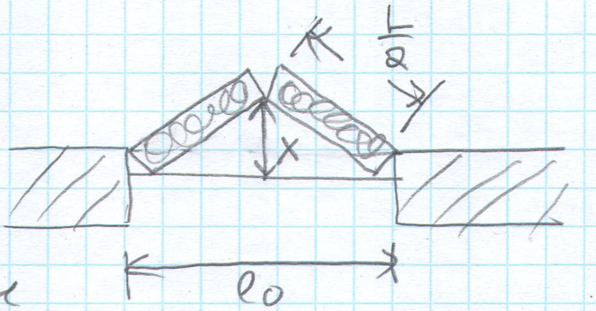


$\alpha_{bar} = 25 \times 10^{-6}/C^\circ$

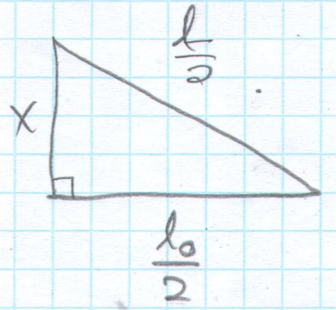
$\Delta T = 32^\circ C$

$l = l_0 \alpha_{bar} \Delta T + l_0$

where L is length after ΔT change



$\frac{l}{2} = \frac{l_0 \alpha \Delta T + l_0}{2}$



$\frac{l}{2}^2 = x^2 + \left(\frac{l_0}{2}\right)^2$

$x^2 = \left(\frac{l}{2}\right)^2 - \left(\frac{l_0}{2}\right)^2 = \left(\frac{1}{4}\right)(l^2 - l_0^2)$

$x = \frac{1}{2}\sqrt{l^2 - l_0^2}$

$= \frac{\sqrt{(l_0 \alpha \Delta T + l_0)^2 - l_0^2}}{2}$

$= \frac{l_0}{2}\sqrt{(\alpha \Delta T + 1)^2 - 1}$

$= \frac{l_0}{2}\sqrt{\alpha^2 \Delta T^2 + 2\alpha \Delta T}$ //

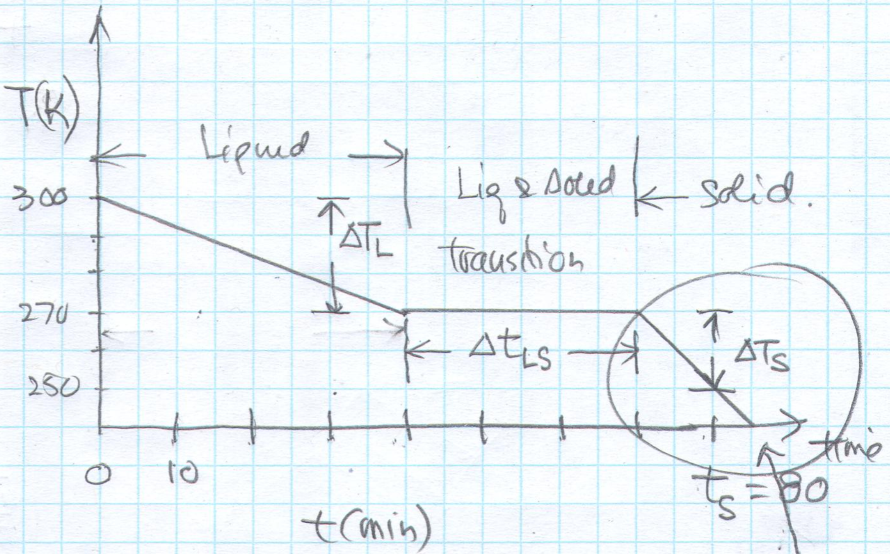
$l_0, \alpha, \Delta T$ are given

18-30

Mass of liquid $= m = 0.4 kg$

$C_L = 3000 \, J/kg\,K$

(a) Find Heat of fusion



$$Q = mC_L \Delta T_L$$

where Q is heat flow from liquid to cooling apparatus

$$\frac{\Delta Q}{\Delta t} = mC_L \frac{\Delta T_L}{\Delta t}$$

where $\frac{\Delta Q}{\Delta t}$ is heat removal rate and is constant, $\frac{\Delta T}{\Delta t}$ is temp drop rate.

