Studies of the Rossby Wave Instability Using the Athena++ Code

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Lopsided Structure of PPDs

ALMA observations in sub-mm dust continuum have revealed lopsided structures in some of PPDs.

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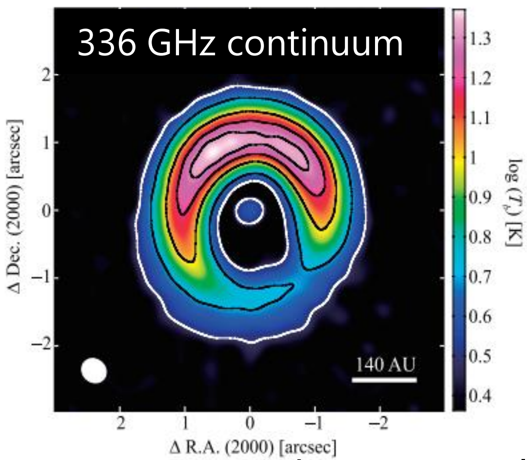
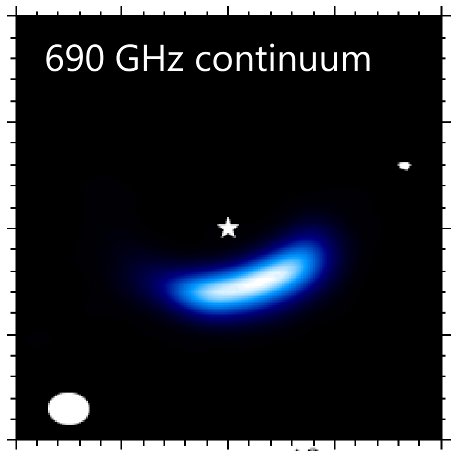


Fukagawa et al. 13

IRS47



HD135344B

van der Marel et al. 2016

<u>My Interests</u>

- How are these structures formed?
- How important are they in the planet formation?

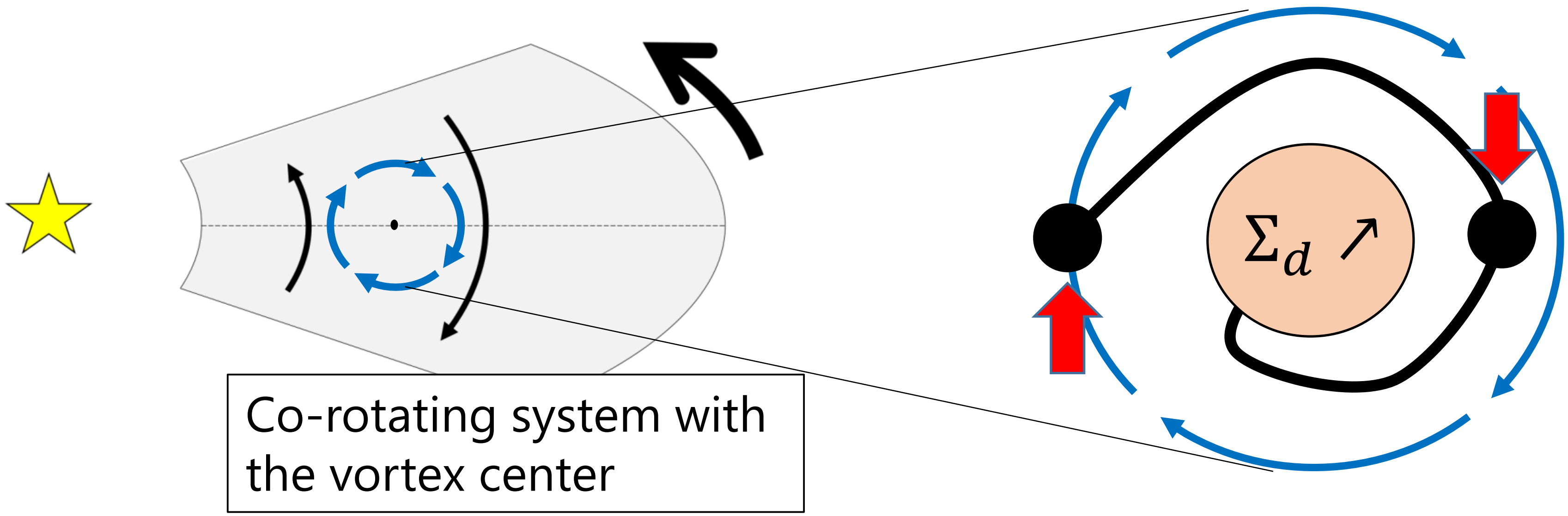
Large-Scale Gas Vortex

Large-scale gas vortex can give us plausible interpretations.

