Introductory Physics Problems

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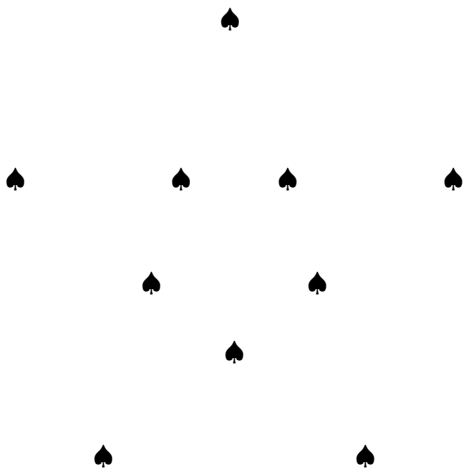

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Introduction

Introductory Physics Problems (IPP) is a source book for calculus-based introductory physics. The book is available in electronic form to instructors by request to the author. It is free courseware and can be freely used and distributed, but not used for commercial purposes.

The problems are grouped by topics in chapters: see Contents below. The chapter ordering follows roughly the traditional chapter/topic ordering in introductory physics books. For each chapter there are two classes of problems: in order of appearance in a chapter they are: (1) multiple-choice problems and (2) full-answer problems. Almost all the problems have complete suggested answers. The answers may be the greatest benefit of IPP. The questions and answers can be posted on the web in pdf format.

The problems have been suggested by many sources, but have all been written by me. Given that the ideas for problems are the common coin of the realm, I prefer to call my versions of the problems redactions.

At the end of the book are three appendices. The first is an equation sheet suitable to give to students as a test aid and a review sheet. The next is a table of integrals. The last one is a set of answer tables for multiple choice questions.

IPP is now sufficiently complete to be a reasonable resource for both semesters of introductory physics: classical mechanics, fluid physics, wave motion, thermodynamics, and electricity and magnetism, optics, special relativity, and quantum mechanics. The coverage for some topics (e.g., geometrical optics) is still marginal. There probably are not quite enough problems at present for every topic, but the author will be updating and adding to them as circumstances allow. User instructors can, of course, add and modify as they list.

Everything is written in plain TEX in my own idiosyncratic style. The questions are all have codes and keywords for easy selection electronically or by hand. A fortran program for selecting the problems and outputting them in quiz, assignment, and test formats is also available. Note, the quiz, etc. creation procedure is a bit clonky, but it works. User instructors could easily construct their own programs for problem selection.

I would like to thank the Department of Physics & Astronomy of the University of Nevada, Las Vegas for its support for this work. Thanks also to the students who helped flight-test the problems.

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Chapt. 1 Measurement, Units, Conversions, Significant Figures, and Uncertainties

Multiple-Choice Problems

001 qmult 00001 1 1 1 easy memory: attendance

1. Are you attending the whole class period or if not, have you given or will you give good cause to the instructor in person outside of the class period for being late or leaving early?

   a) Yes. b) No.

   An answer of no leads to zero on the quiz/test. A false answer of yes is academic dishonesty.

---

001 qmult 00003 1 1 1 easy thinking: present for quiz

2. Are you present for this quiz?

   a) Yes. b) Yessss! c) Oui. d) Si. e) Jawohl.

---

001 qmult 00005 1 4 1 easy deducto-memory: reading done

3. Did you complete the chapter reading (either the chapter in the textbook or the corresponding latex-ed notes if they are complete or nearly complete) before the **SECOND DAY** on which the chapter was lectured on?

   a) Yes! b) No. c) No! d) No, sir! e) OMG no!

---

001 qmult 00007 1 4 1 easy deducto-memory: reading done 2

4. Did you complete the chapter reading (either the chapter in the textbook or the corresponding latex-ed notes if they are complete or nearly complete) before the **SECOND DAY** on which the chapter was lectured on?

   a) YYYessss! b) Jawohl! c) Da! d) Si, si. e) OMG no!

---

001 qmult 00009 1 4 1 easy deducto-memory: reading done 3

5. The readings are an essential part of the course. You will not get the most out of the homeworks without having done the readings first. Have you done the reading for this homework?

   a) YYYessss! b) No! c) Maybe! d) Nyet. e) OMG no!

---

001 qmult 00010 1 1 1 easy memory: name written down

6. Have you written your name in the name place at the upper right of this paper?

   a) Yes. b) No. c) Nope. d) Nein. e) Never.

---

001 qmult 00012 1 1 2 easy memory: group names, find groups

7. Introduce yourselves to the people around and form groups of 2 to 4. You can change groups later, dump your friends, or become lone wolves. But for the moment, you will have a group. Write down the names and emails of group mates below the questions. Have you done so?

   a) No. b) Yes. c) No, merci. d) Nyet. e) Verily no.

---

001 qmult 00014 1 1 1 easy memory: locate answer line

8. Just below your name write the correct answer letters in a row in order. Just to make sure you get it, they should read "aba". You'll lose 1 mark unless you've done so. Have you done so?

a) Yes. b) No. c) Maybe. d) The instructions are too complicated.
e) Yes/no.

---

001 qmult 00016 1 1 5 easy memory: instructor name and office

9. The instructor's name is _____ and his office is _____.

a) David Jeffreys; MCCL 315 b) David Jefferies; MCCL 311
c) David Jeffrey; MCCL 311 d) David Geffry; EP 315 e) David Jeffery; EP 315

---

001 qmult 00018 1 1 4 easy memory: quiz and chapter test

10. In this course, there will a quiz **OR** chapter test nearly every:

a) minute. b) week. c) month. d) class day. e) biweek.

---

001 qmult 00100 1 4 5 easy deducto-memory: physics defined

11. "Let's play *Jeopardy*! For $100, the answer is: In shorthand, it is the science of matter and motion, space and time."

What is _____, Alex?

a) biology b) phrenology c) economics d) rocket science e) physics

---

001 qmult 00110 1 1 3 easy memory: theory of everything

12. The ultimate goal of physics—or at least one of the ultimate goals—is a theory of:

a) something (TOS). b) nothing (TON). c) everything (TOE).
d) woe (TOW). e) Fred (TOF).

---

000 qmult 00120 1 4 1 easy deducto-memory: Presocratics

13. "Let's play *Jeopardy*! For $100, the answer is: The most obvious choice for the first natural philosophers or physicists. They attempted to explain the natural world from simple general principles: from these principles complex structures could arise. In mythology on the other hand, usually deities with human-like personalities are invoked to explain structures. But there is probably no hard line between natural philosophy and mythology: one shades into the other in the early days."

Who are _____, Alex?

a) Presocratic philosophers of ancient Greece b) Babylonian diviners
c) Roman augurs d) Mongolian shamans e) Irish druids

---

001 qmult 00140 1 1 2 easy memory: mathematical physics origin

14. Mathematical physics in a sense began with the ancient Babylonian astronomers. Later the ancient Greeks added a bit to mathematical physics, particularly Archimedes (circa 287–212 BCE) with his work on statics and hydrostatics. But idea of physics as a largely mathematical science is largely attributable to _____, _____, and their contemporaries in the 17th century.

a) William Shakespeare (1564–1616); John Milton (1608–1674)
b) Galileo (1564–1642); Isaac Newton (1643–1727)
c) Bernini (1598–1680); Borromini (1599–1667)
d) Caravaggio (1571–1610); Poussin (1594–1665)
e) Wallenstein (1583–1634); Turenne (1611-1675)

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001 qmult 00150 1 1 4 easy memory: true exact emergent theory

15. Because there are no doubts about its validity in the classical limit (scales larger than microscopic, but smaller than cosmic, speeds much less than that of light, gravity much less strong than near a black hole) and because it is the exact limit of more general theories, Newtonian physics can be regarded as a/an:

a) utterly false theory. b) negligible theory c) useless theory.
d) true exact emergent theory. e) phantasm

---

001 qmult 00192 1 1 5 easy memory: clearly in physics, a joke question

16. In physics jargon, the word "clearly" means:

a) clearly. b) unclearly. c) after 4 pages of algebra. d) wrongly.
e) all of the above.

---

001 qmult 00194 1 1 5 easy memory: must be in physics, a joke question
Extra keywords: Not a serious question.

17. In physics jargon, the phrase "must be" means:

a) is. b) just accept it that. c) not necessarily so. d) can't be.
e) all of the above.

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001 qmult 00260 1 4 2 easy deducto-memory: scientific idealization

18. "Let's play *Jeopardy*! For $100, the answer is: In scientific theorizing and model building, it is the elimination/neglect of complicating secondary factors in order to understand the main factors. In reality, the secondary factors may or may not be eliminated/neglected to one degree or another. If they cannot be eliminated/neglected, they have to be modeled in order to test the understanding of the main factors."

What is _____, Alex?

a) Platonic idealization b) scientific idealization c) the scientific method
d) experiment e) observation

---

001 qmult 00300 1 4 2 easy deducto-memory: scientific notation defined
Extra keywords: physci

19. "Let's play *Jeopardy*! For $100, the answer is: It is a notation in which one expresses a number by a coefficient decimal number multiplied explicitly by 10 to the appropriate power. If the coefficient is in the range 1 to 10, but not including 10, the notation is called normalized."

What is _____, Alex?

a) British notation b) scientific notation c) metric notation d) tensy notation
e) Irish notation

---

001 qmult 00310 1 3 4 easy math: hundred million billion in sci. not.
Extra keywords: physci

20. Write a hundred million billion miles in scientific notation.

a) 10^2 mi. b) 10^6 mi. c) 10^9 mi. d) 10^{17} mi. e) 10^{-9} mi.

---

001 qmult 00320 1 3 1 easy math: show in scientific notation

21. Express 4011 and 0.052 in normalized scientific notation form.

a) 4.011×10^3 and 5.2×10^{-2}. b) 40.11×10^3 and $52. \times 10^{-2}$.
c) 40.11×10^2 and $52. \times 10^{-3}$. d) 4.011×10^{-2} and 5.2×10^3. e) 4011 and 0.052.

---

001 qmult 00410 1 4 5 easy deducto-memory: units needed
Extra keywords: physci

22. "Let's play *Jeopardy*! For $100, the answer is: In any measurements of quantities, they are conventionally agreed upon standard things."

What are _____, Alex?

a) unities b) dualities c) duplicities d) quantons e) units

---

001 qmult 00420 1 1 3 easy memory: SI or metric units

Extra keywords: physci

23. The modern standard set of units for science, most engineering, and much of everyday life (except in the 2nd largest country in North America) is the International System of Units (Système International d'Unités or SI) which is often called the:

a) English units. b) Mesopotamian system. c) metric system.
d) Paraguayan system. e) United States customary units.

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001 qmult 00422 1 5 3 easy thinking: US Customary Unit System 1

Extra keywords: the horror, the horror

24. Why is the US Customary Units system (loosely called English Units) hard to use in calculations in comparison with the metric system?

a) The US Customary Units system has no unit of mass. Consequently, when you need to use mass, you have to mentally work around the lack of a unit.
b) The US Customary Units system is completely unknown by scientists and engineers, and so, of course, can scarcely be used.
c) The US Customary Units system has units in a host of irregular sizes: 16 ounces to the pound, 8 pints in a gallon, 12 inches to the foot, and 45 inches to the ell (fact!). This makes calculations and conversions tortuous. In particular, it is awkward that cubic units of length (e.g., cubic inches, feet, etc.) are not simply related to standard volume units (e.g., a U.S. gallon is defined as 231 cubic inches). In everyday life the irregularities of the US Customary Units system don't cause much of a problem and maybe even have mnemonic value. But that hardly helps people who want to send probes to Mars.
d) The complete regularity of the US Customary Units system means that you never know if a number is 10, 100, 1000, etc., except by context.
e) It's the revenge for Bunker Hill.

---

001 qmult 00424 1 1 5 easy memory: US Customary Unit System 2

25. The 2nd largest country in North America in terms of land area for most everyday purposes uses the system of:

a) brutish units—brut, not brit. b) Finnish units. c) Pictish units.
d) kludgy units. e) US Customary Units.

---

001 qmult 00430 1 4 3 easy deducto-memory: basic quantities

26. Three quantities usually adopted as basic (i.e., not reducible to other kinds of quantities by convention) are:

a) length, area, volume. b) mass, weight, heft. c) length, mass, time.
d) time, duration, age. e) length, mass, density.

---

001 qmult 00432 1 1 1 easy memory: the 7 base units

27. The 7 base units are meter, kilogram (gram would have been more logical), second, ampere,

a) kelvin, mole, and candela. b) calvin, mold, and candeleria.
c) kelvin, mouse, and candace. d) melvin, moose, and cantrip.
e) kludge, moor, and mountain.

---

001 qmult 00450 1 4 3 easy deducto-memory: MKS acronym for what?

28. MKS stands for:

a) meters, kilometers, centimeters. b) meters, kilometers, seconds.
c) meters, kilograms, seconds. d) millimeters, kilometers, seconds.
e) millimeters, kilograms, seconds.

---

001 qmult 00452 1 5 3 easy thinking: consistent MKS units in calculations

29. If one used **ONLY** MKS units (i.e., units of meters, kilograms, seconds, coulombs, amperes, kelvins, moles, etc., plus MKS units derived from these) in calculations, then one will get answers in:

    a) CGS units only. b) MKS or CGS units. c) MKS units only.
    d) English units only. e) any old units.

---

001 qmult 00454 1 5 5 easy thinking: consistent and convenient units, natural units

30. In scientific calculations, it is best to stick to one complete, consistent set of units (MKS or CGS):

    a) always. b) half the time. c) never. d) whenever. e) except when its not convenient. In most specialized fields, there are natural units that are convenient in actual calculations, but overwhelmingly most often only in human readable outputs. Natural units are most useful for understanding not calculation. Astronomers, for example often use the solar mass ($1.9891 \times 10^{30}\,\mathrm{kg}$) as a unit for the masses of stars since the Sun for Earthlings is basic standard of reference for stars.

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001 qmult 00460 1 1 1 easy memory: metric kilo and centi
Extra keywords: physci

31. In SI, the prefixes kilo and centi indicate, respectively, multiplication by:

    a) 1000 and 0.01. b) 0.01 and 1000. c) 1000 and 100. d) 60 and 0.01.
    e) π and e.

---

001 qmult 00462 1 4 2 easy deducto-memory: SI prefix symbols M- and m-

32. The metric or SI unit prefix symbols M- and m- stand for:

    a) mega (factor of 10^6) and milli (factor of 10^{-6}). b) mega (factor of 10^6) and milli (factor of 10^{-3}). c) kilo (factor of 10^6) and milli (factor of 10^{-3}). d) kilo (factor of 10^6) and milli (factor of 10^{-6}). e) merger (factor of 10^9) and melba (factor of 10^{-6}).

---

001 qmult 00500 1 1 1 easy memory: conversion in general, factors of unity
Extra keywords: physci

33. In conversions, one can just treat units as variables whose values are never specified. One can do algebra with them and cancel them. One also knows a set of equalities relating units, and so can write down factors of unity or conversion factors. For example, $1000\,\mathrm{m} = 1\,\mathrm{km}$, and so a factor of unity (or conversion factor) is

$$1 = \frac{1000\,\mathrm{m}}{1\,\mathrm{km}} \ .$$

    This factor would be used to convert an amount in kilometers to meters by:

    a) multiplication. b) division. c) addition. d) subtraction. e) squaring.

---

001 qmult 00510 1 4 1 easy deducto-memory: conversion factor=unity factor
Extra keywords: physci

34. A units conversion factor is sometimes called a factor of:

    a) unity. b) 2. c) a few. d) 10. e) fact.

---

001 qmult 00520 1 5 1 easy thinking: meter to centimeters

35. Using compact one-number conversion factors is often trickier than just using explicit factors of unity in doing calculations. But if one has to do repeated conversions, it is convenient to have them. Some are straightforward to find. For example, what is the conversion factor for converting meters **TO** centimeters?

    a) $100\,\mathrm{cm/m}$. b) $(1/100)\,\mathrm{cm/m}$. c) $(1/10)\,\mathrm{cm/m}$. d) $10\,\mathrm{cm/m}$. e) $10\,\mathrm{m/cm}$.

---

001 qmult 00530 2 3 1 moderate memory: m/s to km/h

36. Using compact one-number conversion factors is often trickier than rather than just using explicit factors of unity in doing calculations. But if one has to do repeated conversions, it is convenient to have them. Some are straightforward to find and others a little less so. For example, what is the conversion factor for converting meters per second **TO** kilometers per hour? **HINT:** Start with the $1\,\mathrm{m/s} = 1\,\mathrm{m/s} \times 1 \times 1$ and replace the 1's by simple factors of unity and find a factor of unity for required conversion and then the compact one-number conversion factor.

    a) 3.6 (km/h)/(m/s). b) 1/3.6 (km/h)/(m/s). c) 1000 (km/h)/(m/s).
    d) 3600 (km/h)/(m/s). e) 1000/3600 (km/h)/(m/s).

---

001 qmult 00540 1 4 1 easy deducto-memory: kilogram to grams
Extra keywords: physci

37. A kilogram is:

    a) 1000 grams. b) 1×10^{-3} grams. c) 3.1416 grams. d) 2 grams. e) 0 grams.

---

001 qmult 00550 1 4 4 easy deducto-memory: conversion of an inch to cm
Extra keywords: physci

38. One inch is:

    a) 1 m. b) 1 km. c) 1 cm. d) 2.54 cm. e) 1 ft.

---

001 qmult 00560 1 4 3 easy deducto-memory: conversion mile to km

39. 1 mile is nearly exactly:

    a) 1 m. b) 1 km. c) 1.609 km. d) 2.54 cm. e) 1 ft.

---

001 qmult 00570 2 4 4 moderate deducto-memory: 10 m/s to mph
Extra keywords: physci

40. A human (a very speedy human) can run 10 m/s. What is this speed in miles per hour (mi/h)? **HINT:** You do not need to do any explicit calculation (although that will work too). Just think about everyday reality. Can you outrun a car? Yes/no/maybe?

    a) 100 mi/h. b) 1 mi/h. c) 11.2 km/s. d) 22.37 mi/h. e) 22.37 miles.

---

001 qmult 00572 1 1 4 easy memory: c into ft/ns

41. Convert approximately $c = 2.99792458 \times 10^8$ m/s into feet per nanosecond.

    a) 100 ft/ns. b) 30 ft/ns. c) 10 ft/ns. d) 1 ft/ns. e) 0.3 ft/ns.

---

001 qmult 00580 1 3 4 easy math: seconds in a day
Extra keywords: physci

42. Exactly how many seconds are there in a day? How many seconds are there in a day to order of magnitude? **HINT:** "To order of magnitude" means that the number is rounded off to the nearest power of 10. There is no universal rule about the dividing line between rounding down and rounding up. However, using $\sqrt{10} \approx 3.162$ is a reasonable choice. Thus, the prefix number in normalized scientific notation is rounded down to 1 if it is less than $\sqrt{10}$, rounded up to 10 if it is larger than $\sqrt{10}$, and rounded to make an even power of 10 if it is exactly $\sqrt{10}$. For example, 2.2 rounds off to $10^0 = 1$, 991 rounds off to 10^3, and $\sqrt{10000} = \sqrt{10} \times 10^2$ rounds off to 10^2.

    a) 86400 s and 10^4 s. b) 1440 s and 10^4 s. c) 1440 s and 10^5 s.
    d) 86400 s and 10^5 s. e) $\pi \times 10^7$ s.

---

001 qmult 00582 1 3 4 easy math: convert hundred million billion miles to km

43. **CONVERT** a hundred million billion miles into kilometers.

a) 1.6×10^2 km. b) 1.6×10^6 km. c) 1.6×10^9 km. d) 1.6×10^{17} km.
e) 1.6×10^{-9} km.

---

001 qmult 00590 1 1 3 easy memory: density conversion

44.* Convert $1\,\mathrm{g/cm^3}$ into kilograms per cubic meter.

a) $1\,\mathrm{kg/m^3}$. b) $10^2\,\mathrm{kg/m^3}$. c) $10^3\,\mathrm{kg/m^3}$. d) $10^4\,\mathrm{kg/m^3}$. e) $10^6\,\mathrm{kg/m^3}$.

---

001 qmult 00710 1 3 3 easy math: sig.fig. multiplication

45. Multiply experimental values 3.27 and 9.9, and give the answer to correct significant figures.

a) 30. b) 34. c) 32. d) 32.3. e) 32.4.

---

001 qmult 00720 1 3 4 easy math: sci. not. and sig. fig.

46. Add and multiply 3.01×10^2 and 1.1×10^{-1} rounding off to significant figures.

a) 3.0111×10^2 and 3.311×10^1. b) 3.01×10^2 and 3.311×10^1.
c) 3.0111×10^2 and 3.3×10^1. d) 3.01×10^2 and 3.3×10^1.
e) 3.0×10^2 and $3. \times 10^1$.

---

001 qmult 00730 1 3 2 easy math: sigfig addition

47. Add experimental values 3.27 and 9.9, and give the answer to correct significant figures.

a) 13.17. b) 13.2. c) 13.1. d) 13. e) 14.

---

001 qmult 00850 1 2 2 moderate memory: relative error

48. You have traveled a distance 10 ± 1 km. The relative uncertainty (or less exactly, but rather commonly, the relative error) in your travel distance is:

a) 1. b) 0.1. c) 0.5. d) 2. e) 10.

---

001 qmult 01092 1 1 3 easy memory: world commercial power usage

49. Here's a Fermi problem. What is the total commercial power (i.e., energy per unit time) used in the world to order of magnitude or so? **HINT:** There are about 7×10^9 people on Earth. Power in MKS units is measured in watts (W) which are joules per second. A 100-watt light bulb uses 100 W.

a) 10^5 W. b) 10^7 W. c) 10^{13} W. d) 10^{20} W. e) 10^{24} W.

---

001 qmult 01510 1 1 3 easy math: experimentally measuring π

50. To determine π experimentally:

a) convert miles per hour into meters per second.
b) measure the ratio of the circle area to circle circumference.
c) measure the ratio of circle circumference to circle diameter
d) measure the ratio of the circle girth to circle heft.
e) measure the acceleration near the Earth's surface due to gravity.

---

001 qmult 01520 2 4 1 easy math: sigfig and error

51. You have just measured a length in centimeters and estimated its uncertainty to **ONE** significant figure (which is usually all that can be done). You write that datum down in the simplest, proper experiment reporting style you know. That datum is:

a) $(4.17 \pm 0.08) \times 10^3$ cm. b) $(4.2 \pm 0.08) \times 10^3$ cm. c) $(4.17 \pm 0.082) \times 10^3$ cm.
d) $4.17 \times 10^3\,\mathrm{cm} \pm 0.082 \times 10^3$ cm. e) $4.177 \times 10^3\,\mathrm{cm} \pm 0.08 \times 10^3$ cm.

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001 qmult 01540 1 3 1 easy math: sigfig and slope

52. You are determining a slope from two points on a best-fit line: in (x, y) format point 1 is $(5.50, 42.1)$ and point 2 is $(10.6, 62.3)$. What is the slope to correct significant figures? **HINT:** Significant figures can always be lost in subtractions.

a) 4.0. b) 3.96. c) 3.961. d) 4. e) 3.9.

---

001 qmult 01610 2 3 5 moderate math: fractional uncertainty and speed

53. You have just calculated a distance 25.7 m from a speed of 3.7 ± 0.5 m/s and a time of 2.7 ± 0.1 s. Using the Montemayor fractional uncertainty method, calculate the uncertainty of the distance and write the distance and its uncertainty to correct significant figures. **HINT:** Calculated uncertainties can differ slightly depending on where you round off in the calculation. Allow for that.

a) 25.7 ± 3.5 m.
b) 25.7 ± 1.0 m.
c) 25.7 ± 1 m.
d) 30 ± 3 m.
e) 26 ± 3 m.

---

001 qmult 01630 2 3 3 moderate math: FU friction coefficient

54. You have just calculated a coefficient of friction of 0.175 from a mass of 110 ± 20 kg and a time of 2.7 ± 0.3 s. Using the Montemayor fractional uncertainty method, calculate the uncertainty of the coefficient and write the coefficient and its uncertainty to correct significant figures. **HINT:** Calculated uncertainties can differ slightly depending on where you round off in the calculation. Allow for that.

a) 0.175 ± 0.03.
b) 0.175 ± 0.02.
c) 0.18 ± 0.03.
d) 0.18 ± 0.01.
e) 0.2 ± 0.03.

---

001 qmult 01640 2 3 4 moderate math: FU momentum

55. You have just calculated the magnitude of a linear momentum to be 6.01 kg m^2 s^{-2} from a mass of 3.0 ± 0.3 kg and speed of 2.00 ± 0.03 m s^{-1}. Using the Montemayor fractional uncertainty method, calculate the uncertainty of the magnitude of linear momentum and write it and its uncertainty to correct significant figures. **HINT:** Calculated uncertainties can differ slightly depending on where you round off in the calculation. Allow for that.

a) 0.175 ± 0.03.
b) 0.175 ± 0.02.
c) 0.18 ± 0.03.
d) 6.0 ± 0.6.
e) 0.2 ± 0.03.

---

001 qmult 01650 2 5 2 moderate math: FU plotting

56. You have a series of data (all with uncertainty 0.2): 1.0, 3.4, 5.6, 7.1, 8.7, and 10.1. You are to plot the square root of these numbers. How do you obtain the appropriate (absolute) uncertainty for the square roots using the Montemayor fractional uncertainty method and what is it that uncertainty?

a) Take the fractional uncertainty of a typical datum, assign this fractional uncertainty to the datum's square root, find the square root's absolute uncertainty using its newly assigned fractional uncertainty, and use this absolute uncertainty for all square rooted data. A reasonable square root absolute uncertainty is 4.
b) Take the fractional uncertainty of a typical datum, assign this fractional uncertainty to the datum's square root, find the square root's absolute uncertainty using its newly assigned fractional uncertainty, and use this absolute uncertainty for all square rooted data. A reasonable square root absolute uncertainty is 0.1.
c) The absolute uncertainty is the square root of 0.2: i.e., 0.45.
d) The absolute uncertainty is the square root of 0.2: i.e., 0.4.

e) The absolute uncertainty is the square root of 0.2: i.e., 3.

---

001 qmult 01750 1 5 2 easy thinking: plotting a power linearly

57. You have taken a set of data pairs (t, z). There is a theoretical formula relating t and z:

$$z^2 = at^3 + b \ ,$$

where a and b are constants. You want to test this formula using your data and linear graphical analysis. Now t and z are **NOT** predicted to be linearly related by theory. So you plot functions of the t and z data that are predicted to be linearly related.

a) The functions are t^3 and z^2. If you obtain a line within uncertainty, the theory is **NOT CONFIRMED**.
b) The functions are t^3 and z^2. If you obtain a line within uncertainty, the theory is **CONFIRMED**.
c) The functions are t^2 and z^3. If you obtain a line within uncertainty, the theory is **NOT CONFIRMED**.
d) The functions are t^2 and z^3. If you obtain a line within uncertainty, the theory is **CONFIRMED**.
e) The functions are $\sin(t)$ and z^3. If you obtain a line within uncertainty, the theory is **NOT CONFIRMED**

---

001 qmult 01770 1 5 3 easy thinking: plotting $\tan(x)$ linearly

58. Given $y^2 = a\tan(x) + b$, identify the new variables y_{new} and x_{new} that reduce the equation to the equation of a line, $y_{\text{new}} = ax_{\text{new}} + b$.

a) y and $\cos(x)$.
b) y^3 and $\cos(x)$.
c) y^2 and $\tan(x)$.
d) y and $\tan(x)$.
e) y and x.

---

001 qmult 01772 2 5 1 moderate thinking: plotting a square root linearly

59. Given $y = 2\pi\sqrt{ax/h} + b$ (where a, h, and b are constants), identify the new variables y_{new} and x_{new}, and the new constants c_1 and c_2 that reduce the equation to the equation of a line, $y_{\text{new}} = c_1 x_{\text{new}} + c_2$.

a) $y_{\text{new}} = y$, $x_{\text{new}} = \sqrt{x}$, $c_1 = 2\pi\sqrt{a/h}$, and $c_2 = b$.
b) $y_{\text{new}} = y^3$, $x_{\text{new}} = \sqrt{x}$, $c_1 = 2\pi\sqrt{a/h}$, and $c_2 = b^3$.
c) $y_{\text{new}} = y$, $x_{\text{new}} = 1/x$, $c_1 = 2\pi\sqrt{a/h}$, and $c_2 = b$.
d) $y_{\text{new}} = y$, $x_{\text{new}} = 1/\sqrt{x}$, $c_1 = 2\pi\sqrt{a/h}$, and $c_2 = b$.
e) $y_{\text{new}} = y$, $x_{\text{new}} = \sqrt{x}$, $c_1 = 2\pi$, and $c_2 = b$.

---

001 qmult 01774 2 5 1 moderate thinking: plotting an inverse with singularity

60. Given $y = a/(x-b)$ (where a and b are constants), identify the new variables y_{new} and x_{new}, and the new constants c_1 and c_2 that reduce the equation to the equation of a line, $y_{\text{new}} = c_1 x_{\text{new}} + c_2$.

a) $y_{\text{new}} = y$, $x_{\text{new}} = 1/(x-b)$, $c_1 = a$, and $c_2 = 0$.
b) $y_{\text{new}} = y^3$, $x_{\text{new}} = \sqrt{x}$, $c_1 = a$, and $c_2 = b$.
c) $y_{\text{new}} = y$, $x_{\text{new}} = 1/(x+b)$, $c_1 = a$, and $c_2 = 0$.
d) $y_{\text{new}} = y$, $x_{\text{new}} = x - b$, $c_1 = 1/a$, and $c_2 = 0$.
e) $y_{\text{new}} = y$, $x_{\text{new}} = x + b$, $c_1 = 1/a$, and $c_2 = 0$.

---

001 qmult 01776 2 5 3 moderate thinking: plotting power 1/3 linearly

61. Given $y = ax^{1/3}$ (where a is a constant), identify the new variables y_{new} and x_{new}, and the new constants c_1 and c_2 that reduce the equation to the equation of a line, $y_{\text{new}} = c_1 x_{\text{new}} + c_2$.

a) $y_{\text{new}} = y$, $x_{\text{new}} = x$, $c_1 = a$, and $c_2 = 0$.

b) $y_{\text{new}} = y^3$, $x_{\text{new}} = x^{1/3}$, $c_1 = a$, and $c_2 = a$.
c) $y_{\text{new}} = y$, $x_{\text{new}} = x^{1/3}$, $c_1 = a$, and $c_2 = 0$.
d) $y_{\text{new}} = y$, $x_{\text{new}} = \sqrt{x}$, $c_1 = 1/a$, and $c_2 = 0$.
e) $y_{\text{new}} = y^3$, $x_{\text{new}} = x^2$, $c_1 = 1/a$, and $c_2 = 0$.

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001 qmult 10510 2 3 5 moderate math: acre-foot conversion fullmult

62.** American hydraulic engineers often use acre-feet to measure volume of water. An acre-foot is the amount of water that will cover an acre of flat land to 1 foot. Say 4.00 in of rain fell on a plain of 30.0 km^2. How many acre-feet of water fell? Note 1 square mile equals 640 acres and 1 km = 0.6214 mi. **HINTS:** First, find the volume in the hybrid units of inch-km^2 and then use a separate factor of unity for each unit conversion: divide and conquer.

a) 1066 acre-feet. b) 1492 acre-feet. c) 1776 acre-feet. d) 1850 acre-feet.
e) 2470 acre-feet.

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001 qmult 10512 2 3 4 moderate math: acre-foot conversion fullmult 2

63.** American hydraulic engineers often use acre-feet to measure volume of water. An acre-foot is the amount of water that will cover an acre of flat land to 1 foot. Say 5.00 in of rain fell on a plain of 50.0 km^2. How many acre-feet of water fell? Note 1 square mile equals 640 acres and 1 km = 0.6214 mi. **HINTS:** First, find the volume in the hybrid units of inch-km^2 and then use a separate factor of unity for each unit conversion: divide and conquer.

a) 1066 acre-feet. b) 1492 acre-feet. c) 1776 acre-feet. d) 5150 acre-feet.
e) 2470 acre-feet.

---

001 qmult 10520 1 4 5 easy deducto-memory: Fermi on lectures and the micro-century fullmult
Extra keywords: mathematical physics

64.* "Let's play *Jeopardy*! For $100, the answer is: Nearly the time period of a standard 50-minute lecture period as noted by Italian-American physicist Enrico Fermi (1901–1954)."

What is _____, Alex?

a) an eternity b) a deci-century (i.e., a tenth of a century) c) 360 seconds d) a centi-century (i.e., a hundredth of a century) e) a micro-century (i.e., a millionth of a century)

---

001 qmult 10950 3 3 2 tough math: leap seconds fullmult

65.*** The mean solar day is known to increase secularly (i.e., in a long-term way) by about 1.70×10^{-3} seconds per century due mainly to the tidal interaction of the Moon and Sun (Wikipedia: Leap second: Rationale). Currently, the mean solar day is about 86400.002 seconds long (Wikipedia: Solar time). **HINT:** Both parts (i) and (ii) are questions of how long at a given rate does a given amount take to accumulate.

i) About how often does a leap second need to be introduced in standard time in order to keep standard and solar time synchronized?

ii) If the current rate of increase in the day continues, about how long in years will it take before a leap second is needed daily: i.e., how long until the mean solar day is about 86401.0 seconds?

a) (i) 100 days (ii) 4 years.
b) (i) 500 days (ii) 60000 years.
c) (i) 4 years (ii) 60000 years.
d) (i) 2 months (ii) 4 years.
e) (i) 60000 years (ii) never.

Full-Answer Problems

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001 qfull 00510 1 3 0 easy math: acre-foot conversion

Extra keywords: an exercise in conversion only: there is no useful purpose

66. American hydraulic engineers often use acre-feet to measure volume of water. An acre-foot is the amount of water that will cover an acre of flat land to 1 foot.

    a) Say 3.00 in of rain fell on a plain of 30.0 km^2. How many acre-feet of water fell? Note 1 square mile equals 640 acres and 1 mi = 1.609344 km exactly by the mile definition. **HINTS:** First, find the volume in the hybrid units of inch-km^2 and then use a separate factor of unity for each unit conversion: divide and conquer.

    b) Now do a perhaps more useful calculation. Find the conversion factor from acre-feet to cubic meters. Note that 1 in = 2.54 cm exactly by the modern inch definition.

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001 qfull 00520 1 3 0 easy math: Fermi on lectures and the micro-century

Extra keywords: Fermi, micro-century, microcentury

67. Italian-American physicist Enrico Fermi once noted that a standard 50 minute university lecture was nearly a micro-century. **NOTE:** A Julian year, which is exactly 365.25 days, is exactly 3.15576×10^7 seconds. Actually, a convenient mnemonic is that a Julian year is $\pi \times 10^7$ s which is too small by only 0.5 %. It just a coincidence that the Julian year is almost this number of seconds. More exactly a Julian year is $1.0045096\pi \times 10^7$ s.

    a) How long is a micro-century in minutes actually?

    b) What is the percentage difference between a standard lecture period and a micro-century.

---

001 qfull 00740 1 3 0 easy math: pulsar periods in a day, sigfig

68. Pulsar PSR 1937+21 has rotation period 1.55780644887275(3) ms, where the bracketed number is the uncertainty in the last digit. Note "ms" is millisecond, not microsecond which has symbol μs.

    a) How many rotations does the pulsar do in 1.0 days keeping track of significant figures?

    b) How many rotations does the pulsar do in exactly 1 day (i.e., 1.0000 . . . days) keeping track of significant figures insofar as your calculator allows?