$$\frac{\Delta Q}{\Delta t} = mC_L \frac{(270-300)}{40} \quad \left(\frac{J}{min}\right)$$

This is also the heat removal when the liquid starts to free

The Total ΔQ required to transform liquid at 270°K to solid

$$\Delta Q_{LS} = \left(\frac{\Delta Q}{\Delta t}\right) \Delta t_{LS}$$

Heat of fusion (change liquid phase to solid phase, $L_F = \frac{\Delta Q_{LS}}{m}$

$$L_F = \left(\frac{\Delta Q}{\Delta t} \Delta t_{LS}\right) / m$$

(b) Specific heat in frozen state

$$Q = mC_S \Delta T_S$$

$$\frac{\Delta Q}{\Delta t} = mC_S \frac{\Delta T_S}{\Delta t}$$

where $\frac{\Delta T_S}{\Delta t}$ is the slope of Temp vs. time below 270K

$\frac{\Delta Q}{\Delta t}$ is the heat removal rate of the apparatus and is the same through out

$$C_S = \frac{\frac{\Delta Q}{\Delta t}}{m\left(\frac{\Delta T_S}{\Delta t}\right)} = \frac{mC_L\left(\frac{\Delta T_L}{\Delta t}\right)}{m\left(\frac{\Delta T_S}{\Delta t}\right)}$$

$= C_L$ ratio of Temp vs time slope in liquid and solid phase

18-34 Consider the heat flow equation

$\Delta Q = m c \Delta T$ where ΔQ is heat flow between objects

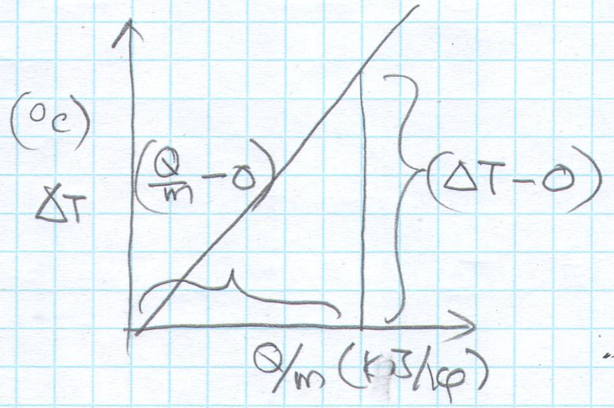
m and c are mass and specific heat cap

$\left(\frac{\Delta Q}{m}\right) = c\,\Delta T$ ΔT is temperature change.

using information in Fig. 18-33 (b) ΔT vs. Q/m plot to find c_B

$\frac{1}{c_B}$ = slope of ΔT vs. $\frac{Q}{m}$ plot

$= \frac{\Delta T}{\Delta\left(\frac{Q}{m}\right)}$

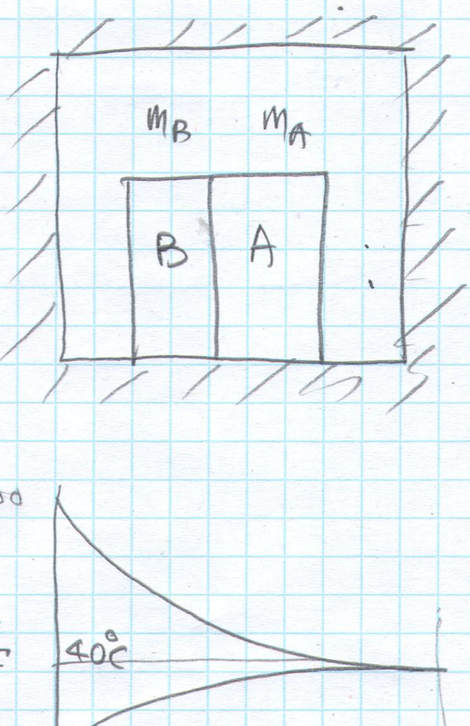


$c_B = \frac{\Delta \frac{Q}{m}}{\Delta T} = \frac{16 \text{ kJ/kg}}{1.6\ ^\circ C}$

