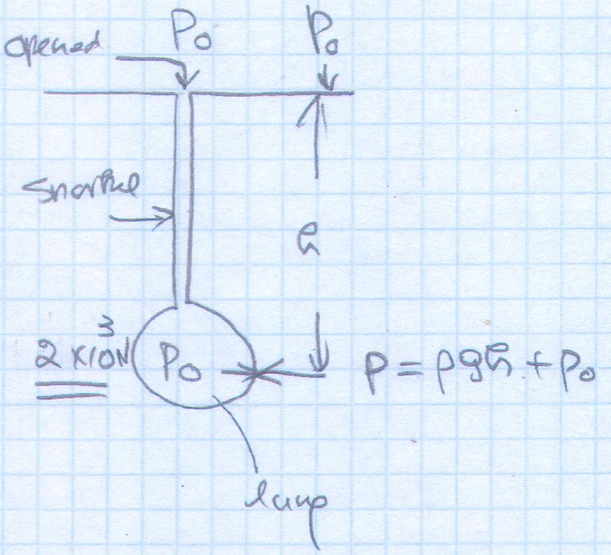
14–16. (a) $\Delta p = \rho g h$



$\rho = 998\ \mathrm{kg\,m^{-3}}$

$g = 9.81\ \mathrm{m s^{-2}}$

$h = 0.2\mathrm{m}$

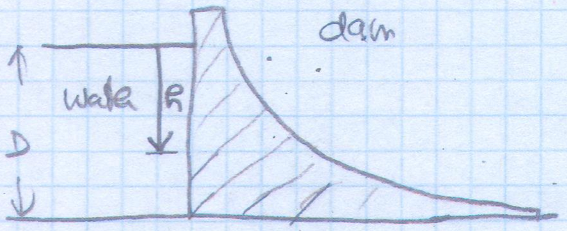
$\Delta p = (998)(9.81)(0.2)\mathrm{N} = \underline{\underline{2 \times 10^{3}\mathrm{N}}}$

(b) $h = 4\mathrm{m}$

$\Delta p = (998)(9.81)(4)\mathrm{N}$

$= \underline{\underline{3.9 \times 10^{4}}}\ \mathrm{N}$

14–24 $D = 35\mathrm{m}$ width $w = 314\mathrm{m}$



Since $p = p_0 + \rho g h$

where p_0 is the atmosphere pressure

h is the height of the water column below its surface

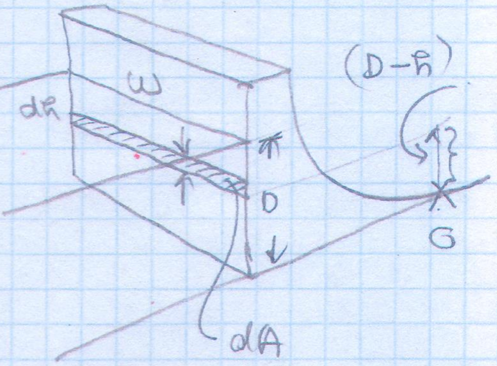
gauge pressure refer to $p - p_0 = \rho g h = P_G$

The pressure is $\propto$ to the depth of the water (height of the water column)

Since $dF = p\,dA$ where $p(h)$ acts on area dA and dF is the corresponding force acting on dA at pressure $p(h)$

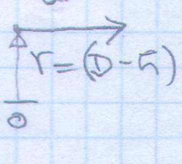
$dF = \rho g h (w\,dh)$

Total force $= \int_{h=0}^{h=D} dF = \rho g w \int_0^D h\,dh = \rho g w \frac{h^2}{2} = \underline{\rho g \frac{w D^2}{2}}$



To calculate the net torques one has to find the $d\tau$ due to the dF acting on the dam

$d\tau = (\rho g h w\,dh)(D-h)$



14-24

$$\int_{h=0}^{h=D} d\tau = \int_{h=0}^{D} \rho g W D h \, dh - \int_{0}^{D} \rho g W h^2 \, dh$$

$$= \rho g W \left(\frac{D^3}{2} - \frac{D^3}{3} \right)$$

$$= \frac{\rho g W D^3}{6}$$

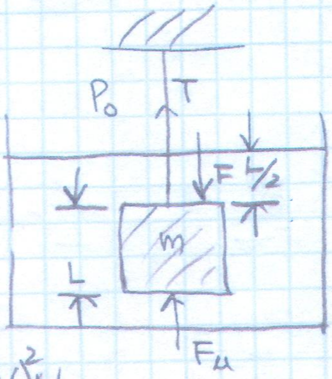
Since $\tau = r \times F = rF\sin\theta = rF\sin(90) = rF$ with r = moment arm

$$r = \frac{\tau}{F} = \frac{\cancel{\rho g W D^{3}}\,/\,\cancel{6}\,3}{\cancel{\rho g W D^{2}}\,/\,\cancel{2}} = \underline{\underline{\frac{D}{3}}}$$