---

001 qfull 00810 3 3 0 tough math: simple error propagation

69. Say you have two experimental values A and B, and these differ from the true values by unknown deviations δA and δB respectively. Of course, you don't know what the deviations are, but you estimate their maximum absolute values to be a and b: i.e., $|\delta A| \leq a$ and $|\delta B| \leq b$ ideally. The a and b quantities are your estimated uncertainties or less correctly (but very commonly) your errors. We assume $a << |A|$ and $b << |B|$: this is often the case especially in cooked-up intro physics labs. We will now prove some simple error propagation formulae.

    a) Show that the uncertainty in $A \pm B$ is $a + b$. **HINT:** Add/subtract the true (but unknown) values first.

    b) Show that the **RELATIVE** error in AB is $a/|A| + b/|B|$ to 1st order in small a/A and b/B quantities. The relative errors add on multiplication: this is a general and very useful rule. **HINT:** To 1st order means that you can neglect higher order terms like $(a/A)^2$, $(b/B)^2$, and $[ab/(AB)]^2$ as being negligibly small.

    c) What is the 1st order formula for error in $f(A)$, where $f(x)$ is any differentiable function? **HINT:** Expand $f(A + \delta A)$ in a Taylor's series about A and truncate to 1st order.

    d) What is the 1st order formula for the relative error in $1/B$? **HINT:** Use the result of the part (c) answer.

    e) What is the 1st order formula for the relative error in A/B? **HINT:** Combine the part (b) and (d) answers.

---

001 qfull 00820 1 3 0 easy math: example error propagation

70. Answer the following error propagation questions using the 1st order error propagation formulae and without using a calculator. (You are in a lab *mise en scène* and don't want to bother with a lot of fussing.) Retain only significant figures. Remember errors are typically only 1-digit accurate only.

    a) What is the sum of 1.7 ± 0.3 and 0.81 ± 0.1 and its error?

    b) What is the product of 1.7 ± 0.3 and 0.81 ± 0.1 (to correct significant figures) and the error in the product? **HINT:** Estimate the relative errors crudely.

    c) What is 1.7 ± 0.3 divided by 0.81 ± 0.1 (to correct significant figures) and the error in this value? **HINT:** Estimate the relative errors crudely.

    d) You are multiplying together six values each with 1 % error. What is the percentage error in the product?

---

001 qfull 00930 1 3 0 easy math: light travel time to Sun, amount rate problem

71. The mean Sun-Earth distance, 1.496×10^{13} cm, is a convenient natural unit in astronomy. (All astronomy is CGS, not MKS by the by.) This unit is called the astronomical unit. Given the speed of light is approximately 3.00×10^8 m/s, what is the light travel time from the Sun in minutes?

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001 qfull 00950 2 3 0 moderate math: leap seconds, amount rate problem

72. The mean solar day is known to increase secularly (i.e., in a long-term way) by about 1.70×10^{-3} seconds per century due mainly to the tidal interaction of the Moon and Sun (Wikipedia: Tidal acceleration; Wikipedia: Leap second: Rationale). Currently, the mean solar day is about 86400.002 seconds long (Wikipedia: Solar time). **HINT:** Both parts (a) and (b) are questions of how long at a given rate does a given amount take to accumulate.

    a) About how often does a leap second need to be introduced in standard time in order to keep standard and solar time synchronized?

    b) If the current rate of increase in the day continues, about how long in years will it take before a leap second is needed daily: i.e., how long until the mean solar day is about 86401.0 seconds?

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001 qfull 00960 2 3 0 moderate math: national debt, amount rate

73. Circa year 2010, the US gross debt is/was estimated to be about 14 trillion dollars (i.e., $\$14 \times 10^{12}$).

    a) If payments were made at a rate of \$1000 per second and no interest accrued, how many **YEARS** would it take to pay off the debt?

    b) If 14 trillion one-dollar bills were laid end to end, how many times would they encircle the Earth? **HINT:** The Earth's mean radius is 6371.0 km and a dollar bill is 15.5 cm by actual measurement.

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001 qfull 01030 1 3 0 easy math: woodpeckers in Latah county, Moscow Idaho

74. Find to order of magnitude the number of woodpeckers in Latah County, Idaho. You are **NOT** permitted to look up any numbers. You must estimate them from what you know already. There is no exactly right answer—well all right, there is an exaclty right answer, but the instructor doesn't know what it is. You must justify the values you estimate.

---

001 qfull 01040 2 3 0 easy math: dandelions in US in summer

75. Find to order of magnitude the number of dandelion plants in the US at one time at about midsummer. You are **NOT** permitted to look up any numbers. You must estimate them from

what you know already. There is no exactly right answer—well all right, there is an exaclty right answer, but the instructor doesn't know what it is. You must justify the values you estimate.

---

001 qfull 01130 2 3 0 moderate math: Taylor's series

76. Some have seen Taylor's series, some have not. We won't prove it: we'll just write it down. Any sufficiently differentiable function in the neighborhood of a point x_0 can be approximated by a Taylor's series expansion about x_0 for a sufficiently small neighborhood. The general formula is

$$f(x) = \sum_{\ell=0}^{\infty} \frac{(x-x_0)^\ell}{\ell!} f^{(\ell)}(x_0) \ ,$$

where $f^{(\ell)}(x_0)$ is the ℓth derivative of f evaluated at x_0. The terms in the series are labeled by their power of $(x-x_0)$: $(x-x_0)^0$ is 0th (zeroth) order, $(x-x_0)^1$ is 1st order, $(x-x_0)^2$ is 2nd order, and so on. The Taylor's series is convergent (i.e., doesn't explode to infinity) for sufficiently small $(x-x_0)$. The smaller $(x-x_0)$ is, the less important the higher order terms. In physics (and other fields no doubt), one is often interested in the behavior near some important point x_0, and so truncates the Taylor's series to find an simple approximate expression for the neighborhood of x_0. For example

$$f(x) = \begin{cases} f(x_0) \ , & \text{0th order or constant approximation;} \\ f(x_0) + (x-x_0) f'(x_0) \ , & \text{1st order or linear approximation;} \\ f(x_0) + (x-x_0) f'(x_0) + \dfrac{(x-x_0)^2}{2} f''(x_0) \ , & \text{2nd order or quadratic approximation.} \end{cases}$$

The 0th order expansion, approximates the function as a constant which is the function value at x_0; the 1st order expansion approximates the function by a line that is tangent to the function at x_0; the 2nd order expansion approximates the function by a quadratic that is tangent to the function x_0.

a) Draw a general function on the Cartesian plane and at a general point x_0 schematically show how the 0th, 1st, and 2nd order approximations to the function behave.

b) Taylor expand

$$f(x) = \frac{1}{1+x}$$

to 2nd order about $x_0 = 0$. (Here $x - x_0$ is just x, of course.)

c) Taylor expand

$$f(x) = \sqrt{1+x}$$

to 2nd order about $x = 0$.

d) Taylor expand

$$f(x) = \frac{1}{\sqrt{1+x}}$$

to 2nd order about $x = 0$.

e) Taylor expand

$$f(x) = \sin(x)$$

to 3rd order about $x = 0$. Note that x must be in radians for the Taylor's expansion to work for the trigonometric functions. This is because the derivatives of these functions are derived using radians.

f) Taylor expand

$$f(x) = \cos(x)$$

to 2nd order about $x = 0$.

Chapt. 2 One-Dimensional Kinematics

Multiple-Choice Problems

002 qmult 00050 1 4 5 easy deducto-memory: calculus limited definition

77. "Let's play *Jeopardy*! For $100, the answer is: It is the branch of mathematics that deals with limits, derivatives, differentiation, integrals, and integration."

    What is _____, Alex?

    a) geometry b) algebra c) number theory d) set theory e) calculus

002 qmult 00051 1 1 2 easy memory: Delta use defined

78. It is common to specify a change in a quantity (specified by a symbol) by a prefixed capital Greek letter:

    a) Alpha A. b) Delta Δ. c) Lambda Λ. d) Psi Ψ. e) Omega Ω.

002 qmult 00052 1 1 3 easy memory: derivative definition

79. The formula

$$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x} \ ,$$

    where $f(x)$ is a general function of x is the not-explicit definition of the:

    a) integration. b) integral. c) derivative. d) differentiation. e) differential.

002 qmult 00054 1 1 2 easy memory: differentials

80. The quantity dx in calculus is the:

    a) derivative of x. b) differential of x. c) integral of x. d) integratial of x. e) product of d and x.

002 qmult 00056 1 1 2 easy memory: approximate derivative

81. We can approximate derivative df/dx to some degree of accuracy by:

    a) $\Delta f \Delta x$. b) $\dfrac{\Delta f}{\Delta x}$. c) $\dfrac{\Delta x}{\Delta f}$. d) $\dfrac{d^2 f}{dx^2}$. e) $\dfrac{df^2}{\Delta x}$.

002 qmult 00058 1 1 4 easy memory: power-law derivative

82. The derivative of

$$Ax^p \ ,$$

    where A is a general constant and p is a general power, is:

    a) $\dfrac{Ax^{p+2}}{p+2}$. b) $\dfrac{Ax^p}{p}$. c) $\dfrac{Ax^{p+1}}{p+1}$ or $A \ln(x)$ if $p = -1$.
    d) pAx^{p-1} or 0 if p and x are both zero. e) Ax^p .

002 qmult 00060 1 1 1 easy memory: definite and indefinite integrals

83. The expression

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

is a/an _____ and the expression

$$\int f(x)\,dx = F(x) + C$$

is a/an _____.

a) definite integral; indefinite integral b) indefinite integral; definite integral
c) antiderivative; derivative d) differential equation; antidifferential equation
e) differential; antidifferential

---

002 qmult 00062 1 1 3 easy memory: power-law integral

84. The indefinite integral or antiderivative (without any additive undetermined constant) of

$$Ax^p\ ,$$

where A is a general constant and p is a general power, is:

a) $\dfrac{Ax^{p+2}}{p+2}$. b) $\dfrac{Ax^p}{p}$. c) $\dfrac{Ax^{p+1}}{p+1}$ or $A\ln(x)$ if $p = -1$.
d) pAx^{p-1} or 0 if p and x are both zero. e) Ax^p .

---

002 qmult 00100 1 2 1 moderate memory: kinematics definition 1

Extra keywords: physci

85. Kinematics is:

a) the description of motion. b) the techniques of the cinema. c) dynamics by another name. d) the study of the causes of motion in terms of physical quantities, most prominently force and mass. e) the rate of change of acceleration with time.

---

002 qmult 00102 1 1 4 easy memory: kinematics definition 2

86. The area of physics dealing with just the description of motion without considering forces is called:

a) statics. b) dynamics. c) kinesiology. d) kinematics. e) cinematics.

---

002 qmult 00110 1 1 1 easy memory: quantities of kinematics

87. The three quantities that are of most obvious interest in kinematics are displacement, velocity, and:

a) acceleration. b) deceleration. c) mass. d) force. e) inertia.

---

002 qmult 00112 1 1 3 easy memory: magnitudes of x,v,a

88. The magnitudes of displacement, velocity, and acceleration are usually called distance, speed, and:

a) acceleration speed. b) deceleration. c) acceleration. d) accelmag.
e) the unnameable.

---

002 qmult 00120 1 1 3 easy memory: one-dimension vectors

89. Displacement, velocity, and acceleration are quantities that have both magnitude and direction, and so are:

a) scalars. b) unities. c) vectors. d) multiplicities. e) Templars.

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002 qmult 00122 1 4 5 easy deducto-memory: direction in 1-d

90. "Let's play *Jeopardy*! For $100, the answer is: All that is used to indicate direction in 1-dimensional kinematics."

What is _____, Alex?

a) symbol b) image c) vision d) vision quest e) sign

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002 qmult 00140 1 1 3 easy memory: definitions of v(avg), etc.

91. The following equations

$$v_{\rm avg} = \frac{\Delta x}{\Delta t} \ , \qquad v = \frac{dx}{dt} \ , \qquad a_{\rm avg} = \frac{\Delta v}{\Delta t} \ , \qquad a = \frac{dv}{dt}$$

are, respectively, the 1-dimensional definitions of average velocity, velocity, _____, acceleration.

a) average deceleration b) deceleration c) average acceleration d) speed
e) average speed

---

002 qmult 00142 1 1 2 easy memory: average velocity sensible

92. There are different ways of defining average quantities. But one wants definitions that are useful. For example, the conventional definition for average velocity is

$$v_{\rm avg} = \frac{\Delta x}{\Delta t} \ ,$$

where Δx is the displacement that occurred in time Δt. This definition is **USEFUL** since $v_{\rm avg}\Delta t$ is:

a) a random number. b) the displacement that takes place in time Δt.
c) a quantity with the dimension of length.
d) a quantity with the dimension of velocity. e) a time.

---

002 qmult 00150 1 1 2 easy memory: motion diagrams

93. Graphs of kinematic variables (i.e., displacement, velocity, acceleration, jerk) versus time can all be called:

a) emotion diagrams. b) motion diagrams. c) velocity diagrams.
d) acceleration diagrams. e) graphoi

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002 qmult 00152 1 1 3 easy memory: constant acceleration gives quadratic displacement

94. If acceleration is constant, then velocity is linear with time and displacement is _____ with time.

a) constant b) linear c) quadratic d) cubic e) quartic

---

002 qmult 00154 1 1 4 easy memory: singularity

95. Cusps, discontinuities, and infinities of functions are all:

a) anomies. b) autonomies. c) anomalies. d) singularities. e) multiplicities.

---

002 qmult 00210 1 1 4 easy memory: kinematic equations by antidifferentiation

96. Given that 1-dimensional acceleration a is a constant, one obtains the equations

$$v = at + v_0$$

and

$$x = \frac{1}{2}at^2 + v_0 t + x_0 \ ,$$

where v_0 is an initial velocity and x_0 is an initial displacement. The equations are obtained by:

a) algebra. b) distinction. c) differentiation. d) antidifferentiation.
e) antialgebra.

---

002 qmult 00220 1 1 4 easy memory: constant accel. kinematic equations

97. For 1-dimensional, constant acceleration cases there are ____ **INDEPENDENT** equations. Using these equations, 1-dimensional, constant acceleration problems can be solved. Only ____ unknowns can be solved in general from the ____ **INDEPENDENT** equations. One can derive by **ALGEBRA** extra kinematic equations that speed the solution of problems in some cases. But one can still only solve for ____ unknowns in general.

a) 5 b) 4 c) 3 d) 2 e) 1

---

002 qmult 00250 1 1 5 easy memory: timeless equation

98. The constant-acceleration kinematic equation without the time variable (which can be called the timeless equation for mnemonic reasons) is:

a) $v^2 = v_0^2 - 2a\Delta x.$ b) $v = v_0 + 2a\Delta x.$ c) $v^2 = v_0^2 + 2a/\Delta x.$ d) $v^2 = v_0^2 + a\Delta x$ e) $v^2 = v_0^2 + 2a\Delta x.$

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002 qmult 00310 1 3 3 easy math: d=vt calculation
Extra keywords: physci

99. You move at speed 3 m/s. How far do you go in 9 s?

a) 3 m. b) 9 m. c) 27 m. d) 1/3 m. e) 0 m.

---

002 qmult 00320 1 1 3 easy math: travel time from distance/speed: Knoxville 1
Extra keywords: physci

100. You have just traveled the back roads from Knoxville to Nashville. Your average speed was 60 mi/h, but you occasionally hit an instantaneous speed of 130 mi/h. (Could be you're hauling white lightning.) Your odometer travel distance is 250 miles. How long have you been on the road?

a) 1/4 hours. b) 10 hours. c) 4.17 hours. d) 6 hours. e) about 2 hours.

---

002 qmult 00322 1 3 2 easy math: average speed on round trip: Knoxville 2
Extra keywords: a round trip to Knoxville

101. You have just traveled a total distance of 400 km on a trip to Knoxville and back. It took 8 hours. Your average **SPEED** was:

a) 0 km/h. b) 50 km/h. c) 100 km/h. d) 200 km/h. e) 400 km/h.

---

002 qmult 00324 1 3 1 easy memory: average velocity: Knoxville 3

102. You have just traveled 400 km on a trip to Knoxville and back. Knoxville is due east of your starting point. It took 8 hours. Your average **VELOCITY** (with velocity definitely meaning a vector here) was:

a) 0 km/h with an indeterminate direction. b) 50 km/h west. c) 100 km/h east. d) 200 km/h west. e) 400 km/h north.

---

002 qmult 00330 1 3 5 easy math: average speed over two phases
Extra keywords: physci

103. You move 3 meters due west and then, **WITHOUT** a discontinuous change in direction, go onto a circular path (circle **RADIUS** 2 meters) bending to the left until you are headed due east. This has taken you 10 seconds. Your average speed is approximately _____. **HINT:** Draw a diagram.

a) 3 m/s. b) 1.55 m/s. c) 0.3 m/s. d) 15.5 m/s. e) 0.9 m/s.

---

002 qmult 00340 1 1 3 easy memory: lightning and thunder
Extra keywords: physci KB-60-5

104. The speed of sound in air (at 1 atm pressure and 20°C) is 343 m/s. A lightning flash occurs 1.5 km away. How long until you hear thunder?

a) 228.7 s. b) 0.0044 s. c) 4.4 s. d) 515 s. e) 1.5 s.

---

002 qmult 00392 1 1 4 easy memory: useful average speed

105.* You make a journey using a set of different speeds v_i for time different time intervals t_i. There are N time intervals. What is your average speed (i.e., average speed using the conventional meaning of average speed? **HINT:** The symbol $\sum$ means sum the quantities to the right. The symbol $\sum_{i=1}^{N}$ means sum the indexed quantities to the right from $i = 1$ to $i = N$.

a) $\sqrt{\frac{\sum_{i=1}^{N} v_i^2}{N}}$. b) $\frac{N}{\sum_{i=1}^{N} 1/v_i}$. c) $\frac{\sum_{i=1}^{N} v_i}{N}$. d) $\frac{\sum_{i=1}^{N} v_i t_i}{\sum_{i=1}^{N} t_i}$. e) $\sum_{i=1}^{N} v_i$.

---

002 qmult 00400 1 1 2 easy memory: acceleration and speed
Extra keywords: physci

106. If an object's speed changes, the object:

a) stops. b) accelerates. c) starts. d) goes forward. e) hesitates.

---

002 qmult 00410 2 5 2 moderate thinking: acceleration opposite velocity
Extra keywords: physci

107. Say you are moving in the positive x-direction, but your speed is decreasing.

a) Your acceleration points in the **POSITIVE** x-direction.
b) Your acceleration points in the **NEGATIVE** x-direction.
c) You are **NOT** accelerating at all. You are decelerating.
d) Your acceleration points **PERPENDICULAR** to the x-axis.
e) Your acceleration points in the **POSITIVE** x-direction. Acceleration always points in the same direction as velocity.

---

002 qmult 00420 2 5 5 moderate thinking: acceleration and instantaneous acc.
Extra keywords: physci

108. At time zero you are moving at 10 m/s in the positive y-direction. At time 10 s, you are moving at 15 m/s in the positive y-direction. What is your average acceleration over the 10 s of travel? What is your instantaneous acceleration at time 5 s?

a) The average acceleration and the 5 s instantaneous acceleration are both $0.5\,\mathrm{m/s^2}$ in the **POSITIVE** y-direction.
b) The average acceleration and the 5 s instantaneous acceleration are both $0.5\,\mathrm{m/s^2}$ in the **NEGATIVE** y-direction.
c) The average acceleration is $0.5\,\mathrm{m/s^2}$ in the **NEGATIVE** y-direction. There is **NOT** enough information to determine the 5 s instantaneous acceleration.
d) The average acceleration and the 5 s instantaneous accelerations are both $5\,\mathrm{m/s^2}$ in the **NEGATIVE** y-direction.
e) The average acceleration is $0.5\,\mathrm{m/s^2}$ in the **POSITIVE** y-direction. There is **NOT** enough information to determine the 5 s instantaneous acceleration.

---

002 qmult 00500 1 1 2 easy memory: surface gravity points down
Extra keywords: physci

109. The acceleration due to gravity near the Earth's surface (whose magnitude is denoted by g and whose value is nearly constant and is near fiducial value $9.8\,\mathrm{m/s^2}$):

a) points east. b) points down toward the Earth's center more or less. c) points straight up more or less. d) has no direction at all. e) points toward the Moon.

---

002 qmult 00510 1 1 1 easy memory: free-fall speed and acceleration
Extra keywords: physci KB-58-4

110. As an object falls freely under gravity. Neglecting air drag, its speed _____ and its acceleration is _____.

a) increases; constant. b) decreases; increases. c) increases; increases.
d) decreases; decreases. e) is constant; constant.

---

002 qmult 00520 1 1 2 easy memory: zero vertical velocity at top of trajectory

111. An object is thrown straight up. At the top of its trajectory, it's vertical velocity is:

a) always completely indeterminate. b) zero. c) infinite. d) g.
e) unknown with the given information.

---

002 qmult 00530 1 1 3 easy memory: free fall independent of mass: Galileo
Extra keywords: physci

112. It is reported that Galileo (circa 1590) dropped balls of different mass at the same time from the top of the Leaning Tower of Pisa in order to demonstrate that:

a) the heavier ball hit the ground first by a large margin.
b) the lighter ball hit the ground first by a large margin.
c) both balls hit the ground at more or less the same time.
d) the balls would levitate toward the Moon.
e) the Leaning Tower was leaning.

---

002 qmult 00540 1 3 2 easy math: ball thrown down
Extra keywords: physci KB-60-17

113. A ball is thrown downward at 12 m/s. About what is its **SPEED** 2.0 s later assuming no air drag?

a) 20 m/s. b) 32 m/s. c) 22 m/s. d) -2 m/s. e) 12 m/s.

---

002 qmult 00550 2 3 4 moderate math: kinematic equations: arrow flight 1
Extra keywords: physci

114. A tall archer with her longbow shoots an arrow straight up at 100 m/s. The arrow rises, slows, holds for an instant, and then descends picking up speed. The rise time, neglecting air drag, is:

a) 100 s. b) 100.2 s. c) 9.8 s. d) 10.2 s. e) 980 s.

---

002 qmult 00552 2 3 5 moderate math: kinematic equations: arrow flight 2

115. A tall archer with her longbow shoots an arrow straight up at 100 m/s. The arrow rises, slows, holds for an instant, and then descends picking up speed. The speed of the arrow when it reaches the level from which it was shot is, neglecting air drag:

a) 9.8 m/s^2. b) 9.8 m/s. c) 200 m/s. d) 50 m/s. e) 100 m/s.

---

002 qmult 00560 1 3 1 easy math: free-fall speed in 3 seconds

116. How fast is a person falling after 3 s starting from rest? Recall the acceleration due to gravity is $g = 9.8$ m/s^2 (which is the fiducial value). Neglect air drag.

a) 29.4 m/s. b) 44.1 m/s. c) 9.8 m/s. d) 88.2 m/s. e) At the speed of light.

---

002 qmult 00562 1 3 1 easy math: kinematic equations: free fall distance
Extra keywords: in 3 seconds

117. A human falls off some high scaffolding. About how far do they fall in 3 seconds? (Neglect air drag.)

a) 44 m. b) 88 m. c) 22 m. d) 9.8 m. e) 4.9 m.

---

002 qmult 00564 1 3 2 easy math: falling in 4 seconds

118.* How far does a person fall in 4 s starting from **REST**? (Neglect air drag.)

a) 39.2 m. b) 78.4 m. c) 9.8 m. d) 156.8 m. e) From the Earth to the Moon.

---

002 qmult 00610 1 4 3 easy deducto-memory: terminal velocity defined

Extra keywords: physci

119. "Let's play *Jeopardy*! For $100, the answer is: It occurs to a dense falling object falling near the Earth's surface when the force of gravity and the force of air drag (AKA air resistance) cancel to give no net force on an object."

What is _____, Alex?

a) acceleration upward b) acceleration downward c) terminal velocity
d) initial velocity e) parabolic motion

---

002 qmult 00620 3 1 2 easy math: travel time, human terminal velocity 1

Extra keywords: physci

120. The terminal velocity of a human in air is about 120 mi/h. At this speed how long does it take to fall 2 miles.

a) 2 minutes. b) 1 minute. c) 1 hour. d) 2 hours. e) 1 second.

---

002 qmult 00630 1 2 3 easy deduction: human terminal velocity 2

Extra keywords: physci

121. What is approximately the terminal velocity of a human in air? **HINT:** You don't have to know the answer; you can deduce it.

a) 10 km/h. b) 1 km/h. c) 200 km/h. d) 0.1 km/h. e) 3×10^5 km/s.

---

002 qmult 00640 1 4 5 easy deducto-memory: cat fall

Extra keywords: mathematical physics

122. "Let's play *Jeopardy*! For $100, the answer is: These features allow cat victims of the high-rise syndrome (i.e., the propensity to taking flying leaps into oblivion—cats being so darn smart you know) to survive falls of $\gtrsim 20$ m without major injuries—sometimes that is."

What are _____, Alex?

a) feline insouciance, savoir-faire, panache, et je-ne-sais-quoi.
b) the cat **WRONGING** reflex and relatively **LOW** terminal velocity when spread-eagled
c) the cat **WRONGING** reflex and relatively **HIGH** terminal velocity when spread-eagled
d) the cat **RIGHTING** reflex and relatively **HIGH** terminal velocity when spread-eagled
e) the cat **RIGHTING** reflex and relatively **LOW** terminal velocity when spread-eagled

---

002 qmult 10210 2 3 3 moderate math-memory: linear acceleration with time fullmult

Extra keywords: mathematical physics

123.** "Let's play *Jeopardy*! For $100, the answer is: It is displacement as a function of time t for the case where the **DERIVATIVE** of a 1-dimensional acceleration, $b = da/dt$ is a constant."

What is _____, Alex?

a) $x = x_0 + v_0 t$ b) $x = x_0 + v_0 t + (1/2)at^2$ c) $x = x_0 + v_0 t + (1/2)a_0 t^2 + (1/6)bt^3$
d) $x = x_0 + v_0 t + (1/2)a_0 t^2 + (1/6)b_0 t^3 + (1/24)ct^4$ e) $x = x_0 \cos(\omega t)$

---

002 qmult 10410 2 3 1 moderate math: easy car motion fullmult

124.* A car starts from **REST** with constant acceleration 1 m/s^2.

i) What is its velocity after 10 s from its **START POSITION**?

ii) What is its displacement traveled after 10 s from its **START POSITION**?

iii) What is its velocity when it has traveled 200 m from its **START POSITION**?

a) (i) 10 m/s (ii) 50 m (iii) 20 m/s. b) (i) 10 m/s (ii) 50 m (iii) 14.1 m/s.
c) (i) 10 m/s (ii) 10 m (iii) 10 m/s. d) (i) 10 m/s (ii) 10 m (iii) 14.1 m/s.
e) (i) 10 m/s (ii) 10 m (iii) 20 m/s.

---

002 qmult 10440 1 1 5 easy memory: two-object car passing problem fullmult

125.** At time zero, there is a car stopped at a light and a truck moving at a constant velocity of 30.0 m/s is 500 m behind the car. Also at time zero, the car starts accelerating forward at 1.0 m/s^2. At what time does the truck first pass the car?

a) 7.6 s. b) 52 s. c) 312 s. d) 520 s. e) It never passes the car.

---

002 qmult 10520 2 3 1 mod math: Super-Armadillo on Mars fullmult

126.** Super-Armadillo is on Mars where the acceleration due to gravity is $g = 3.69\,\mathrm{m/s^2}$. Starting from **REST** on the surface—smack dab in Valles Marineris: the giant Martian canyon—he accelerates straight upward at 10.0 m/s^2 for 4.00 s and then turns off his super-thrust and lets the Martian gravity determine the rest of his trajectory. What is his crash speed? Neglect air drag which is small, but not completely negligible in reality, on Mars.

a) 46.8 m/s. b) 31.8 m/s. c) 40.0 m/s. d) 80.0 m/s. e) 1020 m/s.

Full-Answer Problems

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002 qfull 00052 2 3 0 moderate math: differentiation power law

127. Differentiation is an extremely important mathematical operation in physics.

a) Differentiate the power-law formula

$$y = Ax^p \ ,$$

where A is a constant and p is a general power.

b) Find the derivatives of

$$y = A \ , \qquad y = Ax \ , \qquad y = Ax^2 \ .$$

---

002 qfull 00210 2 3 0 moderate math: linear acceleration with time, constant jerk

Extra keywords: calculus-based question

128. Jerk is derivative of acceleration. The term jerk is not used much by a lot of people. Say jerk for a one-dimensional case is a constant b. This means that $b = da/dt$, where a is acceleration. Using antidifferentiation (i.e., integration), write down the expressions for acceleration, velocity, and position remembering that a constant of integration arises at every integration.

---

002 qfull 00410 1 3 0 easy math: easy car motion

129. A car starts from **REST** with constant acceleration 1 m/s^2.

a) What is its velocity after 10 s from its **START POSITION**?

b) What is its displacement traveled after 10 s from its **START POSITION**?

c) What is its velocity when it has traveled 100 m from its **START POSITION**?

---

002 qfull 00440 2 3 0 moderate math: two-object car passing problem

130. At time zero, there is a car stopped at a light and a truck moving at a constant velocity of 30.0 m/s is 200 m behind the car. Also at time zero, the car starts accelerating forward at 1.00 m/s^2.

a) Draw a **QUALITATIVE** plot of position x versus time t showing the trajectory curves of both vehicles. What are the three qualitatively different ways the curves can intersect?

b) At what time or times do the car and truck pass? Is it possible for there to be only one pass with the given conditions?

---

002 qfull 00442 2 3 0 moderate math: Merc-Jag kinematic eqn. problem

Extra keywords: Mercedes and Jaguar

131. At time zero, there is a car (a Jaguar) stopped at a red light and a Mercedes (that has just run the red) is moving at a constant 50.0 m/s and is 20.0 m ahead of the Jag. Also at time zero, the stoplight turns green and the Jag (driven by an atavistic categorical imperative) starts accelerating forward at 5.00 m/s^2.

    a) What is 50.0 m/s in miles per hour? Note, the modern mile is defined to be exactly 1609.344 meters.

    b) Given the conditions, must the Jag pass the Mercedes? Why? (If your answer is no, you can skip the rest of the question.)

    c) Calculate the time from time zero to when the Jag's velocity equals that of the Mercedes.

    d) Calculate the time from time to when the Jag passes the Mercedes. **HINT:** Draw position x versus time t plot with the position curves for both cars.

    e) Calculate the distance from the stoplight to the pass?

    f) How fast is the Jag going at the pass in meters per second **AND** miles per hour?

---

002 qfull 00444 2 3 0 moderate math: geo yaris two object problem

132. A 1992 GM Geo Metro is stopped at a red light. A 2007 Toyota Yaris passes the Geo (and runs the red light). At the time of this pass, the Yaris has velocity 30.0 m/s and constant acceleration of 1.00 m/s^2. Two seconds (more precisely 2.00 s) after the pass, the "law-abiding" Geo accelerates from rest with constant acceleration 2.00 m/s^2. **TIME ZERO** is when the Geo starts to move. **HINT:** Draw a displacement motion diagram to understand what is going on.

    a) Will the Geo ever pass the Yaris? Explain your answer?

    b) What is the **YARIS'S** velocity **AND** displacement from the stoplight at **TIME ZERO** which we **DEFINE** to be the time when the Geo starts to accelerate?

    c) When will the cars be moving at the **SAME VELOCITY**? Is this the time when the Geo passes the Yaris? Explain.

    d) When will the Geo pass the Yaris?

    e) What is the Geo's velocity and displacement at the time of the pass? Convert the Geo's velocity in miles per hour.

---

002 qfull 00510 1 3 0 easy math: raindrops sans air drag

133. Say that raindrops fall 1800 m to the ground starting from rest.

    a) Assuming there is no air drag (AKA air resistance) what is touch-down velocity?

    b) Would it be safe to walk in the rain without air drag? Compare the drop speed to bullet speed? You will find some useful information about raindrop speeds at

        http://hypertextbook.com/facts/2007/EvanKaplan.shtml

    and about bullet speeds at

        http://hypertextbook.com/facts/1999/MariaPereyra.shtml .

    Of course, this is no help on a test.

---

002 qfull 00520 2 3 0 moderate math: Super-Armadillo on Moon

134. Super-Armadillo is on the **MOON** where the free-fall acceleration $g = 1.6$ m/s^2 downward and there is really no air drag (AKA air resistance).

    a) He flies **STRAIGHT UP** from the surface starting from **REST** with an **ACCELERATION** of 5 m/s^2 upward for 20 s. Then his superpower stalls and he can no longer accelerate, but he still has the velocity he had at the end of his self-powered

acceleration phase for an instant. What is his **HEIGHT** and his **VELOCITY** when his superpower failure occurs?

b) After stalling, his acceleration is due to gravity. How high above the **LUNAR GROUND** does he rise before beginning to fall?

c) Eventually Super-Armadillo must fall back to the lunar surface. What is his crash **SPEED**? (Fortunately, his invulnerability is still operative.)

---

002 qfull 00530 1 3 0 moderate math: Irish sprinters nuala and sinead

135. Irish sprinters Nuala O'Blarney and Sinead O'Donnybrook both run the 100-meter sprint in 10.6 s—which is good, but not world class.

a) What are their average velocities?

b) Nuala accelerates from **REST** at a constant acceleration for some time period and then runs the rest of the race at 10.0 m/s in time period 9.3 s. (Recall her total race time was 10.6 s.) What was her displacement from the start at the end of her acceleration phase? What was her acceleration in the acceleration phase? What was her final velocity in the acceleration phase? Nuala's displacement must be continuous as a function of time, but her velocity may **NOT** be.

c) Sinead runs her race in two phases too. There is an acceleration phase with constant acceleration a for $t_1 = 1.00\,\text{s}$ and then a constant-velocity phase with constant velocity v_2 for the rest of the way. She starts from **REST** and her velocity as a function of time is **CONTINUOUS** at all times. Find her constant-acceleration phase acceleration a, how far she runs Δx_1 during the constant-acceleration phase, and her constant-velocity phase velocity v_2. **HINT:** Write down a set of equations relating the knowns and unknowns. Solve for the unknowns from the set of equations.

---

002 qfull 00610 2 5 0 moderate thinking: relative kinematic equations

136. The 5 standard 1-dimensional, constant-acceleration kinematic equations were derived assuming motion with respect to some fixed reference frame. But what if you had say a two-object 1-dimensional, constant-acceleration that you wished to reduce to a one-object problem by changing reference frames: i.e., changing to the rest frame of one of the objects or in other words to a relative frame.

Note, the relative position (or displacement), velocity, and acceleration for object 2 in the frame of object 1 are given by, respectively,

$$\begin{aligned} x_{\text{rel}} &= x_2 - x_1 \ , \\ v_{\text{rel}} &= v_2 - v_1 \ , \\ a_{\text{rel}} &= a_2 - a_1 \ . \end{aligned}$$

The first formula is a definition—a very reasonable one—and the others follow from differentiation of the first. The formulae just express what one means by relative position, velocity, and acceleration in one dimension.

a) Show that the first two kinematic equations apply to relative motion if the objects involved both have constant acceleration (relative to some standard frame of reference like the ground). Recall these equations are:

$$v = at + v_0 \qquad \text{and} \qquad x = \frac{1}{2}at^2 + v_0 t + x_0 \ ,$$

where the subscript "0" indicates initial value.

b) Show that the other 3 kinematic equations apply to relative motion. **HINT:** This isn't very hard if you know the trick.