$= 10 \text{ kJ/kg}^\circ C$

$\Delta Q_A = m_A c_A \Delta T_A$

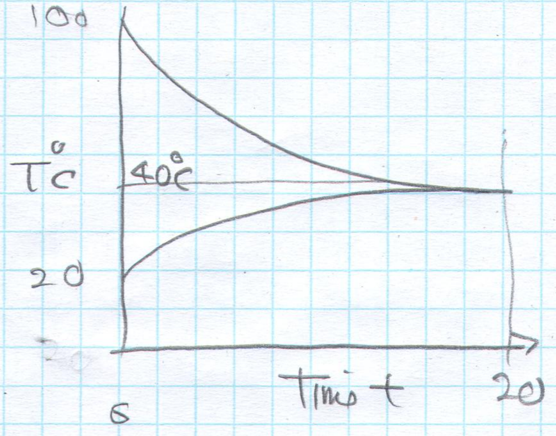
$\Delta Q_B = m_B c_B \Delta T_B$



$\Delta Q_A = \Delta Q_B$, Energy flows from A to B at equilib condition @ 40°C

$m_A c_A \Delta T_A = m_B c_B \Delta T_B$

$c_A = \frac{m_B c_B \Delta T_B}{m_A \Delta T_A}$ //

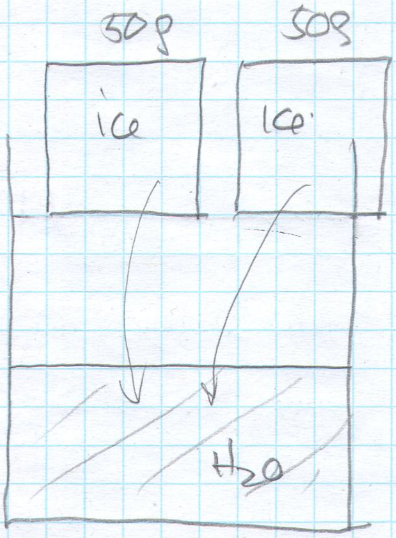


$\Delta T_B = 20^\circ C$, $\Delta T_A = 60^\circ C$

$m_A = 5 \text{ kg}$ $m_B = 1.5 \text{ kg}$

c_B from above $= 10 \text{ kJ/kg}^\circ C$

18-41 Find the final temp of Wate if two 50g cubs of ice at $-15^\circ C$ is add to 200 g of water initially at $25^\circ C$.



There are two senicals:

(a) Heat from water with temp change $= 25^\circ C$ is insufficieal to Melt the two ice cube and then the final mixture of ice and water at $0^\circ C$.

(b) Heal from water with initial temp at $25^\circ C$ is sufficiaal to melt the ice and raise the water melt from ice + 200g of water to a final temp $T > 0$.

Check (a) Amount of heat available to ice when 200g of $25^\circ C$ water is cooled down to $0^\circ C$ ∴ $(m_L) c_L \Delta T = \Delta Q_{H_2O}$ where $m_L = 200g$, $c_L = 4.187 \frac{J}{gK}$

$\Delta T = 25^\circ C$

$$\Delta Q_{H_2O} = (200)(4.2)(25) = 21\ KJ.$$

Heal required to lower the two ice cube from $-15^\circ C$ to $^\circ C$ + Latent heat to melt the ice into liquid.

$$\Delta Q_{ice} = 2 m_{ice} C_{ice} \Delta T + 2 m_{ice} L_F$$

$m_{ice} = 50g$, $C_{ice} = 2.22 J/gK$

$L_F = 333 J/g$

$$= 100g \left[\left(2.22 \frac{J}{gK}\right)(15)K + 333 J/g \right]$$

$$= 36{,}630\ J = \underline{36\ KJ}$$

$\Delta Q_{ice} > \Delta Q_{H_2O}$ means that amount of heat from 200 g of water at $25^\circ C$ is insufficient to melt all the ice. The result is a mixture of water and ice @ $0^\circ C$.

P7

Suppose, only one ice cube is used

$$\Delta Q_{ice} = m_{ice} c_{ice} \Delta T + m_{ice} L_F$$
$$= \frac{36.6\,KJ}{2} = \underline{\underline{18.8\,KJ}}$$

Now $\Delta Q_{H_2O} > \Delta Q_{ice}$ $\Rightarrow$ heat from water can melt one ice cube and raise the temp of the water from ice to above 0°C.