14-32

(a) $F = P_0 A + \rho\left(\frac{L}{2}\right) g (L)^2$

$= P_0 L^2 + \rho \frac{L^3}{2} g$



$P_0 = 10^5$ pascal $\left(\frac{N}{m^2}\right)$

$g = 9.8\ ms^{-2}$

$L = .6\ m$

$\rho = 998\ kg\ m^{-3}$

$F = (10^5)(0.6)^2 + (998)\left(\frac{(0.6)^3}{2}\right)(9.8)(0.6)^2 N$

$=$

(b) mag of upward force on the cube bottom

$F_u = P_0 A + \rho g \left(L + \frac{L}{2}\right) A$

(c) Tension on rope

$T + F_u = F + mg$

$T = F + mg - F_u$

$= \cancel{P_0 A} + \rho\left(\frac{L}{2}\right) g L^2 + mg - \cancel{P_0 A} - \rho g \left(L + \frac{L}{2}\right) A$

$= \cancel{\rho \frac{L^3}{2} g} + mg - \rho g L^3 - \cancel{\rho g \frac{L^3}{2}}$

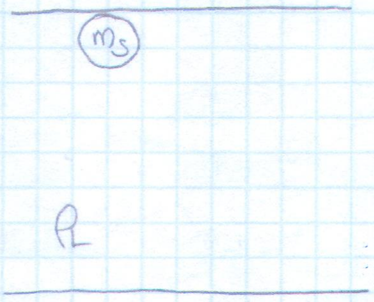
$= \underline{mg - \rho g L^3}$

(d) Buoyancy force

$F_B = \rho g L^3$ = wt of fluid displaced.

14-38

Let ρ_s, m_s, V_s be the density, mass and vol of the ball



The buoyance force due to liquid with density ρ_L is the weight of the liquid displaced

so $F_b = \rho_L V_L g$ and $V_L = V_s = V$ since vol of liquid displaced is the same as sphere.

Net force on the sinking ball

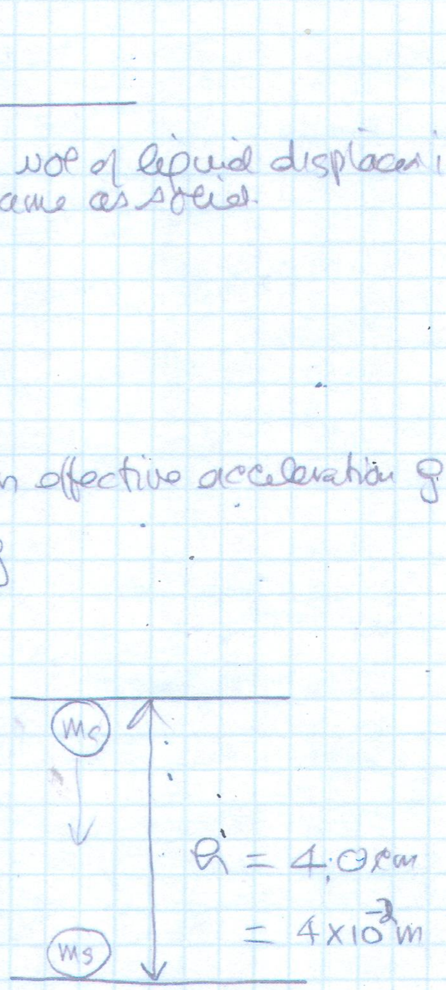
$$F_{Net} = m_s g - m_L g = (m_s - m_L) g$$
$$= (\rho_s - \rho_L) V g$$

this net force F_{Net} that acts on m_s give an effective acceleration g'