---

002 qfull 00620 3 3 0 hard math: colliding trains

137. Train 1 is moving east at 60 m/s and train 2 is moving west at 50 m/s on a collision course with train 1. At 1000 meters apart, the two train engineers notice each other and slam on the brakes simultaneously.

a) What must the relative acceleration be (assuming it is constant) for the trains to just touch at relative velocity zero (i.e., to have a innocuous marginal collision)? Is velocity of the trains (i.e., their velocity with respect to the ground) necessarily zero when they collide? **HINT:** Getting your conventions straight about which direction is negative and which is positive, and about which object is chosen as defining the relative reference frame is almost the whole problem. For the last question, ask yourself do **BOTH** of the trains have to accelerate relative to the ground in order to arrange an innocuous collision?

b) What is the magnitude of relative acceleration in g's (i.e., in units of the gravitational acceleration)? Acceleration expressed in g's is called the g-force which is misnomer since it is an acceleration, not a force.

c) Fundamentally unaccelerated frames of reference are called inertial frames. It's hard to find an exactly inertial frame actually. But approximate ones are common. The ground is approximately an inertial frame of reference for most, but not all human, purposes. In non-inertial frames, there are what are called inertial forces (or fictitious forces). Consider a frame moving in a straight line and accelerating relative to an inertial frame: e.g., the ground. If no real forces act on an object observed in this accelerated frame, the body will undergo an acceleration equal in magnitude, but opposite in direction to the frame acceleration. This acceleration is attributed to an inertial force. The inertial force is a field force actually: it pulls on the object atom by atom. To keep at rest in the accelerated frame, actual forces have to be used to balance the inertial force. Although our descripton has been rather general and abstract, people actually know all about inertial forces in accelerated frames. If you accelerate forward in your car, it seems like there is force pushing you back into your seat: this is an inertial force. The back of your seat must exert a force to balance the inertial force and your body layer by layer must exert internal forces to balance it. The inertial force is like gravity. If you are in free fall, you feel weightless: gravity pulls on you atom by atom, but overall your body is relaxed. Only when you resist gravity does your body exert internal forces and those cause the sense of heaviness. The same is true for an inertial force. Don't resist an inertial force and you are unstressed.

Now to the question. Do the passengers in the train feel an inertial force set by the relative acceleration? Consult your own experience: do you notice inertial forces caused by accelerations relative to other cars when you drive?

d) And by the way, what is 60 m/s in kilometers per hour? Is this fast for a train?

---

002 qfull 01010 2 5 0 moderate thinking: dripping shower

Extra keywords: a cute question, but not so relevant

138. It's been one of those days and now you have just lain down to sleep—and everything is dead quiet—except there's a regular drip-drip from your shower nozzle. Nothing can stop it: it looms in the silence like Poe's pendulum. You toss, you turn, you are going insane—finally you come to physics. It is 2 m from the nozzle to the floor: you prudently measured this distance once anticipating a dark night of the soul. You count 26 drips in 13 pulse beats: this feat of tremendous double concentration only goes to show how far you are from the "death of each day dying." And you ask yourself what is the most and least number of drops in the air at the same time if air drag is neglected?

Chapt. 3 Vectors and Trigonometry

Multiple-Choice Problems

003 qmult 00100 1 4 5 easy deducto-memory: vector defined
Extra keywords: physci

139. "Let's play *Jeopardy*! For $100, the answer is: An incomplete definition is that it is a quantity with both a magnitude and a direction. The direction is in ordinary physical space (i.e., space space). The extent is in ordinary physical space for displacement only and in abstract spaces otherwise (e.g., velocity space)."

What is a/an _____, Alex?

a) director b) aviator c) bisector d) scalar e) vector

003 qmult 00110 1 1 5 easy memory: vector defined 2

140. A vector is an entity with:

a) only magnitude. b) only direction.
c) magnitude, direction, and extension in physical space.
d) magnitude, direction, and temperature. e) magnitude and direction.

003 qmult 00120 1 4 4 easy deducto-memory: physics scalar defined
Extra keywords: physci

141. "Let's play *Jeopardy*! For $100, the answer is: It is a physical quantity expressible by a single real number whose value is independent of the coordinate system (of ordinary physical space). A _____ can have a direction in a 1-dimensional abstract space. For example, temperature on the Celsius scale: it can have positive or negative value. So _____ can be regarded as a 1-dimensional vector with the 1 direction being in an abstract space, but in the usual way of speaking one would say it is **NOT** a vector."

What is a/an _____, Alex?

a) director b) aviator c) bisector d) scalar e) vector

003 qmult 00130 1 1 1 easy memory: scalar defined 2

142. A scalar in physics is an quantity expressible by a single real number that is:

a) independent of the coordinate system. b) dependent on the coordinate system.
c) always positive. d) always negative. e) always zero.

003 qmult 00150 1 4 3 easy deducto-memory: field in physics defined

143. "Let's play *Jeopardy*! For $100, the answer is: It is quantity with a value at every point in space or at least some region of space. The quantity can be a scaler, vector, or tensor."

What is a/an _____, Alex?

a) area b) electric area c) field d) electric space e) zone

003 qmult 00160 1 4 1 easy deducto-memory: true vector

144. A vector (i.e., a true vector, not just something called a vector since it has some vector-like aspects) has the properties that the components are _____ and the magnitude

of the vector (in its space abstract or otherwise) is determined using the _____ with components acting as "sides". Also vectors have certain coordinate transformation properties. Pseudovectors (e.g., angular momentum and torque) act like vectors in most respects, but do not have the exactly the same coordinate tranformation properties.

a) scalars; Pythagorean theorem b) scalars; absolute value operation
c) vectors; Pythagorean theorem d) vectors; absolute value operation
e) integers; Pythagorean theorem

---

003 qmult 00200 1 4 3 easy deducto-memory: displacement defined
Extra keywords: physci

145. "Let's play *Jeopardy*! For $100, the answer is: It is a vector that is specified by giving the direction and a straight-line distance in that direction."

What is _____, Alex?

a) velocity b) acceleration c) displacement d) force e) distance

---

003 qmult 00210 1 1 3 easy memory: displacement prototype vector

146. The prototype vector in physics (i.e., the one that is usually thought of setting what the standard physical vector properties are) is _____. One main reason for this is that of the physical vectors, only _____ has extension in ordinary physical space.

a) force b) mass c) displacement d) speed e) the electric field vector

---

003 qmult 00220 1 1 5 easy memory: velocity is a vector

147. Velocity is:

a) speed. b) a scalar. c) displacement. d) acceleration. e) a vector.

---

003 qmult 00222 1 1 4 easy memory: speed

148. Because it is the magnitude of velocity, speed has:

a) one angle. b) an angle. c) a direction. d) no direction.
e) only negative values.

---

003 qmult 00230 1 1 4 easy memory: definition of acceleration
Extra keywords: physci

149. Acceleration is:

a) speed.
b) velocity.
c) the rate of change of velocity with time. It is a **SCALAR**.
d) the rate of change of velocity with time. It is a **VECTOR**.
e) the rate of change of displacement with time. It is a **VECTOR**.

---

003 qmult 00232 1 1 3 easy memory: vector uniform circular motion
Extra keywords: physci

150. You are going in a circle at a uniform speed (i.e., a constant speed). Is your **VELOCITY** ever changing?

a) No, the speed is constant.
b) Yes, it is constantly changing since the motion is in a **STRAIGHT LINE**.
c) Yes, it is constantly changing since the motion is in a **CIRCLE** and direction is constantly changing.
d) No, the velocity is constant.
e) Yes, on **EVERY OTHER LEFT** bend.

---

003 qmult 00280 1 1 2 easy memory: identify a vector
Extra keywords: physci

151. Which of the following quantities is vector?

a) mass. b) force. c) energy. d) speed. e) temperature.

---

003 qmult 00282 1 1 1 easy memory: identify a non-vector
Extra keywords: physci KB-56-1

152. Which of the following quantities is not a vector?

a) mass. b) force. c) acceleration. d) velocity. e) displacement.

---

003 qmult 00284 1 1 3 easy memory: identify a unit that can be for a vector
Extra keywords: physci

153. Which of the following can be the unit of a vector?

a) kilogram. b) second. c) meter/second. d) liter. e) acceleration.

---

003 qmult 00300 1 1 2 easy memory: trig and trig functions

154. By the ancient traditional meaning trignonometry (abbreviated to trig) is the branch of mathematics dealing with triangles. The word trignonometry is derived from trigonon (Greek for triangle) and metron (Greek for measure). Actually trignonometry is generalized beyond triangles to deal with the components of a radius of a circle in a Cartesian coordinate system. This generalization gives the definitions of the _____ functions.

a) polynomial b) trignonometric c) transcendental d) quadratic e) cubic

---

003 qmult 00302 1 1 2 easy memory: trig abbreviation
Extra keywords: too simple-minded even for tests

155. Trigonometry is often abbreviated to

a) triggy. b) trig. c) gono. d) metro. e) monstro.

---

003 qmult 00304 1 1 5 easy memory: division of circle into 360 degrees

156. For reasons they never bothered to record for posterity, the ancient Babylonians of circ 500 BCE divided the circle into 360 units (i.e., 360 degrees). One likely reason is that they didn't like dealing with decimal fractions (actually sexagesimal fractions in their system), and so chose a number of divisions to be one with a lot of whole number factors. How many whole number factors does 360 have?

a) 6. b) 8. c) 12. d) 16. e) 24.

---

003 qmult 00306 1 1 3 easy memory: radians

157. Nothing requires us to divide the circle into a whole number or even a rational number of divisions. In fact, it gives a natural angular unit to divide the circle into 2π divisions. The division or angular unit obtained is $1/(2\pi)$ of a circle and is called a radian (rad): thus

$$\frac{1\,\text{rev}}{2\pi} = 1\,\text{rad}\ .$$

The radian is the natural unit of angular measure since arc length s is given by _____, where r is the circle's radius and θ is angle in radians. Also all of calculus with trigonometric functions is simplified by using radians.

a) $s = \theta/r$ b) $s = [\theta/(2\pi)]r$ c) $s = \theta r$ d) $s = \theta r^2$ e) $s = r/\theta$

---

003 qmult 00310 1 4 5 easy deducto-memory: transcendental functions

158. "Let's play *Jeopardy*! For $100, the answer is: They are functions that cannot be exactly evaluated by a finite sequence of the algebraic operations of addition, multiplication, and root taking. Examples are the trigonometric, logarithm, and exponential functions."

What are the _____ functions, Alex?

a) algebraic b) polynomial c) real d) emergent e) transcendental

---

003 qmult 00320 1 4 4 easy deducto-memory: trigonometric functions
Extra keywords: mathematical physics

159. "Let's play *Jeopardy*! For $100, the answer is: They are functions whose argument is an angle and which yield the ratios of sides of a right triangle (or right-angled triangle). The functions are extended so that the argument angle can be any value in the range $[0°, 360°]$. The "sides" of the triangle in the extension are the magnitude of a radius vector (which is the hypotenuse) and the components of the radius vector along the x and y axes of the Cartesian plane. The component "sides" can be positive or negative depending on quadrant where the radius lies."

What are the _____ functions, Alex?

a) vector b) eigen c) venial d) trigonometric e) bodily

---

003 qmult 00330 1 1 1 easy memory: basic trig function names

160. The three basic trigonometric functions have abbreviated names:

a) sin, cos, tan. b) sly, crow, tawn. c) slip, crape, toon. d) slop, crip, troop. e) snood, croon, troon.

---

003 qmult 00332 1 1 3 easy memory: trig function definitions

161. The 3 basic trigonometric functions are defined by

$$\sin\theta = \frac{y}{r}\ , \qquad \cos\theta = \frac{x}{r}\ , \qquad \tan\theta = \frac{y}{x}\ ,$$

where r is the magnitude of a radius vector $\vec{r}$, θ is the angle of the radius vector measured counterclockwise from the positive x axis, and (x, y) are the ordered pair that locate the head of the radius vector. Just to be clear, the argument of the functions is θ and the trigonometric functions are the ratios. Immediately, one sees that:

a) $\tan\theta = \cos(\theta)\sin(\theta)$. b) $\tan\theta = \dfrac{\cos\theta}{\sin\theta}$. c) $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$. d) $\tan\theta = \dfrac{1}{\cos\theta}$. e) $\tan\theta = \dfrac{1}{\sin\theta}$.

---

003 qmult 00340 1 1 5 easy memory: Pythagorean theorem

162. Given a right triangle with sides of length x and y being adjacent to the right angle and a side of length r being opposite to the right angle (this side is the hypotenuse), one has the exact relationship

$$r^2 = x^2 + y^2\ .$$

This relationship is called the:

a) Anaxagorean theorem. b) Euclidean theorem. c) Sumerian theorem. d) Taurean theorem. e) Pythagorean theorem.

---

003 qmult 00350 2 1 1 moderate memory: trig functions with 0 degrees

163. Sin, cos, and tan of 0° are, respectively:

a) 0, 1, 0. b) 1/2, $\sqrt{3}/2$, $1/\sqrt{3}$. c) $1/\sqrt{2}$, $1/\sqrt{2}$, 1. d) $\sqrt{3}/2$, 1/2, $\sqrt{3}$. e) 1, 0, ∞.

---

003 qmult 00354 2 1 3 moderate memory: trig functions with 45 degrees

164. Sin, cos, and tan of 45° are, respectively:

a) 0, 1, 0. b) 1/2, $\sqrt{3}/2$, $1/\sqrt{3}$. c) $1/\sqrt{2}$, $1/\sqrt{2}$, 1. d) $\sqrt{3}/2$, 1/2, $\sqrt{3}$. e) 1, 0, ∞.

---

003 qmult 00356 2 1 4 moderate memory: trig functions with 60 degrees

165. Sin, cos, and tan of 60° are, respectively:

a) 0, 1, 0. b) 1/2, $\sqrt{3}/2$, $1/\sqrt{3}$. c) $1/\sqrt{2}$, $1/\sqrt{2}$, 1. d) $\sqrt{3}/2$, 1/2, $\sqrt{3}$.
e) 1, 0, ∞.

---

003 qmult 00380 1 1 3 easy memory: sinusoidal motion
Extra keywords: sinusoidal motion from circular motion.

166. An object is undergoing uniform circular motion (i.e., revolving in a circular in a circle at a uniform speed). If the object's motion is projected on a line in the plane of the circle and passing through the center of the circle, the motion along the line is _____ as a function of time.

a) a sawtooth wave b) time-like c) sinusoidal d) a square wave
e) a lissajous curve

---

003 qmult 00400 1 1 2 easy memory: vector components

167. Vector components are computed by multiplying the magnitude (or length) of a vector by the cosines of the angles the vector makes with the positive coordinate directions of a:

a) circle. b) coordinate system. c) rotation. d) square. e) wheel.

---

003 qmult 00402 1 1 3 easy memory: 2-d vectors

168. In Cartesian coordinates, a two-dimensional vector $\vec{a}$ is given by

$$\vec{a} = (a_x, a_y) \ ,$$

where (a_x, a_y) given by

$$a_x = a\cos\theta \qquad \text{and} \qquad a_y = a\cos\theta_y \ ,$$

where θ is the standard angle measured from the positive x axis and θ_y is the angle measured from the y axis. From trigonometry, we know that

$$\frac{a_y}{a} = \cos\theta_y$$

and that

a) $\frac{a_y}{a} = \tan\theta$. b) $\frac{a_y}{a} = \cos\theta$. c) $\frac{a_y}{a} = \sin\theta$. d) $\frac{a_y}{a} = \cot\theta$.
e) $\frac{a_y}{a} = \csc\theta$.

---

003 qmult 00410 1 1 4 easy memory: vector component non-uniqueness

169. The components of multi-dimensional physical vectors:

a) are unique.
b) can be chosen only two ways: the two ways will lead to different physical behavior.
c) can be chosen in infinitely many ways: each way leads to a different physical behavior.
d) can be chosen in infinitely many ways. However, the physics of the vector remains the same and in any problem the choice of components (i.e., the choice of a coordinate system) is arbitrary. But some choices make the problem a lot easier.
e) cannot be determined at all in principle.

---

003 qmult 00510 1 1 1 easy memory: vector addition with components

170. Vector addition is defined to be done by adding the vector components by the:

a) ordinary real number addition rule. b) ordinary real number multiplication rule.
c) extraordinary real number multiplication rule.
d) super-unusual real number multiplication rule. e) law of cosines.

---

003 qmult 00530 1 1 2 easy memory: vector addition

171. You can add vectors:

a) geometrically or adding their magnitudes. b) geometrically or by components. c) adding their magnitudes or by components. d) adding their magnitudes or by division. e) adding their magnitudes or by integration.

---

003 qmult 00540 1 1 1 easy memory: vector addition commutative

172. Vector addition is:

a) independent of the order of addition: i.e., it is commutative.
b) depends on the order of addition.
c) not possible.
d) only possible for displacement vectors.
e) only possible for velocity vectors.

---

003 qmult 00580 1 5 5 easy thinking: vector addition to zero

173. Say you had two vectors of equal magnitude A, but with opposite directions. What can you say about the vector sum of these vectors? **HINT:** A diagram might help.

a) $2A$ is the magnitude, but the direction cannot be determined without more information.
b) $2A$ is the magnitude. The direction is the direction of the **FIRST** vector in the order of addition. Order of addition is important in vector addition. Say you went 3 kilometers north and 10 kilometers east, you would not be anywhere near where you would be if you had gone 10 kilometers north and 3 kilometers east. The **FIRST** stage clearly dominates where you would be.
c) $2A$ is the magnitude. The direction is the direction of the **SECOND** vector in the order of addition. Order of addition is important in vector addition. Say you went 3 kilometers north and 10 kilometers east, you would not be anywhere near where you would be if you had gone 10 kilometers north and 3 kilometers east. The **SECOND** stage clearly dominates where you would be.
d) **ZERO** is the magnitude. The direction is the direction of the **FIRST** vector in the order of addition. Order of addition is important in vector addition. Say you went 3 kilometers north and 10 kilometers east, you would not be anywhere near where you would be if you had gone 10 kilometers north and 3 kilometers east. The **FIRST** stage clearly dominates where you would be.
e) **ZERO** is the magnitude. The direction of a zero magnitude vector is **UNDEFINED AND UNNEEDED**. Say you added a zero magnitude vector to a non-zero magnitude vector. In any reasonable mathematical system, the vector sum should just be the original non-zero vector and any direction defined for the zero magnitude vector would have no use.

---

003 qmult 00600 1 1 5 easy memory: dot product formula for components, dot product defined 1

174. The formula for the component form of dot product of general vectors $\vec{A}$ and $\vec{B}$ is:

a) $\vec{A} \cdot \vec{B} = A_x B_x A_y B_y A_z B_z$. b) $\vec{A} \cdot \vec{B} = \dfrac{A_x A_y A_z}{B_x B_y B_z}$. c) $\vec{A} \cdot \vec{B} = \dfrac{B_x B_y B_z}{A_x A_y A_z}$.
d) $\vec{A} \cdot \vec{B} = \dfrac{A_x B_y A_z}{B_x A_y B_z}$. e) $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$.

---

003 qmult 00610 1 4 2 easy deducto-memory: dot product defined 2

175. "Let's play *Jeopardy*! For $100, the answer is: It is the product of the magnitudes of two vectors multiplied by the cosine of the angle between them."

What is _____, Alex?

a) trot product b) dot product c) vector product d) cross product
e) angry product

---

003 qmult 00612 1 1 2 easy memory: primary dot product formula

176. The coordinate-system-independent or non-component formula for the dot product of general vectors $\vec{A}$ and $\vec{B}$ with angle θ between is:

a) $\vec{A} \cdot \vec{B} = AB \sin\theta \hat{n}$. b) $\vec{A} \cdot \vec{B} = AB \cos\theta$. c) $\vec{A} \cdot \vec{B} = \dfrac{A}{B} \cos\theta$.
d) $\vec{A} \cdot \vec{B} = \dfrac{A}{B} \sin\theta \hat{n}$. e) $\vec{A} \cdot \vec{B} = -AB \sin\theta \hat{n}$.

---

003 qmult 00620 1 1 3 easy memory: dot product commutes

177. For general vectors $\vec{A}$ and $\vec{B}$, the dot product $\vec{A} \cdot \vec{B}$ equals:

a) $\vec{B} \cdot \vec{B}$. b) $\vec{A} \cdot \vec{A}$. c) $\vec{B} \cdot \vec{A}$. d) $\vec{A} \cdot \vec{B}^{\,2}$. e) $\vec{A}^{\,2} \cdot \vec{B}$.

---

003 qmult 00630 1 3 1 easy math: standard dot products

178. For vectors $\vec{A}$ and $\vec{B}$ with the angle between them θ equal to $0°$, $90°$, $180°$, one has, respectively, the dot products:

a) AB, 0, $-AB$. b) $-AB$, 0, AB. c) AB, $-AB$, 0. d) 0, AB, $-AB$.
e) 0, $-AB$, AB.

---

003 qmult 00810 1 3 5 easy math: walk and elevator displacement
Extra keywords: physci KB-60-9

179. A man walks 40 m on level ground to an elevator and then rises 70 m. What approximately is the magnitude of his displacement from the starting point?

a) 8 m. b) 110 m. c) 30 m. d) 6500 m. e) 80 m.

---

003 qmult 00820 1 3 3 easy math: displacements in Vegas 1
Extra keywords: physci

180. You are in Las Vegas at the intersection of the Strip and Tropicana (where the MGM Grand, New York, New York, Excalibur, and Tropicana are). You go about **1 mile north** on the east side of the Strip to the Harley-Davidson Cafe, cross the Strip to the west side, and go about **half a mile south** to the Monte Carlo and there lose **most** of your of $100 stake at the roulette table.

a) Your total travel distance is about **1.5 miles**, total displacement about **1 mile north**, and you have **more** than $50 left.
b) Your total travel distance is about **1.5 miles**, total displacement about **0.5 miles north**, and you have **more** than $50 left.
c) Your total travel distance is about **1.5 miles**, total displacement about **0.5 miles north**, and you have **less** than $50 left.
d) Your total travel distance is about **1.5 miles**, total displacement about **1.5 miles north**, and you have **more** than $50 left.
e) Your total travel distance is about **0.5 miles**, total displacement about **1.5 miles north**, and you have havn't got **bus fare** left.

---

003 qmult 00822 1 3 2 easy math: displacements in Vegas 2
Extra keywords: physci

181. You are in Las Vegas where the streets are almost all laid out in a rectangular grid. Assume the grid is exactly aligned with the cardinal directions. You drive 2 miles north on the Strip, turn right on Flamingo at the Harley-Davidson Cafe, drive east 3 miles to Maryland, turn south on Maryland and drive 1 mile to Tropicana, and, finally, turn right on Tropicana and drive 3 miles. Where are you? How many miles have traveled? What is your total **DISPLACEMENT**? **HINT:** Draw a diagram.

a) On the Strip (at the MGM Grand for what it's worth), 9 miles, and **1 MILE**.
b) On the Strip (at the MGM Grand for what it's worth), 9 miles, and **1 MILE NORTH**.
c) On the Strip (at the MGM Grand for what it's worth), 1 mile, and **9 MILES NORTH**.

d) On the Strip (at the MGM Grand for what it's worth), 1 mile, and **9 MILES**.
e) On the Strip (at the Hard Rock Cafe), 9 miles, and **9 MILES**.

---

003 qmult 00824 1 1 1 easy memory: displacements in Vegas 3

182. You are in Vegas again. You start from 965 E. Cottage Grove and drive 0.50 mi east to S. Maryland Parkway and then drive 1.00 mi south to Tropicana. What is your displacement?

a) 1.1 mi, 63° south of east. b) 0.9 mi, 63° north of east.
c) 1.3 mi, 63° south of east. d) 1.1 mi, 27° north of east.
e) 1.3 mi, 27° south of east.

---

003 qmult 00830 1 3 2 easy math: ground speed of an airplane
Extra keywords: physci KB-57-23

183. An airplane flying horizontally has an air speed of 250 km/h and is flying in a horizontal wind of 70 km/h. The ground speed of the airplane must be somewhere in the **RANGE**:

a) 250–320 km/h. b) 180–320 km/h. c) 180–250 km/h. d) 240–260 km/h.
e) 70–250 km/h.

---

003 qmult 10780 2 3 1 easy memory: coordinate transformation fullmult

184.* You have a set of Cartesian axes with unit vectors $\hat{x}$ and $\hat{y}$. Another set of Cartesian axes is rotated with respect to the first set by θ. The unit vectors for the second set in terms of those for the first set are

$$\hat{x}' = \cos\theta\hat{x} + \sin\theta\hat{y} \qquad \text{and} \qquad \hat{y}' = \cos(\theta + 90^\circ)\hat{x} + \sin(\theta + 90^\circ)\hat{y} = -\sin\theta\hat{x} + \cos\theta\hat{y} \ .$$

Now consider a general vector $A = A_x\hat{x} + A_y\hat{y}$. It's components in the second set of coordinates are:

a) $A_{x'} = \vec{A}\cdot\hat{x}' = A_x\cos\theta + A_y\sin\theta$ and $A_{y'} = \vec{A}\cdot\hat{y}' = -A_x\sin\theta + A_y\cos\theta$.
b) A_x and A_y.
c) $A_{x'} = \vec{A}\cdot\hat{x}' = A_x\cos\theta\hat{x} + A_y\sin\theta\hat{y}$ and $A_{y'} = \vec{A}\cdot\hat{y}' = -A_x\sin\theta\hat{x} + A_y\cos\theta\hat{y}$.
d) $A_x\cos\theta$ and $A_y\sin\theta$.
e) $A_x\sin\theta$ and $A_y\cos\theta$.

---

003 qmult 10810 3 3 2 tough math: hurricane fullmult

185.* The eye of a hurricane passes over Bermuda moving 10.0° north of west at 50 km/h for 3 hours and then turns due north moving at 30 km/h. What is its displacement relative to Bermuda after 6 hours from being at Bermuda: give distance from Bermuda and angle relative to north? Neglect the curvature of the Earth.

a) 101 km at 48° west of north. b) 188 km at 52° west of north.
c) 157 km at 69° west of north. d) 139 km at 31° west of north.
e) 202 km at 81° west of north.

Full-Answer Problems

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003 qfull 00350 1 3 0 easy math: prove law of sines

186. The law of sines is

$$\frac{\sin\theta_a}{a} = \frac{\sin\theta_b}{b} = \frac{\sin\theta_c}{c} \ ,$$

where a, b and c are the sides of a general triangle and θ_a, θ_b, θ_c are the angles opposite those sides. Prove the law of sines. **HINT:** Use trigonometry and draw an illustrative diagram.

---

003 qfull 00510 1 3 0 easy math: adding two vectors

187. You are given two vectors in component form:

$$\vec{A} = (3.2, 4.2) \qquad \text{and} \qquad \vec{B} = (-10.5, 3.0) \ .$$

a) Give the vector sum $\vec{A} + \vec{B}$ and vector difference $\vec{A} - \vec{B}$ in component form.

b) What is the magnitude of $\vec{A} + \vec{B}$?

c) What is the angle of $\vec{A} + \vec{B}$ relative to the positive x-axis? The positive x-axis is the conventional reference direction on the Cartesian plane.

---

003 qfull 00640 2 3 0 moderate math: vector identity: sum and difference

188. If the sum of two vectors is perpendicular to their difference, prove that the vectors have equal magnitude. **HINT:** Use the dot product of sum and difference. A diagram might make the result look plausible.

---

003 qfull 00650 2 3 0 moderate math: prove law of cosines

189. Say one has a general triangle with sides a, b, and r with angle θ opposite side r. The law of cosines relates the sides and angle:

$$r^2 = a^2 + b^2 - 2ab\cos\theta \ .$$

The law of cosines can be viewed as a generalization of the Pythagorean theorem.

One can prove the law of cosines using elementary geometrical means, but one can also easily prove it using the dot product. The trick is the seeing where to start. And the trick is to see side r as a vector $\vec{r}$ that is the sum of vectors $\vec{a}$ and $\vec{b}$ made out of sides a and b. Take the dot product of $\vec{r}$ with itself and carry on with the proof. **HINT:** Draw a diagram.

---

003 qfull 00750 1 3 0 easy math: symmetry argument for a zero vector

190. You have a set of n vectors labeled by index i. Vector $\vec{A}_i$ is the representative ith vector. The vectors all extend from an origin in a **PLANE**, have **EQUAL** magnitude, and are **EVENLY** spaced in **ANGULAR** direction. The set is placed all around the origin: they are not confined to one quadrant. a test *mise en scène*. There are parts a,b,c,d.

a) Draw a schematic diagram of the system. **HINT:** The vectors should all radiate from the origin. Their tails are on the origin.

b) Say that n is an even number. Find the numerical value of vector sum of the vectors

$$\vec{A} = \sum_i \vec{A}_i$$

and prove why it is that value.

c) Say that n is any number greater than 1. Find the numerical value of vector sum of the vectors

$$\vec{A} = \sum_i \vec{A}_i$$

and prove why it is that value. **HINT:** Imagine rotating the set of vectors by $360°/n$. The sum vector should rotate too—but shouldn't it also be in the same place as before?

d) Now say that $n = 1$. The only vector in the set is $\vec{A}_1$. What is the vector sum in this case? Why is it not the same in general as the vector sum found in the part (c) answer?

---

003 qfull 00790 3 5 0 tough thinking: vector rotational transformations

Extra keywords: One needs a lot of hints for 20-minute test situation

191. Say you have a vector specified by $\vec{r} = (x, y)$ in relative to Cartesian x-y axes. Now create new x'-y' axes by rotating the old axes by θ counterclockwise. The vector $\vec{r}$ has **NOT** moved: it is still the same magnitude and physical direction. But relative to the new axes it is specified by (x', y').

a) Draw a diagram illustrating the situation. Show the primed axes rotated relative to the unprimed axes. There should be just **ONE** origin on the diagram. Let ϕ be the angle of $\vec{r}$ from the x axis: it's positive direction is counterclockwise. Let ϕ' be the angle of $\vec{r}$ from the x' axis: it's positive direction is counterclockwise.

b) Find the transformation relations $x' = f(x, y, \theta)$ and $y' = g(x, y, \theta)$? You have to specify functions f and g in terms of standard functions. **HINT:** There are more than one way to find the transformations, but trigonometry comes into all of them. One way is to write x' and y' in terms of r and ϕ', e.g.,

$$x' = r\cos\phi' \ ,$$

write ϕ' in terms of ϕ and θ, and then make use of the trig identities:

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b) \ ,$$
$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b) \ .$$

c) Show algebraically that the magnitude of the vector is the same in the rotated system as in the first system: i.e., the magnitude is rotation invariant. Of course, it's clear the magnitude is invariant: you used that condition in finding the transformation relations. This part is essentially a consistency check. **HINT:** Take the dot product of $\vec{r}' = (x', y')$ with itself and use the transformation relations to show that $r' = r$.

---

003 qfull 00810 1 3 0 easy math: hurricane

192. The eye of a hurricane passes over Bermuda moving 20.0° north of west at 40 km/h for 2 hours and then turns due north moving at 20 km/h. What is its displacement relative to Bermuda after 4 hours being at Bermuda: give distance from Bermuda and angle relative to north? Neglect the curvature of the Earth.

Chapt. 4 Two- and Three-Dimensional Kinematics

Multiple-Choice Problems

004 qmult 00100 1 4 4 easy deducto-memory: kinematic quantities in multi-dimensions

193. "Let's play *Jeopardy*! For $100, the answer is: The vectors

$$\vec{r}\,, \qquad \vec{v} = \frac{d\vec{r}}{dt}\,, \qquad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}\,.$$"

What are the most obvious quantities of _____, Alex?

a) time b) light c) dynamics d) multi-dimensional kinematics e) rest

004 qmult 00110 1 1 3 easy memory: unit vectors in Cartesian coordinates

194. In Cartesian coordinates, kinematics is very simple if $x(t)$, $y(t)$, and $z(t)$ are specified because the unit vectors of Cartesian coordinates are:

a) variables. b) dependent on time. c) constants. d) dependent on position. e) fewer than two.

004 qmult 00120 1 1 1 easy memory: unit vectors in polar/spherical coordinates

195. The unit vectors of polar and spherical polar coordinates are dependent on:

a) the angular coordinates of the displacement vector.
b) the magnitude of the displacement vector. c) nothing. d) time explicitly. e) mass.

004 qmult 00130 1 5 4 easy think: independent orthogonal motion

196. In Newtonian physics, the component motions of a particle in _____ directions are independent of each other. This means that motion variables in one direction are **NOT** intrinsic functions of motion variables in _____ directions. For example, a particular velocity in the x direction does **NOT** intrinsically set the velocity in the y direction. Now the initial or continuing conditions will set up relationships between motions in _____ directions. But those conditions can be anything allowed by physics and relationships between the motions can be anything allowed by the conditions and physics. For example, the conditions could set the x direction velocity to be a constant $v_x = 1\,\text{m/s}$ and the y direction velocity to a constant $v_y = 2\,\text{m/s}$. Then one has, of course, that $v_y = 2v_x$, but the physically allowed conditions could have set the two velocity components to any constant values or anything else allowed by physics. There is **NO** intrinsic relationship between those velocity components. The independence of motions in _____ directions is one of the amazing facts about the physical world and a vast simplification in dealing with it.

a) the same. b) opposite. c) both negative d) orthogonal e) dependent

004 qmult 00200 1 4 2 easy deducto-memory: projectile motion

197. "Let's play *Jeopardy*! For $100, the answer is: Without qualifications, one usually means the non-powered flight of an object in the air or through space. The simplest in-air case is the one in which air drag is neglected. The science of such motions is ballistics.

What is _____, Alex?

a) apparent motion b) projectile motion c) 1-dimensional motion
d) trigonometric motion e) unstoppable motion

---

004 qmult 00210 2 1 1 mod. deducto-mem.: gravity is always downward on Earth
Extra keywords: physci

198. A ball is tossed into the air and falls to the ground some distance away. Consider its motion in the vertical direction only and neglect air drag.

a) The ball has a constant acceleration downward.
b) The ball first accelerates **UPWARD** on its rising path and then accelerates **DOWNWARD** on its falling path.
c) The ball first accelerates **DOWNWARD** on its rising path and then accelerates **UPWARD** on its falling path.
d) The ball does not accelerate at all.
e) The ball is always accelerating in the upward direction.

---

004 qmult 00220 1 5 2 moderate thinking: projectile parabolic arc

199. Which best describes the path of a ball thrown on level ground at an angle 30° above the horizontal as seen from a side view.

a) Two straight lines that meet at an apex: one for the rising phase; one the declining phase. The rising phase line is **TWICE** the length of the declining phase line.
b) A smooth curve that rises and falls with distance. As far as the eye can tell, the curve could be parabolic.
c) Two straight lines that meet at an apex: one for the rising phase; one the declining phase. The rising phase line is **HALF** the length of the declining phase line.
d) A smooth curve that rises and falls with distance, but suddenly breaks off and descends vertically.
e) A smooth curve that rises and falls with distance and then rises and falls again with distance. A Bactrian camel curve.

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004 qmult 00230 1 1 4 easy memory: horizontal range formula

200. The horizontal range formula for projectile motion near the Earth's surface and neglecting air drag is:

a) $x_{\max} = \dfrac{8v_0^2}{g}\sin(2\theta)$. b) $x_{\max} = \dfrac{v_0^2}{g}\cos(2\theta)$. c) $x_{\max} = \dfrac{v_0^2}{g}$.
d) $x_{\max} = \dfrac{v_0^2}{g}\sin(2\theta)$. e) $x_{\max} = \dfrac{4v_0^2}{g}\sin(2\theta)$.

---

004 qmult 00240 1 1 2 easy memory: horizontal range maximum angle

201. The maximum horizontal range value for projectile motion near the Earth's surface for a given launch speed v_0 and neglecting air drag is obtained for launch angle:

a) 30°. b) 45°. c) 60°. d) 82.245°. e) 90°.

---

004 qmult 00250 1 1 2 easy memory: height formula for projectile motion

202. The maximum height formula for projectile motion near the Earth's surface, neglecting air drag, and measuring from the launch height is:

a) $y_{\max} = \dfrac{v_0^2}{g}\sin^2\theta$. b) $y_{\max} = \dfrac{v_0^2}{2g}\sin^2\theta$. c) $y_{\max} = \dfrac{v_0^2}{4g}\sin^2\theta$.
d) $y_{\max} = \dfrac{v_0^2}{g}\sin(2\theta)$. e) $y_{\max} = \dfrac{v_0^2}{2g}\sin(2\theta)$.