Let the final temp be T_f

- Heat supplied by water @ 25°C to T_f

$$\Delta Q_{H_2O} = m_L c_L (25 - T_f)$$

- Heat need to melt the ice, change phase and heat up the ice water to final temp.

$$\Delta Q_{ice} = m_{ice} c_{ice}(15) + m_{ice} L_F + m_{ice} c_L (T_f - 0)$$

$$m_L c_L (25 - T_f) = m_{ice} c_{ice}(15) + m_{ice} L_F + m_{ice} c_L T_f$$

$$m_L c_L (25) - m_{ice} c_{ice}(15) + m_{ice} L_F = (m_{ice} c_L + m_L c_L) T_f$$

$$T_f = \frac{25 m_L c_L - 15 m_{ice} c_{ice} + m_{ice} L_F}{m_{ice} c_L + m_{H_2O} c_L}$$

with
$$\left.\begin{aligned} m_L &= 200\,g \\ m_{ice} &= 50\,g \\ c_L &= 4.2\,J/gK \\ c_{ice} &= 2.22\,J/gK \\ L_F &= 333\,J/g \end{aligned}\right\}$$

18-43

Work done $\Delta W = \vec{F} \cdot \vec{\Delta X}$

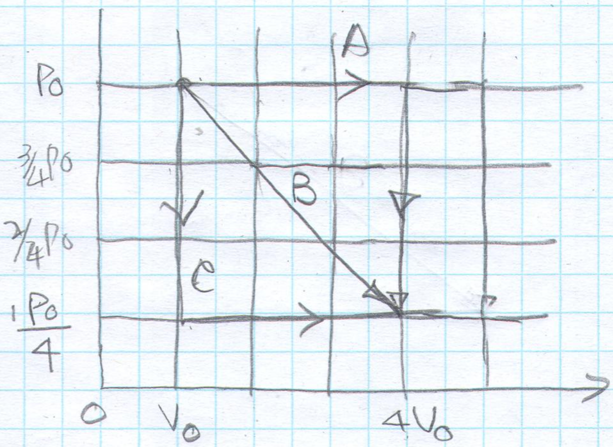
$= F \Delta X \cos\theta$

$= F \Delta X \qquad \theta = 0$

$= (P A) \Delta X$

$= P \Delta V$

$\lim_{\Delta V \to 0} \Delta W = \lim_{\Delta V \to 0} P \Delta V = P dV$



$W = \int dW = \int_{V_i}^{V_f} P dV$

For path A. $\quad W = \int_{V_0 \, A}^{4V_0} P dV = (P_0)(4V_0 - V_0) = 3 \underline{\underline{P_0 V_0}}$

For path B $\quad W = \int_{V_0 \, B}^{4V_0} P \, dV = \text{Area under curve B} = \underline{\underline{\frac{15 P_0 V_0}{8}}}$

For path C $\quad W = \int_C P dV = \text{Area under curve C} = \left(\frac{P_0}{4}\right) 3V_0 = \underline{\underline{\frac{3}{4} P_0 V_0}}$

(cycle of an engine)

18-49

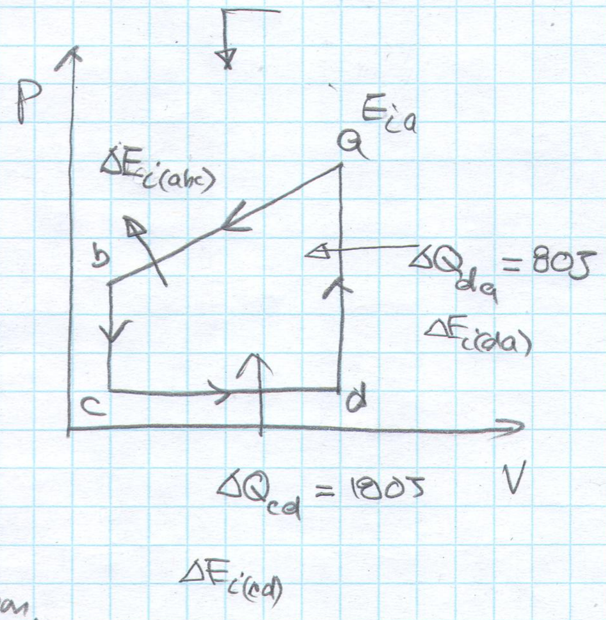
First law of thermodynamic

$dQ = dE_i + dW$

the amount of heat, dQ, flows into the system changes the internal energy dE_i of the system and causes the system to do work dW.

dQ is $-ve$ if heat flows out the system

dW is $-ve$ if work is done to the system.