If vortices are anti-cyclonic,

i. Go with the Keplerian shear.
  ➔ survival for a long time.

ii. Trap dust particles
  ➔ dust concentration

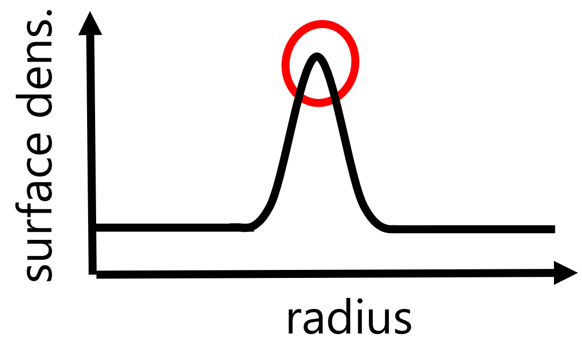


➢ How are the gas vortices formed?

Evolution of RWI and Gas Vortices

The Rossby wave instability [RWI] (e.g., Lovelace+99 & Li+00) can form the gas vortices when disks have a rapid radial variation.

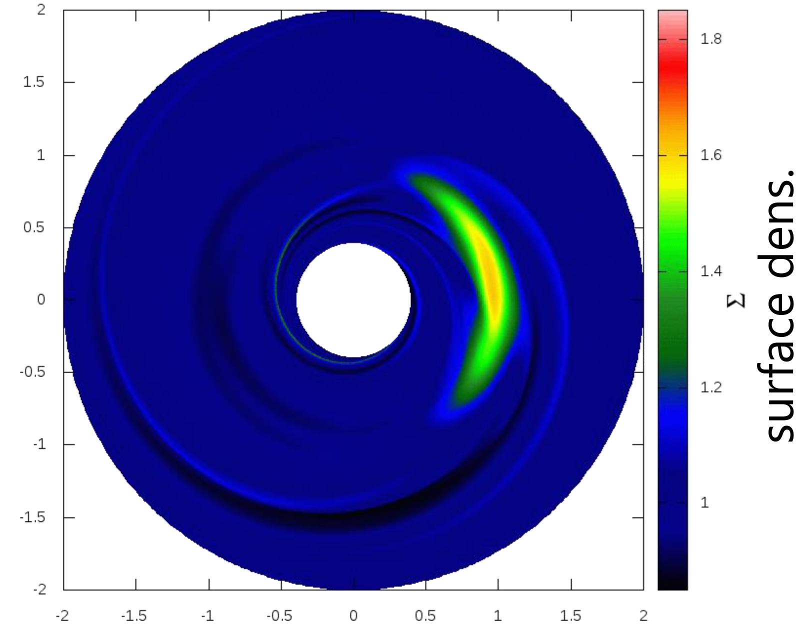
ex)



1. Initial ring
2. Vortices formation
3. Vortex merger
4. Quasi-stationary vortex
5. Vortex migration

RWI

800 rev. @r=1



#Co-rotating system
with the vortex center

Initial Conditions

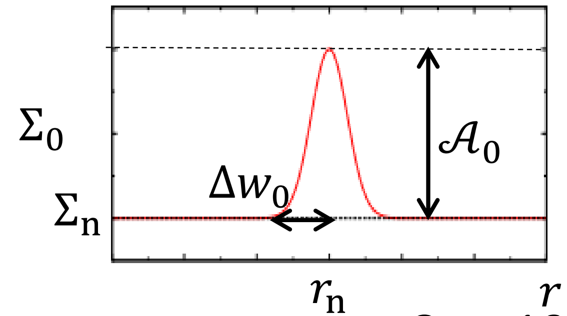
[Disks]

- 2D ideal fluid ← to compare with linear analyses
- barotropic flow ← to avoid potential vorticity generation.

[Initial Conditions]

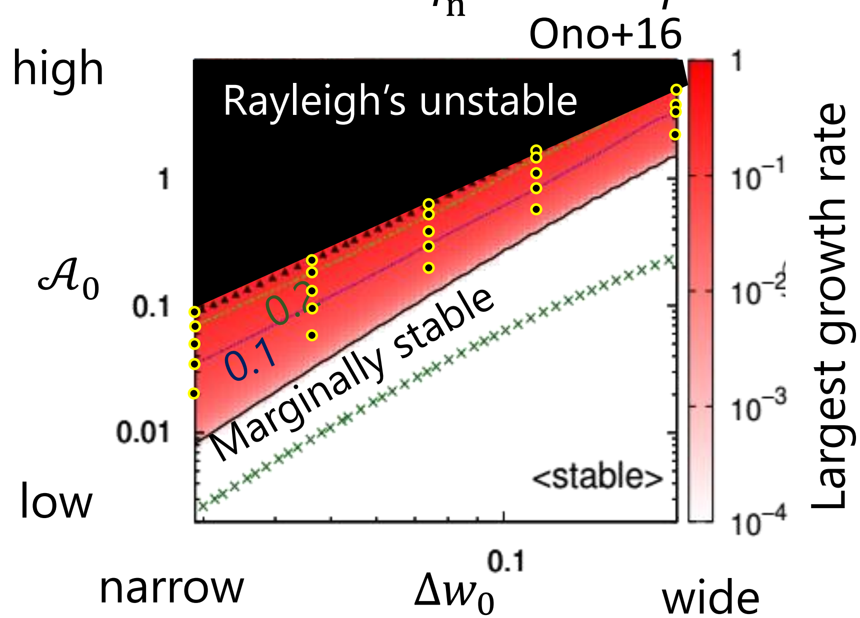
- Axisymmetric stationary disks.
- $v_{r0} = 0$
- Give Σ_0 as a Gaussian bump