---

004 qmult 00260 1 1 1 easy memory: ratio of height to range formulae

203. The ratio of the maximum height formula (for projectile motion near the Earth's surface, neglecting air drag, and measuring from the launch height) to the horizontal range formula (for projectile motion near the Earth's surface and neglecting air drag) is _____ and its value for the maximum range is _____.

a) $\frac{y_{\max}}{x_{\text{range}}} = \frac{1}{4}\tan\theta;\ \frac{1}{4}$ b) $\frac{y_{\max}}{x_{\text{range}}} = \frac{1}{4}\tan\theta;\ 1$ c) $\frac{y_{\max}}{x_{\text{range}}} = \tan\theta;\ 1$
d) $\frac{y_{\max}}{x_{\text{range}}} = \tan\theta;\ \frac{1}{4}$ e) $\frac{y_{\max}}{x_{\text{range}}} = \frac{1}{4}\tan\theta;\ \infty$

---

004 qmult 00300 1 4 5 easy deducto-memory: relative motion explained

204. "Let's play *Jeopardy*! For $100, the answer is: Motion of something with respect to something else. To be a bit more explicit, say you have two objects. The relative displacement of object 2 from object 1 is defined to be

$$\vec{r} = \vec{r}_2 - \vec{r}_1 \ .$$

From this definition, the relative velocity and acceleration follow from differentiation:

$$\vec{v} = \vec{v}_2 - \vec{v}_1 \qquad \text{and} \qquad \vec{a} = \vec{a}_2 - \vec{a}_1 \ .$$

The description of the motion in these terms clarifies the initial answer statement."

What is _____, Alex?

a) variable motion b) relativistic motion c) no motion d) abosulte motion
e) relative motion

---

004 qmult 00350 2 5 5 moderate thinking: inertial frame motion is relative

205. You are in a featureless narrow room playing catch with a friend. How can you tell if the room is in a building or is a sealed compartment on super-smoothly running, non-accelerating-with-respect-ground-frame train (or plane)? **HINT:** Review your whole life experience; try an experiment (but not while you are driving).

a) When you throw the ball **ALONG** the long axis of the room, it would have different speeds (relative to the room) in the two possible directions if you were on a train.
b) When you thow the ball **PERPENDICULAR** to the long axis of the room it would curve off a straight line (relative to the room) if you were on a train.
c) On a train the thrown ball would zigzag wildly in flight.
d) On a train the thrown ball would do loops in flight.
e) There is no way to tell as long as the train motion is very smooth.

---

004 qmult 00360 2 3 1 moderate math: ground speed from Pyth. theorem
Extra keywords: physci

206. You are flying a plane. Air velocity (i.e., plane velocity relative to the air) is 40 mi/h due north. Wind velocity is 30 mi/h due west. What is the magnitude of ground velocity (i.e., the ground speed)?

a) 50 mi/h. b) −50 mi/h. c) 40 mi/h. d) 10 mi/h. e) 2500 mi/h.

---

004 qmult 00400 1 1 2 easy memory: dividing a circle

207. A circle can be divided into:

a) 360 divisions only. b) any number of divisions you like. c) 2π divisions only.
d) π divisions only. e) 360 or 2π divisions only.

---

004 qmult 00410 1 1 2 easy memory: radians in a circle 1

208. How many radians are there in a circle?

a) π. b) 2π. c) 3π. d) 360°. e) 360.

---

004 qmult 00412 1 4 2 easy deducto-memory: radians in a circle 2

Extra keywords: mathematical physics

209. "Let's play *Jeopardy*! For $100, the answer is: 2π."

What is _____, Alex?

a) the number of degrees in a circle b) the number of radians in a circle c) twice the number of radians in a circle d) twice the number of degrees in a circle e) cherry and blueberry

---

004 qmult 00420 1 5 1 easy thinking: 24 factors in 360

210. The division of the circle into 360° was an arbitrary choice—and we don't know why. We just know the ancient Mesopotamian mathematicians and astronomers did it this way—you know Mesopotamia—ancient Iraq: "the cradle of civilization". Their choice was just adopted by the ancient Greeks and got passed on to us. In the French Revolutionary epoch, the decimal system was adopted for most measures, but the revolutionaries didn't get around (you might say) to the circle. We can guess that one reasons is that the ancient Mesopotamians had a preference for whole number arithmetic particularly in division and 360 has a lot of whole number factors. How many whole number (i.e., integer) factors does 360 have counting 1 and 360 itself?

a) 24. b) 360. c) 6. d) 7. e) 12.

---

004 qmult 00430 1 3 4 easy math: radian to degree conversion

211. What is the approximate conversion factor from radians to degrees?

a) 1/60 degrees/radian. b) π degrees/radian. c) 2π degrees/radian.
d) 60 degrees/radian. e) 360 degrees/radian.

---

004 qmult 00440 1 5 5 easy thinking: the 2 pi unit ti

212. There are 2π radians in a circle. It's rather inconvenient that this means that there are $2\pi = 6.2831853\ldots$ radians in a circle which is an irrational number. For convenience, we could use the revolution (with sympbol Rev: vocalized rev) as a new unit: $1\,\text{Rev} = 2\pi$. One hundredth of an Rv would be a:

a) exaRev. b) megaRev. c) kiloRev. d) deciRev. e) centiRev.

---

004 qmult 00442 1 5 5 easy thinking: the 2 pi unit ti

213. There are 2π radians in a circle. It's rather inconvenient that this means that there are $2\pi = 6.2831853\ldots$ radians in a circle which is an irrational number. For convenience, we might invent a new base unit: the 2π with symbol "ti" (for **T**wo p**I** and pronounced tie). One hundredth of a ti would be a:

a) exati. b) megati. c) kiloti. d) deciti. e) centiti.

---

004 qmult 00450 1 3 1 easy math: hand angular measure

214. Approximately, at arm's length a finger subtends 1°, a fist 10°, and a spread hand 18°. These numbers, of course, vary a bit depending on person and exactly how the operation is done. What are these angles approximately in radians?

a) 1/60, 1/6, and 1/3 radians. b) 60, 600, and 1800 radians.
c) $\pi/12$, $\pi/3$, and $\pi/2$ radians. d) $\pi/12$, $\pi/3$, and π radians.
e) $\pi/12$, $\pi/3$, and 2π radians.

---

004 qmult 00460 1 5 5 easy thinking: covering the Moon

215. Can you cover the Moon with your finger held at arm's length? **HINT:** You could try for yourself if you are not in a a test *mise en scène*.

a) No. The Moon is much larger in angle than a finger. Just think how huge the Moon looks on the horizon sometimes.

b) It depends critically on the size of one's finger and arm. People with huge hands can to it and those without can't.
c) Yes. A finger at arm's length typically subtends about 10° and the Moon subtends 0.01°.
d) No. The Moon's diameter is about 3470 km and a finger is about a centimeter or so in width.
e) Usually yes. A finger at arm's length typically subtends about 1° and the Moon subtends 0.5°.

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004 qmult 00470 1 1 5 easy memory: small angle approximations

216. For small angles θ measured in radians and with increasing accuracy as θ goes to zero (where the formulae are, in fact, exact), one has the small angle approximations:

a) $\sin\theta \approx \cos\theta \approx 1 - \frac{1}{2}\theta^2$. b) $\cos\theta \approx \tan\theta \approx 1 - \frac{1}{2}\theta^2$. c) $\sin\theta \approx \cos\theta \approx \theta$.
d) $\cos\theta \approx \tan\theta \approx \theta$. e) $\sin\theta \approx \tan\theta \approx \theta$.

---

004 qmult 00500 1 1 5 easy memory: polar coordinates

217. In 2-dimensional Cartesian coordinates, a displacement vector $\vec{r}$ is given by

$$\vec{r} = (x, y) = x\hat{x} + y\hat{y} \ ,$$

where the unit vectors $\hat{x}$ and $\hat{y}$ are constants. In polar coordinates,

$$\vec{r} = (r, \theta) = r\hat{r} \ ,$$

where the unit vector

$$\hat{r} = \cos\theta\hat{x} + \sin\theta\hat{y} \ .$$

The polar coordinates are obtained from the Cartesian ones by the formulae

$$r = \sqrt{x^2 + y^2} \qquad \text{and} \qquad \theta = \tan^{-1}\left(\frac{y}{x}\right) \ .$$

In calculational work one must be aware that a negative argument of $\tan^{-1}$ is treated by calculators and computers as implying that $y > 0$ and $x < 0$. If the reverse is true, one must explicitly add or subtract 180° from the calculated result. The Cartesian components are obtained from the polar coordinates by the formulae

$$x = r\cos\theta \qquad \text{and} \qquad y = r\sin\theta \ .$$

As well as $\hat{r}$, one needs another unit vector for polar coordinates that is perpendicular to the $\hat{r}$ and that is used for velocity and acceleration vectors and changes in the displacement vector. This is the unit vector _____ given by

$$\text{_____} = \vec{r}(\theta + 90°) = -\sin\theta\hat{x} + \cos\theta\hat{y} \ ,$$

a) $\hat{\alpha}$ b) $\hat{\omega}$ c) $\hat{n}$ d) $\hat{z}$ e) $\hat{\theta}$

---

004 qmult 00530 1 4 1 easy deducto-memory: acceleration in polar coordinates

218. "Let's play *Jeopardy*! For $100, the answer is: The formulae

$$\begin{aligned}
\vec{r} &= r\hat{r} \ , \\
\vec{v} &= \frac{dr}{dt}\hat{r} + \frac{d\theta}{dt}\hat{\theta} \ , \\
\vec{a} &= \left[\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right]\hat{r} + \left(r\frac{d^2\theta}{dt^2} + 2\frac{dr}{dt}\frac{d\theta}{dt}\right)\hat{\theta} \ ,
\end{aligned}$$

where r is magnitude of displacement from the origin (or radial component of the displacement $\vec{r}$), $\hat{r}$ is the unit vector of the radial component, θ the angular component of $\vec{r}$, $\hat{\theta}$ is the unit vector of the angular coordinate, and one often writes $d\theta/dt$ is as ω which is called the angular velocity. The unit vectors are functions of the angular component:

$$\hat{r} = \cos\theta\hat{x} + \sin\theta\hat{y} \qquad \text{and} \qquad \hat{\theta} = -\sin\theta\hat{x} + \cos\theta\hat{y} \ .$$

What are displacement, velocity, acceleration in _____ coordinates, Alex?

a) polar b) Cartesian c) spherical polar d) elliptical e) hyperbolical

---

004 qmult 00550 1 4 3 easy deducto-memory: centripetal acceleration defined sort of
Extra keywords: physci

219. "Let's play *Jeopardy*! For $100, the answer is: It is the acceleration in a case of circular motion."

What is _____, Alex?

a) net acceleration b) centrifugal (center-fleeing) acceleration
c) centripetal (center-pointing) acceleration d) deceleration e) zero

---

004 qmult 00552 1 1 3 easy memory: centripetal acceleration formula 1

220. The radial component of acceleration in polar coordinates

$$a_r = \frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 = \frac{d^2r}{dt^2} - r\omega^2$$

specializes to _____ if the motion is circular and centered on the origin. In this case, the radial component of acceleration is called centripetal (meaning center pointing) since, in fact, it is always negative (i.e., the radial component of acceleration always points toward the origin). The radial component of velocity for circular motion is zero naturally and the angular component, often called the tangential velocity is given by

$$v_\theta = r\omega \ .$$

Usually, one drops the subscripts r and θ on a_r and v_θ if the quantities are identified by context.

a) $a_r = -r\omega = -\dfrac{v_\theta}{r}$ b) $a_r = r\omega^2 = \dfrac{v_\theta^2}{r}$ c) $a_r = -r\omega^2 = -\dfrac{v_\theta^2}{r}$ d) $a_r = \omega^2 = v_\theta^2$
e) $a_r = -\omega^2 = -v_\theta^2$

---

004 qmult 00554 1 5 3 easy thinking: centripetal acceleration formula 2
Extra keywords: physci

221. From dimensional analysis (i.e., by checking for correct dimensions or units) or otherwise, one identifies the formula for centripetal acceleration to be:

a) $\vec{a}_{\rm cen} = -v^2 r\hat{r}$. b) $\vec{a}_{\rm cen} = -v^2 r^2\hat{r}$. c) $\vec{a}_{\rm cen} = -\dfrac{v^2}{r}\hat{r}$. d) $\vec{a}_{\rm cen} = vr\hat{r}$.
e) $\vec{a}_{\rm cen} = vr^2\hat{r}$.

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004 qmult 00556 2 5 2 moderate thinking memory: cent. accel. behavior

222. The formula for centripetal acceleration magnitude for circular motion is

$$a_{\rm cen} = \frac{v^2}{r} \ ,$$

where v is the tangential velocity of the motion and r is the radius of the circle. The centripetal acceleration _____ with v and _____ with r.

a) increases linearly; decreases inverse linearly b) increases quadratically; decreases inverse linearly c) decreases inverse linearly; increases quadratically d) increases quadratically; decreases inverse quadratically e) constant; decreases inverse linearly

---

004 qmult 00560 1 1 5 easy memory: uniform circular motion

223. In uniform circular motion, position, velocity, and acceleration are continually changing. The velocity and acceleration magnitudes are:

a) continually changing too. b) continually changing and constant, respectively.
c) constant and continually changing, respectively.
d) constant and undefined, respectively. e) both constant.

---

004 qmult 01020 1 1 1 easy memory: non-uniform motion acceleration

224. In circular motion, an object is moving in a circle about some center. The acceleration of an object continuously in **NON-UNIFORM** circular motion:

a) does **NOT** point directly toward the center. b) points directly toward the center.
c) points directly outward from the center.
d) maybe points directly toward the center. e) is zero.

---

004 qmult 01030 1 1 4 easy memory: radius of curvature

225. Many, but not all, sections of a smooth curved paths in a plane can be approximated to good accuracy by section of circle. The radius of the circle is called the radius of curvature. A circle is fitted to a curved path over some region by adjusting its position and orientation. In many, but not all cases, a fitted circle can be made to agree to 1st order with a curved path about some point common to both. Agreement to 1st order means that both curved path and circle can be approxiated by the same quadratic function without any linear terms in the neighborhood of the common point. The function's argument is the coordinate of an axis which is tangent to both curves. Say one has a case where one does a fit and gets 1st order agreement. If the curved section is made straighter, the radius of curvature:

a) increases and then decreases to zero as the curve section becomes straight.
b) decreases to a non-zero value. c) decreases to zero.
d) increases and goes to infinity as the curve section becomes straight.
e) stays constant.

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004 qmult 01130 1 1 3 easy memory: Amazon crossing qualitatively

226. On a rotten log, you are paddling across the Amazon—with piranha and crocodiles swarming—with your bare hands. The river flows due east at your location. You aim due north and paddle in that direction—very quickly. Relative to your start point, you arrive on the far shore

a) due north of where you started. b) upstream. c) downstream.
d) west of where you started. e) south of where you started.

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004 qmult 10230 3 3 4 tough math: projectile motion in volleyball fullmult

227.*** On a jump serve, a volleyball player spikes the ball in the **HORIZONTAL** direction with initial velocity v_0 from a height y_0 above the ground and from horizontal location $x = 0$. The horizontal direction is the x direction and the vertical direction is the y direction. Neglecting air drag, what is the expression for v_0 in terms of g, y_0, and the location of the ball at any time after launch (i.e., the ball's x and y coordinates)?

This is a problem in which you want to find the conditions that lead to the desired result. It's really a pretty common kind of problem—in life as well as in physics.

a) $v_0 = y_0\sqrt{\dfrac{g}{2(x-y)}}$. b) $v_0 = (y_0 - y)\sqrt{\dfrac{g}{2x}}$. c) $v_0 = x\sqrt{\dfrac{g}{2(y-y_0)}}$.

d) $v_0 = x\sqrt{\dfrac{g}{2(y_0-y)}}$. e) $v_0 = x\sqrt{\dfrac{2(y_0-y)}{x}}$.

004 qmult 10320 3 5 3 tough thinking: Amazon crossing 1 fullmult

228.** Deep in the Amazonian jungle you wish to cross a river livid with piranha and crocodiles. Let x be the coordinate **ALONG** the river and y the coordinate **PERPENDICULAR** to the river. The river width is $y_{\max} = 40\,\text{m}$. You are going to paddle across (on an unstable rotting log—using your bare hands) and your paddling speed is $v_{\text{paddle}} = 0.50\,\text{m/s}$.

Now let's make the not-too-likely assumption that the river velocity in the x-direction is a linear function of y, the distance in the y-direction. Let

$$v_x(y) = v_{x\,\max}\frac{y}{y_{\max}} \ ,$$

where $y = 0$ at the starting shore and $v_{x\,\max} = 1.5\,\text{m/s}$. How far downstream (i.e., in the x direction) do you move?

a) 20 m. b) 40 m. c) 60 m. d) 80 m. e) 100 m.

Full-Answer Problems

004 qfull 00200 2 3 0 moderate math: quadratic parabolic I

229. The formula for a general quadratic is

$$f(x) = ax^2 + bx + c \ .$$

The shape of a quadratic is actually a simple parabola with the vertical symmetry axis offset from the origin.

a) Show this by completing the square in the function

$$f(x) = ax^2 + bx + c$$

and find the x coordinate of the symmetry axis. The completed square form is called the vertex form.

b) What is the formula for a transformed coordinate x' that explicitly turns the quadratic function into a simple parabola?

004 qfull 00202 2 3 0 moderate math: quadratic parabolic 2

Extra keywords: This question is too stupid to use.

230. The formula for a general quadratic is

$$f(x) = ax^2 + bx + c \ .$$

The shape of a quadratic is actually a simple parabola with the vertical symmetry axis offset from the origin. We will show this in this problem.

a) Differentiate $f(x)$ and find the stationary point x_{st} by setting $df/dx = 0$.

b) Differentiate df/dx and give the conditions for the stationary point being a maximum and being a minimum.

c) By a transformation of the x-coordinate from x to x' one can change the equation of the quadratic to that of a simple parabola: i.e.,

$$f(x') = a'x'^2 + c' \ .$$

Find the transformation equation from x to x' and find the expressions for a' and c' in terms of the a, b, and c.

d) Now having done the proof the hard way, show that the function describes a simple parabola the easy way by completing the square in the function

$$f(x) = ax^2 + bx + c \ .$$

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004 qfull 00210 3 5 0 tough thinking: horizontal range formula algebra

Extra keywords: Suitable for the algebra-based course.

231. A projectile is launched from x-y origin which is on the ground level of large level plain. The x direction is the horizontal and the y direction is the vertical: upward is the positive y direction. The projectile is launched in the positive x direction. The initial launch speed is v_0 at angle θ above the horizontal. Air drag is neglected.

a) Find the expressions for x and y position as functions of time t entirely in symbols and with any dependences on θ shown explicitly. Drop any symbols that stand for known zeros.

b) Now find y as a function of x by eliminating t. Now find the horizontal range formula: i.e., the expression for x when the projectile returns to $y = 0$. Simplify the formula as much as is reasonably possible. **HINT:** The trigonometric identity $\sin(2\theta) = 2\sin\theta\cos\theta$ helps simplifying formula.

c) For the horizontal range formula, find by any means the angle θ that gives the maximum range holding all the other variables constant. Briefly explain how you arrived at your answer. **HINT:** The sine function has only one maximum in the domain of its argument $[0°, 180°]$.

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004 qfull 00220 3 3 0 tough math: projectile motion in general

Extra keywords: only suitable for the calculus-based course. Sort of too much.

232. In answering this problem, you obtain all the really useful expressions for projectile motion in the absence of air drag. Otiose generality is eschewed. Let x and y be the horizontal and vertical coordinates. Assume the projectile is launched from $x = 0$ and $y = y_0$ at $t = 0$ at an angle to the horizontal of $\theta \in [-\pi/2, \pi/2]$. The only acceleration is $-g$: i.e., gravity in the negative y direction. The launch speed is v_0

a) What are the x and y expressions as a function of time.

b) Since x and t are linearly related we only need to find expressions for either one: the other being obtained always by a simple multiplication. Often in elementary projectile problems, one is not so interested in t. Use the expressions from the answer to part (a) to obtain y as function of x and not t. Find the x value that gives $y_{\max}$ and give the expression for $y_{\max}$.

c) Solve for x as a function of y. This is the general range formula. Why are there two solutions?

d) An important special case for the general range formula occurs when $y = y_0$. This formula is the horizontal range formula. Find the horizontal range formula. Find the θ that gives $x_{\max}$ and give the $x_{\max}$ expression.

e) Finally, what is the general range formula specialized to the case for x if $\theta = 0$.

---

004 qfull 00230 3 3 0 tough math: projectile motion in volleyball

233. The women's volleyball court has a net height of 2.24 m and extends 9.0 m on either side of the net. On a jump serve, a player spikes (I think that's the word) the ball at 3.00 m above the court in a direction perpendicular to the net. The initial velocity is **HORIZONTAL** and the net is 8.0 m away from the server. Neglect air drag.

This is a problem in which you want to find the conditions that lead to the desired result. It's really a pretty common kind of problem—in life as well as in physics.

a) Sketch a cross-section diagram of the system: launch, court, net. Then sketch a general trajectory that lands before the net and one that lands on the other side of then (i.e., that clears the net).

b) Now solve in **SYMBOLS** for initial velocity v_0 as a function of **ONLY** the variables x (horizontal position measured from the launch point) and y (height), and the constants y_0 (launch height) and g. The time variable t should be eliminated. For a general point (x, y) on a general trajectory (that had a horizontal launch velocity recall), the formula gives the initial velocity v_0 which will get the ball to that point.

Sketch a graph of v_0 as a function of y for constant x: indicate on sketch the v_0 for $y = 0$ in symbols and the location of the maximum of v_0. What is the meaning of this maximum? Then sketch v_0 as a function of x for constant y.

c) What is the minimum velocity v_0 needed for the ball to clear the net? Assume the ball is a point mass for this part.

d) What is the maximum velocity v_0 allowed if the ball is to stay in court (i.e., does not go farther than 17 m)? Assume no one touches the ball and assume the ball is a point mass for this part.

---

004 qfull 00232 3 3 0 tough math: Lefty O'Doul homer

234. Lefty O'Doul—bats left, throws left—hits a home run that lands in the stands of Baker Bowl. The ball left the bat with initial horizontal velocity v_{x0} and inital vertical velocity v_{y0}. Neglect air drag.

a) What are the x and y coordinates of the ball as **FUNCTIONS** of time t during the ball's flight? Assume the initial position is the **ORIGIN** and that the ball flies off in the positive x direction.

b) Solve for v_{x0} and v_{y0} as functions of **ONLY** x, y, t and g.

c) Solve for the launch speed v_0 and launch angle θ from the horizontal as functions of **ONLY** x, y, t, and g. Note **ONLY** x, y, t, and g.

d) The ball lands 125 m in horizontal distance from the launch pad (i.e., homeplate) and 25 m above the launch height. The flight time is 3.98 s. What are launch speed and angle?

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004 qfull 00240 3 3 0 tough math: orient express

235. You are on the last run of the historic Orient Express train in 1939 traveling from Paris to Istanbul. Somewhere between Belgrade and Sofia, the train ominously starts accelerating in the reverse direction—which we will call the negative x direction. The **MAGNITUDE** of this acceleration is $a_{\rm tr}$. The train is on a **STRAIGHT**, level line of track. Neglect **AIR DRAG**.

a) To make this problem clear, let the ground frame be the primed frame and the train frame be the unprimed frame. The coordinate transformation for any object (i.e, the general transformation) from the ground frame (the primed frame) to the train frame (the unprimed frame) in the x direction is

$$x = x' - x'_{\rm train}$$

and in the y direction is

$$y = y' \ .$$

For the x direction, differentiate to obtain the general transformation for v_x (train frame velocity) and a_x (train frame acceleration). Note, the given $a_{x,\rm train} = -a_{\rm tr}$.

b) Inside a train car, a projectile is launched in the positive x direction (i.e., in the forward direction). What is the horizontal acceleration in the train frame in terms of variables? What is the vertical acceleration in the train frame in terms of variables? Take the **UP** direction as the positive y direction. **HINT:** No elaborate calculations are needed. We are just looking for simple, short, symbol answers. Remember, there is no x acceleration for a projectile in the ground frame, but there is for the y acceleration in the ground frame.

c) You and your mysterious *compagnon de voyage* M. Achille find yourselves locked in your train car—and attempt some two-dimensional kinematics. What are the x and y positions relative to the car as functions of time t for the projectile launched from the origin (which is fixed to the car) at time zero with launch speed v_0 (relative to the car) and at an angle θ to the positive x direction: the angle is the range 0° to 90° relative to the train. Express these positions using the acceleration formulae found in part (b) and using time t, v_0 and θ. **HINT:** Remember that motion in orthogonal directions (i.e., perpendicular directions) is independent in the sense that only forces in those orthogonal directions affect the motion. Of course, the forces may have some dependencies among themselves, but that is at another level of description.

d) For reasons known to himself alone, M. Achille insists that you find the horizontal range formula for the projectile in the train frame: i.e., a formula giving the horizontal range (the x displacement from launch height to launch height) in terms of variables **NOT** including time t. Find that range formula and simplify it as much as reasonably possible.

e) What is the horizontal range formula for the train frame in the case that a_{tr} goes to zero?

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004 qfull 00310 2 3 0 moderate math: swimming English channel

236. You have decided to swim the English channel from Gris-Nez to Folkestone: a distance of about 40 km roughly to the north-west. In this problem, treat the distances and directions as exact: e.g., 40 km is the exact distance from Gris-Nez to Folkestone, north-west is exactly 45° west of north, and south-west is exactly 135° west of north. Also treat the Earth as flat. **HINT:** Smear your body all over with fats to prevent hypothermia.

a) If you swim at 4.0 km/h, how long will it take you?

b) As it turns out you land at New Romney about 20 km south-west of Folkestone. What is the average channel current velocity (note velocity, not just speed) during your swim? **HINT:** Velocity requires a direction specification too.

c) What was your average velocity (note velocity, not just speed) during the swim relative to the fixed Earth? As noted in the preamble, assume that Folkestone is exactly north-west of Gris-Nez and New Romney is exactly south-west of Folkestone. Give the velocity in magnitude-direction format. **HINT:** Velocity requires a direction specification too.

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004 qfull 00320 3 5 0 tough thinking: Amazon crossing 1

237. Deep in the Amazonian jungle you wish to cross a river livid with piranha and crocodiles. Let x be the coordinate **ALONG** the river and y the coordinate **PERPENDICULAR** to the river. The river width is $y_{\text{max}} = 20\,\text{m}$. You are going to paddle across (on an unstable rotting log—using your bare hands) and your paddling speed is $v_{\text{paddle}} = 0.50\,\text{m/s}$.

a) How long does it take you to cross assuming that you (wisely) aim straight in the y-direction?

b) Assuming the river flows at a steady velocity $v_x = 0.20\,\text{m/s}$ how far downstream (i.e., in the x direction) have you gone while crossing?

c) What is your velocity relative to the shore during the crossing? Give this velocity in magnitude-direction format. **HINT:** Remember to specify direction.

d) Now let's make the not-too-likely assumption that the river velocity in the x-direction is a linear function of y, the distance in the y-direction. Let

$$v_x(y) = v_{x\ \max}\frac{y}{y_{\max}} \ ,$$

where $y = 0$ at the starting shore and $v_{x\ \max} = 0.8\,\mathrm{m/s}$. Given that everything, except the river velocity, is the same as in the earlier parts of the question, how far downstream (i.e., in the x direction) do you move?

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004 qfull 00330 3 3 0 tough math: Amazon crossing 2

Extra keywords: Suitable only for the calculus-based course.

238. Deep in the Amazonian jungle you wish to cross a river livid with piranha and crocodiles. Let x be the coordinate along the river and y the coordinate perpendicular to the river. The y-origin is at the river center. The water velocity of the river (which is in the positive x-direction) is a parabolic function of the distance from the center of the river:

$$v_x(y) = v_{\mathrm{cen}}\left[1 - \left(\frac{y}{r}\right)^2\right] \ ,$$

where $r = 25\,\mathrm{m}$ is half the river width and $v_{\mathrm{cen}} = 2\,\mathrm{m/s}$ is the center water speed. You have motor-boat with a $5\,\mathrm{m/s}$ speed. Say you point the boat directly in the y-direction and cross the river.

a) How long does it take you to cross? **HINT:** This is easy.

b) How far downstream (i.e., in the positive x-direction) have you traveled? **HINT:** You are going to have to do an integration.

---

004 qfull 00500 2 3 0 moderate math: v and a in polar coordinates 1

239. Let's do a general treatment—like it or not—of circular motion in polar coordinates.

a) The radial and angular unit vectors of polar coordinates are, respectively,

$$\hat{r} = \cos\theta\hat{x} + \sin\theta\hat{y} \qquad \text{and} \qquad \hat{\theta} = -\sin\theta\hat{x} + \cos\theta\hat{y} \ ,$$

where the Cartesian unit vectors $\hat{x}$ and $\hat{y}$ are constants and θ is angular position. Note that $\hat{r}$ is the unit vector that points in the θ direction. Both polar coordinate unit vectors are position dependent, unlike the Cartesian unit vectors $\hat{x}$ and $\hat{y}$. Show $\hat{\theta}$ points $\pi/2$ counterclockwise from $\hat{r}$. **HINT:** Evaluate $\hat{r}$ for $\theta + \pi/2$ (i.e., evaluate the unit vector that points in the $\theta + \pi/2$ direction) and make use of standard trig identities

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b) \qquad \text{and} \qquad \sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b) \ .$$

b) Differentiate $\hat{r}$ and $\hat{\theta}$ with respect to time and write the results in terms of $\hat{r}$ and $\hat{\theta}$ and the angular velocity $\omega = d\theta/dt$. **HINT:** Use the chain rule.

c) The position vector for circular motion in polar coordinates is

$$\vec{r} = r\hat{r} \ ,$$

where r is a constant (since the motion is cirular). Derive the velocity and acceleration formulae making use of the results of the part (b) answer. Note that angular acceleration is given the symbol α: thus,

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} \ .$$

d) Write down the formulae for the tangential and radial components of velocity and acceleration. The radial component of the acceleration is called the centripetal acceleration which means center seeking acceleration since for circular motion it always points toward the center (i.e., is always negative). Write the centripetal acceleration in terms of ω and tangential velocity component.

e) The position vector for in polar coordinates in general (i.e., for varying direction and magnitude) is

$$\vec{r} = r\hat{r} \ ,$$

Derive the general velocity and acceleration formulae making use of the results of the part (b) answer and using the symbols for angular velocity (ω) and angular acceleration (α) for simplification.

---

004 qfull 00502 2 3 0 moderate math: v and a in polar coordinates 2

Extra keywords: calculus-based course. Kind of stupid?

240. It is not all that hard to derive the general vector expressions for velocity and acceleration in polar coordinates of some object. So let's do it. Remember that the unit vectors of polar coordinates are $\hat{r}$ and $\hat{\theta}$. These unit vectors are in fact functions of the polar coordinate θ. The $\hat{r}$ vector always points radially away from the origin is given by

$$\hat{r} = \cos\theta\hat{x} + \sin\theta\hat{y} \ .$$

The $\hat{\theta}$ vector is $\pi/2 = 90^\circ$ counterclockwise from $\vec{r}$ and is given by

$$\hat{\theta} = \cos(\theta + \pi/2)\hat{x} + \sin(\theta + \pi/2)\hat{y} = -\sin\theta\hat{x} + \cos\theta\hat{y} \ .$$

These expressions for $\hat{r}$ and $\hat{\theta}$ are not, in fact, needed for this problem although one could use them too if one liked.

a) The displacement vector $\vec{r}$ in polar coordinates is

$$\vec{r} = r\hat{r} \ ,$$

where r is the magnitude of $\vec{r}$ and $\hat{r}$ is unit vector pointing in the direction of $\vec{r}$. Differentiate $\vec{r}$ with respect to time t to find $\vec{v}$ using the product rule, but don't for the moment try to determine what $d\hat{r}/dt$ is.

b) There is a non-rigorous, but valid and convincing way of evaluating $d\hat{r}/dt$. Draw a diagram with $\hat{r}$ and $\hat{r} + \Delta\hat{r}$ with angle $\Delta\theta$ between them. The $\Delta\hat{r}$ and $\Delta\theta$ are the changes in, respectively, radial unit vector $\hat{r}$ and angular of position θ of the object in time Δt. With $\Delta\theta$ in radians show that

$$\Delta\hat{r} \approx \Delta\theta\hat{\theta} \ ,$$

where recall $\hat{\theta}$ is always $\pi/2 = 90^\circ$ clockwise from $\hat{r}$. Now show

$$\frac{d\hat{r}}{dt} = \omega\hat{\theta} \ ,$$

where $\omega = d\theta/dt$ is the angular velocity. The angular velocity is usually in units of radians per unit time.

c) Now write $\vec{v}$ substituting for $d\hat{r}/dt$.

d) Now write the expression for $\vec{a} = d^2\vec{r}/dt^2$, but without trying to evaluate $d\hat{\theta}/dt$.

e) There is a non-rigorous, but valid and convincing way of evaluating $d\hat{\theta}/dt$. Draw a diagram with $\hat{\theta}$ and $\hat{\theta} + \Delta\hat{\theta}$ with angle $\Delta\theta$ between them. The $\Delta\hat{\theta}$ and $\Delta\theta$ are the changes in,

respectively, angular unit vector $\hat{\theta}$ and angular of position θ of the object in time Δt. The angle $\Delta\theta$ (in radians) is the same angle as in the diagram of the part (b) answer since $\hat{\theta}$ is alway $\pi/2 = 90^\circ$ clockwise of $\hat{r}$. With $\Delta\theta$ in radians show that

$$\Delta\hat{\theta} \approx -\Delta\theta\hat{r} \ .$$

Now show

$$\frac{d\hat{\theta}}{dt} = -\omega\hat{r} \ ,$$

where $\omega = d\theta/dt$ is the angular velocity.

f) Now write $\vec{a}$ substituting for $d\hat{\theta}/dt$.

g) Specialize the expression for $\vec{v}$ and $\vec{a}$ to the case of uniform circular motion: i.e., the case where r and ω are constants.

---

004 qfull 00530 2 3 0 moderate math: uniform circular motion and TGV

241. Consider a TGV (Train à Grande Vitesse).

a) Say it has a speed of 216 km/h and is moving in a circle. What is the minimum radius of the circle if the centripetal acceleration is to be kept less than $0.05g$? Explain why it must be the minimum radius. **HINT:** Schematically plot centripetal acceleration versus r.

b) If the radius is 1.00 km, what is the maximum speed (in m/s and km/hr) before the acceleration exceeds $0.05g$? Explain why it is the maximum speed. **HINT:** Schematically plot centripetal acceleration versus v.

NOTE: Here we use $g = 9.8\,\mathrm{m/s^2}$ as a unit of acceleration. An accelerated frame is exactly like being gravitational field so far a mechanical experiments internal to the frame are concerned. This is actually a profound observation that started Einstein on his way to his general theory of relativity. The pseudo-gravitational force points opposite to the acceleration and has magnitude ma. This pseudo or inertial force is sometimes called the g-force and is specified as a force per unit mass in units of g. Thus circling with acceleration $0.05g$ is just like being in gravitational field of $0.05g$ where the gravitational force points radially outward.

Chapt. 5 Newton's Laws and Dynamics I

Multiple-Choice Problems

005 qmult 00100 1 4 2 easy deducto-memory: dynamics defined 1

242. "Let's play *Jeopardy*. For $100, the answer is: The branch of physics that explains motion and acceleration in terms of forces and masses."

What is _____, Alex?

a) kinematics b) dynamics c) statics d) economics e) cinematics

005 qmult 00110 1 4 5 easy deducto-memory: dynamics defined 2

243. Dynamics is that branch of physics that:

a) explains motion and acceleration in terms of the kinematic equations.
b) explains motion and acceleration in terms of error analysis.
c) treats dynamos.
d) treats electricity and magnetism or electromagnetism.
e) explains motion and acceleration in terms of forces and masses.

