16-6

Let the traveling sinusoidal wave be represented by

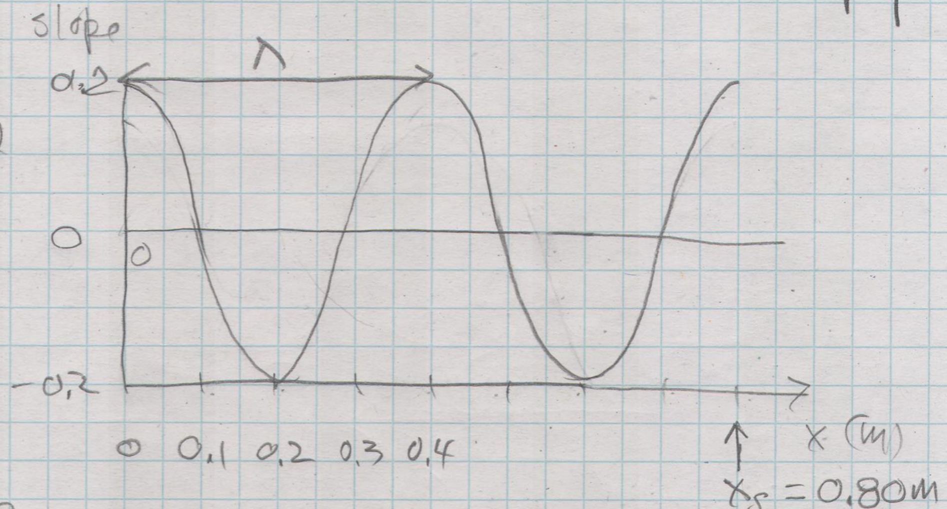
$y(x,t) = A\sin(kx - \omega t)$

At $t=0$

$y(x, t=0) = A\sin kx$

Slope $= \dfrac{dy}{dx} = Ak\cos kx$

with $k = \dfrac{2\pi}{\lambda}$



λ & Ak can be obtained from the plot of the slope $\left(\dfrac{dy}{dx}\right)$ v.s. x

$\lambda = 0.4$m

$(Ak) = A\left(\dfrac{2\pi}{\lambda}\right) = 0.2$

Amplitude $A = \dfrac{(0.2)(\lambda)}{2\pi} = \dfrac{(0.2)(0.4)}{2\pi}$

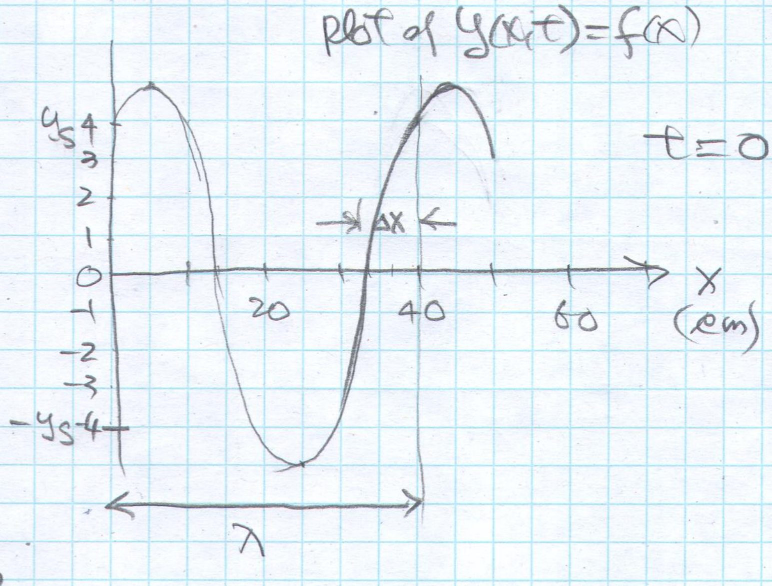
16-23

Let the equation representing a sinusoidal transverse wave traveling in $-\hat{x}$ be:

$y(x,t) = A\sin(kx + \omega t)$

Tension on the string $T = 3.6N$

Linear density $\mu = 25g/m$



(a) Find Amplitude A

$A = 5$ cm.

(b) $\lambda = 40$ cm

(c) $V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{3.6N}{25\times10^{-3}kg/m}} = 12ms^{-1}$

(d) period T:

Since $\lambda f = v$ and $f = \frac{1}{T}$ $\therefore T = \frac{\lambda}{V} = \frac{40\times10^{-2}m}{12ms^{-1}}$ //

(e) max transverse speed.