$$\frac{\Sigma_0}{\Sigma_n} = 1 + \mathcal{A}_0 \exp\left[-\frac{1}{2}\left(\frac{r - r_n}{\Delta w_0}\right)^2\right]$$



[Parameters]

① Amplitude ($\mathcal{A}_0$)
② Width (Δw_0)
③ Disk aspect ratio ($h \equiv H/r$)
④ Effective adiabatic index (Γ)



Settings for Numerical Simulations

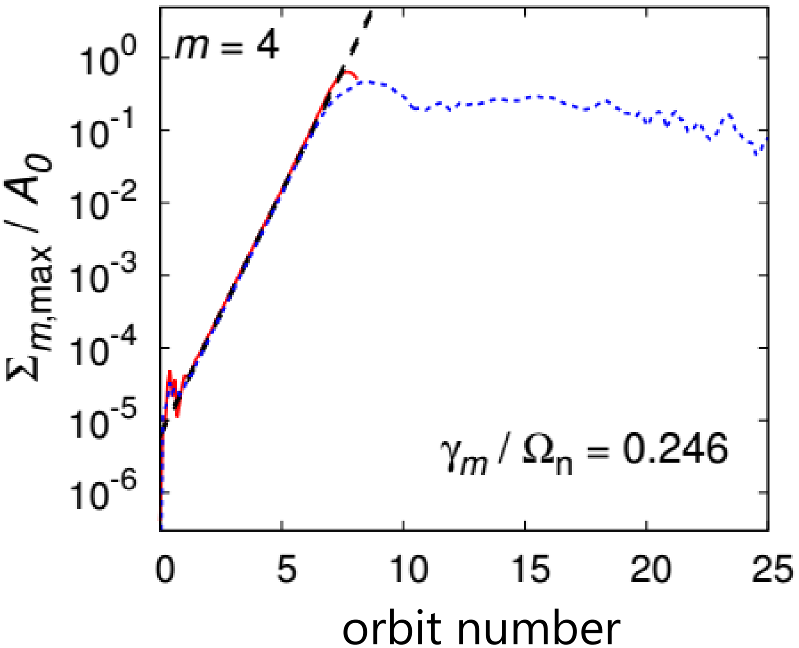
- Code: Athena++ (Stone+ in prep.)
- Parameters
  h=[0.05, 0.1, 0.15, 0.2], $\Gamma = 5/3$
  change $\mathcal{A}_0$ and Δw_0 54models in total
- Mesh
  (r) [0.3 : 2.5] (φ) [-π : π]
  resolve $1H$ with at least 25 grids
- Boundary Condition
  (r) outgoing (Godon96), (φ) periodic
- Calculations
  A) Single mode noise + single mode filter
  B) White noise perturbation ($m \leq 128$)

Remove other modes using FFT in every step.