005 qmult 00120 1 1 2 easy memory: dynamics defined 3

244. The area of physics dealing with the motions and forces is called:

a) statics. b) dynamics. c) kinematics. d) kinesiology. e) cinema.

005 qmult 00130 1 5 1 easy thinking: statics defined

245. The area of physics dealing with **ONLY** cases of balanced forces (or equilibrium) is called:

a) statics. b) dynamics. c) kinematics. d) kinesiology. e) cinema.

005 qmult 00310 1 5 3 easy thinking: what forces do

246. Forces can cause accelerations relative to inertial frames or cancel other forces. Another manifestation (which actually follows from their property of causing acceleration) is that they can cause:

a) velocity (without causing acceleration).
b) mass.
c) bodies to distort: i.e., flex, compress, stretch, etc.
d) bodies to live
e) bodies to rule.

005 qmult 00400 1 1 4 easy memory: inertial frames defined

247. Accelerations with respect to _____—which we will call natural frames for the nonce—though no one calls them that—and only they require forces as causes as prescribed by Newton's 2nd law ($\vec{F}_{\rm net} = m\vec{a}$) in the classical limit. What are natural frames? They have been elusive historically. Newton hypothesized that a primary natural frame defined by the mean position of the fixed stars—absolute space as he called it. But the fixed stars move—as Newton knew himself—and revolve around the center of the Milky Way in complex orbits—as Newton did not know himself. The Milky Way and other galaxies are also in complex orbits in galaxy clusters or otherwise in complex relative motions. In modern cosmological theory, natural frames are

frames of reference attached to points in space that participate in the mean expansion of the universe. Space is growing—just accept it. Not all space—not space within bound systems like you, me, and the Milky Way—but the space in between bound systems like galaxy clusters. To every point participating in the mean expansion of the universe attach the origin of a local pimary natural frame. It is called local because sufficiently close to the origin, the frame has the behavior given above. As you move away from the origin, there is a progressive departure from the behavior, but you have to move over distance scales larger than a galaxy cluster for that to become very noticeable. Now any frame in uniform motion (i.e., unaccelerated) with respect to a local primary natural frame is also a local natural frame. Say the pimary frame is unprimed and the non-primary is primed. Then we have

$$\vec{r}\,' = \vec{r} - \vec{r}_{\text{prime}} \ ,$$

where is $\vec{r}_{\text{prime}}$ is the position of the primed frame in the unprimed frame. Differentiate twice and you get $\vec{a}\,' = \vec{a}$. So accelerations in the unprimed frame are exactly those of the primed frame. So Newton's 2nd law ($\vec{F}_{\text{net}} = m\vec{a}$) must be obeyed for accelerations relative to non-primary local frames. Forces themselves are frame-independent in classical mechanics. If you need relativistic physics the story changes.

Actually, Newton's 2nd law can be generalized to non-natural frames by introducing what are called inertial forces which are not real forces, but force-like terms that account for using non-natural frames. In fact, using inertial forces is usually the best approach to non-natural frames.

By the by, we can actually identify natural frames in the universe and our own local one very precisely using astronomical measurements. However, for many purposes we can find non-natural frames that are sufficiently close to being natural frames that they can be used as natural frames to some degree of approximation. The local Earth surface (i.e., the ground) is natural enough for many purposes: not long-range gunnery or large-scale weather phenomena. If you need a more natural natural frame, you can use the fixed stars. For highest accuracy, we can use the local primary natural frame using cosmological knowledge.

a) rotating frames. b) accelerated frames. c) non-inertial frames.
d) inertial frames. e) picture frames.

---

005 qmult 00410 2 4 3 moderate deducto-memory: inertial frames

248. Velocity and acceleration in, respectively, Newton's 1st and 2nd laws of motion are referenced to:

a) rotating frames. b) accelerated frames. c) inertial frames.
d) non-inertial frames. e) picture frames.

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005 qmult 00510 1 1 3 easy memory: number of Newton's laws

249. How many laws of motion did Newton posit?

a) 1. b) 2. c) 3. d) 4. e) 5.

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005 qmult 00512 1 1 3 easy memory: the number of Newton's laws 2
Extra keywords: physci

250. Newton once thought about having six laws of motion (or so the instructor seems to recall), but eventually settled on:

a) one. b) two. c) three. d) four. e) three and half.

---

005 qmult 00520 3 5 3 tough thinking: 1st law redundant

251. Newton's 1st law is.

a) **PHYSICALLY INDEPENDENT** of the other two laws of motion and **CANNOT** be dispensed with as an axiom of Newtonian physics.

b) **PHYSICALLY INDEPENDENT** of the other two laws of motion, but nonetheless it **CAN** be dispensed with as an axiom of Newtonian physics.
c) actually a **SPECIAL CASE** of the **2ND LAW**. The case when the net force is zero. Therefore logically we need only two laws of motion. Perhaps for clarity Newton formulated his explicit 1st law and perhaps for the same reason physicists have retained it.
d) actually a **SPECIAL CASE** of the **3RD LAW**. The case when the net force is zero. Therefore logically we need only two laws of motion. Perhaps for clarity Newton formulated his explicit 1st law and perhaps for the same reason physicists have retained it.
e) is **INCORRECT**, but is kept in the books for historical reasons.

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005 qmult 00530 1 1 3 easy memory: Newton's 2nd law: 1

252. Newton's 2nd law is:

a) $m = \vec{F}_{\rm net}\vec{a}$.
b) $\vec{a} = m\vec{F}_{\rm net}$.
c) $\vec{F}_{\rm net} = m\vec{a}$.
d) For every force there is an equal and opposite force.
e) For every acceleration there is an equal and opposite acceleration.

---

005 qmult 00532 1 1 1 easy memory: Newton's 2nd law: 2

253. Newton's 2nd law is:

a) $\vec{F} = m\vec{a}$. b) $m\vec{F} = \vec{a}$. c) $E = mc^2$. d) $E = mc^3$. e) $m = Ec^2$.

---

005 qmult 00534 1 5 1 easy memory: Newton's 2nd law class mantra 1

254. From here on in this course, a key thing to remember (to recite to yourself) when faced with any force problem is that Newton's 2nd law ($\vec{F}_{\rm net} = m\vec{a}$) is:

a) **ALWAYS VALID**. And it is a **VECTOR** equation, and so is always **VALID** component by component. And $\vec{F}_{\rm net}$ is the **VECTOR** sum of all forces acting on the body of mass m. It is not any particular force. If all the forces sum to zero vectorially, $\vec{F}_{\rm net} = m\vec{a} = 0$. If you are given the acceleration, then you can often use $\vec{F}_{\rm net} = m\vec{a}$ to solve for an unknown force.
b) **ALWAYS VALID**. And it is a **SCALAR** equation. And $\vec{F}_{\rm net}$ is the **SCALAR** sum of all forces acting on the body of mass m. It is not any particular force. If all the forces sum to zero, $\vec{F}_{\rm net} = m\vec{a} = 0$. If you are given the acceleration, then you can often use $\vec{F}_{\rm net} = m\vec{a}$ to solve for an unknown force.
c) **ONLY VALID** when there is a **NON-ZERO** net force. Because the 2nd law is a **VECTOR** equation, it is valid (when it is valid) component by component. And $\vec{F}_{\rm net}$ is the **VECTOR** sum of all forces acting on the body of mass m. It is not any particular force. If you are given the acceleration, then you can often use $\vec{F}_{\rm net} = m\vec{a}$ to solve for an unknown force.
d) **ALWAYS INVALID**.
d) **NEVER VALID**.

---

005 qmult 00536 1 4 5 easy deducto-memory: class mantra 2
Extra keywords: mathematical physics

255. "Let's play *Jeopardy*! For $100, the answer is: In this class, Newton's 2nd law is always true and it is always true component by component."

What is the _____, Alex?

a) class myth b) class modality c) class marriage d) class monster
e) class mantra

---

005 qmult 00542 1 5 1 easy thinking: acceleration and third law
Extra keywords: also physci KB-59-15

256. If Newton's 3rd law is true, why then does anything accelerate at all?

a) The equal and opposite forces **DO NOT** have to be on the same body.
b) The equal and opposite forces **DO** have to be on the same body.
c) Nothing moves at all as Parmenides argued in the 5th century BC. Motion is but seeming. Anyway Parmenides seems to have been a pretty smart guy since he's credited with the spherical Earth theory and the discovery that the Moon shines by reflected light.
d) Acceleration has nothing do with forces.
e) Forces have nothing do with acceleration.

---

005 qmult 00540 1 4 5 easy deducto-memory: force laws needed

257. "Let's play *Jeopardy*! For $100, the answer is: Laws that prescribe forces for physical systems. They must exist independent of Newton's 3 laws of motion in order for Newtonian physics to be useful."

What are _____, Alex?

a) Newton's 3 laws b) accelerations c) velocities d) force inequalities
e) force laws

---

005 qmult 00550 1 1 4 easy memory: internal force pairwise cancellation

258. Why do internal forces not affect the center of mass acceleration of a system? Because:

a) they cancel out in threesomes. b) they are all zero. c) we just ignore them.
d) they cancel out pairwise. e) the external force cancels them out.

---

005 qmult 00560 1 4 5 easy deducto-memory: center of mass in Fnet=ma

259. "Let's play *Jeopardy*! For $100, the answer is: The mass-weighted average position of a system of particles: a system being any specified set of material mass elements. This position's acceleration obeys $\vec{F}_{\text{net}} = m\vec{a}$, where $\vec{F}_{\text{net}}$ is the net force (and also the net external force) acting on the system."

What is the _____, Alex?

a) center b) bottom end c) top end d) left end e) center of mass

---

005 qmult 00562 1 1 5 easy memory: net external forces and center of mass
Extra keywords: anticipates later chapters, but that's OK

260. If you know nothing about the internal forces of a body and only know the net external force that acts on the body and not where on the body the particular external forces act, then, by itself, Newton's 2nd law for a non-point mass only allows you to predict:

a) the **VELOCITY** of the body.
b) the **VELOCITY** of the center of mass of the body. You can know nothing about internal motions of the body or its rotational behavior.
c) the **ACCELERATION** of the top point of the body.
d) the **ACCELERATION** of the bottom point of the body.
e) the **ACCELERATION** of the center of mass of the body. You can know nothing about internal motions of the body or its rotational behavior.

---

005 qmult 00570 1 1 5 easy memory: newton defined

261. The base SI unit of force is the:

a) farad (F); $1\,\text{F} = 1\,\text{kg}\,\text{m/s}^2 \approx 0.22481\,\text{lb} \approx 1/5\,\text{lb}$.
b) henry (H); $1\,\text{H} = 1\,\text{kg}\,\text{m/s}^2 \approx 0.22481\,\text{lb} \approx 1/5\,\text{lb}$.
c) watt (W); $1\,\text{W} = 1\,\text{kg}\,\text{m/s}^2 \approx 0.22481\,\text{lb} \approx 1/5\,\text{lb}$.
d) joule (J); $1\,\text{J} = 1\,\text{kg}\,\text{m/s}^2 \approx 0.22481\,\text{lb} \approx 1/5\,\text{lb}$.
e) newton (N); $1\,\text{N} = 1\,\text{kg}\,\text{m/s}^2 \approx 0.22481\,\text{lb} \approx 1/5\,\text{lb}$.

---

005 qmult 00600 1 1 4 easy memory: center of mass definition

262. The center of mass is the:

a) position-weighted mean mass of an object.
b) object-weighted mean mass of the position.
c) mean of mass an weighted object position of.
d) mass-weighted mean position of an object.
e) simple center of the object.

---

005 qmult 00610 1 4 2 easy deducto-memory: center of mass, reference frame

263. The center of mass (i.e., the actual physical position of the center of mass in space relative to the physical system it is the center of mass of) is:

a) a function of the coordinate system.
b) independent of the coordinate system.
c) dependent on the coordinate system.
d) both independent of and a function of the coordinate system.
e) neither independent of nor a function of the coordinate system.

---

005 qmult 00620 1 4 5 easy deducto-memory: center of mass, 3-d symmetry 1

264. If an object is symmetric in 3 dimensions about some point (i.e., its geometric center), its center of mass must be:

a) outside of the object.
b) neither inside nor outside the object.
c) at the point about which the object is symmetric in 2 of the dimensions, but not in the 3rd.
d) at the point about which the object is symmetric in 1 of the dimensions, but not in the other 2.
e) at the geometric center.

---

005 qmult 00630 2 5 1 moderate thinking: center of mass, 3-d symmetry 2

Extra keywords: Not a brilliant problem

265. The center of mass of an object is easily found if the object is:

a) symmetric in **THREE** mutually perpendicular directions like, for example, a rectangular block.
b) symmetric in **TWO** mutually perpendicular directions, but not in the third direction perpendicular to the first two.
c) symmetric in **ONE** direction, but not in two mutually perpendicular directions that are perpendicular to the first one.
d) completely irregular.
e) made out of whipped cream.

---

005 qmult 00640 1 5 3 easy thinking: center of mass, symmetric sphere

266. The center of mass of sphere of radius R with a density given by $\rho = Cr^2$ (where r is the radial coordinate) is at:

a) $r = R/2$. b) $r = R/3$. c) $r = 0$. d) $r = 3R$. e) $r = 2R$.

---

005 qmult 00650 1 1 5 easy memory: odd function over even region

Extra keywords: Only for the calculus-based course

267. The integral of an odd function over a symmetric region about the origin is always:

a) an odd function. b) a non-zero even function. c) non-zero. d) negative. e) zero.

---

005 qmult 00652 1 4 2 easy deducto-memory: odd function over even region 2

Extra keywords: a jeopardy problem

268. "Let's play *Jeopardy*! For $100, the answer is: Zero."

a) What is less than, Alex?
b) What is the integral of an odd function over a symmetric region about the origin, Alex?
c) What is the integral of an even function over a symmetric region about the origin, Alex?
d) What is one, Alex?
e) What is not a cognate word to cipher, Alex?

---

005 qmult 00654 1 5 5 easy thinking: hoop center of mass

269. Where is the center of mass of a hoop?

a) At the end of the hoop.
b) At the top of the hoop.
c) At the left side of hoop.
d) Nowhere since a center of mass must be physically inside an object to be a center of mass.
e) On the axis of the hoop at the geometrical center of the hoop.

---

005 qmult 00660 1 4 5 easy deducto-memory: hanging center of mass

270. "Let's play *Jeopardy*! For $100, the answer is: If one hangs a rigid object from a freely turning pivot point and lets it come to stable static equilibrium, the center of mass is directly below the pivot point. Thus, center of mass can be found from the intersection of two lines through the object that start at two points used as pivot points and that go in the direction through the object that was downward when each of the points was the pivot point. The method fails if the two pivot points and the center of mass happen to be collinear."

What is a/an _____, Alex?

a) **EMPIRICAL** method for finding gravitational torque
b) **THEORETICAL** method for finding gravitational torque
c) gravitational torque
d) center of mass
e) an **EMPIRICAL** method for finding the center of mass of a rigid object

---

005 qmult 00670 1 5 5 easy thinking: hanger center of mass

271. Where, roughly speaking, is the center of mass of a coat hanger? **HINT:** Imagine letting it hang from two different free pivot points: this is called a Gedanken (thought) experiment in physics speak. If you arn't in a test *mise en scène*, you could actually do the experiment.

a) At the end of the hook.
b) At the top of the hook.
c) At the left end of the triangular loop.
d) Nowhere since a center of mass must be inside the material of an object to be a center of mass.
e) Oh, somewhere not so far from the middle region of the triangular loop.

---

005 qmult 00710 2 3 5 moderate math: stopping a bike

Extra keywords: physci KB-60-21

272. A bicycle-rider system has a mass of 80 kg. The bike is traveling on level and has initial velocity 6 m/s north. What is the constant force needed to stop the bike in 4 s?

a) 80 N south. b) 80 N north. c) 80 N east. d) 100 N south. e) 120 N south.

---

005 qmult 00720 1 3 5 easy math: F=ma to find a brick's mass to find a force

Extra keywords: physci KB-60-23

273. A 50 N net force gives a brick an acceleration of 5 m/s. What net force is need to give it an acceleration of 10 m/s?

a) 50 N. b) 5 N. c) 10 N. d) 200 N. e) 100 N.

---

005 qmult 00900 1 1 1 easy memory: force law for gravity

274. The magnitude of the gravitational force on an object of mass m for a uniform gravitational field (such as the gravitational field near the Earth's surface for human-size and somewhat larger objects) is given by the formula:

a) $F = mg$. b) $F = m/g$. c) $F = g/m$. d) $F = ma$. e) $F = m/a$.

---

005 qmult 00910 1 1 4 easy memory: solving mg=ma equation of motion

275. Newton's 2nd law applied to the vertical direction with only the gravity force acting and down defined as positive leads to the scalar equation of motion:

a) $g = m/a$. b) $g = ma$. c) $mg = a$. d) $mg = ma$. e) $m/g = m/a$.

---

005 qmult 00920 1 3 3 easy math: person's weight

Extra keywords: physci

276. If you have a mass of 60 kg and $g = 9.8\,\mathrm{m/s^2}$, you weigh about:

a) 10 N. b) 60 N. c) 600 N. d) 500 N. e) 20 N.

---

005 qmult 00930 2 5 2 easy math: gravity field force and contact forces

277. The force of gravity reaches out across space and pulls on each bit of your body independently of every other bit. We call a force like this a **FIELD FORCE** or a **BODY FORCE**. Why don't you accelerate downward, except when off the ground.

a) The **GROUND FORCE** reaches out across space and pushes upward on each bit of your body independently of every other bit. The ground force is also a **FIELD FORCE**.
b) The ground exerts a force on the soles of your feet and the soles of your feet on the next layer of your body and the next layer of your body on the next layer of your body and so on until the top of your head. Each layer pushes up with only enough force to balance the gravity force on the mass above. The ground force and the forces exerted by the layers of our bodies are **CONTACT FORCES**. A **CONTACT FORCE** acts over a very short range: so short that if the distance between the two objects exerting equal and opposite contact forces on each other is more than microscopic there is no contact force at all.
c) Since you are always off the ground, the question has no answer.
d) Since you are always off the ground, the question is hypothetical and the answer, speculative.
e) In orbit, you don't accelerate downward and you are certainly off the ground. So being on the ground may have nothing to do with why you don't accelerate downward.

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005 qmult 00940 1 4 4 easy deducto-memory: mass and weight

Extra keywords: physci KB-13

278. "Let's play *Jeopardy*! For $100, the answer is: they are, respectively, the resistance of a body to acceleration and the magnitude of the force of gravity on a body."

What are _____ and _____, Alex?

a) acceleration; normal force b) mass; normal force c) force; weight
d) mass; weight e) gravity; momentum

---

005 qmult 00950 2 5 4 moderate thinking: diving woman and gravity

Extra keywords: physci KB-60-27

279. What is the approximate mass of a woman who weighs 500 N? What is gravitational force that Earth exerts on her. After she jumps **UPWARD** from a diving board, what is her acceleration in the absence of air drag?

a) About 50 kg, 500 N, and $9.8\,\mathrm{m/s^2}$ downward once she starts moving downward, but **ZERO** before that.

b) About 50 kg, 50 N, and 9.8 m/s^2 downward once she starts moving downward, but **ZERO** before that.
c) About 50 kg, 50 N, and 9.8 m/s^2 downward at **ALL** times.
d) About 50 kg, 500 N, and 9.8 m/s^2 downward at **ALL** times.
e) None of these questions can be answered with the given information.

---

005 qmult 01000 2 1 1 moderate memory: normal force
280. The normal force is:

a) a repulsive contact force exerted by a surface that points perpendicularly outward from that surface. The force turns out to resist compression. In principle, the force can be calculated from the compressional displacement of the surface from equilibrium, but in elementary problems one usually calculates it from Newton's 2nd or 3rd law assuming the surface to be completely rigid.
b) $\vec{F}_{\rm net}$ in $\vec{F}_{\rm net} = m\vec{a}$.
c) the tension force in a rope.
d) the tension force in a rope that allows you to push on a rope.
e) an ordinary, run-of-the-mill force, a pedestrian force, a force without pretensions or airs, a downright force, a regular-guy force, just a plain salt-of-the-earth force.

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005 qmult 01040 3 5 2 tough thinking: dynamics and kinematics on slope 1
281. An object of mass m is on a rigid, frictionless slope of angle θ from the horizontal. What is the magnitude of normal force on the object? What is the component of the gravitational force in the positive x direction which is down the slope? What is the expression for the position x of the object as a function of time t when it starts from rest at $t = 0$ and $x = 0$?

a) $mg\cos\theta$; $mg\cos\theta$; $x = (1/2)(g\cos\theta)t^2$.
b) $mg\cos\theta$; $mg\sin\theta$; $x = (1/2)(g\sin\theta)t^2$.
c) $mg\sin\theta$; $mg\sin\theta$; $x = (1/2)(g\sin\theta)t^2$.
d) $mg\sin\theta$; $mg\sin\theta$; $x = (1/2)(g\cos\theta)t^2$.
e) $mg\sin\theta$; $mg\sin\theta$; $x = (g\cos\theta)t$.

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005 qmult 01042 2 3 1 mod. math: dynamics and kinematics on a slope 2
282. A rigid frictionless slope has angle 30°. An object slides from rest for 10 s. About how far does it travel down the slope?

a) 245 m. b) 424.35 m. c) 24.5 m. d) 42.44 m. e) 490 m.

---

005 qmult 01070 1 1 1 mod. thinking: reaction forces of a book
Extra keywords: physci
283. A book sits at rest on a table. The reaction force that follows from Newton's 3rd law to the gravitational force of the Earth on the book is the:

a) gravitational force of the book on the Earth.
b) normal (i.e., perpendicular upward) force of the table on the book.
c) table friction force on the book.
d) book friction force on the table.
e) book normal force on the table.

---

005 qmult 01080 2 5 1 moderate thinking: elevator acceleration of woman
Extra keywords: physci KB-61-31
284. A woman who has a mass of 50 kg is in an elevator that is accelerating downward at 2 m/s^2. What is the force the floor exerts on her? What is the force she exerts on the floor?

a) 390 N upward; 390 N downward.
b) 390 N downward; 390 N upward.
c) 490 N downward; 490 N upward.
d) 490 N upward; 490 N downward.
e) 100 N upward; 100 N downward.

---

005 qmult 01100 1 1 1 easy memory: tension defined
285. Tension is the magnitude of the force in an object that resists:

a) extension. b) compression.
c) shearing (i.e., the deformation of the object without change in volume).
d) creaking (i.e., the deformation of the object with noise). e) concession.

---

005 qmult 01110 1 4 4 easy deducto-memory: ideal rope

286. "Let's play *Jeopardy*! For $100, the answer is: It has zero thickness and only resists extension along its length. In fact, resists extension completely. Usually, but not always, it is assumed to have zero mass and be unbreakable"

What is a/an _____, Alex?

a) ideal monkey b) ideal rigid rod c) ideal surface d) ideal rope
e) real rope

---

005 qmult 01120 1 1 2 easy memory: ideal rope results 1

287. The normal force magnitude per unit length exerted by the curved surface on an _____ at any general point s is

$$f_{\text{nor}} = \frac{T}{r} \ ,$$

where s is measured from the start of the _____, T is the tension at point s r is the radius of curvature at s, and the normal force per unit length points radially outward from the center of curvature. The center of curvature is the center of a circle that approximates the curve at s to first order. The normal force per unit length exerted by the rope on the curved surface is equal in magnitude to f_{nor}, but points radially inward by the 3rd law.

a) ideal rigid rod b) ideal rope c) unreal rigid rod d) uaenrl riigd rod
e) ideal door

---

005 qmult 01122 1 1 2 easy memory: ideal rope results 2

288. The normal force magnitude per unit length exerted by the curved surface on an _____ at any general point s is

$$f_{\text{nor}} = \frac{T}{r} \ ,$$

where s is measured from the start of the _____, T is the tension at point s, r is the radius of curvature at s, and the normal force per unit length points radially outward from the center of curvature. The center of curvature is the center of a circle that approximates the curve at s to first order. The normal force per unit length exerted by the rope on the curved surface is equal in magnitude to f_{nor}, but points radially inward by the 3rd law.

The tension at any point s along the _____ is given by

$$T = T_0 - F_{\text{par}}(s) \ ,$$

where T_0 is the tension at the start of the _____ at $s = 0$ and F_{par} is the integrated applied parallel force per unit length along the _____ from the start to s.

a) ideal rigid rod b) ideal rope c) unreal rigid rod d) uaenrl riigd rod
e) ideal door

---

005 qmult 01130 1 1 2 easy memory: ideal rope with constant tension

289. A taut ideal, massless rope should have _____ tension (i.e., constant magnitude of tension force) between two endpoints provided no external forces parallel to the rope act on it **BETWEEN** the endpoints: there will in general be external applied forces to hold it taut at the endpoints. The rope does not have to be straight. It can be wrapped around constraints as long as their surfaces exert no parallel forces on it.

a) wildly varying b) constant c) complexly varying d) zero e) 9.8 N/kg

---

005 qmult 01150 1 3 4 easy math: example tension fore of a rope

290. A **MOTIONLESS** mass of 10 kg is suspended from a rope. What is the tension force that the rope exerts on the mass?

a) 100 N downward. b) 200 N downward. c) 200 N upward. d) 100 N upward.
e) 200 N horizontally.

---

005 qmult 01170 1 1 2 easy memory: no motion ordinary room implies net force zero

291. A **MOTIONLESS** mass of 10 kg is suspended from a rope in an ordinary room in an ordinary building. What is the **NET** force on the mass? It is:

a) about 100 N downward. b) 0 N. c) about 200 N upward.
d) about 100 N upward. e) about 200 N horizontally.

---

005 qmult 01200 1 4 5 easy deducto-memory: friction defined

292. "Let's play *Jeopardy*! For $100, the answer is: The macroscopic binding force between smooth surfaces that is parallel to the surfaces."

What is _____, Alex?

a) the linear restoring force b) the spring force c) the tension force
d) the normal force e) friction

---

005 qmult 01220 1 2 2 moderate memory: kinetic friction force

293. The magnitude of the kinetic friction force between a body and a surface equals a coefficient of friction times:

a) the area of macroscopic contact between the body and the surface.
b) the magnitude of the normal force acting on the body.
c) the mass of the body.
d) the density of the body.
e) the density of the air surrounding the body.

---

005 qmult 01230 1 2 5 moderate memory: friction coefficient sizes

294. Which is larger: the coefficient of static or kinetic friction?

a) They are always equal.
b) Neither. The larger depends on the materials involved and its about a 50-50 split on which is larger.
c) The kinetic coefficient is always larger.
d) The kinetic coefficient is usually larger.
e) The static coefficient is almost always (always?) larger.

---

005 qmult 01240 1 1 3 easy memory: friction to heat
Extra keywords: physci related to KB-94-9 but not really that

295. The friction between sliding surfaces tends to change macroscopic kinetic energy into:

a) potential energy. b) rest mass energy. c) thermal or heat energy.
d) magnetic energy. e) nothing.

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005 qmult 01280 1 5 4 easy thinking: balanced forces on a coffee cup

296. You are pushing a coffee cup across a table (just an ordinary table, not an imaginary frictionless table) at a **CONSTANT** velocity. The magnitudes of the push force and frictional force are $|F_{\rm p}|$ and $|F_{\rm f}|$, respectively.

a) $|F_{\rm f}| > |F_{\rm p}|$ and this is why the cup does not accelerate.
b) The $|F_{\rm f}| < |F_{\rm p}|$, but nevertheless the frictional force prevents any acceleration.
c) There is no frictional force when you push the cup at a constant velocity. Thus, $|F_{\rm f}| = 0$ and clearly then $|F_{\rm p}| > |F_{\rm f}|$.

d) The $|F_{\rm f}|$ must **EQUAL** $|F_{\rm p}|$ in order for there to be no acceleration.
e) The $|F_{\rm f}|$ must be **TWICE** $|F_{\rm p}|$ in order for there to be no acceleration.

---

005 qmult 01410 1 1 1 easy memory: elevator acceleration, non-inertial frame, jokiness

297. An elevator just starts moving upward.

a) You feel slightly heavy for a moment.
b) You feel slightly light for a moment.
c) You feel slightly light and carefree for a moment.
d) You feel totally carefree and ethereal.
e) You come to understand that there are no forces in foxholes.

---

005 qmult 10610 1 3 1 easy math: center-of-mass integration of a rod fullmult

298.*** You have a 1-dimensional object. It is a non-uniform thin rod. It's linear density (i.e., mass per unit length) is given by

$$\lambda = Cx^{-2} \ ,$$

where C is a constant and the x coordinate of the rod extends from $x = a$ to $x = b$: both a and b are **GREATER THAN ZERO!!!**. The center of mass of the rod relative to the **ORIGIN** (i.e., $x = 0$) is:

a) $a + \dfrac{ab\ln(b/a)}{b-a}$. b) $a + ab\ln(b/a)$. c) $\dfrac{ab\ln(b/a)}{b-a}$. d) $ab\ln(b/a)$. e) $\dfrac{a+b}{2}$.

---

005 qmult 11010 2 3 3 mod. math: frictionless incline fullmult

299.** There is a 3.00 kg block on a frictionless incline that is at $\theta = 40.0°$ from the horizontal.

i) What is the normal force on the block? **HINTS:** Draw a free body diagram and remember the class mantra: "$\vec{F}_{\rm net} = m\vec{a}$ is always true and it's true component by component".

ii) What is the net force down the slope?

iii) What is the acceleration down the slope?

iv) Starting from rest how far does the block slide in 5 s?

a) (i) 17.0 N (ii) 9.8 N (iii) 4.9 m/s^2 (iv) 245 m.
b) (i) 18.9 N (ii) 22.5 N (iii) 6.30 m/s^2 (iv) 78.7 m.
c) (i) 22.5 N (ii) 18.9 N (iii) 6.30 m/s^2 (iv) 78.7 m.
d) (i) 9.8 N (ii) 17.0 N (iii) 4.9 m/s^2 (iv) 245 m.
e) (i) 17.0 N (ii) 9.8 N (iii) 9.8 m/s^2 (iv) 490 m.

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005 qmult 11110 2 3 2 moderate math: fuzzy dice at angle fullmult

300.** You are in a car accelerating at a constant 3.0 m/s^2 with constant direction. The car is on level ground. There are fuzzy dice hanging by a cord from the mirror. The dice cord is a (massless) ideal rope. Assuming the dice are a point mass, what is the angle of the dice cord from the **VERTICAL**? **HINT:** Draw a free body diagram for the dice. Remember the class mantra: "$\vec{F}_{\rm net} = m\vec{a}$ is always true and it's true component by component".

a) 3.0°. b) 17°. c) 22°. d) 45°. e) 90°.

---

005 qmult 11112 2 3 3 moderate math: fuzzy dice at angle 2 fullmult

301.** You are in a car accelerating at a constant 7.5 m/s^2 with constant direction. The car is on level ground. There are fuzzy dice hanging by a cord from the mirror. The dice cord is a (massless) ideal rope. Assuming the dice are a point mass, what is the angle of the dice cord from the **VERTICAL**? **HINT:** Draw a free body diagram for the dice. Remember the class mantra: "$\vec{F}_{\rm net} = m\vec{a}$ is always true and it's true component by component".

a) 27°. b) 32°. c) 37°. d) 42°. e) 47°.

---

005 qmult 11210 2 3 5 moderate math: pig on slide: with and sans friction fullmult
Extra keywords: HRW-113-14p

302.*** A free-spirited pig (let's call him Waldo Pepper) loves sliding down chutes (pronounced shoots).

i) Say there is a frictionless chute and Waldo accelerates down at $3.00\,\mathrm{m/s^2}$. What is the inclination angle of the chute? **HINT:** Remember that free body diagram.

ii) Now the evil magician of physics turns on kinetic friction and Waldo only accelerates down at $2.00\,\mathrm{m/s^2}$? What is the coefficient of kinetic friction? **HINT:** Update that free body diagram.

a) (i) 30.7° (ii) 0.11. b) (i) 17.8° (ii) 0.24. c) (i) 3.00° (ii) 2.00.
d) (i) 30.7° (ii) 0.24. e) (i) 17.8° (ii) 0.11.

Full-Answer Problems

005 qfull 00610 1 3 0 easy math: center-of-mass integration of a rod

303. You have a 1-dimensional object of length a. It is a non-uniform thin rod. It's linear density (i.e., mass per unit length) is given by

$$\lambda = Cx^p \ ,$$

where C is a constant, $p > -1$ (i.e., p is a real number greater than -1) and the origin is at one end of the rod. Derive the formula for the center of mass of the rod. Sketch a plot of x_{CM} versus p.

005 qfull 00620 2 3 0 moderate math: center-of-mass integration of cube

304. You have cube with sides of length a. Take one corner to be the origin of a standard x-y-z coordinate system. For convenience, snug the cube into the all positive coordinate octant.

a) Assuming density is given by the formula

$$\rho(x, y, z) = Cxyz$$

(where C is a constant) calculate the location of the center of mass of of the cube. **HINT:** The solution involves triple integrations, but nice triple integrations, not a nasty ones. Calculate the total mass first.

b) Now that you've cut your teeth, let's go for the curse of generality as my old friend Francesco used to say. Assuming density is given by the formula

$$\rho(x, y, z) = Cx^\ell y^m z^n$$

(where C is a constant and ℓ, m, and n are positive or zero integers) calculate the location of the center of mass of the cube.

005 qfull 00630 2 5 0 moderate thinking: superimposed symmetry CM

305. You have a sphere of radius R completely containing a spherical hollow of radius aR. The center of the hollow is located bR from the center of the sphere. Since the hollow is completely contained $a + b \leq 1$. The density of the sphere is a constant ρ (except of course in the hollow where its zero). Locate the center of mass.

005 qfull 00950 2 3 0 moderate math: rocket pod descent on Callisto
Extra keywords: David Bowman and 2001: A Space Odyssey

306. As this is (or was within living memory) 2001, let's say you are David Bowman and you've just arrived at Jupiter. Before going off to investigate that monolith (and go beyond humankind), you decide on a little excursion to Callisto, one of Jupiter's 4 major moons. Assume you are so close to Callisto's surface throughout the maneuvers of this question the gravitational field g_{Cal} can be approximated as a constant.

a) As your landing pod descends straight down to the Callisto surface and when your are relatively close to touchdown, your rocket thrust is 3260 N and your descent velocity is **CONSTANT**. What is the gravitational force on your pod? Take the upward direction as the positive direction.

b) Say you reduce thrust to 2200 N and find that the pod has a downward acceleration of $0.39\,\text{m/s}^2$. What is the mass of your pod including yourself?

c) What's the free-fall acceleration magnitude due to gravity near the Callisto surface (i.e., g_{Cal}, the analog to g for gravity near Earth's surface)? The free-fall acceleration magnitude is also the gravitational field magnitude.

d) Say you have a mass of 70 kg. What's your **WEIGHT** on Callisto and what is your Callisto weight divided by your Earth weight (i.e., what is the weight **RATIO**)?

e) Now the hard part. After finishing your excursion on the icy surface, you launch and go into uniform circular motion, low-Callisto orbit. The gravitational acceleration is approximately the same as the surface gravitational acceleration and the radius of the orbit is approximately just Callisto's radius of 2400 km. Calculate the **ORBITAL SPEED**. Then find the **ORBITAL PERIOD** P (i.e., the time to orbit once) in seconds and in hours. **HINT:** Remember centripetal acceleration and $\vec{F}_{\text{net}} = m\vec{a}$.