Since $y(x,t) = A\sin(kx+\omega t)$

$\frac{dy(x,t)}{dt} = A\omega\cos(kx+\omega t)$

$\frac{\omega}{2\pi} = f$

$\left.\frac{dy(x,t)}{dt}\right)_{max}$ when $\cos(kx+\omega t) = \pm 1$.

$\therefore A\omega = \left.\frac{dy(x,t)}{dt}\right)_{max} = A(2\pi f) = \frac{2\pi A}{T}$ //

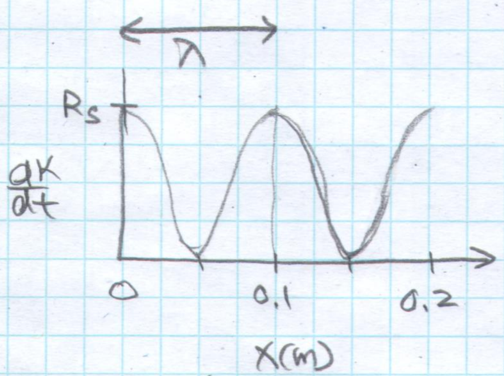
If the wave $y(x,t) = y_m\sin(kx \pm \omega t + \phi)$, the correct sign is (+).

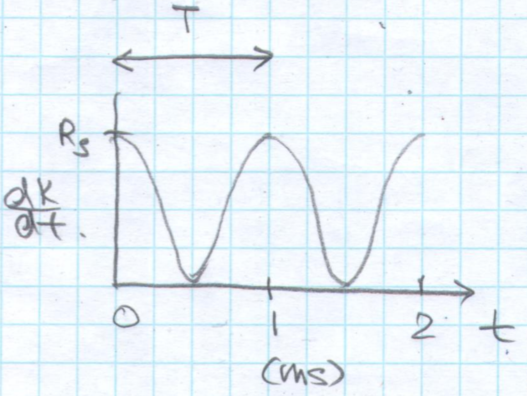
k and ω are the same as above and the phase angle ϕ can be read out from the graph $\phi = \left(\frac{\Delta x}{40}\right)360 = \left(\frac{7}{40}\right)\pi = 0.175\pi$

16-27 given. $\mu = 2g/m$

Also given is $\frac{dK}{dt}$ vs. x and $\frac{dK}{dt}$ vs. t

$R_s = 10W$





Find the wave amplitude

From the graphs, the wave is sinusoidal in nature, and the equation that describe the rate of transmission of kinetic energy is

$$\frac{dK}{dt} = \frac{1}{2} y_m^2 \omega^2 \cos^2(kx - \omega t)\mu v \qquad v - \text{vel of propagation}$$

$\boxed{\text{Find } y_m}$ period $T = 1\,ms = 10^{-3}s$, $f = \frac{1}{T}$, $\omega = 2\pi f = \frac{2\pi}{T}$ rad/sec

Wavelength $\lambda = 0.1\,m$. and $k = \frac{2\pi}{\lambda}$

and $v = \lambda f = (0.1)\left(\frac{1}{10^{-3}}\right) ms^{-1}$

At $\left.\frac{dK}{dt}\right)_{max} = \frac{1}{2} y_m^2 \omega^2 \cos^2(kx - \omega t)\mu v$ with $\cos^2(kx - \omega t) = 1$

$$= \frac{1}{2} y_m^2 \omega^2 \mu v$$

$$y_m = \sqrt{\frac{2\left(\frac{dK}{dt}\right)_{max}}{\omega^2 \mu v}}$$

//

$\frac{dK}{dt} = 10W$, $\omega = \frac{2\pi}{10^{-3}}$

$\mu = 2 \times 10^{-3} kg/m$

$v = 10^2 ms^{-1}$

P4

16-29 $\quad y(x,t) = (2\,mm)\left[20\,m^{-1}x - 4s^{-1}t\right]^{0,5}$

$$y^2(x,t) = (2\,mm)^2\left[20\,m^{-1}x - 4s^{-1}t\right]^1$$

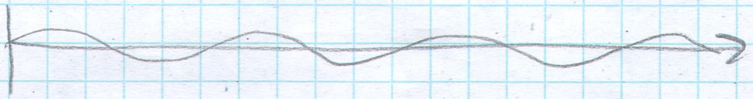
$$\frac{y^2(x,t)}{(2\,mm)^2(20\,m^{-1})} = \left[x - \left(\frac{4\,s^{-1}}{20\,m^{-1}}\right)t\right]$$

cf with the traveling equation

$$x' = x - vt$$

$$v = \frac{4}{20}\left(\frac{m}{s}\right) = \underline{\underline{0,2\,ms^{-1}}}$$