$$F_{Net} = m_s g' = (m_s - m_L) g = (\rho_s - \rho_L) V g$$

work done by this force to bring m_s to a depth of $h = (m_s g')h$

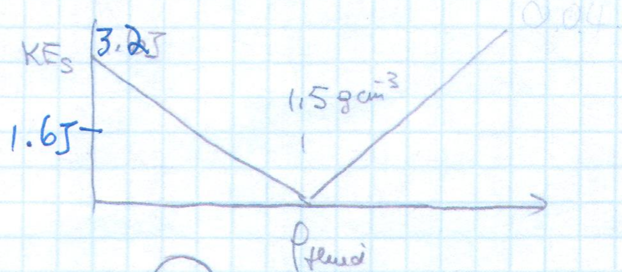
This change in the potential energy is the new kinetic energy of the ball (we assume the ball is stationary initially when it is released at the surface)



$$m_s g' h = KE_s$$

replacing g' by ρ_s, ρ_L and V

$$m_s \frac{(\rho_s - \rho_L)}{m_s} V g h = KE_s$$



$$\rho_s V g h - \rho_L V g h = KE_s \qquad (1)$$

This equation shows KE_s is a linear function of ρ_L and is consistent with the graph in Fig 14-40

Since $\rho_s V$, g and h are consts then

$$A - \rho_L B = KE_s \quad \text{with } A = \rho_s V g h \ , \ B = V g h$$

when $A = \rho_L B$, $KE_s = 0$

and when $\rho_L = 0$ $\quad A = KE_s$

Lets use the values obtained from graph to find ρ_s and V_s

At $\rho_L = 0$, $KE_s = 2(1.65)J = 3.2J$

$$\therefore \quad \rho_s V = \frac{1.65 \times 2}{gh} \quad \text{——} \ (2)$$

Eq (2) has two unknown parameters ρ_s & V_s. Need an add. eq

consider the KE_s at $\rho_L = 1.5\ g\,cm^{-3}$ where $KE_s = 0$

From (1)

$$\rho_s \cancel{Vgh} = \rho_L (1.5\ g\,cm^{-3}) \cancel{Vgh}$$

$$\rho_s = 1.5\ g/cm^3 = 1500\ Kg/m^3$$

and $V_s = V = \dfrac{2 \times 1.65\ J}{g h \rho_s} = \dfrac{1.65 \times 2}{g h (1500)}$ $\qquad h = 0.04\ m$

$g = 9.81\ ms^{-2}$

(use SI units K, M, S.)

14-45 Ignore buoyance force by air. the correction is small since water density is at least 1000 times than that of air.

the actual vol of iron $V_{iron} = \frac{m}{\rho}$ where m is the iron mass and ρ is density of iron

$m = 6000\,N$ and

$\rho = \left(7.87\, g/cm^3 =\right) 7.87 \times 10^3\, Kg/m^3$

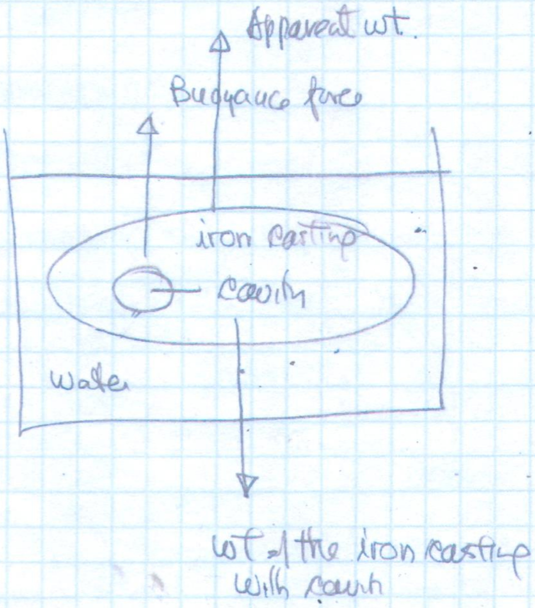
use SI units since the Force is expressed in Newton.

the apparent wt = 4000N

the wt in air = 6000N

the buoyance force = 2000 N

this buoyance force comes from the wt of water displaced by iron casting



$W_{H_2O} = 2000N = m_{H_2O}\, g$

$m_{H_2O} = \rho_{H_2O} V_{H_2O}$ with $V_{H_2O} = V_{iron\ casting} = V_{iron} + \Delta V_{cavities}$

$\therefore\ V_{iron} + \Delta V_{cavity} = \frac{m_{H_2O}}{\rho_{H_2O}} = \frac{W_{H_2O}}{g\,\rho_{H_2O}}$