Let E_{ia} be the total internal energy at a.

After the cyclic process ie the system starting from a then moves through b, c, d and back to a again, the total internal energy returns to E_{ia}.

Total change in internal energy $= 0$

$$\Delta E_{i(abc)} + \Delta E_{i(cd)} + \Delta E_{i(da)} = 0 \quad \text{——— (1)}$$

but $\Delta E_i = \Delta Q_i - \Delta W$

For $\Delta E_{i(cd)} = \Delta Q_{cd} - \Delta W_{cd}$ — (2) and $\Delta E_{i(da)} = \Delta Q_{da} - \Delta W_{da}$ — (3)

However $\Delta W_{da} = \int_d^a p\,dV$ with $dV = 0$.

$$\Delta W_{da} = 0$$

sub (2) & (3) to (1)

$$\Delta E_{i(abc)} + \Delta Q_{cd} - \Delta W_{cd} + \Delta Q_{da} = 0$$

$$\Delta W_{cd} = \Delta E_{i(abc)} + \Delta Q_{cd} + \Delta Q_{da}$$

$$= -200J + 180J + 80J$$

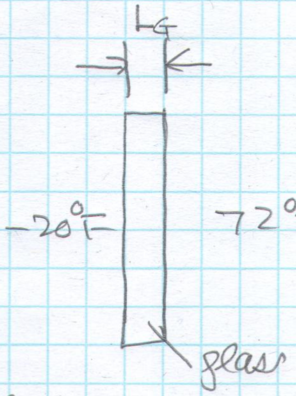
$$= \underline{\underline{+60J}}$$

work done to the outside world

$\Delta E_{i(abc)} = -200J$ flows out of the syst.

$\Delta Q_{cd} = +180J$
$\Delta Q_{da} = +80J$ } flows into the system

18-57. $L_G = 3\,mm$



$\Delta T = 72+20 = 92°F$
$= \left(\frac{92}{9}\right)(5)°C$

$$\frac{\Delta Q}{\Delta t} = KA\frac{(T_H - T_L)}{L}$$

$$\frac{\Delta Q}{\Delta t}\left(\frac{1}{A}\right) = K\frac{(T_H - T_L)}{L}$$

↑ rate of energy loss per meter sq.

K for glass $= 1\,W/m \cdot K$

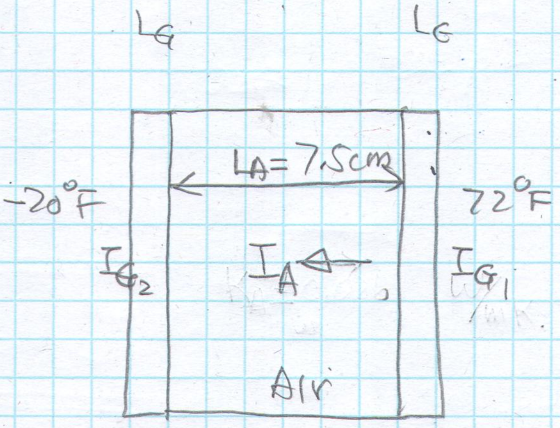
For glass of $L_G = 3 \times 10^{-3}\,m$, $\Delta T = \left(\frac{92}{9}\right)(5)°C = \left(\frac{92}{9}\right)(5)\,K$.

$$\frac{\Delta Q}{\Delta t}\left(\frac{1}{A}\right) = \frac{(1)\left(\frac{92}{9}\right)(5)}{3\times10^{-3}} \frac{W}{m\cancel{K}}\frac{\cancel{K}}{m} \quad \left(\frac{J/t}{m^2}\right) ✓$$

$$= 17{,}037\ \frac{W}{m^2}$$

With storm window
the flow of energy requires that

$$I_{G_1} = I_A = I_{G_2} = \frac{\Delta Q}{\Delta t}\left(\frac{1}{A}\right)$$



Flow of heat per Area is the same through material in series.