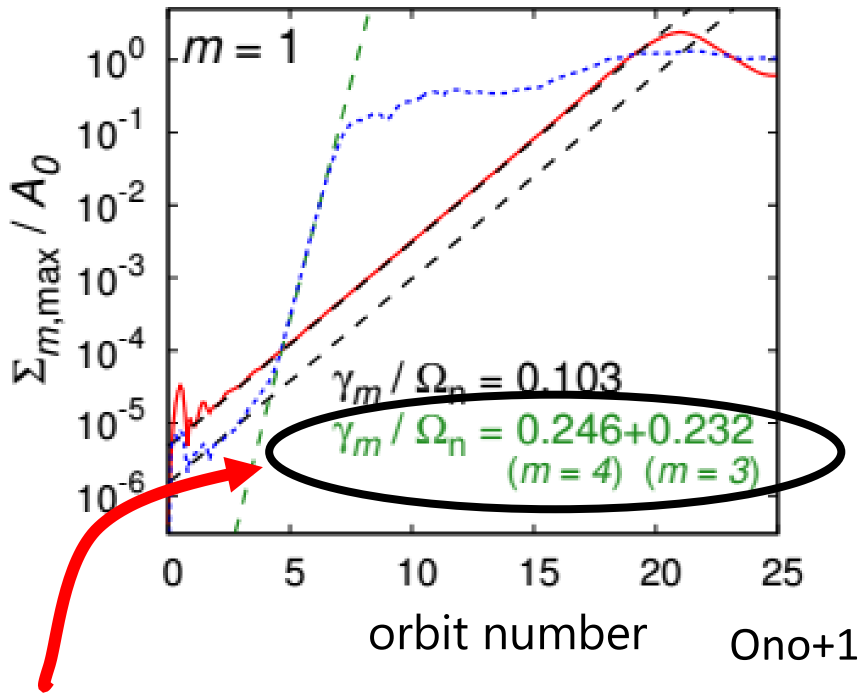
Comparison with the linear analyses 1

Calc. A (single mode), Calc. B (white noise),
Linear growth, Two modes coupling

most unstable mode ($m = 4$)



another mode ($m = 1$)

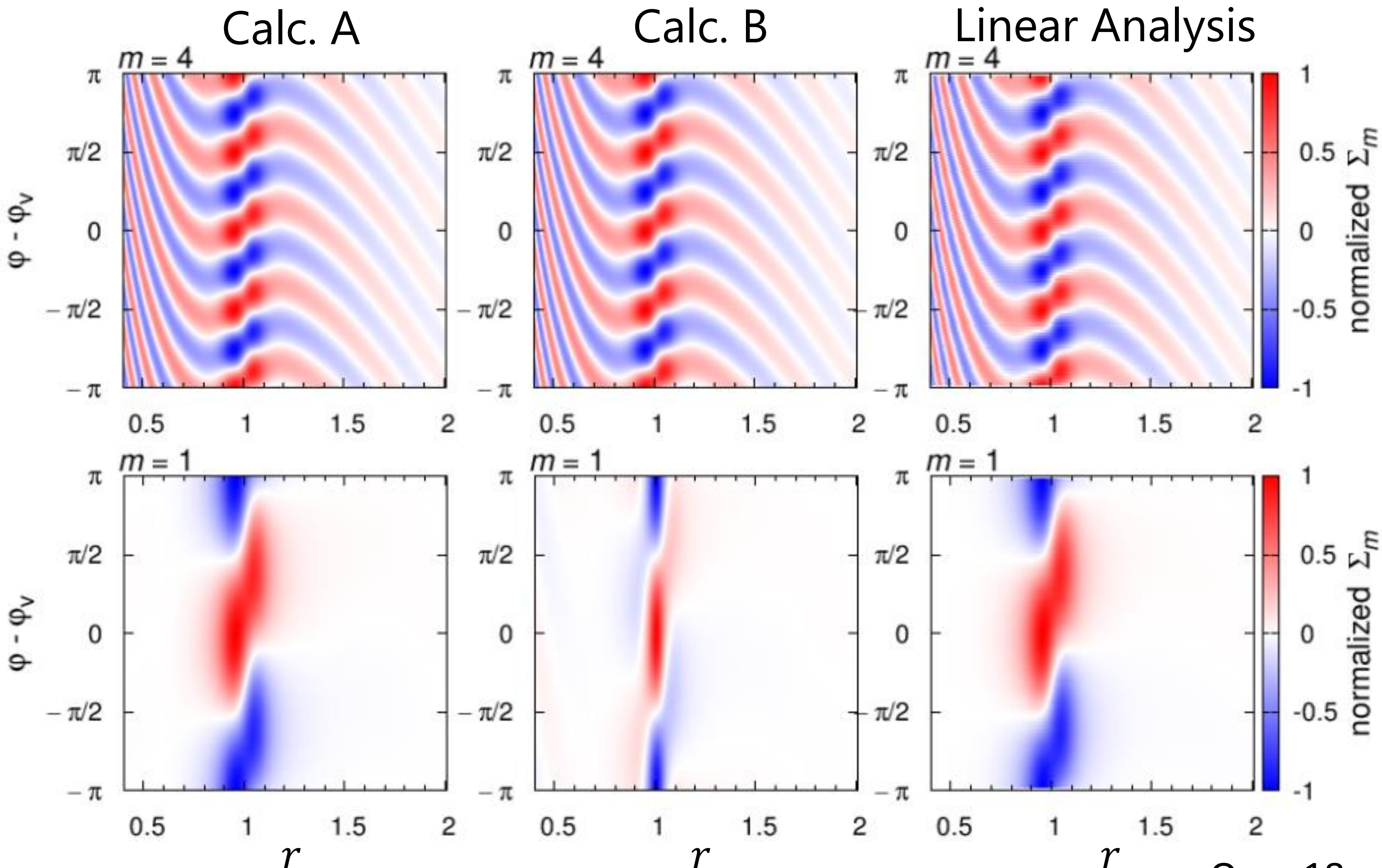


Ono+18

$$\exp[i(4\varphi - \gamma_4 t)] \times \exp[i(-3\varphi - \gamma_3 t)]$$
$$= \exp[i\{1\varphi - (\gamma_4 + \gamma_3)t\}]$$

Comparison with the linear analyses 2

The distribution of surface density perturbation at $\tau = 7$.