---

005 qfull 00952 2 3 0 moderate math: rocket pod descent on Io

Extra keywords: David Bowman and 2001: A Space Odyssey

307. As this is (or was within living memory) 2001, let's say you are David Bowman and you've just arrived at Jupiter. Before going off to investigate that monolith (and go beyond humankind), you decide on a little excursion to Io, one of Jupiter's 4 major moons. Assume you are so close to Io's surface throughout the maneuvers of this question the gravitational field g_{Io} can be approximated as a constant.

a) As your landing pod descends straight down to the Io surface and when your are relatively close to touchdown, your rocket thrust is 2500 N and your descent velocity is **CONSTANT**. What is the gravitational force on your pod? Take the upward direction as the positive direction.

b) Say you reduce thrust to 1800 N and find that the pod has an acceleration of $-0.50\,\text{m/s}^2$ taking upward as the positive direction. What is the mass of your pod including yourself?

c) What's the free-fall acceleration magnitude due to gravity near the Io surface (i.e., g_{Io}, the analog to g for gravity near Earth's surface)? The free-fall acceleration acceleration magnitude is also the gravitational field magnitude.

d) Say you have a mass of 70 kg. What's your **WEIGHT** on Io and what is your Io weight divided by your Earth weight (i.e., the weight **RATIO**)?

e) Now the hard part. After finishing your excursion on the sulforous icy/boiling surface of the solar system's most volcanically active body, you launch and go into uniform circular motion, low-Io orbit. The gravitational acceleration is approximately the same as the surface gravitational acceleration and the radius of the orbit is approximately just Io's mean radius of 1821.3 km. Note, the units are **KILOMETERS**. Calculate the orbital speed. Then find the **ORBITAL PERIOD** (i.e., the time to orbit once) in seconds and in hours.

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005 qfull 01010 1 3 0 easy math: frictionless incline

308. There is a 2 kg block on a frictionless incline that is at $\theta = 30°$ from the horizontal.

a) What is the normal force on the block? **HINTS:** Draw a free body diagram and remember the class mantra: "$\vec{F}_{\text{net}} = m\vec{a}$ is always true and it's true component by component".

b) What is the net force down the slope?

c) What is the acceleration down the slope?

d) Starting from rest how far does the block slide in 10 s?

---

005 qfull 01020 1 5 0 easy thinking: incline plane ad infinitum

309. Physics students frequently flub analyzing forces and accelerations on an inclined plane. Let's get it straight.

a) A Cairn terrier named Bit has dog-rolled to a point on an inclined plane. The plane has an angle from the horizontal of θ. Bit's mass is m. What is the component of gravitational force on Bit parallel to the inclined plane? What is the normal force on Bit? **HINT:** Draw a diagram.

b) Fun-loving pig Waldo Pepper (mass m) is sliding down a frictionless incline (with angle θ form the horizontal). What is his acceleration? What is the normal force on Waldo? **HINT:** Draw a diagram.

c) Underdog has just alighted on an inclined plane from which the Wonder Woofer surveys the world with a flint-hard gaze. The inclined plane has an angle of θ from the horizontal. What is the gravitational force component parallel to the inclined plane on the Caring Canine (mass m)? What is the normal force on the Magnificent Mutt? **HINT:** Draw a diagram.

d) A 1992 GM Geo Metro (mass m) is sliding down a frictionless incline (with angle θ form the horizontal) which is sort of like a hill in Moscow, Idaho in January. What is Baby's acceleration? What is the normal force on Baby? **HINT:** Draw a diagram.

---

005 qfull 01080 3 3 0 tough math: skiing downhill into a head wind

Extra keywords: Save this for test? That integration may rule it out.

310. You are skiing down a 30° incline. The slope is completely frictionless. A horizontal wind—i.e., a wind exactly perpendicular to the direction of gravity, a wind parallel to the surface of resting fluid at this location, a flat, flat wind—is blowing in your face exerts a 100 N force horizontal force on you. Your mass is 100 kg.

a) What is your acceleration? **HINT:** Draw that free body diagram.

b) How far do you travel along the slope in 10 s assuming you start from rest?

c) Now the hard part. Assume that the wind force is not constant, but increases at a constant rate from 100 N to 200 N in the 10 s of travel. How far do you travel along the slope now? **HINT:** You will have to do an integration.

---

005 qfull 01120 2 5 0 moderate thinking: Bob swinging on a rope

Extra keywords: Adapted from *Physics Today,* 2002, May, 55, no. 5, 52; a cent used

311. You have tied your good old pal Bob on a rope from the ceiling—but not around the neck—and have set him swinging. Assume there is negligible air drag, negligible friction at the pivot point, and that the rope stays straight.

a) Draw a schematic diagram illustrating Bob's motion. Mark on the lowest point P and the highest point Q—well one of the highest points.

b) Indicate the velocity and acceleration vectors at points P and Q. If a vector is zero, indicate that and explain why it is zero.

c) What happens if the rope breaks suddenly at point P? at point Q? Explain the subsequent motion qualitatively.

---

005 qfull 01130 2 5 0 moderate thinking: block and 3rd law

312. You have block just sitting on horizontal flat ground. **HINT:** This is a problem for rumination—or perhaps ruminants.

a) Draw a free body diagram for the block. Indicate all the forces acting on the block. What is the cause of these forces and are they contact or field forces?

b) Now by the 3rd law, what forces does the block exert and on what and where exactly does it exert them?

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005 qfull 01110 2 3 0 moderate math: fuzzy dice at angle

313. You are in a car accelerating at a constant $10\,\mathrm{m/s^2}$ in a constant direction. The car is on level ground. A pair of fuzzy dice is hanging by a cord from the mirror at an angle θ from the vertical. The dice cord is a (massless) ideal rope. Assuming the dice are a point mass, what is this angle? **HINT:** Draw a free body diagram for the dice. Remember the class mantra: "$\vec{F}_{\mathrm{net}} = m\vec{a}$ is always true and it's true component by component".

---

005 qfull 01130 3 3 0 tough math: tension in a dangling rope

Extra keywords: Requires integration

314. A rope is dangling from the ceiling. Its mass per unit length is given by

$$\mu(y) = \mu_0 + \mu_1 y + \mu_2 y^2 \ ,$$

where y is measured upward from the dangling end of the rope taken as $y = 0$. What is the expression for the tension at any point y along the rope.

---

005 qfull 01140 2 3 0 moderate math: double incline with two blocks

315. You have a **FRICTIONLESS** triangular block which gives you two inclines: i.e., double incline. Incline 1 is at θ_1 to the horizontal and incline 2 at θ_2. You have an ideal massless pulley (or alternatively and equivalently, a friction-free bend) at the apex and a taut ideal rope connecting two blocks, one on each slope. The rope is parallel to both inclines. The incline 1 block has mass m_1 and the incline 2, mass m_2. The masses of the blocks and the incline angles are the formal knowns of the problem.

a) Write down Newton's 2nd law for each block for the direction along the inclines. Take up as positive for incline 1 and down as positive for incline 2. **HINT:** Draw two free body diagrams on a diagram of the double incline.

b) Derive the explicit formula for acceleration of the two blocks along their respective inclines and the explicit formula for the tension in the rope? **NOTE:** By "explicit" in this context, we mean formulae with $a = \ldots$ and $T = \ldots$ with no a's or T's on the right-hand sides.

c) Specialize the formulae from the part (b) answer for the case of $\theta_1 = \theta_2 = \pi/2$. This case is Atwood's machine.

d) Specialize the formulae from the part (b) answer for the case of $\theta_1 = 0$ and $\theta_2 = \pi/2$.

e) Specialize the formulae from the part (b) answer for the case of $\theta_2 = \pi/2$.

---

005 qfull 01150 3 3 0 tough math: monkey on a pulley

316. An ideal rope (i.e., massless, but it has friction so a monkey can grip it) hangs over ideal tree bough (i.e., frictionless). The rope segments on both sides hang vertically. An ideal monkey of

mass 5 kg (let's call her object 1) climbs up one side of the rope. A block of 10 kg (let's call it object 2) sits on the ground at least initially and is tied to the end of the opposite side of the rope.

How the monkey is supported and can be accelerated is a somewhat difficult to understand—but they are everyday effects. She exerts a friction force on a rope segment that is equal and opposite to the friction force of the rope segment on her needed to support her against gravity and accelerate her. The rope segment isn't moving at all, and so the monkey's force on the segment and the tension force on the rope segment must be equal and opposite. So the tension force of the rope on the rope segment is equal to the friction force on the monkey. But that is all a bit complex. Let's just take the monkey and segment of rope she is holding as all part of one monkey system. So tension and gravity are the external forces acting on the monkey system. You may wonder how the tension force can help accelerate the monkey system if the rope segment doesn't move when the monkey accelerates upward or downward. Well it's the center of mass of monkey system that is accelerated. We will get to the center-of-mass concept sometime if not already. For the moment, just regard the monkey system as a particle to be accelerated by tension and gravity. For simplicity, we will just say monkey rather than monkey system.

a) Draw free body diagrams for both monkey and block. Remember the only forces that appear are those that act **ON** the object and they are drawn with their tails at the origin of the diagram.

b) Write down Newton's 2nd law for the monkey (object 1) and the block (object 1). Use symbols **ONLY**. Masses and g are formally knowns. What the unkowns? Can they be solved for without more information.

c) What is the relationship known before any calculation between the block's acceleration and the normal force on the block? If a solution for the normal force yields a negative number (i.e., the normal force is required by the given values to be attractive), what does this mean?

d) If the monkey accelerates as she climbs, what must be her acceleration in order that the normal force on block goes to zero, but the block stays motionless? **NOTE:** The zero normal force and motionless block are conditions we are imposing.

e) Say that the monkey then starts accelerating downward at $3\,\mathrm{m/s^2}$. The block is initially at rest on the ground. What is the tension in the rope? What is the normal force on the block? Does the block stay at rest on the ground?

f) Say that the monkey then starts accelerating upward at at $12\,\mathrm{m/s^2}$—she's really rocketing. The block is initially at rest on the ground. What is the tension in the rope? What is the normal force on the block? Does the block stay at rest on the ground?

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005 qfull 01160 3 5 0 tough thinking: massive rope and block system

317. You have a uniform-density rectangular block of mass m_b on a frictionless, horizontal surface. You are pulling it along with a uniform-density rope of mass m_r. the total mass of the block-rope system is $m = m_\mathrm{b} + m_\mathrm{r}$. Aside from its mass and weight the rope is ideal. The two ends of the rope are at the same height which is well above the ground. The motion of the system is entirely 1-dimensional. This is an entirely symbolical and thinking question until part (e).

a) Must the rope sag at least a little? Why or why not?

b) Assume any sag of the rope is negligible for the rest of this question and that the block-rope system can be regarded as rigid and non-rotating. Given that there is force F in the positive x-direction pulling on the rope and holding it taut and that no other external forces in the x-direction on the block-rope system, what is the x-direction center-of-mass acceleration a

of the block-rope system? What is the acceleration of every bit of the block-rope system?

c) Say the positive x direction is to the right. Let m_x be the total mass of the system to the left of point x along the length of the block-rope system. What is the internal tension force at x accelerating the mass m_x as a function **ONLY** of F and the relevant masses?

d) What are forces F_x at middle of the rope, the point of block-rope contact, and at the middle of the block as functions **ONLY** of F and the relevant masses?

e) Given the masses $m_\mathrm{b} = 5.0\,\mathrm{kg}$ and $m_\mathrm{r} = 0.1\,\mathrm{kg}$, and the acceleration of the system $a = 2.0\,\mathrm{m/s^2}$, what is F?

f) What is the block's speed at 10 m from the starting point if it starts from rest?

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005 qfull 01200 1 3 0 easy math: determining coefficient of static friction

318. Will will now determine the coefficient of static friction μ_st from an empirical measurement using an ajustable incline. The adjustable angle of incline from the horizontal is θ.

a) Write down Newton's 2nd law for a block sitting at rest on the incline for two directions: perpendicular to the incline and parallel to it. The only forces are the gravity, the normal force, and friction.

b) The angle of adjustable incline is increased just to the slipping point for the block. Give the parallel 2nd law equation just before slipping occurs.

c) Solve for the μ_st.

Chapt. 6 Newton's Laws and Dynamics II

Multiple-Choice Problems

006 qmult 00100 1 1 1 easy memory: five special forces

319. Five special forces in physics are:

a) gravitational force, normal force, tension, friction force, and the linear force (the Hooke's law force).
b) gravitational force, normal force, tension, friction force, and left force (the Captain Hooke's law force).
c) gravitational force, normal force, tension, friction force, and right force.
d) gravitational force, normal force, tension, friction force, and air force.
e) gravitational force, normal force, tension, friction force, and **the Force**.

---

006 qmult 00120 1 1 3 easy memory: linear force formula

320. In one dimension, the linear force formula is:

a) $F = -kx^2$. b) $F = kx$. c) $F = -kx$. d) $F = kx^2$. e) $F = k/x^2$.

---

006 qmult 00160 2 1 4 moderate memory: the linear force law
Extra keywords: spring force law, Hooke's law

321. The ideal spring force is:

a) a **CONSTANT** like the Earth's surface gravity.
b) proportional to the **SQUARE** of the spring stretch from its equilibrium length. It's an example of a quadratic force.
c) proportional to the **CUBE** of the spring stretch from its equilibrium length. It's an example of a cubic force.
d) **PROPORTIONAL** to the spring stretch from its equilibrium length. It's an example of a linear force.
e) is always **ZERO**.

---

006 qmult 00200 1 1 1 easy memory: SHO defined

322. A object acted on only by the linear force is called a:

a) simple harmonic oscillator. b) anharmonic oscillator. c) simple oscillator.
d) linear-force oscillator. e) simple pendulum.

---

006 qmult 00250 1 1 4 easy memory: change force contant change period for harmonic ocsillator

323. If you double the force constant of a simple harmonic oscillator, the period changes by a factor of:

a) 1/2. b) $\sqrt{2}$. c) 2. d) $1/\sqrt{2}$. e) 2/3.

---

006 qmult 00310 1 1 2 easy memory: 2nd law in polar coordinates

324. The formula

$$F_{r,\text{net}}\hat{r} + F_{\theta,\text{net}}\hat{\theta} = m\left[\left(\frac{d^2r}{dt^2} - r\omega^2\right)\hat{r} + \left(r\alpha + 2\frac{dr}{dt}\omega\right)\hat{\theta}\right]$$

is:

a) Newton's 2nd law in spherical polar coordinates.
b) Newton's 2nd law in polar coordinates. c) the centripetal force formula.
d) the centripetal acceleration formula. e) the simple harmonic oscillator formula.

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006 qmult 00400 1 1 4 easy memory: uniform circular motion defined
Extra keywords: physci

325. Uniform circular motion is motion in a/an:

a) circle at a constant **VELOCITY**. b) oval at a constant **VELOCITY**.
c) oval at a constant **SPEED**. d) circle at a constant **SPEED**.
e) circle at a nonconstant **SPEED**.

---

006 qmult 00410 1 1 3 easy memory: centripetal acceleration in UCM
Extra keywords: physci

326. In uniform circular motion, the acceleration has:

a) a constant magnitude and always points **OUTWARD** from the center of motion.
b) a constant magnitude and always points **ALONG** the circular path (i.e., tangent to the circular path).
c) a constant magnitude and always points **INWARD** to the center of motion.
d) a zero value.
e) a nonconstant magnitude, but a constant direction.

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006 qmult 00430 3 1 1 easy math: centripetal acceleration

327. The formula for the magnitude of centripetal acceleration is:

$$a_{\text{centripetal}} = \frac{v^2}{r} \ ,$$

where v is the speed of a uniform circular motion and r is the radius of the motion. Say $v = 10\,\text{m/s}$ and $r = 5\,\text{m}$, what is $a_{\text{centripetal}}$?

a) $20\,\text{m/s}^2$. b) $10\,\text{m/s}^2$. c) $5\,\text{m/s}^2$. d) $100\,\text{m/s}^2$. e) $15\,\text{m/s}^2$.

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006 qmult 00450 1 4 3 easy deducto-memory: centripetal force defined

328. The centripetal force is:

a) a mysterious force that **APPEARS** whenever an object goes into uniform circular motion.
b) a mysterious force that tries to throw you **OFF** playground merry-go-rounds.
c) in fact $m\vec{a}$ of $\vec{F}_{\text{net}} = m\vec{a}$ when this equation is specialized to the case of uniform circular motion. It is **NOT** a mysterious force that appears whenever you have uniform circular motion: it is a force requirement to be satisfied for uniform circular motion. Particular physical forces (e.g., gravity, tension force, and normal force) must act (sometimes in combination) to give a centripetal force which then causes uniform circular motion.
d) in fact $m\vec{a}$ of $\vec{F}_{\text{net}} = m\vec{a}$ when this equation is specialized to the case of uniform circular motion. The force itself is **ALWAYS** a field force emanating from the center of motion that pulls on the circling object atom by atom.
e) a mysterious force that **DISAPPEARS** whenever an object goes into circular motion.

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006 qmult 00460 1 1 5 easy memory: centripetal force
Extra keywords: physci KB-57-13

329. For a body to move in a circle at a uniform speed in an inertial frame it is **NECESSARY** to supply a/an _____ force.

a) centrifugal b) inertial c) gravitational d) tension e) centripetal

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006 qmult 00470 2 3 2 moderate math: centripetal force yo-yo string
Extra keywords: physci KB-61-35

330. A yo-yo of 200 g and string length 80 cm is swirled around by holding the end of the string in the absence of gravity and air drag. The string will break if tension force exceeds 10 N. About how fast can the yo-yo be swung without breaking the string?

a) 3.2 m/s. b) 6.3 m/s. c) 80 m/s. d) 200 m/s. e) 10 m/s.

---

006 qmult 00472 3 5 1 tough thinking: lifting from a hump
Extra keywords: physci KB-61-39

331. There is a hump on the road with a cylindrical shape. The radius of the hump is 14.7 m. In an idealized picture, above about what horizontal speed must a car at the top of the hump lift from the hump?

a) 12 m/s. b) 14.7 m/s. c) 144 m/s. d) 10 m/s. e) 10.4 m/s.

---

006 qmult 00473 3 5 4 tough thinking: lifting from a hump
Extra keywords: physci KB-61-39

332. There is a hump on the road with a cylindrical shape. The radius of the hump is 25.6 m. In an idealized picture, above about what horizontal speed must a car at the top of the hump lift from the hump?

a) 7.2 m/s. b) 10.1 m/s. c) 14.7 m/s. d) 15.8 m/s. e) 17.4 m/s.

---

006 qmult 00474 3 5 3 tough thinking: lifting from a hump
Extra keywords: physci KB-61-39

333. There is a hump on the road with a radius of curvature of 100 m just at the top. In an idealized picture, above about what horizontal speed must a car at the top of the hump lift from the hump?

a) 12 m/s. b) 25 m/s. c) 31 m/s. d) 36 m/s. e) 43 m/s.

---

006 qmult 00550 1 1 4 easy memory: banking angle formula

334. The banking angle formula is _____. **HINT:** Use dimensional analysis. Ask yourself what formula has reasonable limiting behavior when input values go to extremes. Or just derive it using the centripetal force formula and the 2nd law.

a) $\theta = \tan^{-1}\left(\frac{v}{rg}\right)$ b) $\theta = \tan^{-1}\left(\frac{rg}{v^2}\right)$ c) $\theta = \tan^{-1}\left(\frac{v^2 r}{g}\right)$
d) $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$ e) $\theta = \tan^{-1}\left(\frac{g}{v^2 r}\right)$

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006 qmult 00630 1 1 2 easy memory: simple pendulum

335. The period formula for the oscillation of a simple pendulum of arm length r and bob mass m is:

a) $P = 2\pi\sqrt{\frac{g}{r}}$. b) $P = 2\pi\sqrt{\frac{r}{g}}$. c) $P = 2\pi\sqrt{\frac{mr}{g}}$. d) $P = 2\pi\sqrt{gr}$.
e) $P = 2\pi\sqrt{mgr}$.

---

006 qmult 00770 2 5 2 moderate thinking: circular motion in non-inertial frame
Extra keywords: physci KB-59-21 , but I've corrected it

336. In what situations, if any, can a body move in a circular path at constant speed without a centripetal force?

a) None. b) In certain special non-inertial frames. c) In all non-inertial frames.
d) In all inertial frames. e) Always.

---

006 qmult 00700 1 1 4 easy memory: drag force as contact force

337. The drag force (the resistive force of a fluid: e.g., air drag) is an example of a/an:

a) linear (or Hooke's law) force in all cases. b) gravitational force.
c) tension force. d) contact force. e) animal force.

---

006 qmult 00710 1 1 2 easy memory: two drag force laws

338. In general the drag force on an object moving through a fluid is complex, but there are two well-known drag force laws that have wide application in different physical regimes: the drag equation law and the Stokes law. They both depend linearly on a power of the relative velocity opposite the direction of the motion of the object (i.e., the flow speed). The powers for the drag equation law and the Stokes law are, respectively:

a) 1 and 2. b) 2 and 1. c) 2 and 3. d) 1 and 3. e) 3/2 and 5/2

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006 qmult 10550 1 1 5 easy memory: plane banking fullmult

339.** A plane flying at 500 km/h on the level is making making a circular turn with radius 5.0 km. The lift force on a plane is **PERPENDICULAR** to the wings. The gravity force points down as always. What angle is does the plane have to bank at (i.e., at what angle to the horizontal must its wings be tilted) to make the turn? **HINT:** This is really just the road banking problem all over again. Solve for the banking angle using Newton's 2nd law. Draw the free body diagram.

a) 2.1°. b) 4.1°. c) 10.2°. d) 17.4°. e) 21.5°.

Full-Answer Problems

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006 qfull 00220 1 3 0 easy math: simple harmonic oscillator constants

340. The general position formula for a 1-dimensional simple harmonic oscillator is

$$x = A\cos(\omega t) + B\sin(\omega t) \ ,$$

where A and B are general constants.

a) Given that initial position x_0 and initial velocity v_0, solve for A and B as functions of x_0, v_0, and ω only. **HINT:** Recall that "initial" means time zero (i.e., $t = 0$).

b) Any linear of combination of a cosine and sine can be written as single sinusoid, with the appropriate choice of coefficient and phase shift. For example,

$$x = A\cos(\omega t) + B\sin(\omega t)$$

can be written

$$x = C\cos(\omega t + \phi)$$

with the appropriate choice of C and ϕ. Solve for C and ϕ as functions of A and B only.

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006 qfull 00310 2 3 0 moderate math: central force

341. Newton's 2nd law in polar coordinates is:

$$\vec{F}_{\rm net} = F_{r,\rm net}\hat{r} + F_{\theta,\rm net}\hat{\theta} = m\left[\left(\frac{d^2r}{dt^2} - r\omega^2\right)\hat{r} + \left(r\alpha + 2\frac{dr}{dt}\omega\right)\hat{\theta}\right] \ ,$$

where $F_{r,\rm net}$ is the net force in the radial direction, $F_{\theta,\rm net}$ is the net force in the angular direction, and

$$\vec{a} = \left(\frac{d^2r}{dt^2} - r\omega^2\right)\hat{r} + \left(r\alpha + 2\frac{dr}{dt}\omega\right)\hat{\theta}$$

is the general formula for acceleration for in polar coordinates. Recall angular velocity ω and angular acceleration α are given by, respectively,

$$\omega = \frac{d\theta}{dt} \qquad \text{and} \qquad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} \ .$$

a) If a force on an object (i.e., on its center of mass) is purely radial with respect to a central-force origin and the force only depends on the radial distance from the central-force origin, the force is called a central force.

A central-force system is one in which only a central force acts on an object. A particular example of a central force system is the solar system. Except for minor perturbations, the planets are subject to only to the central force of the Sun's gravity.

The motion of an object in a central-force system is confined to a single plane and hence is two-dimensional motion. This single plane is determined by the initial velocity direction and central-force origin. There is no force to cause the object to develop a velocity componet out of the plane. Central force systems are one of the main areas of application for Newton's 2nd law in polar coordinates since they exploit the symmetry of the system if one puts the coordinate origin at the central-force origin and, of course, makes the polar coordinate plane the plane of motion. We assume hereafter we have chosen the polar coordinates to exploit the symmetry our central-force systems.

With our choice of polar coordinates, the central force formula is

$$\vec{F}(\vec{r}) = F(r)\hat{r} \ .$$

Given Newton's 2nd law in polar coordinates, there is one relationship among the kinematic variables that is immediately seen for the central force case. What is it?

b) To make use of the relationship found in the part (a) answer, we need the antiderivative or indefinite integral of

$$\frac{1}{x}\frac{dx}{dt}$$

with respect to t, where x is a general variable greater than zero. What is it? **HINT:** You need to use the chain rule in reverse. Remember that in general there will be a constant of integration.

c) Make use of the results of parts (a) and (b) answers to find a simple algebraic relationship between r and ω for a central-force system.

d) The areal velocity v_{areal} is the rate at which area is swept out by a radius from a polar coordinate origin to an object moving through space in the polar coordinate plane. The formula for areal velocity is

$$v_{\text{areal}} = \frac{1}{2}r^2\omega \ ,$$

where r is the radial position of the object and ω is the angular velocity. Give a non-rigorous proof of this formula. Note, the particle is **NOT** necessarily moving on a circle. It's just moving on a path in the plane of the polar coordinate system.

e) Now use the areal velocity and the part (c) answer to prove Kepler's 2nd law that a planet's orbital radius sweeps out equal areas in equal times.

The law holds regardless of the shape of the swept out portion of the orbit. Kepler discovered his 2nd law semi-empically in the first decade of the 17th century long before Newton derived it from Newtonian physics.

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006 qfull 00410 2 5 0 moderate thinking: Is-Hilda on vinyl record

342. Is-Hilda the ladybug is on a vinyl record spinning at 78 rpm. She starts at the center and six-leggedly walks radially outward relative to the rotating frame of reference. Her radial velocity (caused by her walking) is low enough that the Coriolis force is negligible compared to the centrifugal force. Both these forces are inertial forces present in the rotating frame. At 6 cm from the center, Is-Hilda suddenly slides off the record. The record counterfactually is a smooth surface—and isn't corrugated—and Is-Hilda is an ideal ladybug without sticky feet. What is the static friction coefficient between her and the record? Note, it is static friction that allows her or anyone to walk on a surface.

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006 qfull 00420 1 3 0 easy math: conical pendulum with detailed solution

343. There is a conical pendulum of length ℓ with point mass bob of mass m. The bob is executing uniform circular motion with velocity v and radius r (which is measured from the axis of rotation not along the length of the pendulum). The pendulum has an ideal rope with tension T. Note, an ideal rope is massless and can only exert a tension force which is uniform along the rope and only turns on when the rope is taut. There is no friction, no drag, and no dissipation forces in general. Thus the motion is perpetual. The rope sweeps out a cone and the opening angle of the cone from the vertical is θ.

a) Write down Newton's 2nd law for the horizontal direction (taking inward to the axis of rotation as positive) and vertical direction (taking upward as positive). Make use of the centripetal acceleration formula. Remember, the rope sweeps out a cone and the opening angle of the cone from the vertical is θ. **HINT:** Draw diagram.

b) Solve for θ as a function of g, r and v and for T as a functions of m, g and θ.

c) For a person just swinging a conical pendulum by hand, one can set m and ℓ directly and with some skill period $p = 2\pi r/v$. So those variables are the obvious control parameters of the system. Determine in order formulae for θ in terms of m, ℓ, and p only.

d) Now determine in order formulae for T, r, and v in terms of m, ℓ, and p only.

e) Determine $p_{\max}$, the maximum period for the circular motion allowed by the formulae. For what angle θ does this occur?

f) You swing a crude pendulum with $\ell = 0.5\,\text{m}$ such that $p \approx 1\,\text{s}$. What is the angle θ?

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006 qfull 00510 1 3 0 easy math: level road corner centripetal force

344. Consider an ordinary road corner that is level ground. Such corners are usually not banked very much, in fact.

a) What force supplies the centripetal force that allows a vehicle to make the corner?

b) For a corner turn with radius of curvature r and a vehicle of mass m, derive the formula for the maximum turn speed v before slipping occurs. **HINT:** You will have to consider the upper limit on static friction.

c) Say $r = 10\,\text{m}$ (which seems reasonable for ordinary corners) and $\mu_{\text{st}} = 1.0$ (as for rubber on concrete in dry conditions approximately: Wikipedia: Friction: Approximate coefficients of friction), what is the maximum turn speed before slipping? Now say $\mu_{\text{st}} = 0.30$ (as for rubber on concrete in wet conditions approximately: Wikipedia: Friction: Approximate coefficients of friction), what is the maximum turn speed before slipping? Convert the answers to miles per hour. **HINT:** Note, $1\,\text{mi} = 1.609344\,\text{km}$ exactly (Wikipedia: Mile).

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006 qfull 00550 2 3 0 moderate math: plane making a banked turn

345. An airplane of mass m is making a turn through a circular bend with a speed of v. It moves horizontally only: i.e., it's not moving or accelerating downward or upward. In order to make the turn the plane banks at angle θ from the horizontal (i.e., its wings are tipped at θ to the horizontal with the high side away from the center of the turn). Assume all the force the air can exert on the plane (the aerodynamic lift) acts perpendicularly to the wings. Let F_{lift} be the aerodynamic lift magnitude.

a) Draw a free body diagam for the airplane in the plane perpendicular to the airplane's instantaneous direction of motion.

b) Apply Newton's 2nd law to airplane in the plane perpendicular to the airplane's instantaneous direction of motion. Remember the course mantra:

"Newton's 2nd law is always true and it's always true component by component."

c) Solve for the r in terms of the other variables: i.e., in terms of m, θ, v, g, and F_{lift}.

d) Given $v = 600\,\text{km/h}$ and $\theta = 30°$, what is r?

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006 qfull 00560 3 3 0 tough math: road banking formula

346. Because road friction is variable with weather (dipping low with rain and especially ice and snow) and because relying on friction for turning at high speeds is hard on cars and car control, civil engineers (who are very polite engineers) don't like to rely on friction (and we mean static friction here) to provide much of the centripetal force to make vehicles (or ve-hicles as we used to say in Tennessee) go around bends. Thus they tend to bank the roads so that gravity and the normal force of the road supply all or almost all of the centripetal force if the vehicle goes at the prescribed speed. Gravity is practically invariable as far as cars are concerned and under normal circumstances the road should hardly flex at all no matter what normal force is demanded of it. Thus gravity and the normal force can be relied on.

Now probably all kinds of considerations go into designing real road banking. But in the idealized case of a circular bend with cars moving at a fixed speed and only gravity, friction, and the normal force acting, there is a simple formula for the banking angle that reduces friction (i.e., static friction parallel to the road surface and **PERPENDICULAR** to the vehicle's instantaneous direction of motion) to zero. Here we investigate the banking question and find the ideal banking angle formula.

a) Assume a car is going around a circular bend of radius r at a uniform speed v. Draw **HORIZONTAL** and **VERTICAL** axes to make up a diagram with the car a point mass at the center. Mark the direction of the banking of the road: i.e., a line that runs along the road surface. The banking angle θ is the angle of road from the **HORIZONTAL**. Mark the direction that the centripetal force points in: remember the centripetal force is not any particular force: it is the sum of the actual forces in the plane of the uniform circular motion. Now indicate the gravitational force and another force which we will just call the contact force $\vec{F}_{\text{contact}}$: the contact force points at an angle θ_{contact} from the **VERTICAL**. The contact force on the bottom of the tires is the vector sum of the normal force and the friction force. But the contact force can be any contact force that causes something to go around the bend. Thus the contact force description applies to forces internal to the car that make parts of it go around the bend too: e.g., the contact force of the seat on the driver or the tension force acting on the fuzzy dice.

b) Since we have assumed that the car is going in uniform circular motion around the bend, whatever forces makes up the contact force (e.g., friction, normal force, tension) are up to the job of causing the uniform circular motion. The contact force turns on because otherwise the car would push into the road or slide off or something. Find the components of the contact force in the horizontal direction toward the center of motion and in the vertical direction. Solve for the angle of contact force from the vertical θ_{contact} in terms of v (vehicle speed), r (bend radius), and g. **HINT:** Remember the class mantra: "$\vec{F}_{\text{net}} = m\vec{a}$ is always true and it is always true component by component".

c) Show that the banking angle θ is also the angle of the **PERPENDICULAR** (or normal) to the road surface from the **VERTICAL**.

d) What is the appropriate formula for the banking angle θ such that the friction force is never called upon to help cause the car to go into uniform circular motion? Why is this the case? If friction is never called upon, can it ever fail to supply adequate force to keep the car in uniform circular motion? What force or forces do supply the centripetal force on the car friction is not called on?

e) In this part, assume that banking angle formula has not been used. Thus, θ_{contact} and θ are independent. Now let us assume that the contact force $\vec{F}_{\text{contact}}$ is the vector sum of the normal force and the friction force. Since the normal force only acts perpendicular

to the surface and the friction force parallel to the surface, you can find formulae for the normal and friction forces by taking the components of $\vec{F}_{\text{contact}}$, respectively, along the **PERPENDICULAR** to the surface (take outward as positive) and along the surface (take downhill as positive). First find the normal and friction force formulae in terms of the magnitude of $\vec{F}_{\text{contact}}$ (i.e., F_{contact}) and $\theta_{\text{contact}} - \theta$ and simplify the formulae using trigonometry and the expressions from the part (b) answer for the components of $\vec{F}_{\text{contact}}$ for horizontal and vertical axes. The final formulae should be just in terms of $\sin\theta$, $\cos\theta$, mg and mv^2/r. What condition makes the friction force go to zero? Give the limiting case formulae for $v = 0$.

f) What's the banking angle for 60 mi/h and bend radius 100 m that will eliminate the friction component of the contact force?

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006 qfull 00810 1 3 0 easy math: artificial gravity

347. Rotating frames are non-inertial frames. But Newton's 2nd law can be applied to them by introducing inertial forces. But the full treatment of these inertial forces is beyond the scope of intro physics. Let's just consider motion in two dimensions: i.e., in planar space. You have frame rotating at constant angular frequency ω relative to inertial frames. The net force in the rotating frame is

$$\vec{F}'_{\text{net}} = \vec{F}_{\text{net}} + m\omega^2 r\hat{r} - 2m\omega(\hat{z} \times \vec{v}\,') \ ,$$

where $\vec{F}_{\text{net}}$ is the real net force, $m\omega^2 r\hat{r}$ is the centrifugal force, $-2m\omega(\hat{z} \times \vec{v}\,')$ is the Coriolis force, $\hat{z}$ is the direction of the axis of rotation (found by the thumb with the fingers curled in the direction of rotation using the right hand), and $\vec{v}\,'$ is the velocity relative to the rotating frame. The centrifugal and Coriolis forces are inertial forces. The derivation of the Coriolis force is non-trivial by the standards of intro physics. The complication is that a rotating frame is not a single non-inertial frame, but a continuum of them: every point in the rotating frame viewed from an inertial frame has a different, non-constant acceleration.

The quantity $\hat{z} \times \vec{v}\,'$ is a cross product whose result is a vector that is perpendicular to the plane defined by $\hat{z}$ and $\vec{v}\,'$ and whose direction given by the direction of the thumb sweeping the right hand from the rotation axis to $\vec{v}\,'$ (i.e., by another right-hand rule). The Coriolis force in this case points in the planar space, but we won't worry much about the direction of the Coriolis force. But you should remember that it is perpendicular to the velocity in the rotating frame, and so it would try to drive you off a straight-line path if you went walking a rotating frame.