$K_A = 0.026\,W/mK$

$$I = \frac{\Delta Q}{\Delta t A} = K\left(\frac{T_H - T_C}{L}\right)$$

$$= \left(\frac{K}{L}\right)(T_H - T_C)$$

I is determined by K, L and ΔT.

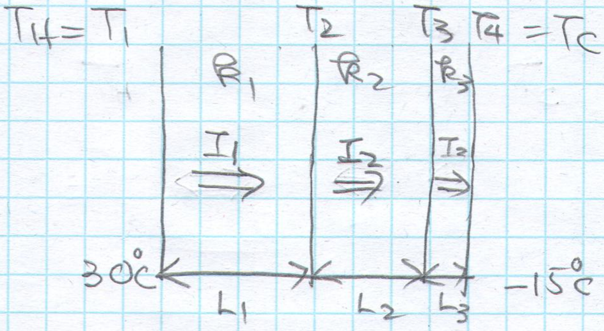
In this case $K_G \gg K_A$ and $L_A \gg L_G$ so the flow of heat through the glass/air/glass gap is determined by the heat flow at the air gap. ie I_A.

P11

$$I_A = K_A \frac{T_H - T_C}{L}$$

$$= 0.026 \frac{W}{mK} \frac{\left(\frac{92}{9}\right)(5)\,K}{7.5\times10^{-2}\,m} = \underline{\underline{17.72}} \quad \frac{W}{m^2}$$

18-60



Layer thickness

L_1, $L_2 = 0.7L_1$, $L_3 = 0.35L_1$

k_1, $k_2 = 0.9k_1$, $k_3 = 0.8k_1$

$T_H = 30°C$ $T_c = -15°C$

Heat current through layer 1 $= I_1 = \frac{\Delta Q_1}{\Delta t}\left(\frac{1}{A}\right)$

Heat " " ... 2 $= I_2 = \frac{\Delta Q_2}{\Delta t}\left(\frac{1}{A}\right)$

... 3 $= I_3 = \frac{\Delta Q_3}{\Delta t}\left(\frac{1}{A}\right)$

At steady conduction. $I_1 = I_2 = I_3$ $\frac{\text{Energy flow through}}{\text{time}}/A$ = constant.

$$I_1 = k_1 \frac{T_1 - T_2}{L_1} = I \quad \text{—— (1)}$$

$$I_2 = k_2 \frac{T_2 - T_3}{L_2} = I \quad \text{—— (2)}$$

$$I_3 = k_3 \frac{T_3 - T_4}{L_3} = I \quad \text{—— (3)}$$

Need to find $T_2 - T_3 = \Delta T$

From (1) $\frac{I L_1}{k_1} = T_1 - T_2$

$$T_2 = T_1 - \frac{I L_1}{k_1} = T_H - \frac{I L_1}{k_1} \quad \text{—— (1a)}$$

From (3) $\frac{I L_3}{k_3} = T_3 - T_4$

$$T_3 = \frac{I L_3}{k_3} + T_4 = \frac{I L_3}{k_3} + T_C \quad \text{—— (3a)}$$

$$\therefore \Delta T = T_2 - T_3 = T_H - \frac{I L_1}{k_1} - \left(\frac{I L_3}{k_3} + T_C\right)$$

$$\Delta T = (T_H - T_C) - I\left(\frac{L_1}{k_1} + \frac{L_3}{k_3}\right) \quad \text{—— (4)}$$

compare with eq (2) where

$$\Delta T = T_2 - T_3 = I\left(\frac{L_2}{k_2}\right) \quad \text{—— (5)}$$

combine ④ & ⑤ to eliminate I

From ⑤ $I = \Delta T\left(\frac{k_2}{L_2}\right)$

Sub to ④

$$\Delta T = (T_H - T_C) - \Delta T\left(\frac{k_2}{L_2}\right)\left(\frac{L_1}{k_1} + \frac{L_3}{k_3}\right)$$