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Numerical calculations agree with linear analyses.

Physical Quantities Characterizing Vortices

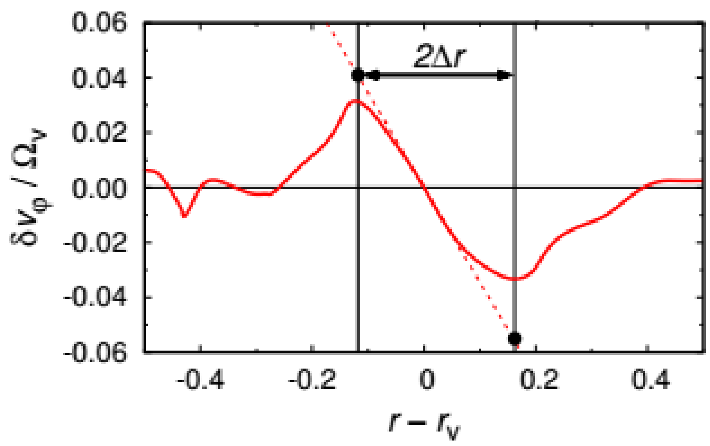
There are 3 physical quantities which are almost constant in time.

① vortex radial size (Δr)
② vortex aspect ratio ($\chi \equiv a/b$)
③ $\Sigma_v / \Omega_v = \Sigma_v \, r_v^{1.5}$ (for $GM = 1$)

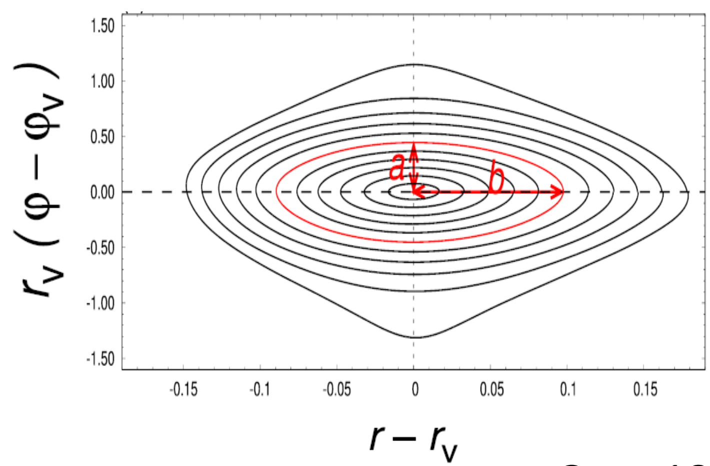
Note:

$$\Omega_v \equiv \Omega(r_v, \varphi_v) = \Omega_K(r_v)$$
$$\Sigma_v \equiv \Sigma(r_v, \varphi_v)$$

Profile of $\delta v_\varphi \equiv v_\varphi - v_K$



Stream lines



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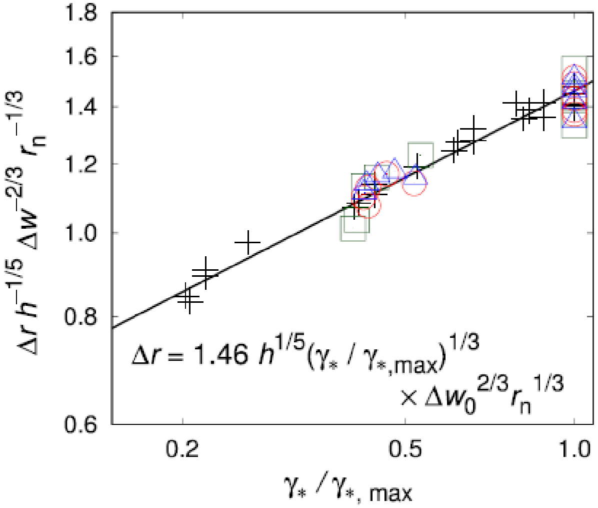
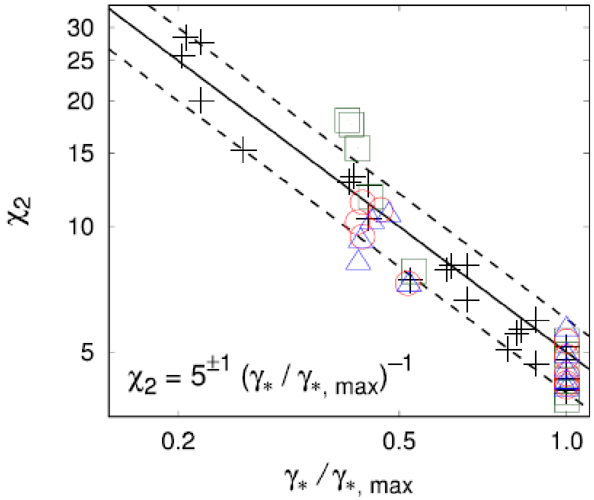
Empirical Relations of the RWI vortex