$\Delta V_{cavity} = \frac{W_{H_2O}}{g\,\rho_{H_2O}} - V_{iron}$

$W_{H_2O} = 2000N$

$g = 9.8\, ms^{-2}$

$\rho_{H_2O} = 998\, Kg\, m^{-3}$

$V_{iron} = \frac{6000\ N}{7.87 \times 10^3\ Kg\, m^{-3}}$

14 – 49 Def of mass density

$$\frac{m}{V} = \rho = \text{mass density}$$

$$m = \rho V$$

$$= \rho A v \qquad v = \text{vel of flow}, \; A = \text{X sectional area}$$

Since fluid is not compressible

conservation of mass req

$$m = \cancel{\rho} A_1 V_1 = \cancel{\rho} A_2 V_2$$

$$A_1 V_1 = A_2 V_2 \quad \text{— Eq of Continuity.}$$

Apply this equation to this problem

At c, X section of channel

$$A_i = HD$$

$$\therefore V_i HD = \text{mass flow}$$

At a, xsection of the water flow is restricted

$$A_a = \cancel{2}\left[(H-h)\frac{(D-d)}{\cancel{2}}\right] + (H-b)d$$

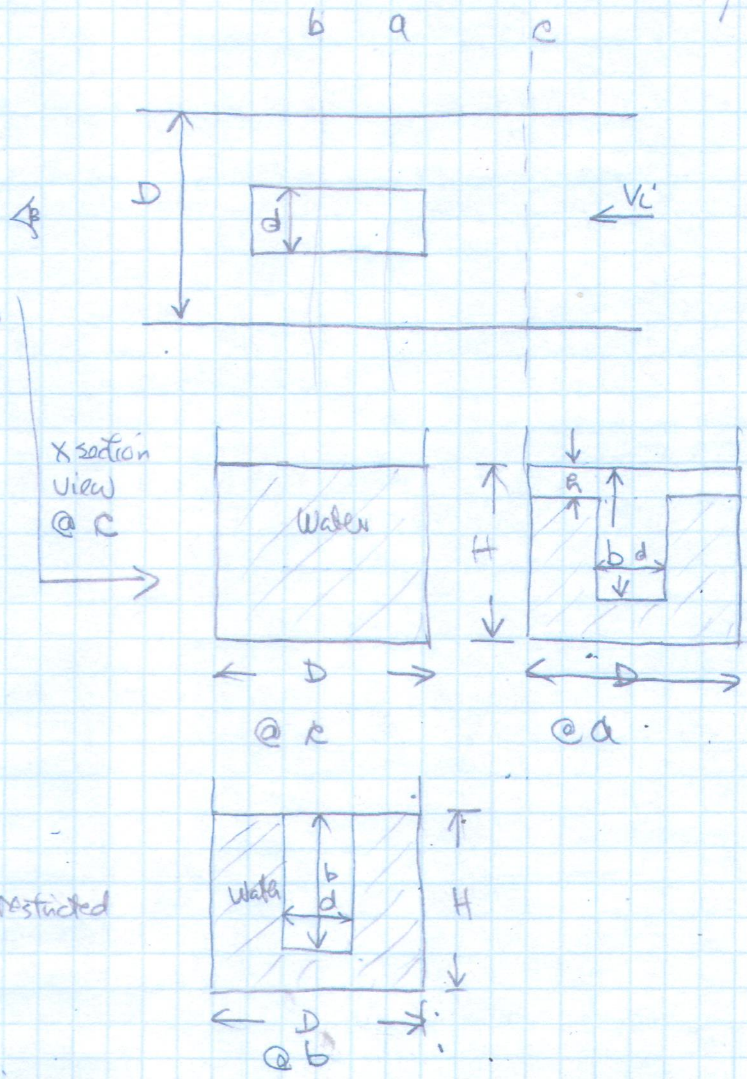
$$V_a A_a = V_i A_i$$

$$V_a = \frac{V_i A_i}{A_a} = \frac{V_i HD}{(H-h)(D-d) + (H-b)d}$$

At b, X section of water flow is less restricted

$$A_b = \cancel{2}\left(H\frac{(D-d)}{\cancel{2}}\right) + (H-b)d$$

$$V_b = \frac{V_i A_i}{A_b} = \frac{V_i HD}{H(D-d) + (H-b)d} \quad //$$



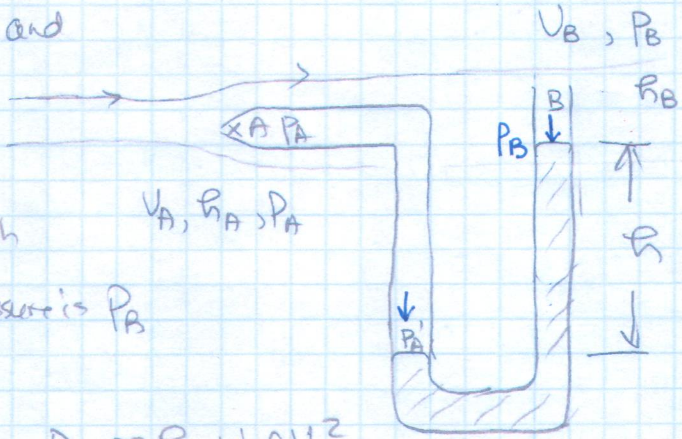
where V_i, H, D, d, b, h are given

14-62 pitot tube.

At pt A, air is stagnant and the vel of air is zero

The static pressure is P_0

At pt B, air is flowing with Vel V_B, the static pressure is P_B



using Bernoulli's eq

$$P_A + \rho g h_A + \frac{1}{2}\rho V_A^2 = P_B + \rho g h_B + \frac{1}{2}\rho V_B^2$$