$$\Delta T\left(1 + \left(\frac{k_2}{L_2}\right)\left(\frac{L_1}{k_1} + \frac{L_3}{k_3}\right)\right) = (T_H - T_C)$$

$$\Delta T = \frac{(T_H - T_C)}{\left(1 + \left(\frac{k_2}{L_2}\right)\left(\frac{L_1}{k_1} + \frac{L_3}{k_3}\right)\right)}$$

$$= \frac{(T_H - T_C)}{\left(1 + \left(\frac{0.9k_1}{0.7L_1}\right)\left(\frac{L_1}{k_1} + \frac{0.35L_1}{0.8k_1}\right)\right)}$$

$$= \frac{45}{\left(1 + \left(\frac{0.9}{0.7}\right)\left(1 + \frac{0.35}{0.8}\right)\right)}$$

$$= \frac{45}{2.85}$$

$$= 15.8\,^\circ C$$

If $k_2 = 1.1k_1$, the rest is the same

$$\Delta T = \frac{45}{1 + \left(\frac{1.1k_1}{0.7L_1}\right)\left(\frac{L_1}{k_1} + \frac{0.35L_1}{0.8k_1}\right)}$$

$$= 13.81\,^\circ C$$

Since $I = \frac{T_H - T_C}{L_1/k_1 + L_2/k_2 + L_3/k_3}$

and if $k_2 = 1.1k_1$ instead of $0.9k_1$

$\Rightarrow \sum L_i/k_i$ is smaller and I will increase.

the rate energy is conducted is greater than previously with $k_2 = 0.9k_1$.

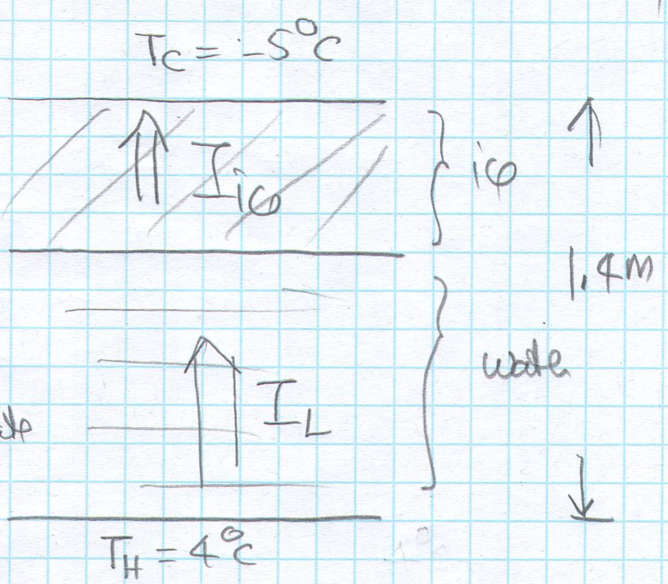
18-65

Bottom of the Pond is warmer than the surface of the ice

Heat is constantly conducted through water then the ice sheet to the open air @ $-5°C$

At steady state, the heat flow rate per unit area will be constant

ie $I_L = I_{ice}$ = heat flow rate through ice and water is same



$$I = \frac{\Delta Q}{\Delta t}\left(\frac{1}{A}\right) = k\frac{\Delta T}{L}$$

For ice, $I_{ice} = k_i \frac{\Delta T_i}{L_i} = \frac{k_i(0 - T_C)}{L_i} =$

For water: $I_L = k_L \frac{\Delta T_L}{L_L} = \frac{k_L(T_H - 0)}{L_L}$

$I_{ice} = I_L \Rightarrow \frac{k_i(5)}{L_i} = \frac{k_L(4)}{L_L}$

$L_i = \frac{5k_i}{4k_L} L_L$ ——— (1)

Since $L_i + L_L = 1.5\,m$. ——— (2)

solve for L_i

From (2) $L_L = 1.5 - L_i$ sub to (1)

$L_i = \left(\frac{5}{4}\right)\frac{k_i}{k_L}(1.5 - L_i)$ $\quad k_i = 0.4 \quad k_L = 0.12$

$= 4.17\,(1.5 - L_i)$

$L_i = \frac{(4.17)(1.5)}{5.17}\,m = \underline{\underline{1.21\,m}}$