16-34 $T = 1200N$, $\omega = 1200 \text{ rads}^{-1}$ $y_m = 3\text{ mm}$, $\mu = 2g/m$



(a) Find the average rate at which energy is transported to the other end of the cord

The equation that describe the total energy transfer rate (KE+P.E)

$$\frac{dE}{dt} = \mu \omega^2 y_m^2 V \cos^2(kx - \omega t)$$

The average

$$\left\langle \frac{dE}{dt} \right\rangle = \text{cycle average} = \text{average} = \mu \omega^2 y_m^2 V \langle \cos^2(kx - \omega t) \rangle$$

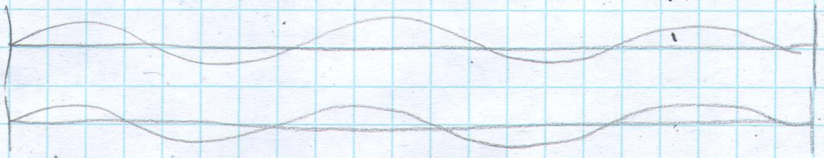
$\frac{1}{2}$ cycle average

$$= \frac{1}{2} \mu \omega^2 y_m^2 V$$

$$= \frac{1}{2} \mu \omega^2 A^2 \sqrt{\frac{T}{\mu}}$$

$$= \frac{1}{2} \omega^2 A^2 \sqrt{\mu T}$$ ω, A, μ, T from above

(b)



Total average for identical cord is $2\left\langle \frac{dE}{dt} \right\rangle$.

(c) two waves through the same cord, but at different phase angle δ

Case (1) $\delta = 0$.

$$\frac{dE}{dt} = \mu \omega^2 y_m^2 V \left[\cos^2(kx - \omega t) + \cos^2(kx - \omega t + \delta)\right]$$

$$= 2\mu \omega^2 y_m^2 V \cos^2(kx - \omega t)$$

$$\left\langle \frac{dE}{dt} \right\rangle = \frac{2}{2} \mu \omega^2 y_m^2 V \cos^2(kx - \omega t)$$

case(2). $\delta = 0.4\pi$

Need to work out the equation of the resultant waves

$y_1 = y_{m_1} \sin(kx - \omega t)$

$y_2 = y_{m_2} \sin(kx - \omega t + \delta)$ with $\delta = 0.4\pi$.

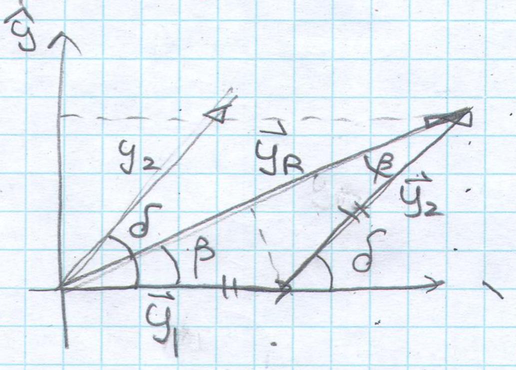
Resultant wave

$$y_R = y_1 + y_2 = y_{m_1} \sin(kx - \omega t) + y_{m_2} \sin(kx - \omega t + \delta)$$

we can find the resultant y_R by treating y_1 and y_2 as two different vectors with magnitude y_{m_1} and y_{m_2} and their relative direction is defined by the phase angle δ. (phasor representation)

Over time, $\vec{y}_R$ will rotate with angular speed ω.

the projection of $\vec{y}_R$ on $\hat{y}$ axis is the sum of y_1 and y_2



phasor diagram

$y_{mR} = y_{m_1} \cos\beta + y_{m_2} \cos\beta \quad = 2y_{m_1} \cos\frac{\delta}{2}$ with $y_{m_1} = y_{m_2}$

and $2\beta = \delta$ or $\beta = \frac{\delta}{2}$

$$y_R = y_{mR} \sin(kx - \omega t + \beta)$$

$$= 2y_{m_1} \cos\frac{\delta}{2} \sin(kx - \omega t + \beta)$$

$$= A \sin(kx - \omega t + \beta) \quad \text{with } A = 2y_{m_1} \cos\frac{\delta}{2}, \ \beta = \frac{\delta}{2}$$