since $h_A \simeq h_B$ and $P_0 = P_A$ = atm. pressure and $V_A = 0$ stagnant air at A.

$$P_0 = P_B + \frac{1}{2}\rho V_B^2$$

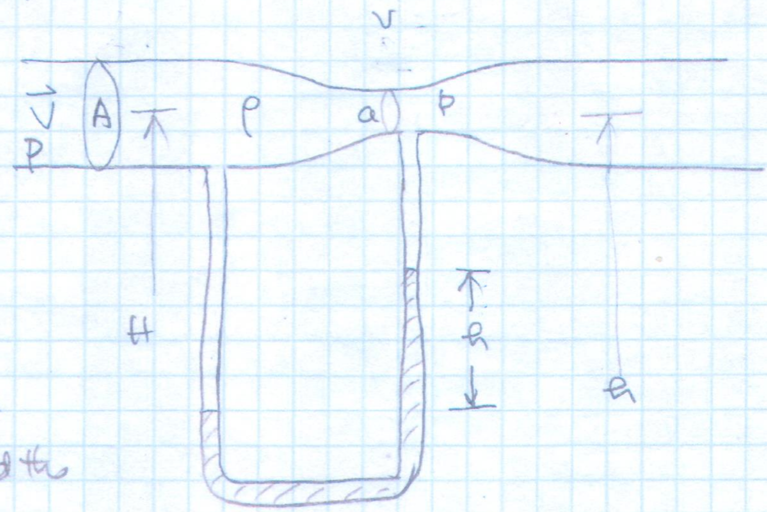
$$\sqrt{\frac{2(P_0 - P_B)}{\rho}} = V_B$$

Consider $(P_0 - P_B)$: $P_A = P_A'$ since this region is stagnant and P_A must be equal to P_A' at the surface of the liquid column

$(P_0 - P_B) = \rho_L g h$ where ρ_L is density of liq column

$V_B = \sqrt{\frac{2\rho_L g h}{\rho}}$ where ρ is density of air

14-65 To show $V = \sqrt{\frac{2a^2 \Delta p}{\rho(a^2 - A^2)}}$ Venturi meter



At Entrance

$p + \rho g H + \frac{1}{2}\rho V^2 = \text{Const.}$

At Venturi meter region

$p + \rho g h + \frac{1}{2}\rho v^2 = \text{const}$

Since there is no differential in mean height of the entrance and the Venturi meter $H = h$

and since that the two region is different region of the same flow, the

(a) Bernulli's eq: will hold.

$$P + \cancel{\rho g H} + \frac{1}{2}\rho V^2 = p + \cancel{\rho g h} + \frac{1}{2}\rho v^2$$

$$P - p = \frac{1}{2}\rho(v^2 - V^2) = \Delta p \quad \text{——— } (1)$$

use equation of Continuity

$$VA = va$$

$$v = V\left(\frac{A}{a}\right) \qquad v^2 = V^2\left(\frac{A^2}{a^2}\right)$$

Sub to (1)

$$\frac{1}{2}\rho\left(V^2\frac{A^2}{a^2} - V^2\right) = \Delta p$$

$$\frac{1}{2}\rho V^2\left(\frac{A^2 - a^2}{a^2}\right) = \Delta p$$

$$V = \pm\sqrt{\frac{2\Delta P}{\rho}\left(\frac{a^2}{A^2 - a^2}\right)}$$

(b) $\Delta p = (55 - 41)\,\text{kPa}$ $\rho_{H_2O} = 998\ \text{kg}\,\text{m}^{-3}$

