

Visualizing Qubits on the Bloch Sphere Surface

In Section 2.1.2 of the text we introduced the Bloch sphere representation of a qubit. In it, a qubit $|\psi\rangle$ is represented by a matrix that is parameterized by the quantities $0 \leq \theta \leq \pi$, $0 \leq \phi \leq 2\pi$, so that

$$|\psi\rangle = \begin{pmatrix} \cos[\theta/2] \\ e^{i\phi} \sin[\theta/2] \end{pmatrix}$$

(up to an overall complex phase)

On the Bloch sphere surface, shown below, each point on the surface (except for the poles) is characterized by a unique value of $\theta\phi$. Therefore, we can represent the ket $|\psi\rangle$ by a vector (the red arrow shown in the figure below). In the visualization, you can adjust the levers to the desired values for θ, ϕ in order to graph the corresponding ket. In the figure the numerical value (up to an overall complex phase) is shown.

```
ClearAll["Global`*"]
ket[θ_, ϕ_] := Format[ ( Chop[Cos[θ / 2]]
                      Exp[I ϕ] Sin[θ / 2] ), TraditionalForm]
vector[θ_, ϕ_] :=
  Arrow[Tube[{ {0, 0, 0}, {Cos[ϕ] Sin[θ], Sin[θ] Sin[ϕ], Cos[θ]}}, 0.025]]

bloch[θ_, ϕ_] := Show[Graphics3D[{Specularity[White, 100],
  Lighting -> {{"Point", Cyan, {1, 0, 1}}, Opacity[0.2], Sphere[{0, 0, 0}]}],
  Graphics3D[{Red, vector[θ, ϕ], Inset[ket[θ, ϕ], {0, 0, -0.5}]}],
  Boxed -> False, ViewPoint -> {Infinity, 0, 0}]
```

```
Manipulate[bloch[ $\theta$ ,  $\phi$ ], { $\theta$ , 0, Pi}, { $\phi$ , 0, 2 Pi}, SaveDefinitions -> True]
```