We obtain empirical relations of Δr, χ, $\Sigma_v\ r_v^{1.5}$

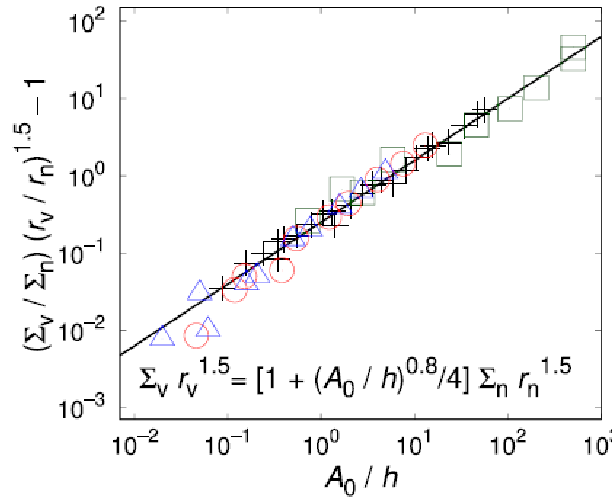
Δr



χ



$\Sigma_v\ r_v^{1.5}$



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Three quantities can be estimated from initial conditions.

$h, \Delta w_0, \gamma \rightarrow \Delta r$

$\gamma \rightarrow \chi$

$h, \mathcal{A}_0 \rightarrow \Sigma_v\ r_v^{1.5}$

Note: γ is the largest linear growth rate of the RWI.

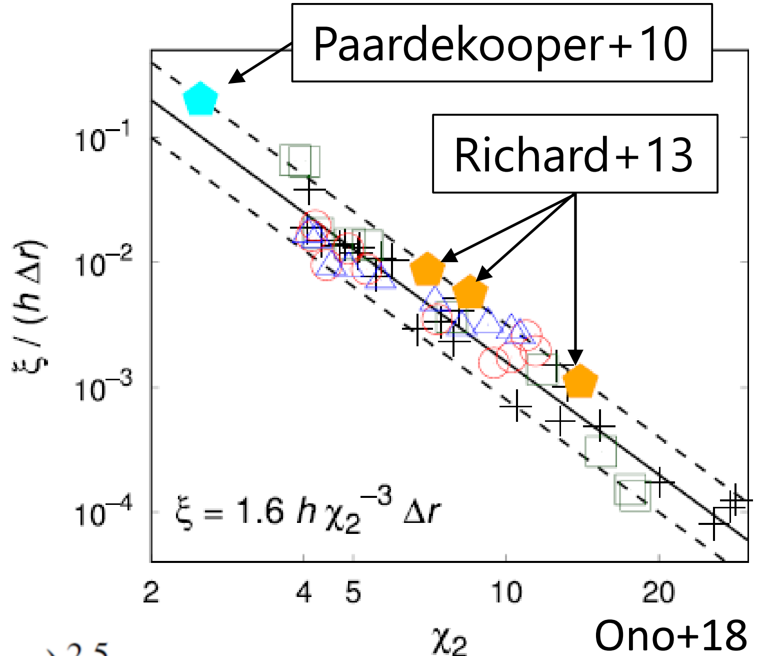
Empirical Law of Vortex Migration

We define a physical quantity ξ by

$$\mathrm{d}r_\mathrm{v}/\mathrm{d}t = -\xi\Omega_\mathrm{v}$$

Empirical formula of ξ is obtained as

$$\xi \approx 1.6 h \Delta r \chi^{-3}$$



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Migration timescale

$$\tau_\mathrm{mig} \approx 2.1 \times 10^3 \left(\frac{h}{0.1}\right)^{-1} \left(\frac{\chi_2}{6}\right)^3 \left(\frac{\Delta r}{0.1\, r_\mathrm{n}}\right)^{-1} \left(\frac{r_\mathrm{v}}{r_\mathrm{n}}\right)^{2.5}$$

$$= 2.1\ \mathrm{Myr} \times \left(\frac{h}{0.1}\right)^{-1} \left(\frac{\chi_2}{6}\right)^3 \left(\frac{\Delta r}{10\ \mathrm{au}}\right)^{-1} \left(\frac{r_\mathrm{v}}{100\ \mathrm{au}}\right)^{2.5}$$

for $M_* = M_\odot$, $r_n = 100$ AU

Unless χ is small and Δr is large,
the migration timescale ~ the life time of PPDs (~1-10 Myr)

(e.g., Haisch+01)