$y_R(x,t)$ is the resultant wave travelling down the cord

$$\frac{dK}{dt} = \mu \omega^2 y_{mR}^2 V \cos^2(kx - \omega t + \beta)$$

$$\left\langle \frac{dK}{dt} \right\rangle = \mu \omega^2 \left(2y_{m_1} \cos\frac{\delta}{2}\right)^2 V \underbrace{\langle \cos^2(kx - \omega t + \beta) \rangle}_{\frac{1}{2}} = \frac{1}{2} \omega^2 \left[2y_{m_1} \cos(0.2\pi)\right]^2 \sqrt{\mu T}$$

(cycle average)

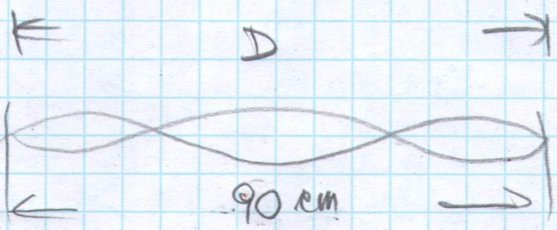
P7

case(3) $\delta = \pi$

$y_{mR} = 2y_{m_1}\cos\left(\frac{\pi}{2}\right)$
$\hookrightarrow 0$

$\left\langle \frac{dP}{dt} \right\rangle = \frac{1}{2}\omega^2\left[2y_{m_1}(0)\right]^2\sqrt{\mu T} \; = 0$

16-49



$\mu = 7.2\ g/m$ $\quad T = 150N$

$D = 90\ cm$

Find speed V, λ, f of the traveling wave forming this standing wave

$$V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{150N}{7.2\times10^{-3}\ kg/m}} \quad //$$

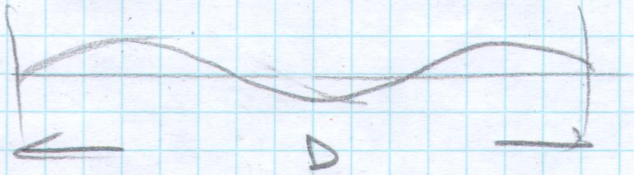
$$\lambda = \frac{2D}{3} \quad // \qquad \text{and} \qquad f = \frac{V}{\lambda} \quad //$$

P9

16-51. $V = 100\,ms^{-1}$ $A = 1$ cm. the simplest standing wave is formed by two travelling waves travelling in opposite direction with same amplitude.

$y_1(x,t) = y_{m_1} \sin(kx + \omega t)$

$y_2(x,t) = y_{m_2} \sin(kx - \omega t)$



with k, ω, and y_{m_i} same

the sign of the freq. ω is negative ie $(-)$.

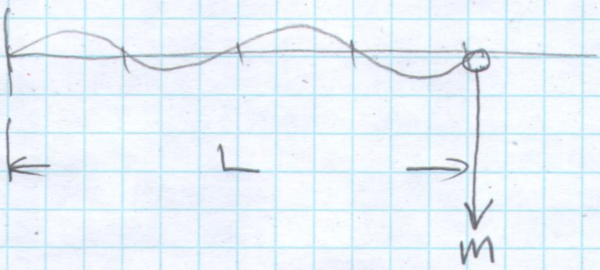
$k = \frac{2\pi}{\lambda}$ and $\lambda = \frac{2D}{3}$, $\lambda f = V$

$f = \frac{V}{\lambda}$.

$\omega = 2\pi f$

16-58 $L = 1.2\,m$ $\mu = 1.6\,g/m$ $f = 120\,Hz$

Find m for the generation of 4th harmonic on the string



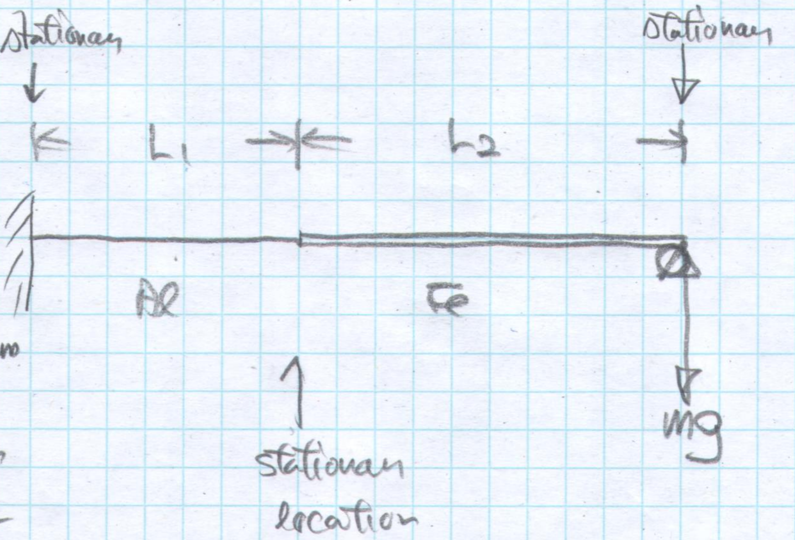
$\lambda f = v$ and $v = \sqrt{\frac{T}{\mu}}$ and $T = mg$