The magnitude of the Coriolis force is $2m\omega v'$ for two-dimensional motion.

a) Derive the centrifugal force $\vec{F}_{\text{cf}}$ by considering a particle that is motionless in the rotating frame. Note that by definition the centrifugal force is the only inertial force that acts on a particle that is motionless in rotating frame. But the centrifugal force acts in any case (motionless or motionful) in the rotating frame. **HINT:** You know that $\vec{F}_{\text{cf}} = m\omega^2 r\hat{r}$ from the preamble, but the formula for centrifugal force is what you have to prove and so start with $\vec{F}'_{\text{net}} = \vec{F}_{\text{net}} + \vec{F}_{\text{cf}} + \vec{F}_{\text{co}}$, where $\vec{F}_{\text{co}}$ is the Coriolis force.

b) Artificial gravity in free fall in space can be created by rotating the spacecraft about some axis. Once rotating at a constant rate with no outside torques acting on the spacecraft, conservation of angular momentum keeps the spacecraft rotating at the constant rate. Let's not worry about torques and angular momentum here. But gravity of large spherical bodies exerts virtually no torque on spacecraft, and so spacecraft in orbit are nearly torque-free. People have often imagined giant pinwheel space stations where people work under artificial gravity conditions on the wheel. Near zero artificial gravity would exist in a module at the center of the wheel connected by spokes to the wheel. What is the artificial gravity force on a motionless object? What way does the artificial gravity point? Which way do human bodies point when standing normally in the wheel? What force stops standing people from "falling" under the artificial gravity?

c) What's the artifical gravitational field magnitudes for any ω and r for the centrifugal force and the Coriolis force? Continue assuming a 2-dimensional system for simplicity. Which way does the centrifugal force gravitational field point?

d) Due to ordinary human motions, the Coriolis force can act on the inner ear causing dizziness, nausea and disorientation for rotation greater than about 2 rpm. How small a outer radius can a pinwheel space station have and give Earth-like gravity (i.e., an centrifugal gravitational field of $g = 9.8\,\mathrm{N/kg}$) and still avoid negative human effects?

Given rotation of 2 rpm and centrifugal gravitational field of $g = 9.8\,\mathrm{N/kg}$, about how fast would a person need to be moving for the Coriolis gravitational field to become significant compared to the centrifugal gravitational? For significant, let's say 10 % of the centrifugal gravitiational field. Is this speed within the range of typical walking-around or head-wagging human motions.

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006 qfull 00770 3 3 0 easy math: tough thinking, the rotating loop with bead problem

348. Consider a circular loop or radius R that is in plane that is **PERPENDICULAR** to the ground. The loop is rotating about an axis that passes through the center of the loop and is also **PERPENDICULAR** to the ground—it's like a coin spinning on its **EDGE**.

The angular velocity of rotation of the loop is a constant and is given the symbol ω_{loop}. The loop is completely rigid and frictionless. A bead slides on the loop.

We consider the system in the **ROTATING FRAME** of the loop. The only forces that have components that act on the bead along the path of the loop are gravity and the centrifugal force: i.e.,

$$\vec{F}_{\mathrm{gr}} = -mg\hat{y} \qquad \text{and} \qquad F_{\mathrm{cf}} = m\omega_{\mathrm{loop}}^2 r\hat{r}\ ,$$

where m is the bead mass, $\hat{y}$ points downward, $\hat{r}$ is **PERPENDICULAR** to the axis of rotation and points directly away from the axis, and r is the perpendicular distance from the axis to the bead (and is not R). The normal force of the loop on the bead and the Coriolis force have zero components along the bead's path on the loop, and so cannot accelerate the bead in the **ROTATING FRAME** of the loop. Because the loop is completely rigid, the normal force cancels all other forces that act perpendicular to the loop.

a) Draw a diagram of the loop. Let θ be the angular position of the bead on the loop with the angle measured about the loop center. Let the zero angle be vertically downward from the loop center. Angle is measured positive in the counterclockwise sense. Draw vector arrows for the gravity and centrifugal forces that act on the bead at about angles $0°$, $\pm 45°$, $\pm 90°$, $\pm 135°$, and $180°$. Leave off arrows for zero vectors. **HINT:** Make the diagram large. No teeny, tiny diagrams.

b) Redraw a diagram of the loop, but now instead of vector arrows representing the gravity and centrifugal forces put on vector arrows representing their components along the loop. Leave off arrows for zero vectors. Identify four places where it is possible for the net force along the loop to be zero. At which of these two is it certain that the net force along the loop is zero. Why is it certain? **HINT:** Make the diagram large. No teeny, tiny diagrams.

c) The equation of motion of the bead (i.e., the 2nd law applied to the bead) for its position coordinate θ is

$$mR\frac{d^2\theta}{dt^2} = F_{\mathrm{net}}$$

where is F_{net} is the net force **ALONG** the path of the loop for any θ. (Note F_{net} is the net force along the path in the rotating frame, and so contains real force and inertial force terms.) Write out F_{net} explicitly in terms of m, g, θ, ω_{loop}, and R (not r). **HINT:** You will have take components of the gravity and centrifugal forces along the loop. Get the signs right. For mental clarity in determining the components, consider θ in the 1st quadrant (i.e., in the range $(0, 90°)$. The expressions are for a general angle when you think

about, but the 1st quadrant is isn't to comprehend. Draw a diagram with the geometry qualitatively correct.

d) From the results of the part (c) answer solve for four angles θ where

$$\frac{d^2\theta}{dt^2} = 0 \ :$$

i.e., where the angular acceleration of the bead on the loop is zero. These are equilibria or equilibrium points: i.e., points where the net force along the loop is zero. **HINT:** Watch out for divisions by zero: you could miss solutions that way. Also note that inverse cosine function is actually double valued: i.e., for argument A

$$\theta = \pm \cos^{-1}(A) \ .$$

e) Discuss the location of the $\omega_{\rm loop}$ equilibria in the cases

$$\frac{g}{\omega_{\rm loop}^2 R} > 1 \ , \qquad \frac{g}{\omega_{\rm loop}^2 R} = 1 \ , \qquad \frac{g}{\omega_{\rm loop}^2 R} < 1 \ , \qquad \frac{g}{\omega_{\rm loop}^2 R} = 0 \ .$$

Note, the cases are ordered in order of increasing $\omega_{\rm loop}$.

f) What would happen if you put a bead at one of the equilibria points with zero initial velocity? Why would this happen.

g) Equilibria are called stable/unstable if when an object is displaced from them by some small amount, there is a net force that pushes the object back to/away from the equilibria. A force that pushes back to equilibrium is called a restoring force. If there is no force on a small displacement, the equilibria are called neutral.

Determine the statuses of the four equilibria of the bead: i.e., are they stable, unstable, or neutral? Consider all cases of $g/(\omega_{\rm loop}^2 R)$: i.e., when it is > 1, 1, < 1, and 0 **HINT:** The figure in the part (b) answer should help in understanding why the equilibria have the status that they do. But for a definitive determinations, one needs to consider $F_{\rm net}$ as a function of angle θ. Decomposing $F_{\rm net}$ into a function times $mR\omega_{\rm loop}\sin\theta$ makes the determinations easier.

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006 qfull 00660 2 3 0 moderate math: Waldo Ecuador, Waldo equator

349. Waldo Pepper's on the equator in Ecuador—"Buenos días senoras y senores"— and thinking about tangential velocity, angular velocity, centripetal acceleration, centripetal force, and non-inertial frames.

a) Waldo remembers that the Earth's equatorial radius is 6378.1 km and that the rotation period of the Earth relative to the fixed stars (which is a more inertial frame than the one relative to the Sun) is one sidereal day (86164.1 s) which is a bit less than the solar day (which is the rotation period relative to the Sun) (Wikipedia 2007nov07). Waldo correctly calculates the magnitudes of **TANGENTIAL VELOCITY** (in m/s) and **ANGULAR VELOCITY** (radians per second) of the Earth at the equator. What values does he get? **HINT:** Waldo does remember about the circumference of a circle.

b) Now Waldo calculates the magnitude of centripetal acceleration for a body on the equator. What does he find?

c) The sagacious swine now reflects on inertial frames. For most purposes the Earth's surface is a pretty good inertial frame. But the Earth is rotating, and so it's not exactly an inertial frame. The fixed stars (as they are called for historical reasons) define a more inertial frame. Say we take—Waldo says to himself—$\vec{a}$ in Newton's 2nd law $\vec{F}_{\rm net} = m\vec{a}$ and decompose it (as nothing forbids us to do) into acceleration relative to the rotating frame of equatorial

Ecuador (i.e., $\vec{a}'$) and centripetal acceleration ($\vec{a}_{\rm c} = -a_{\rm c}\hat{r}$, where $\hat{r}$ is a radially outward pointing unit vector) which is the acceleration of the rotating frame. We obtain

$$\vec{a} = \vec{a}' - a_{\rm c}\hat{r} \ .$$

We can now write—Waldo still speaking to himself—Newton's 2nd law as

$$\vec{F}_{\rm net} = m(\vec{a}' - a_{\rm c}\hat{r})$$

or

$$\vec{F}_{\rm other} - mg\hat{r} = m(\vec{a}' - a_{\rm c}\hat{r})$$

(where $\vec{F}_{\rm other}$ is all forces other than gravity) or

$$\vec{F}_{\rm other} - m(g - a_{\rm c})\hat{r} = m\vec{a}' \ .$$

The last equation is effectively Newton's 2nd law for motion relative to the rotating frame.

In the rotating frame taken as an inertial frame, everything is as if the magnitude of the force of gravity per unit mass is $g - a_{\rm c}$ (provided velocites relative to the rotating frame are much less the tangential velocity of the Earth). The effective decrease of the magnitude of the force of gravity per unit mass by $a_{\rm c}$ is the centrifugal force effect. ($a_{\rm c}\hat{r}$ is the fictitious outward pointing centrifugal force per unit mass The centrifugal force is the "force" which tries to throw you out of rotating frame.) Using the Earth mass ($5.9736\times10^{24}\,{\rm kg}$), the mean Earth radius ($6371.0\,{\rm km}$) and gravitational constant $G = 6.67430(15)\times10^{-11}\,{\rm N\,m^2/kg^2}$ (Wikipedia: Gravitational constant 2015jun25), Waldo calculates the Earth mean g value (i.e., $g_{\rm mean}$) and then the $g_{\rm mean} - a_{\rm c}$ value. What are those values?

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006 qfull 00700 1 3 0 easy math: general drag force in falling from rest case

350. An object falling from rest is subject to a drag force of magnitude $f(v)$, where v is the object's speed. The function $f(v)$ is monotonically increasing with v and $f(0) = 0$, but is otherwise general. The inverse function to $f(v)$ is $f^{-1}(x)$.

a) Apply Newton's 2nd law to the object taking the downward direction as positive. **HINT:** You just write down the Newton's 2nd law for this particular case.

b) What is terminal velocity, what is the condition necessary for it to hold for a falling object, and why should the falling object reach it? What is the formula for the terminal velocity $v_{\rm ter}$? Make use of $f^{-1}(x)$.

c) The evolution to terminal velocity can be crudely divided into two phases: a linear growth phase (when the velocity is growing approximately linearly) and an asymptotic phase (when the velocity is asymptotically approaching terminal velocity). The characteristic time $t_{\rm ch}$ boundary between the two phases is obtained by setting $f(v) = 0$ in the equation of motion and solving for the time when $v = v_{\rm ter}$. Beyond this time, the pure linear growth must be over. **DERIVE** the formula for $t_{\rm ch}$. Then **DERIVE** the formula for the characteristic length $\ell_{\rm ch}$ which is the distance fallen in time $t_{\rm ch}$ assuming $f(v) = 0$.

d) The formula

$$v = v_{\rm ter}(1 - e^{-t/t_{\rm ch}})$$

is a crude approximate solution for velocity in general. **SKETCH** a v-versus-t plot for the approximate solution.

If we have linear drag $f(v) = bv$ (where b) is a constant, then the approximate solution becomes exact. There are cases where linear drag holds: for very low speeds and no turbulence. Find $t_{\rm ch}$ and $\ell_{\rm ch}$ for the linear drag force case. Verify by substitution into the equation of motion (i.e., our Newton's 2nd law application from) that the velocity formula is exact for $f(v) = bv$. A good way to do this is to evaluate with the solution the left-hand

side (LHS) and right-hand side (RHS) of the equation of motion separately. Then observe that LHS and RHS are equal.

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006 qfull 00710 1 3 0 easy math: linear drag force 1

351. The linear drag force magnitude is given by

$$F = bv \ ,$$

where v is the speed of a body and b is constant determined by many things (e.g., body cross sectional area perpendicular to direction of motion, shape, texture). The force points opposite to the direction of motion. Consider a small styrofoam ball of mass m that is falling and is acted on only by **GRAVITY** and the **LINEAR DRAG FORCE**. The styrofoam ball starts falling from **REST** at time zero. Take downward as the positive direction.

a) What is the equation of motion for the styrofoam ball (i.e., what is $\vec{F}_{\text{net}} = m\vec{a}$ applied to the ball)? What is the expression for the acceleration of the styrofoam ball?

b) What is the formula for acceleration at time zero and what is the formula for terminal speed (i.e., the speed when acceleration is zero)?

c) Why is the speed of the styrofoam ball driven to the terminal speed?

d) You measure the terminal speed to be 3.00 m/s. What is the constant b and the time constant (or e-folding time) τ for the styrofoam ball? The styrofoam has mass $m = 0.0100\,\text{kg}$.

e) At what time does the styrofoam ball reach 90 % of its terminal speed?

f) Say that instead of the linear drag force, we had the general power law drag force

$$F = bv^p \ ,$$

where $p > 0$. What is the formula for the terminal speed in this case for the styrofoam ball? What happens to the styrofoam if $p = 0$?

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006 qfull 00720 2 3 0 moderate math: linear drag force 2

Extra keywords: Can't be used on a test until students know exponential derivatives

352. The formula for the velocity of a body accelerating from rest acted on by a constant driving force F_{drive} and the linear drag force (magnitude bv) is

$$v = v_{\text{T}}(1 - e^{-t/\tau}) \ ,$$

where v_{T} is the terminal speed and τ is the time constant or e-folding time. The system is 1-dimensional in the direction of the driving force.

a) What is the formula for v_{T} in this case?

b) What is the general formula for the acceleration of the body as a function of time t in terms of v_{T}, τ, and t?

c) Verify that the formula for v solves the equation of motion (i.e., $\vec{F}_{\text{net}} = m\vec{a}$ applied to the present system) and determine the formula for τ.

d) Although not much used in introductory physics, there is a special word for the derivative of the acceleration with respect to time. It's jerk or jolt or surge or lurch (Wikipedia: Jerk (physics)). At least according to Wikipedia, the symbol for jerk is $\vec{j}$. What is the general formula for the jerk for the body as a function of time t in terms of v_{T}, τ, and t?

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006 qfull 00730 3 3 0 tough thinking: pseudo-rolling friction

Extra keywords: (Ha-114:28p), terminal velocity

353. There is a locomotive of mass $10^5\,\mathrm{kg}$ on a level track. To move itself it can exert a force $F_{\mathrm{loco}} = 5000\,\mathrm{N}$ on the track, and so, of course, the track can exert the same back on the locomotive. These forces are in fact static friction of wheels on track and track on wheels assuming there is no slipping: both forces are parallel to the track: the static friction force on the train points **IN** the direction of train motion. There is also a rolling friction force on the train whose magnitude is by the formula (which I'm just making up)

$$F_{\mathrm{roll\ f}} = Cmv$$

where $C = 2.5 \times 10^{-3}\,\mathrm{s}^{-1}$ is a constant, m is locomotive mass, and v is locomotive speed. Rolling friction is another kind of friction (not static, not kinetic), and it points **OPPOSITE** to the direction of motion. It's sort of due to an object having to roll out of the depression its own weight is causing and to overcome the distortion of the wheels likewise caused by the object's own weight.

a) What is the acceleration of locomotive just at the instant it starts moving from rest? **HINT:** Write down $\vec{F}_{\mathrm{net}} = m\vec{a}$ with the actual forces shown explicitly and specialize for time zero.

b) What is the terminal velocity of the locomotive in the limit that time goes to infinity? **HINT:** Write down $\vec{F}_{\mathrm{net}} = m\vec{a}$ with the actual forces shown explicitly and reflect on what happens as velocity increases.

c) The second law with the forces written explicitly is a differential equation for velocity as function of variable time. A differential equation being an equation for a function that contains derivatives of various orders of the function. To solve a differential equation is to find the function as an explicit function of the variable. Many, many physical problems come down to solving differential equations. In this case, verify that

$$v = v_0 e^{-Ct} + \frac{F_{\mathrm{loco}}}{Cm}(1 - e^{-Ct})$$

is the explicit solution of the differential equation: v_0 is an initial velocity at time zero. What is the differential equation and solution if $F_{\mathrm{loco}} = 0$?

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006 qfull 00740 1 3 0 easy math: drag equation drag solution

354. The drag equation drag law is

$$F_{\mathrm{DE}} = \frac{1}{2}\rho v^2 C_{\mathrm{D}} A = bv^2$$

where v is flow speed, ρ is fluid density, $(1/2)\rho v^2$ is the relative kinetic energy density of the fluid, C_{D} is the drag coefficient (which depends on many factors in general, but the always depends on the shape of the object and probably is often determined empirically), and A is the reference area (for many objects just projected frontal area of the object), and b is a combined coefficient introduced for simplicity.

a) Taking downward as positive, write down Newton's 2nd law for an object of mass m falling under Earth's near-surface gravity with drag acting. Determine the formula for acceleration in the form $a = g[1 - \ldots]$ with v/v_{ter} as one for the terms with a velocity parameter v_{ter} defined appropriately.

b) Now define $z = v/v_{\mathrm{ter}}$ and reformulate the formula for acceleration a as a formula with differential $dt = (v_{\mathrm{ter}}/g)\,dz/(\ldots)$.

c) Integrate the dt expression from part (b) to obtain $t = t(v)$ assuming initial time and velocity are zero. You will need the table integral

$$\int \frac{dz}{1 - z^2} = \mathrm{artanh}(z) \qquad \text{for } z < 1\ .$$

where artanh(z) is the inverse hyperbolic tangent function. Note, artanh is the preferred modern symbol for the inverse hyperbolic tangent function (Wikipedia: Inverse hyperbolic functions: Notation).

d) Invert the expression from part (c) to obtain the function $v(t)$. Note, the hyperbolic tangent function is defined and explicated by

$$\tanh(x) = \begin{cases} \dfrac{e^x - e^{-x}}{e^x + e^{-x}} & \text{in general.} \\ -\tanh(-x) & \text{odd function behavior.} \\ x & \text{for } x \text{ small enough that the function is in the linear regime.} \\ x - \dfrac{1}{3}x^3 + \dfrac{2}{15}x^5 - \dfrac{17}{315}x^7 + \ldots & \text{expansion in small } x. \\ 0.761594\ldots & \text{for } x = 1, \text{ the fiducial scale value between small and large } x \text{ regimes.} \\ \pm(1 - 2e^{-2|x|}) & \text{extreme } \pm x \text{ asymptotic behavior.} \\ \pm 1 & \text{for } x = \pm\infty. \end{cases}$$

What is the terminal velocity (i.e., the velocity as $t \to \infty$)? What is the scale time t_{scale} (i.e, the time when hyperbolic tangent argument is 1)? What is v at the scale time? What is the meaning of the scale time?

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006 qfull 00742 3 3 0 tough math: quadratic drag force, drag equation drag

355. The quadratic drag force magnitude on a body is

$$F_{\text{drag}} = bv^2 = \frac{1}{2}C\rho A v^2 \ ,$$

where C is the dimensionless drag coefficient (which is typically in the range 0.4 to 1 [HRW-104] and is set by all kinds of things like shape and texture of the body), ρ is fluid density, A is the area of body perpendicular to the flow (i.e., the cross sectional area), and v is the magnitude of the relative velocity between fluid and a body. The quadratic drag force is an approximate drag force law that applies to rather unstreamlined bodies moving at relatively high speeds in which the flow downstream the body turns rather turbulent with eddies (HRW-104). The direction of the quadratic drag force is opposite to the direction of motion of the body relative to the fluid. The law can actually be derived from the Bernoulli equation (Serway-403).

a) In solutions for the acceleration of a body subject to the drag force and other forces, the drag force occurs as a term divided by the body mass. Show that the drag force divided by body mass m is inversely proportional to the body density ρ_{body} times a characteristic length ℓ_{ch} for the body along the flow direction. What does this result imply in general?

b) Say there is a constant driving force F_{drive} on a body of mass m in some particular direction in a fluid. The driving force and the drag forces are the only forces acting on the body. Write down the equation of motion (i.e., $\vec{F} = m\vec{a}$) for the body in that direction. What is the speed of the body if the acceleration is 0 in terms of F_{drive} and b? This speed is called the terminal speed v_{T}. Argue that whatever the initial velocity of the body, the body must eventually end up at the terminal speed moving in the direction of the driving force. Given the experience with the linear drag force, when might "eventually" be?

c) Say the system of part (b) starts from time zero with initial velocity v_0. What is the time-zero acceleration a_0 and what is the expression for the velocity as a function of time if a_0 were maintained? What is the time τ when the velocity of the body reaches v_{T} for a_0 maintained?

006 qfull 00750 3 5 0 tough thinking: fluid resistive force

Extra keywords: Fr-153. I hope this is a good final question.

356. Fluid resistive forces are complex and hard to treat in general despite being ubiquitous. Throughout history people have often just dealt with them by relying on experience. An approximate formula with a wide validity for the magnitude of fluid resistive force on rigid bodies is

$$F(v) = Av + Bv^2$$

(Fr-153), where the force always points **OPPOSITE** to the direction of motion, v is body's speed, A and B are constants determined in a very complex way by the fluid and the body's shape, surface nature, and relative density distribution, but not the body's mass. There is no dependence on mass because the fluid resistive force is a contact force. Mass is, of course, important to motion in resistive fluids, but it enters through the gravitational force and $m\vec{a}$ of $\vec{F}_{\rm net} = m\vec{a}$. Note A and B are always greater than or equal to zero.

a) Sketch $F(v)$ as a function of v assuming A and B are both greater than zero. Note, the physical region is where $v \geq 0$: only plot that region. Identify two different velocity regimes. Determine an expression for a characteristic velocity for the transition between small and large velocity regimes. What is the force $F_{\rm tr}$ at the transition velocity? **HINTS:** There is no absolute rule for determining characteristic values. For instance in this case a simple value that allows a yes/no decision about what regime the velocity is in would be appropriate.

b) Say we have a body of mass m in a fluid that obeys our fluid resistive force law. The body is completely surrounded by the fluid and the context of the system is the Earth's surface. There are three forces on the body: the fluid resistive force, gravity, and the buoyancy force $F_{\rm buoy}$. The buoyancy force is an upward contact force caused by the varying pressure in the fluid. A simple Gedanken (thought) experiment allows you to find the formula for the buoyancy force. Imagine a closed surface in the fluid that is exactly shaped like the body in question. The fluid is at rest (e.g., like motionless water in glass): i.e., the fluid is in hydrostatic equilibrium. Gravity pulls down on the fluid inside your closed surface, but that enclosed fluid doesn't move. Now say the body's volume was V and the fluid's density was $\rho_{\rm fluid}$. What is the formula for the buoyancy force on the body? Explain your reasoning.

c) Let us now say that the buoyancy force is negligible—and after all the work in part (b) too. What is the expression for the acceleration a of the body in fluid as a function of **VELOCITY** v under the force of gravity and fluid resistive force? Say the body starts at **REST** at time $t = 0$ and allowed then to move forever as the given forces determine. Sketch its acceleration a and velocity v with as functions of **TIME** t. Take **DOWNWARD** as positive. The sketches are based on your qualitative understanding of what must happen: you don't have any explicit solutions for a and v as functions of time.

d) Continuing with the part (c) case, find the formula for the terminal velocity of the body. You will get two solutions. One is unphysical: which one and exactly why is it unphysical? The unphysical solution certainly satisfies the equation you obtained it from.

e) What are useful formulas for the terminal velocity in cases of very small and very large B values? Note just setting $B = 0$ and $B = \infty$ is too extreme, and doesn't give useful formulas. **HINT:** For $x << 1$,

$$\sqrt{1+x} \approx 1 + \frac{1}{2}x \ .$$

006 qfull 00800 1 3 0 easy math: friction sliding angle and acceleration on incline

357. There is a block of mass m on an incline with inclination angle θ. The incline has static friction coefficient $\mu_{\rm s}$ and kinetic friction coefficient $\mu_{\rm k}$.

a) The block is not sliding. Determine the gravity component along the incline taking downward as positive. Determine the normal force taking outward from the incline as postive. Determine the friction force. **HINT:** Draw a diagram

b) Determine for the angle formula at which the block is reaches the upper limit of the static friction force. What is the angle for $\mu_{\rm s} = 0.5$?

c) Determine for the acceleration formula of the block when sliding. What is the accleration for $\mu_{\rm k} = 0.2$ and $\theta = 60°$?

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006 qfull 00830 2 3 0 moderate math: pig on slide: with and sans friction
Extra keywords: HRW-113-14p

358. A free-spirited pig (let's call him Waldo Pepper) loves sliding down chutes (pronounced shoots).

a) Say there is a frictionless chute and Waldo accelerates down at $5.0\,{\rm m/s^2}$. What is the inclination angle of the chute? **HINT:** Remember that free body diagram.

b) Now the evil magician of physics turns on kinetic friction and Waldo only accelerates down at $3.0\,{\rm m/s^2}$? What is the coefficient of kinetic friction? **HINT:** Update that free body diagram.

c) What's Waldo's mass? Explain your answer. **HINT:** The evil magician of physics is being spiteful.

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006 qfull 01010 1 3 0 easy math: generalized Newton's 2nd law, (Tsiolkovsky) rocket problem

359. Newton's 2nd law in ordinary form is

$$\vec{F}_{\rm net} = m\vec{a}_{\rm cm} \ ,$$

where $\vec{F}_{\rm net}$ is the net force on a system (AKA object, AKA body), the system mass is m, and the sytem acceleration is $\vec{a}_{\rm cm}$. Newton's 2nd law is the basis for determining system's center of mass motion: i.e., its acceleration $\vec{a}_{\rm cm}$, velocity $\vec{v}_{\rm cm}$, and position $\vec{r}_{\rm cm}$ all as functions of time. (Note, we drop the center-of-mass subscript cm hereafter for formula simplicity.) Like all general physical laws, Newton's 2nd law is what is eternally true everywhere: i.e, eternally true everywhere in the classical limit. Other features of the system and its environment that are not determined by physical law are contingent on the history of the system.

In fact, Newton's 2nd law has to be generalized to a rate of momentum change equation since momentum can be changed by a net external force and/or a mass inflow (which is a mass outflow if negative) and usually a change momentum implies a change in acceleration even with zero net external force. Now momentum (i.e., linear momentum, not angular momentum) for a system is defined

$$\vec{p} = m\vec{v} \ .$$

The generalized Newton's 2nd law is

$$\vec{F}_{\rm net} + \vec{v}_{\rm flow}\frac{dm}{dt} = \frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt} = m\vec{a} + \vec{v}\frac{dm}{dt} \ ,$$

where dm/dt is the mass inflow rate which can be positive (i.e., actual inflow) or negative (i.e., actual outflow) and $\vec{v}_{\rm flow}$ is the flow velocity (i.e., the velocity of the inflowing mass). Note, we have assumed there is a single $\vec{v}_{\rm flow}$ for simplicity in our discussion.

Note, the inflow when not part of the system can exert an external force, but it cannot by definition when part of the system since then it can only exert internal forces. In fact, it may be an analysis choice when to consider the inflow as part of the system.

In this problem, we consider the generalized Newton's 2nd law.

a) Under what condition does the generalized Newton's 2nd law reduce to the ordinary Newton's 2nd law?

b) If $\vec{F}_{\text{net}}$ and dm/dt are zero, what can one say about the system momentum and acceleration?

c) If $\vec{F}_{\text{net}}$ is zero, $dm/dt \neq 0$, and $\vec{v}_{\text{flow}} = \vec{v}$, what can one say about the system momentum and acceleration?

d) What is the explicit formula for acceleration?

e) An interesting special case for the generalized Newton's 2nd law is the (Tsiolkovsky) rocket problem (Wikipedia: Tsiolkovsky rocket equation). To explicate, say you have a rocket in empty space with $\vec{F}_{\text{net}} = 0$. It can still be accelerated by thrust: ejecting burnt exhausted fuel opposite the direction of motion. The speed of the exhausted fuel relative to the rocket is called the exhaust speed v_{ex} which is a parameter of the rocket. The mass inflow (i.e., the rate of ejection of exhausted fuel) $dm/dt < 0$ is also a parameter of the rocket. For our analysis of the rocket problem, we will consider only 1-dimensional motion. Specialize the acceleration formula of part (d) for the 1-dimentional case of the rocket problem.

f) What is the velocity of the ejected exhausted fuel in the outside inertial frame (i.e., the inertial frame that the rocket is traveling in) in the formalism of generalized Newton's 2nd law? What is the expression for it this velocity? What does it mean if the velocity is negative/zero/positive?

g) Making use of the parts (e) and (f) results, write down the formula for the acceleration of the rocket simplified as much as possible.

h) Solve the part (f) equation (which is a differential equation) for velocity as a function of mass by integration recalling $a = dv/dt$. Assume the initial mass m_0 and the intial velocity is v_0. What happens to velocity as $m \to 0$? Why must the velocity formula actually fail to be physically real if the velocity gets too large? Why is this failure unlikely in practice.

i) Invert formula found in part (g) to find the amount of fuel $m_{\text{fuel}} = m_0 - m$ ejected as a function of change in velocity $\Delta v = v - v_0$ and m_0 desired.

Chapt. 7 Kinetic Energy and Work

Multiple-Choice Problems

007 qmult 00100 1 4 5 easy deducto-memory: essence energy definition 1

360. "Let's play *Jeopardy*! For $100, the answer is: Admitting that all short definitions of it are inadequate, one can suggest that it is the conserved essence of structure and transformability. 'Converved' means that the quantity that is neither created nor destroyed. In this case, all forms of the quantity must be summed to have conservation in general though in special cases conservation may hold for a subset of forms. 'Essence of structure' means that the quantity is a general measure of structure, but it does not describe details of structure. 'Structure' is used here in a very general sense to mean the ordering of objects, particles, fields, and motions. 'Essence of transformability' means that the quantity can be transformed into different forms of itself and that all changes in the amounts of the forms result in changes in structure. If you have some of the quantity, transformations are possible. Knowing which ones occur requires more detailed information."

What is _____, Alex?

a) force b) momentum c) angular momentum d) potential e) energy

007 qmult 00110 1 4 4 easy deducto-memory: change energy definition 2

361. "Let's play *Jeopardy*! For $100, the answer is: A useful, but incomplete definition, is the quantified capacity for change."

What is _____, Alex?

a) momentum b) force c) potential d) energy e) aspiration

007 qmult 00120 1 4 5 easy deducto-memory: work energy definition 3
Extra keywords: physci KB-66

362. "Let's play *Jeopardy*! For $100, the answer is: It has been described as that property of something that enables that something to do work."

What is _____, Alex?

a) inevitability b) noise c) tension d) gravity e) energy

007 qmult 00130 1 1 3 easy memory: conservation of energy 1

363. The principle of conservation of energy is that energy is never:

a) adequately defined. b) destroyed, but can be created. c) created or destroyed.
d) created, but can be destroyed. e) destroyed.

007 qmult 00132 1 1 4 easy memory: conservation of energy 2
Extra keywords: physci

364. In the physics, the conservation of energy means energy:

a) shouldn't be wasted on cars. b) is never destroyed. c) is never created.
d) is never created or destroyed. e) is perpetually created.

007 qmult 00134 1 1 1 easy memory: energy conservation implication

Extra keywords: physci KB-72

365. The total energy of a closed system (i.e., a system which nothing enters or leaves) is:

a) conserved. b) annihilated. c) created. d) combusted. e) eviscerated.

---

007 qmult 00136 1 1 2 easy memory: universe and conservation of energy

Extra keywords: physci KB-92-5

366. If the universe as a whole is a closed system, then current physical law tells us that its total energy is:

a) 30 J. b) conserved. c) not conserved. d) only kinetic energy.
e) sometimes conserved.

---

007 qmult 00140 1 1 1 easy memory: dimensions of energy

367. The physical dimensions of energy are:

a) ML^2/T^2. b) ML/T^2. c) ML^2. d) ML^2/T. e) M/T^2.

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007 qmult 00142 1 1 1 easy memory: unit of energy, the joule

Extra keywords: physci

368. The standard SI unit of energy and of work is the:

a) joule (J). b) newton (N). c) kelvin (K). d) bassingthorp (B).
e) trufflehunter (T).

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007 qmult 00180 2 1 1 moderate memory: energy necessity and sufficiency

Extra keywords: physci

369. That one has enough energy for a certain job or transformation that requires energy E is a _____ condition, but **NOT** a _____ condition for the job or transformation.

a) necessary; sufficient b) sufficient; necessary c) inevitable; necessarily so
d) harmonious; ceremonious e) forbidden; given

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007 qmult 00182 1 4 5 easy deducto-memory: energy and money

Extra keywords: physci

370. "Let's play *Jeopardy*! For $100, the answer is: Because of its protean nature, energy is very much like this thing in everyday human life which, however, unlike energy is not conserved."

What is/are _____, Alex?

a) furs b) assignats c) shells d) gold e) money

---

007 qmult 00200 1 4 2 easy deducto-memory: work defined

371. "Let's play *Jeopardy*! For $100, the answer is: In physics, it is a macroscopic process of energy transfer."

What is _____, Alex?

a) energy b) work c) force d) weight e) sloth

---

007 qmult 00210 1 1 2 easy memory: differential work formula

372. The differential work formula is:

a) $dW = \vec{F}\,d\vec{s}$. b) $dW = \vec{F} \cdot d\vec{s}$. c) $dW = \vec{F}/d\vec{s}$. d) $dW = F\,ds$.
e) $dW = \vec{F} \times d\vec{s}$.

---

007 qmult 00212 1 1 4 easy memory: integral work formula

373. The formula

$$W = \int \vec{F} \cdot d\vec{s}$$

is:

a) never zero. b) always zero for a closed path. c) always zero.
d) the integral work formula. e) the differential work formula.

---

007 qmult 00220 1 1 3 easy memory: work formula for a constant force 1
Extra keywords: This was for weak-on-vectors class

374. A constant force $\vec{F}$ (with magnitude F) acts on a body while that body moves a through a displacement $\vec{S}$ (with magnitude S). Let $F_{\parallel}$ and $F_{\perp}$ be the force components parallel and perpendicular, respectively, to the direction of motion. The work W done on the body by the force is given by:

a) $W = \vec{F}S$. b) $W = F_{\perp}S$. c) $W = F_{\parallel}S$. d) $W = F_{\parallel}F_{\perp}S$.
e) $W = \vec{F}F_{\parallel}F_{\perp}S$.

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007 qmult 00222 1 1 3 easy memory: work formula for a constant force 2
Extra keywords: For the vector and dot product literate

375. A constant force $\vec{F}$ acts on a body while that body moves a distance $\Delta\vec{r}$. The work W done on the body by the force is given by:

a) $W = \vec{F}/\Delta\vec{r}$. b) $W = \vec{F}$. c) $W = \vec{F} \cdot \Delta\vec{r}$. d) $W = \vec{F} \cdot \vec{F} \cdot \Delta\vec{r}$.
e) $W = \vec{F} \cdot \Delta\vec{r} \cdot \Delta\vec{r}$.

---

007 qmult 00280 1 3 1 easy math: work lifting 100 kg load
Extra keywords: physci KB-95-3

376. How much work is done by a weightlifter lifting a 100 kg load straight upward 10 m without acceleration?

a) 9800 J. b) 100 J. c) 1000 J. d) 10 J. e) 980 J.

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007 qmult 00282 1 3 1 easy math: work lifting ostrich
Extra keywords: physci KB-93-21

377. The work done by the lifting force of a person lifting a 30 kg ostrich to a height of 30 m without acceleration is about:

a) 9000 J. b) 900 J. c) 300 J. d) 3 J. e) 4500 J.