Tracer Particles in Vortices

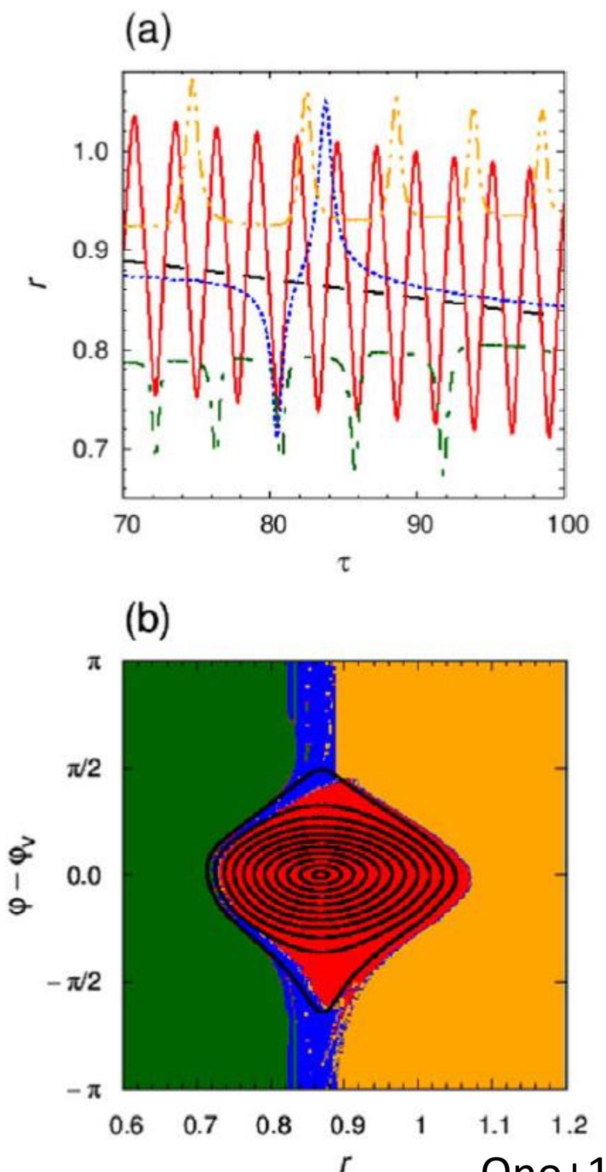
We calculate the motion of
about 0.1 M tracer particles for 30 rev.

There are 4 groups of tracer particles.

i. Remain in the vortex part
ii. Move from the inner part to the outer part
iii. Remain in the inner part
iv. Remain in the outer part

✓The RWI vortex can be considered as a large fluid particle.

✓Particles of Group ii contributes to about 20% of the ang. mom. transport.



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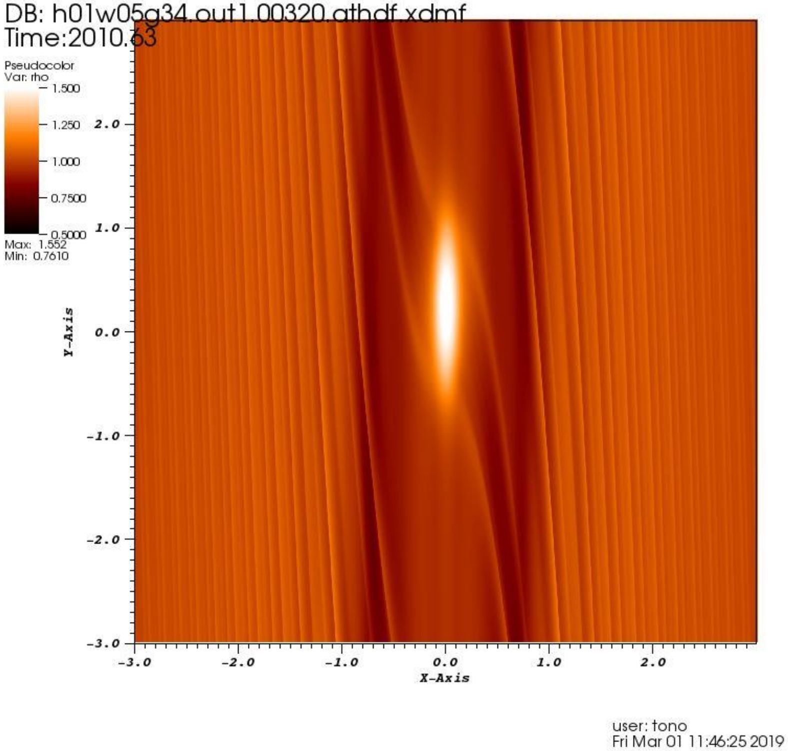
Next Project

[1] Dust concentration in various vortex
using the dust module (being developed by Chao-Chin).