$\therefore \lambda f = \sqrt{\frac{mg}{\mu}} \Rightarrow \frac{\mu \lambda^2 f^2}{g} = m$ //

$\lambda = \frac{L}{2}$, $f = 120\,s^{-1}$, $\mu = 1.6 \times 10^{-3}\,kg/m$. Find m

$g = 9.81\,ms^{-2}$

16-59



For Al: $L_1 = 60\,cm$
$A = 10^{-2}\,cm^2$
$\rho = 2.6\,g/cm^3$

$\mu_1 = \rho A = 2.6 \times 10^{-2}\,g/cm$

For Steel: $L_2 = 86.6\,cm$
$A = 10^{-2}\,cm^2$
$\rho = 7.8\,g/cm^3$

$\mu_2 = 7.8 \times 10^{-2}\,g/cm$

$T = mg$ with $m = 10\,kg$.

Let the allowed freq be f. This is the same freq for both material with same tension $T = mg$

For Aluminum

$L_1 = \frac{n \lambda_1}{2}$ where n is # of node that oscillate between the wall and the joint between two wires

and $v_1 = \sqrt{\frac{T}{\mu_1}} = \lambda_1 f$

$$\sqrt{\frac{T}{\mu_1}} = \frac{2L_1}{n} f \quad \text{—— (1)}$$

For Steel. Let m be # of node between pulley and joint of wires

$$\sqrt{\frac{T}{\mu_2}} = \frac{2L_2}{m} f \quad \text{—— (2)}$$

Take ratio of (1) to (2)

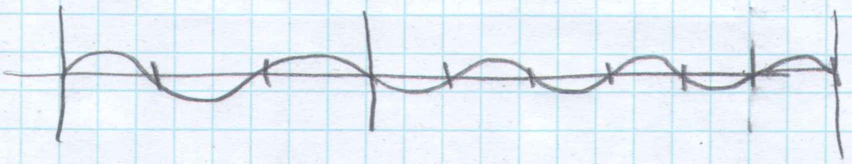
$$\sqrt{\frac{\mu_2}{\mu_1}} = \frac{m L_1}{n L_2}$$

$$\frac{m}{n} = \left(\frac{L_2}{L_1}\right)\sqrt{\frac{\mu_2}{\mu_1}} = \frac{5}{2}$$

or the min. value of n is 2 and the corresp value for m is 5

n & m must be an integer since they reflect # of node on the string that free to oscillate.

$n=2$ for Alum and 5 for Steel



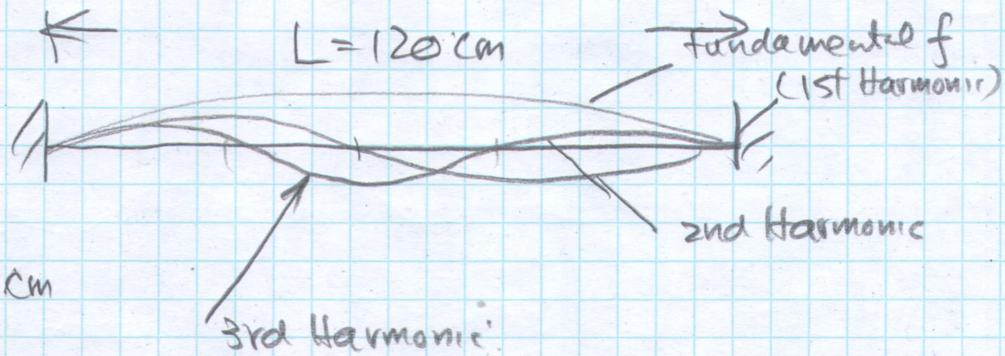
Total # of node including the node on the joint is

$2+5+1 = \underline{\underline{8}}$

The lowest freq f :

$$f = \frac{n}{2L_1}\sqrt{\frac{mg}{\mu_1}}$$

16-85



$\lambda_{Fund} = 2(120)$ cm

$\lambda_{2nd\,Harmonic} = 120\left(\frac{2}{2}\right)$ cm

$\lambda_{3rd\,Harmonic} = 120\left(\frac{2}{3}\right)$ cm

The wavelength of the standing wave matches the wavelength of the traveling waves that generate the standing waves.