$A = 64\ \text{cm}^2$

$a = 32\ \text{cm}^2$

the rate of water flow is $\frac{\Delta m}{\Delta t} = \frac{\rho \Delta V}{\Delta t} = \rho A \frac{\Delta L}{\Delta t}$

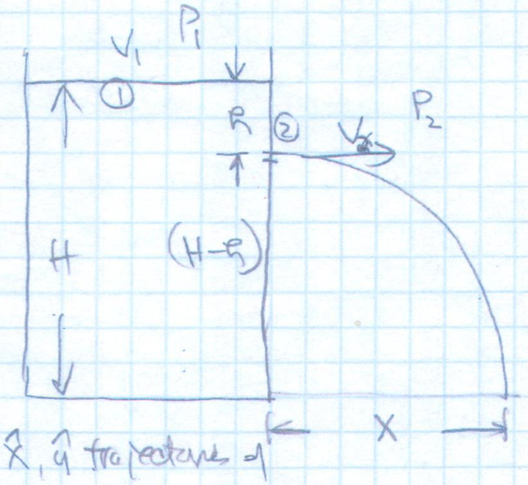
In diff form, Rate $= \frac{dm}{dt} = \rho A \frac{dl}{dt} = \rho A U$ where U is the vel

$$= \rho A V$$

$$= \rho_{H_2O} A \sqrt{\frac{2 \Delta p \, a^2}{\rho_{H_2O} (A^2 - a^2)}}$$

$$= \boxed{\sqrt{\frac{2 \Delta p \, a^2 A^2 \rho_{H_2O}}{(A^2 - a^2)}}}$$

K4-71 Apply Bernoulli equation to ① & ②



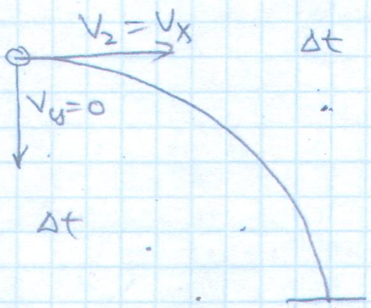
$$P_1 + \rho g H + \frac{\rho V_1^2}{2} = P_2 + \rho g (H-h) + \frac{\rho V_2^2}{2}$$

$P_1 = P_0 = P_2 = 1\,\text{atm}$, $V_1 \doteq 0$

$$\cancel{\rho} g h = \frac{\cancel{\rho} V_2^2}{2}$$

(a) $$\underline{V_2 = \sqrt{2gh}}$$

V_2 is in $\hat{x}$ direction. To find X, consider the $\hat{x}, \hat{y}$ trajectories of a single water drop at the orifice at ②



consider $\hat{x}$: $V_x = \dfrac{X}{\Delta t} = V_2$

consider $\hat{y}$: $y = \frac{1}{2} g (\Delta t)^2 + V_{0y} \Delta t$

$$(H-h) = \frac{1}{2} g \Delta t^2$$

$$\Delta t = \sqrt{\frac{2(H-h)}{g}}$$

$$\therefore \quad X = V_x \Delta t = V_x \sqrt{\frac{2(H-h)}{g}} = V_2 \sqrt{\frac{2(H-h)}{g}}$$

$$= \left(\sqrt{2gh}\right) \sqrt{\frac{2(H-h)}{g}}$$

$$\underline{\underline{X = \sqrt{4h(H-h)}}} \quad //$$

$H = 40\,\text{cm}$, $h = 10\,\text{cm}$

(b) There could be different h that gives the same X.

Square X $\therefore$ $X^2 = 4h(H-h) = 4Hh - 4h^2$

$$4h^2 - 4Hh + X^2 = 0$$

$$\left[\begin{array}{l} \text{As. } ay^2 + by + c = 0 \\ y = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ h = \dfrac{4H \pm \sqrt{16H^2 - 16X^2}}{(2)(4)} \end{array}\right]$$

solve for quadratic equation with $X = \sqrt{4h(H-h)}$

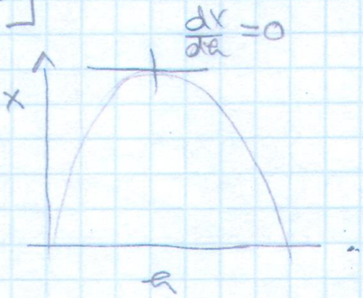
//

(c) To determine the depth of the hole where x is max
ie at what h that x is max.