[2] Density waves induced by the vortex
To understand
- vortex migration
- observability

➔We perform simulations
in a local shearing box
to avoid vortex migration.



High resolution & wide domain
are required to resolve density wav

➔ Numerical cost is large
even in 2D calculations.

Implementation of FARGO Scheme in Athena++

I started to implement the FARGO scheme (Masset00)
in the Athena++ two weeks ago in response to Jim's advise.

FARGO scheme for HD in the cartesian coordinates

If $v = v_K \boldsymbol{e}_y + \tilde{v}$,

$$\frac{\partial U}{\partial t} + \nabla F = S \iff \frac{\partial \tilde{U}}{\partial t} = \underbrace{\left(-v_K \frac{\partial \tilde{U}}{\partial y} + \acute{S}\right)}_{\text{advection}} + \underbrace{\left(-\nabla \tilde{F} + \tilde{S}\right)}_{\text{hydro}}$$

Solve them separately.
If v_K is static, the advection part can be solved analytically.

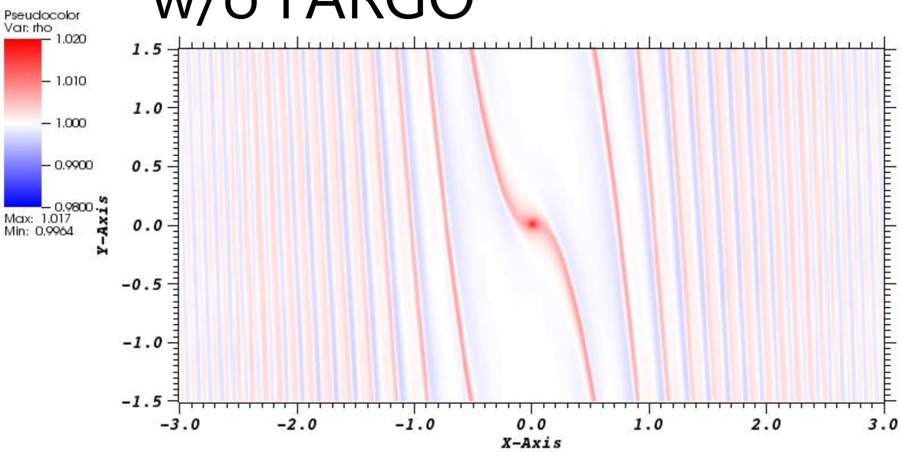
➔ We are not annoyed with the CFL condition
due to the orbital motion.

Test for FARGO in Athena++

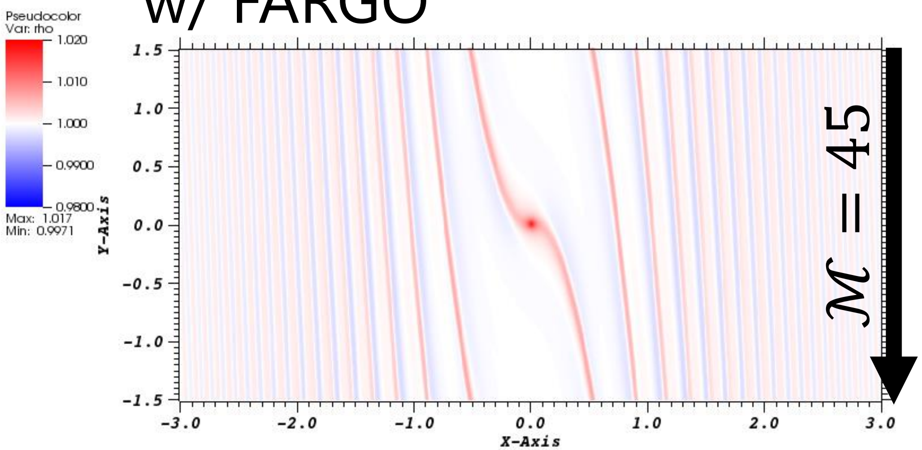
Test calculation of a disk with a planet ($M_p = M_\oplus$) for 10 rev. in the 2D local shearing box.

Domain: x[-30H:30H] 1280 grids,
y[-15H:15H] 600 grids
MPI 80 core

w/o FARGO



w/ FARGO



476 sec. → 28 sec.

× 1/17

FARGO in Athena++

My implementation FARGO in Athena++

Done

- in 2D/3D cartesian coordinates
- in 2D/3D cylindrical coordinates

Doing

- in 3D spherical_polar coordinates
- with hydro_diffusion

NOT yet

- for MHD
- with SMR in the direction of the orbital motion.
- with AMR

Summary

- We did a parametric study of the RWI vortices using the Athena++.
- We obtain some empirical relations between initial conditions and the RWI vortices
- As a result of tracer particle analyses, the RWI vortex can be considered as a large fluid particle.
- To investigate density waves induced by vortices, I've implemented the FARGO scheme in Athena++