Consider eq for (a) $x = [4h(H-h)]^{1/2}$

$$\frac{dx}{dh} = \left(\frac{1}{2}\right)[4h(H-h)]^{-1/2} \frac{d[4h(H-h)]}{dh}$$

$$= \frac{1}{2(4h(H-h))^{1/2}}\left[4H - (4)(2)h\right]$$



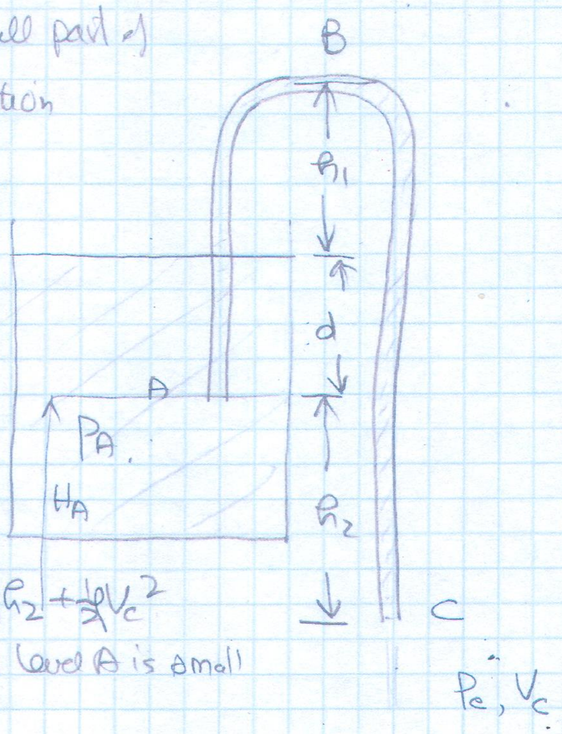
For max x. $\frac{dx}{dh} = 0$

$(4H - 8h) = 0$

$$\underline{\underline{h = \frac{1}{8}H}}$$

14-83 The fluid in A, B and C are all part of the fluid flow that satisfies the assumption made in Bernulli's equation.

Apply Bernulli's equation to flow region A and flow region C



$$P_A + \rho g H_A + \frac{1}{2}\rho V_A^2 = P_C + \rho g (H_A - h_2) + \frac{1}{2}\rho V_C^2$$

But $P_A = P_0 + \rho g d$, $P_C = P_0 = 1\,atm$

$$\cancel{P_0} + \rho g d + \cancel{\rho g H_A} + \frac{1}{2}\cancel{\rho V_A^2} = \cancel{P_0} + \cancel{\rho g H_A} - \rho g h_2 + \frac{1}{2}\rho V_C^2$$

vel on Area at level A is small

(a) $\frac{1}{2}\rho V_C^2 = \rho g (d + h_2)$

$$V_C = \sqrt{2g(d+h_2)}$$

(b) Consider Bernulli's eq at A and B.

$$P_A + \rho g H_A + \frac{1}{2}\rho V_A^2 = P_B + \rho g (H_A + d + h_1) + \frac{1}{2}\rho V_B^2$$

Since the fluid flow through B and C at the same rate and the x-section of the tube at B and C are the same $V_B = V_C$

$$P_0 + \cancel{\rho g d} + \cancel{\rho g H_A} + \frac{1}{2}\underbrace{\rho V_A^2}_{0} = P_B + \cancel{\rho g H_A} + \cancel{\rho g d} + \rho g h_1 + \frac{1}{2}\rho V_C^2$$

$$P_0 = P_B + \rho g d + \rho g h_2 + \rho g h_1$$

$$P_B = P_0 - \rho g d - \rho g h_2 - \rho g h_1$$

(c) For $P_B = 0 \Rightarrow P_0 = \rho g d + \rho g h_2 + \rho g h_1$

ie static pressure P_B at $B = 0$ mean that no fluid is present at that region. The max h_1 that achieve this is $P_0 \leq \rho g h_1$. The pressure P_0 that cannot sustain the fluid column in the tube at a height of h_1.