---

007 qmult 00284 1 5 5 easy thinking: no macroscopic work done
Extra keywords: physci KB-95-1

378. A person holds 10 kg grouse at 2.0 m above the ground for 30 s. How much macroscopic net work is done by the person on the grouse?

a) 600 J. b) 20 J. c) 300 J. d) 60 J. e) 0 J.

---

007 qmult 00300 1 1 3 easy memory: kinetic energy definition
Extra keywords: physci

379. Kinetic energy is:

a) the energy of **POSITION** with formula $K = mgy$. b) the energy of **MOTION** with formula $K = mgy$. c) the energy of **MOTION** with formula $K = (1/2)mv^2$.
d) the energy of **POSITION** with formula $K = (1/2)mv^2$. e) heat energy.

---

007 qmult 00310 1 1 3 easy memory: work-kinetic-energy theorem

380. The work-kinetic-energy theorem is:

a) $K = \frac{1}{2}mv^2$. b) $\Delta E = W_{\text{non}}$. c) $\Delta K = W$. d) $\Delta K = \frac{1}{2}W$.
e) $\Delta K = \frac{1}{2}mv^2$.

---

007 qmult 00370 2 5 4 moderate math: friction killing K
Extra keywords: physci KB-95-9

381. A moving object has initial $K = 100\,\text{J}$ and is subjected to a friction force of magnitude 2 N and no other forces. How far does the object go before coming to a stop?

a) 100 m. b) 2 m. c) 1000 m. d) 50 m. e) 0 m.

---

007 qmult 10410 1 3 5 easy math: work done pushing a block on frictionless surface fullmult

382.* You are pushing a 50 kg block across a frictionless surface—which is no mean feat in itself.

i) You exert 100 N of force in the direction of motion and push the block for 10 m. How much work have you done on the block?

ii) Assuming you start the block from **REST** and no other forces do work, what is the final speed of the block?

a) (i) 1000 J (ii) 4 m/s. b) (i) 10 J (ii) 5 m/s. c) (i) 10 J (ii) 10 m/s.
d) (i) 1000 J (ii) 10 m/s. e) (i) 1000 J (ii) 6.3 m/s.

Full-Answer Problems

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007 qfull 00100 1 3 0 easy math: work defined fully

383. The differential expression that defines work in physics is

$$dW = \vec{F} \cdot d\vec{s}\,,$$

where dW is the differential amount of work done on a object by the force $\vec{F}$ (which not net force in general) dot-producted with a differential displacement of the object's center of mass $d\vec{s}$ while the force is acting.

Work is actually an energy transfer process. The component of the force $\vec{F}$ in the direction of $d\vec{s}$ has helped move the object in that direction. The energy from the force has thereby helped accelerate the object giving it velocity (and thereby kinetic energy) and/or helped move it in a field of force thereby giving the object field of force energy.

Why is work a energy transfer process? Recall Newton's 3rd law (or some generalization of that as needed): for every force there is an equal and opposite force. The object does equal and opposite work on the force source (another object or field of force) causing force $\vec{F}$. The force source loses kinetic energy and/or field energy. Losing field energy can be vaguely described as relaxing from a state of "tension" (e.g., a spring being sprung). Given the 3rd law, the work done the force source is the negative of the work done on the object. Of course, if the work done on the object is negative, the work done on the force source is positive. The upshot is that in the work process energy is conserved. If energy is the same stuff everywhere (and it is as far as we know), since it is conserved in every work process it is conserved overall if you account for all forms of energy.

However, it has to said that general relativity does not guarantee energy conservation in an ordinary sense in processes far from the classic limit and there are cases where energy is not converved in an ordinary sense as far as we know (e.g., perhaps gravitational waves and certainly in expansion of the universe: e.g., Penrose-464–469; Carroll-120). General relativity (our best theory of gravity and motion under gravity) guarantees itself and not our classical and quantum mechanical notions of energy conservation. It does have generalization for energy conservation (the energy-momentum conservation equation: e.g., Carroll-120), but it does not seem nearly as useful as the classical and quantum mechanical energy conservation concept.

Work can be macroscopic or microscopic. For example, if you push on a block that moves, you (i.e., the force source) have done work on the block which could become block kinetic energy usually with some downstream friction work on the ground transferring some of the

orginal transferred energy into waste heat (energy). What if you push on a wall and using up body chemical energy? You are not transferring macroscopic energy to the wall, but there are all kinds of transfer of chemical energy and kinetic energy to waste heat in your body upstream from the hand-wall interface. And since the wall is microspically flexing, microscopic energy is being transferred to the wall where downstream from the hand-wall interface all kinds of transfer processes are converting microscopic kinetic energy into waste heat.

Do inertial forces (forces due relations of motions to spacetime structure) obey the 3rd law and do they conserve energy? Actually, this is a matter of perspective it seems. If you treat an inertial force that does work as due to a field that has the potential energy property, then that inertial force obeys the 3rd law and conserves energy. This seems to yours truly as the good perspective since it unifies the treatment of ordinary forces and inertial forces. But if you take the perspective that inertial forces do not obey the 3rd law and are not due to any field, then energy conservation holds in a true inertial frame perspective and not in the perspective of a non-inertial frame converted to an inertial frame using inertial forces. Your truly does not know which perspective is used in practice. Maybe both.

a) What is the integral definition of work done. **HINT:** Use an integral sign and leave the endpoints unspecified.

b) What is the unit of energy (and therefore of work) and what is its relation to the base units?

c) Forces that do not do work can still do things. They can be constraint forces: forces that contrain the motion of an object to a particular path, but are always perpendicular to the direction of motion: i.e., $\vec{F} \cdot d\vec{s} = 0$ always. Such forces can change momentum (though they do not do this always) but they do no work. A constraint force is a contraint force by what it does, not by what it is intrinsically. What is the contraint force on a block sliding down an incline.

d) There are forces that are not contraint forces that do not do work and still do things. For example the force of a magnetic field on a charged particle (i.e., the magnetic part of the Lorentz force)

$$\vec{F} = q\left(\vec{v} \times \vec{B}\right) \ ,$$

where q is the particle charge, $\vec{v}$ is the particle velocity, and $\vec{B}$ is the magnetic field. Another example is the Coriolis force (an inertial force used in a rotating frame converted to an inertial frame by using inertial forces)

$$\vec{F} = -2m\left(\vec{\omega} \times \vec{v}\right) \ ,$$

where m is the mass of the object the force is exerted on, $\vec{\omega}$ is the angular velocity of the rotating frame of reference (it points along the axis of rotation in the direction given by a right-hand rule: curl your fingers of your right hand around the axis and your thumb points in the axis direction), and $\vec{v}$ is the velocity of the object relative the rotating frame. Both the Lorentz force and the Coriolis force can change momentum, but cannot do work. Why not?

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007 qfull 00200 1 3 0 easy math: derivation of work-kinetic energy theorem

384. We will now the derive the work-energy theorem and kinetic energy little-bitty steps. The work-energy theorem is

$$\Delta K = W_{\rm net} \ ,$$

where the net $W_{\rm net}$ (or just W when one knows one means net work) is the work done on an object by the net external force acting on the object and ΔK is the change in the object's center-of-mass kinetic energy.

a) Write down the differential definition of work.

b) Write down the integral definition of work with unspecified endpoints.

c) Write down the formula for the work between points 1 and 2 done by net external force on an object and make use of Newton's 2nd law to rewrite this formula in terms of the object's center-of-mass acceleration.

d) Prove that

$$\vec{a} \cdot \vec{v} = \frac{1}{2}\frac{dv^2}{dt} \ .$$

e) Since the object moves in time $d\vec{s} = \vec{v}\,dt$. Use this to prove the work-energy theorem and define kinetic energy by an appropriate formula.

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007 qfull 00300 1 3 0 easy math: relative kinetic energy and center of mass 1

385. Recall the kinetic energy formula for an object

$$K = \frac{1}{2}mv^2 \ ,$$

where m is object mass and v is object center-of mass velocity. The object could have internal motions and therefore internal kinetic energy, but we are not concerned with that at the moment. However, kinetic energy is a reference frame dependent quantity because it depends on the velocity of the reference frame relative to other reference frames.

However, what is important for the evolution of a system of objects that are to some degree independent of the their environment is their kinetic energy relative to an origin that is the system center of mass or is at rest with respect to the system center of mass. To make the last statement plausible, consider a system of objects confined to rigid box which is fixed to an inertial frame. The objects inside only interact with each other and the box. Only their kinetic energy relative to the box is important for their motions relative to the box. The box could be moving any velocity relative an outside inertial frame. So the kinetic energy associated with the overall motion of the box is irrelevant and, in fact, could be set an arbitrarily large size and most that kinetic energy would have no physical relevance at all since it mostly cannot be changed. To give a concrete example, say you are in train car moving at 20 m/s relative to the ground in a straight line. That velocity relative to the ground is physically relevant for train-ground intereactions, but is probably mostly irrelevant motions inside the train car. But say you decide to measure the train car velocity relative to an arbitrary origin moving away from the direction of the motion of the train car at 1000 km/s. You can calculate the kinetic energy of the train car and of you with respect to that origin, but except for contribution from the 20 m/s motion with respect to the ground, it is physically meaningless energy that you create out of nothing and it can do nothing.

In astrophysics, the center of mass of system of gravitationally-bound interacting astro-bodies (e.g., planet-moon systems, planetary systems, star clusters, galaxies, galaxy clusters) is a good choice for the origin of reference frame because the center of mass is close the center of the gravitational attraction and the gravitational attraction dominates structure. In the terrestrial environment, a convenient origin at rest with respect the center of mass is usually chosen since the center of mass is usually not the center of attraction/repulsion for any force. In a terrestrial environment, the down force of gravity and the up force of pressure tend to dominate structure. In our train car example, the train car has a center of mass, but for motions inside the train car, you could set the origin at one end of the train car on the floor.

a) Consider a system of particles (which could be astro-bodies). Their total kinetic energy relative to an arbitrary origin is

$$K = \sum_i \frac{1}{2}m_i v_i^2 = \sum_i \frac{1}{2}m_i(\vec{v}_{\rm cm} + \Delta\vec{v}_i) \cdot (\vec{v}_{\rm cm} + \Delta\vec{v}_i) \ ,$$

where the sum is over all particles, m_i is particle mass, $\vec{v}_i$ is particle velocity relative to the arbitrary origin, $\vec{v}_{\rm cm}$ is the center of mass velocity relative to the arbitrary origin, and $\Delta\vec{v}_i$ is particle velocity relative to the center of mass. Prove that

$$K = K_{\rm cm} + K_{\rm rcm} = \frac{1}{2}mv_{\rm cm}^2 + \sum_i \frac{1}{2}m_i\Delta v_i^2 \ ,$$

where $K_{\rm cm}$ is the center of mass kinetic energy relative to the arbitrary origin, $K_{\rm rmc}$ is kinetic energy relative to the center of mass, and $m = \sum_i m_i$ is the total mass of the system. The result shows that using the center of mass as the origin for the system of particles allows a clean separation external and internal kinetic energies.

b) What is the significance of $K_{\rm cm}$ to the internal motions of the system if the system does not interact with environment at all or the arbitary origin is moving at some fantastic arbitrary velocity compared to all the velocities of the environment relative to the center of mass. What if the significance of $K_{\rm cm}$ to the internal motions of the system if the system interacts moderately with the environment and the arbitrary origin is relatively nearby by and is, in fact, moving at the center of mass velocity of the combined environment and system.

c) What is the kinetic energy of the center of mass $K_{\rm cm}$ if the arbitrary origin is at rest with respect to the center of mass? When would arbitrary origins like this be useful.

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007 qfull 00302 1 3 0 easy math: relative kinetic energy and center of mass 2

386. In this problem, we consider relative kinetic energy.

a) Consider a system of particles (which could be astro-bodies). Their total kinetic energy relative to an origin that is physically significant for the system and environment of the system is

$$K = \sum_i \frac{1}{2}m_i v_i^2 = \sum_i \frac{1}{2}m_i(\vec{v}_{\rm cm} + \Delta\vec{v}_i)\cdot(\vec{v}_{\rm cm} + \Delta\vec{v}_i) \ ,$$

where the sum is over all particles, m_i is particle mass, $\vec{v}_i$ is particle velocity relative to the origin, $\vec{v}_{\rm cm}$ is the system center of mass velocity relative to the origin, and $\Delta\vec{v}_i$ is particle velocity relative to the system center of mass. Prove that

$$K = K_{\rm cm} + K_{\rm rcm} = \frac{1}{2}mv_{\rm cm}^2 + \sum_i \frac{1}{2}m_i\Delta v_i^2 \ ,$$

where $K_{\rm cm}$ is the center of mass kinetic energy relative to the origin, $K_{\rm rmc}$ is kinetic energy relative to the center of mass, and $m = \sum_i m_i$ is the total mass of the system. The result shows that using the center of mass as the origin for the system of particles allows a clean separation external and internal kinetic energies.

b) Consider an arbitrary origin 1 that is moving at arbitrary velocity that is completely unrelated to any physical process in the universe. The velocity of the system center of mass to origin 1 is $\vec{v}_{\rm cm1}$. What is the formula total kinetic energy K_1 using origin 1? **HINT:** Make use of part (a).

c) Consider an arbitrary origin 2 that is moving at arbitrary velocity that is completely unrelated to any physical process in the universe. The velocity of the system center of mass to origin 2 is $\vec{v}_{\rm cm2}$. What is the formula total kinetic energy K_2 using origin 2? **HINT:** Make use of part (a) again.

d) What is formula for the energy difference betwen K_2 and K_1? What physical significance does it have? Why does it have this significance?

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007 qfull 00310 1 3 0 easy math: absolute velocity and kinetic energy

387. Is there in any sense an absolute velocity and an absolute kinetic energy? Yes. Take any point in the observable universe and imagine a large sphere surrounding it. If the sphere is large enough, the overall density can be well approximated as constant and the selected point is the center of mass of the sphere. This kind of center of mass participates in the mean expansion of the universe. So at every point in the observable universe an inertial frame exists that is said to be comoving with the mean expansion of the universe: these inertial frames are called comoving frames. (The comoving frames are also not rotating with respect to the bulk matter of observable universe since that kind of rotation is absolute rotation and gives rise to rotational inertial forces.) The upshot is that velocity and kinetic energy of an object measured with respect to the local comoving frame (the one centered right where the object is) can be called absolute velocity and kinetic energy.

In large-scale structure calculations those absolute velocities and kinetic energies are always known. Modeling of the local observable universe is able to determine the actual values for local universe fairly well (e.g., Tully et. al. 2014).

The absolute kinetic energy of the observable universe is all important for the formation of the large scale structure down to the smallest structures. But oddly enough it makes a negligible contribution (via $E = mc^2$) to the overall gravitational/inertial mass that determines the overall expansion of the universe along with the dark energy (whatever that actually is). The overall gravitational/inertial mass is pimarily the rest mass energy of the baryonic matter (i.e., the rest mass energy of protons, neutrons, and electrons) and the dark matter (whatever that actually is).

Another odd thing about the absolute kinetic energy is that it decreases in an abolute sense: it vanishes without being converted into any other form of energy. So the observable universe does not actually conserve energy overall in an ordinary sense. This absolute non-conservation of energy has no (or almost no effect) on gravitationally bound systems (e.g., galaxy clusters and anything smaller), but it important for large-scale structure and it is included in calculations of that. The absolute non-conservation is allowed by general relativity which does not guaranteed conservation of energy in an ordinary sense. General relativity just guarantees itself.

How the does the absolute kinetic energy just vanish? First, note that the distance $r(t)$ between any two points participating in the mean expansion of the universe (comoving points which are just the centers of comoving frames defined above) scales according to

$$r(t) = a(t)r_0 \ ,$$

where r_0 is the distance at cosmic present and $a(t)$ is the cosmic scale factor which is 1 at cosmic present by convention. The cosmic scale factor is the solution of the mean expansion of the universe from general relativity: it grows with time: $a(t) < 1$ in cosmic past, a(t)=1 at cosmic present; $a(t) > 1$ in cosmic future. Note, $a(t)$ is not acceleration: it just has the same conventional symbol. The peculiar velocities of galaxies etc. are superimposed on the mean expansion. Note,

$$1) \quad r(t) = a(t)r_0 \qquad 2) \quad \dot{v} = \dot{a}(t)r_0 \qquad 3) \quad \frac{v}{r} = \frac{\dot{a}}{a} \equiv H \qquad 4) \quad v = Hr \ ,$$

where the last formula is Hubble's law where v and r are measured at one instant in cosmic time and H is the Hubble parameter (which is the relative rate of expansion of the universe) at that instant in cosmic time. Now v is recession velocity which is a velocity between inertial frames not relative to an inertial frame. Recession velocities grow as you go to the remote oservable universe and exceed the speed of light eventually, but this is not a violation of special relativity since they are not velocities relative to local inertial frames. Now imagine a particle free streaming through the observable universe unaffected by any forces other than the average of those that cause the expansion of the obserable universe itself. The particle has ordinary velocity $v_{\rm par}$ relative to a local comoving frame. Say that velocity takes differential distance dr to an new local comoving frame. In the new comoving frame it has velocity

$$v'_{\rm par} = v_{\rm par} - dv = v_{\rm par} - H\,dr$$

where we have used Hubble's law. Note, we can mix ordinary and recession velocities in the classical limit. Now

$$1)\quad dv_{\rm par} = -H\,dr \qquad 2)\quad dv_{\rm par} = -\frac{\dot a}{a}\,dr = -\frac{da}{a}\frac{dr}{dt} = -\frac{da}{a}v_{\rm par} \qquad 3)\quad \frac{dv_{\rm par}}{v_{\rm par}} = -\frac{da}{a}\ .$$

The last expression is a differential equation for $v_{\rm par}$ in terms of a taken as in independent variable. The solution gives gives how $v_{\rm par}$ evolves as the particle free streams through space. Solving the differential equation is part of the problem for the student below. The upshot of the solution is that the particle loses velocity and kinetic energy as it free streams. Where does that kinetic energy go? It is just gone. Some people argue that it has gone somewhere, but where is that and what does it do there and can it ever be recovered? The are no answers to those questions. In the expansion of the universe dictated by general relativity plus various assumptions, that kinetic energy has just vanished. Particles in gravitationally-bound systems do not free stream. However, the largest gravitationally bound systems themselves often free stream (i.e., their centers of mass do), and thus their center of mass kinetic energy decreases with the last energy vanishing.

a) Solve the differential equation for the velocity of a free streaming particle:

$$\frac{dv}{v} = -\frac{da}{a}\ ,$$

where we have dropped the subscript par for simplicity. Only give a proportionality solution and recall $a = a(t)$. **HINT** The solution is easy. Just integrate both sides of the equation and simplify.

b) Given the solution to part (a), what is the proportionality solution for the kinetic energy of a free streaming particle.

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007 qfull 00410 1 3 0 easy math: work done pushing a block on frictionless surface

388. You are pushing a 60 kg block across a frictionless surface—which is no mean feat in itself.

a) You exert 100 N of force in the direction of motion and push the block for 30 m. How much work have you done on the block? Show how you got your answer.

b) Assuming you start the block from **REST** and no other forces do work, what is the final kinetic energy and velocity of the block? Show how you got your answer.

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007 qfull 00510 1 3 0 easy math: work done and power on block on incline

Extra keywords: (Ha-157:16e)

389. Power in physics is the rate of energy transferred or transformed per unit time. This is a general definition independent of the where the energy is coming from and where it is going too. If work is the energy transfer process, the differential work formula $dW = \vec F \cdot d\vec s$ (where dW is differential work done on a object by external force F and $d\vec s$ is differential center-of-mass displacement) leads immediately to the power formula for work:

$$P = \frac{dW}{dt} = \vec F \cdot \frac{d\vec s}{dt} = \vec F \cdot \vec v\ ,$$

where $\vec v$ is the velocity (i.e., the center-of-mass velocity) of a displaced object. The MKS unit of work is the joule per second (J/s) which has its own name: the familar watt (with symbol W).

Now that we've preambled, there is a frictionless block of ice of 50 kg on an incline with inclination angle $\theta = 30°$. A worker pushes the block straight up the incline at a **CONSTANT** velocity of 3.0 m/s.

a) What is the power output (i.e., rate of work done) by the worker?

b) What is the power output by gravity?

c) What is the power output by the normal force? Why?

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007 qfull 00610 3 3 0 tough math: variable force, path integral

390. There is a force (a force field) given by

$$\vec{F} = (a_x x^2, a_y y^3) \ ,$$

where a_x and a_y are constants. Through this field an object moves following a path given by

$$\vec{r} = (b_x t, b_y t^3) \ ,$$

where b_x and b_y are constants and t is a path parameter that we might as well call time. What is the general expression for work done starting from time zero. **HINT:** You will have to do an integral. But to do it, you'll have to change the integration variable from $\vec{r}$ to t.

Chapt. 8 Potential Energy and Mechanical Energy

Multiple-Choice Problems

008 qmult 00100 1 1 2 easy memory: potential energy definition
Extra keywords: physci

391. Potential energy is:

a) the energy of position: it exists for nonconservative forces.
b) the energy of position: it exists for conservative forces.
c) the energy of motion: its formula is $U = (1/2)mv^2$.
d) the energy of position: its formula is $U = (1/2)mv^2$.
e) heat energy.

008 qmult 00110 2 1 3 moderate memory: paths and a conservative force

392. The work done by a conservative force on an object while the object moves on a path between two endpoints is:

a) **INDEPENDENT** of the path and endpoints.
b) **DEPENDENT** on the path.
c) **INDEPENDENT** of the path between the endpoints.
d) **DEPENDENT** on the path, but **NOT** on the endpoints.
e) equal to the path length.

008 qmult 00120 1 4 1 easy deducto-memory: general potential energy formula

393. "Let's play *Jeopardy*! For $100, the answer is: $\Delta U = -W$."

a) What is the formula relating **POTENTIAL** energy change in a conservative force field to work done by the conservative force (i.e., what is the general potential energy formula), Alex?
b) What is Faraday's law, Alex?
c) What are capacitors, Alex?
d) What is ... no, no wait ... what is unicorn circular motion, Alex?
e) What is the formula relating **KINETIC** energy change in a conservative force field to work done by the conservative force (i.e., what is the work-kinetic-energy theorem), Alex?

008 qmult 00130 1 4 5 easy deducto-memory: potential energy

394. "Let's play *Jeopardy*! For $100, the answer is: Energy X for a force is an energy type defined, not by its particular intrinsic nature, but because its value for a body is set by the body's location in space. So energy X is a position energy—and probably should have been called that—but it's too late to change centuries of tradition. It is always true that in any real physical case of energy X, the energy is by its nature some kind of field energy: e.g., electric field energy, magnetic field energy, gravitational field energy. (Actually, gravitational field energy is a very special case as told to us by general relativity: it is encoded in spacetime in such way that is non-localizable: you cannot say there is so much energy here or there and it does not gravitate or provide inertia in an ordinary way. This is very different from the electromagnetic field for which there is definite energy density: there is always definite amount of field energy in any volume.) Now a particular field energy may be a potential energy or not depending on the actual

system considered. For example, the magnetic field depending on the system may or may not have an associated potential energy, but it always has a definite field energy density. Energy X for some unreal imagined kind of force does not have any fundamental explanation—unless one imagines one."

What is _____, Alex?

a) polecat b) pole energy c) potentate energy d) potent energy
e) potential energy

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008 qmult 00140 1 1 5 easy memory: conservative force and system

395. Whether a force is conservative or not depends not only on the fundamental nature of the force, but also on the _____ which is being considered. For example, the electric force is usually mentioned as a conservative force, but there are _____ in which it is not: for example, those in which it is generated by the Maxwell-Faraday's law. For another example, the magnetic force is often mentioned as a non-conservative force, but there _____ in which it is: for example, magnetic dipoles are subject to a conservative magnetic force.

a) force/forces b) horse/horses c) law/laws d) motion/motions
e) system/systems

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008 qmult 00142 1 1 4 easy memory: conservative force examples

396. Two forces that are conservative in ordinarily-thought-of systems are:

a) power and might. b) gravity and kinetic friction. c) gravity and work.
d) gravity and the linear (or spring) force. e) work and the linear (or spring) force.

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008 qmult 00144 1 1 1 easy memory: nonconservative force example

397. An example of nonconservative force is:

a) kinetic friction. b) gravity. c) the linear force. d) work. e) power.

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008 qmult 00180 1 4 2 easy deducto-memory: energy and heat
Extra keywords: physci KB-73

398. British American Benjamin Thompson (1753–1814)—ennobled as Count Rumford—while employed as director of the Bavarian arsenal, noticed that in boring cannon—but not in causing cannon ennui—that the boring motion and friction seemed to produce unlimited amounts of heat. He concluded:

a) heat was a substance of which there could only be so much of in any object.
b) that heat was somehow generated by motion and friction. This conclusion eventually led to the recognition of heat as another form of energy that could be converted from or converted into other forms of energy (e.g., mechanical or chemical energy) and to the concept of conservation of energy.
c) that heat had no relation to motion and friction and was somehow spontaneously generated by cannon.
d) that cannon could be the plural of cannon.
e) that the biergartens in Munich were much better than the taverns in Boston and that Sam Adams, patriot-founding-father notwithstanding, could have learnt a thing or two about brewing beer.

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008 qmult 00200 1 4 1 easy deducto-memory: work-mechanical-energy theorem

399. "Let's play *Jeopardy*! For $100, the answer is: $\Delta E = W_{\text{nonconservative}}$."

What can be described as the _____, Alex?

a) work-mechanical-energy theorem b) work-kinetic-energy theorem
c) potential-energy-work formula d) work-potential-energy theorem
e) kinetic energy formula

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008 qmult 00210 1 1 4 easy memory: mechanical energy conservation
Extra keywords: physci

400. Mechanical energy is the sum of kinetic energy and potential energy. It is a conserved quantity:

a) always.
b) whenever it has both kinetic and potential energy components.
c) if all the forces that do work are **NONCONSERVATIVE**.
d) if all the forces that do work are **CONSERVATIVE**.
e) whenever it is positive.

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008 qmult 00214 1 1 1 easy memory: conservation of mechanical energy 2

401. Interpreting the symbols in the most **STRAIGHTFORWARD WAY** and without mental qualifications (e.g., this is really the inverse or the absolute value of a quantity or a change in a quantity or this is for a special case), the formula for conservation of mechanical energy for a particle or a system that can be treated as a particle (i.e., is described by its center of mass behavior) is:

a) $\Delta K = -\Delta U$. b) $\Delta K = \Delta U$. c) $K = U$. d) $K = 1/U$. e) $\Delta K = -1/\Delta U$.

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008 qmult 00220 1 1 4 easy memory: workless constraint forces

402. Frequently, in conservation-of-mechanical energy problems, one encounters non-conservative forces that guide the motion and cause accelerations. Mechanical energy is conserved because these _____ do work because they are always _____ to the direction of motion. Actually, conservative forces can also be _____ when they are _____.

a) work-doing constraint forces; parallel
b) work-doing constraint forces; perpendicular
c) workless constraint forces; parallel
d) workless constraint forces; perpendicular
e) worthless unconstrained forces; peculiar

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008 qmult 00270 1 3 5 easy math: dog drops brick mech. energy conserved
Extra keywords: physci

403. A brick has mass 1 kg. A dog—from a joke that yours truly will tell you someday—drops the brick (which it was holding in its mouth or, one might say, with its jowl) 1 m to the ground. What is the kinetic energy of the brick just before it hits the ground? **HINT:** The calculator is superfluous.

a) 9.8 watts. b) 9.8 gems. c) 9.8 newtons. d) 9.8 jowls. e) 9.8 joules.

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008 qmult 00272 1 3 3 easy math: dog drops brick mech. energy conserved 2
Extra keywords: physci

404. A brick has mass 2.0 kg. A dog—from a joke that I'll tell you someday—drops the brick (which it was holding in its mouth or, one might say, with its jowl) 2.0 m. The brick started from rest and air drag is negligible. What is the kinetic energy of the brick just before it hits the ground?

a) 9.8 J. b) 19.6 J. c) 39.2 J. d) about 50 J. e) about 160 J.

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008 qmult 00274 2 5 5 moderate thinking: girl on a swing K/U
Extra keywords: physci KB-95-19

405. A girl on a swing oscillates between being 2 m off the ground where she is _____ and 1 m off the ground where her speed is a _____. Non-conservative forces do **NO** work. What is her maximum speed?

a) moving; minimum; 0 m/s.
b) at rest; minimum; 0 m/s.
c) at rest; maximum; 1 m/s.
d) at rest; maximum; 2.4 m/s.
e) at rest; maximum; 4.4 m/s.

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008 qmult 00280 1 1 3 easy memory: bead on wire calculation

406. A bead is strung on a rigid, frictionless wire which is the near-Earth environment. At time zero, the bead height is $y_0 = 10.0\,\mathrm{m}$ and the bead speed is $v_0 = 15.0\,\mathrm{m/s}$. The bead then moves along the wire. What is the bead spead when its height is $y = 17.0\,\mathrm{m}$?

a) 8.96 m/s. b) 9.06 m/s. c) 9.37 m/s. d) 9.57 m/s. e) 9.76 m/s.

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008 qmult 00330 1 1 1 easy memory: turning points in a potential well

407. An object is trapped and moving around in some kind of potential well: it's a bound particle we'd say. What are those special points called where the kinetic energy of the particle momentarily goes to zero? Why are they so called?

a) Turning points—so called because when a particle reaches one, it must come to rest and reverse direction.
b) Stable static equilibrium points—so called because when a particle reaches one it stops.
c) Stable static equilibrium points—so called because when a particle reaches one it accelerates.
d) Unstable static equilibrium points—so called because when a particle reaches one it accelerates.
e) Turning points—so called because of the film "The Turning Point" starring Shirley Maclaine, Anne Bancroft, and Mikhail Barishnykov.

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008 qmult 00350 1 4 5 easy deducto-memory: equilibria defined

408. "Let's play *Jeopardy*! For $100, the answer is: In mechanics, it is a position where the net external force on a body is zero. Thus, the body has zero acceleration. If the body is at rest in an inertial reference frame of interest, the equilibrium is static relative to that frame."

What is an _____, Alex?

a) equipotential b) equipoise c) equus d) equator e) equilibrium

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008 qmult 00352 1 1 3 easy memory: types of equilibria

409. Usually one defines four types of equilibria: stable, unstable, neutral, and

a) fallible. b) infallible. c) metastable. d) meta. e) metaphorical.

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008 qmult 0364 1 1 4 easy memory: restoring force of table and gravity

410. A block sits at rest on table. Gravity pulls it down and a normal force pushes it up. The two forces cancel each other. The block is in static equilibrium—a stable static equilibrium. If a tiny force pushes the block up, the normal force turns off since it is a contact force. If a tiny force pushes the block down, the normal force strengthens to cancel it since it is a force of reaction—as Marx would say. Together, gravity and the normal force constitute a:

a) strong force. b) friction force. c) linear or Hooke's law force.
d) restoring force. e) weak force.

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008 qmult 00370 1 1 1 easy memory: SHO for small perturbations

Extra keywords: maybe should be moved to oscillations chapter

411. If one makes a sufficiently small displacement from stable static equilibrium of almost any system with a smooth potential energy function and then lets the system evolve in isolation, the system will approximate a/an:

a) simple harmonic oscillator. b) anharmonic oscillator. c) traveling wave.
d) unstable equilibrium system. e) a vector field.

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008 qmult 00380 1 5 3 easy thinking: perfect balance scale

Extra keywords: seems a miscellaneous question in chapter 8

412. Why can't a practical balance scale ever be in perfect unstable static equilibrium when balancing?

a) Nothing is perfect.
b) Everything is perfect.
c) There is always some frictional force that makes the balance position, however, slightly metastable or the scale never really balances: the perturbations just grow so slowly that one can make a measurement before they become obvious. Of course, the less the stabilizing friction and the less the perturbations, the **MORE** exactly a mass can be determined.
d) There is always some frictional force that makes the balance position, however, slightly metastable or the scale never really balances: the perturbations just grow so slowly that one can make a measurement before they become obvious. Of course, the less the stabilizing friction and the less the perturbations, the **LESS** exactly a mass can be determined.
e) No one wants to.

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008 qmult 00400 1 1 1 easy memory: power definition
Extra keywords: physci

413. Work per unit time or energy transfered or transformed per unit time is:

a) power. b) might. c) oomph. d) strength. e) pay.

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008 qmult 00470 1 5 3 easy thinking: sunlight power
Extra keywords: physci KB-99-17

414. If you could capture it all for useful work, the energy sunlight delivers to a square meter of ground would run one or two ordinary incandescent light bulbs. The power delivered by the Sun to a square meter of ground on average is to order or magnitude:

a) 1 W. b) 10 W. c) 100 W. d) 10^6 W. e) 1 MW.

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008 qmult 00472 2 5 2 moderate thinking: boy running up stairs
Extra keywords: physci KB-93-23

415. A 50 kg boy runs up a flight of stairs of 5 m in height in 5 s at a constant rate. His power output just to work against gravity is:

a) 50 W. b) 490 W. c) 980 W. d) 10^6 W. e) 1 MW.

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008 qmult 00474 2 5 2 moderate thinking: mountain climber power output
Extra keywords: physci KB-95-7

416. A 100 kg mountain climber climbs 4000 m in 10 hours. What is his average power output going into gravitational potential energy? What is his total average power output?

a) 3.92×10^6 W and 3.92×10^6 W.
b) The power going into gravitational potential energy is 109 W. His total power output cannot be exactly calculated since a lot of power goes into just living (which ends up as waste heat in the environment) and a lot more must go into waste heat due to frictional forces. All one can easily say is that 109 W is a **LOWER BOUND** on the total power output.
c) The power going into gravitational potential energy is 3.92×10^6 W. His total power output cannot be exactly calculated since a lot of power must go into waste heat due to frictional forces and into the body heat which is lost to the environment. All one can easily say is that 3.92×10^6 W is a **LOWER BOUND** on the total power output.
d) The power going into gravitational potential energy is 3.92×10^6 W. His total power output cannot be exactly calculated since a lot of power must go into waste heat due to frictional forces and into the body heat which is lost to the environment. All one can easily say is that 3.92×10^6 W is an **UPPER BOUND** on the total power output.
e) The power going into gravitational potential energy is 109 W. His total power output cannot be exactly calculated since a lot of power must go into waste heat due to frictional forces and and into the body heat which is lost to the environment. All one can easily say is that 109 W is an **UPPER BOUND** on the total power output.

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008 qmult 10210 1 3 2 easy math: gravity U to K, spanner fullmult

417.* You let a 5.0 kg spanner slide from rest down a frictionless loopy playground slide. A friend catches it at 5 m below the drop height. Air drag is negligible. The speed of the spanner just before she catches it is about:

a) 5 m/s. b) 10 m/s. c) 15 m/s. d) 20 m/s. e) 25 m/s.

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008 qmult 10280 1 1 2 easy memory: monster cricket Jerry slide and spring fullmult

418.*** A cricket named Jerry of mass $m = 0.1\,\text{kg}$—he's a monster—slides down a **FRICTIONLESS CURVE** to the level ground. He starts from **REST** (i.e., his initial kinetic energy is zero). The total drop in height is $y_0 = 2.50\,\text{m}$. On the level ground, Jerry slides $s_0 = 3.00\,\text{m}$. Jerry then encounters an ideal spring with force constant $k = 1.00\,\text{N/m}$. The spring is massless and initially at its equilibrium extension. The spring is aligned with the ground and the Jerry's path. Jerry compresses the spring by a displacement s at which point Jerry is instantaneously at **REST** (i.e., his final kinetic energy is zero). The **GROUND** is not frictionless: it has kinetic coefficient for friction $\mu_{\text{ki}} = 0.300$. The whole problem is 2-dimensional. What is s? **HINT:** Draw a diagram.

a) 1.25 m. b) 1.50 m. c) 1.75 m. d) 2.00 m. e) 2.25 m.

Full-Answer Problems

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008 qfull 00200 1 3 0 easy math: work-energy theorem proven

419. We will now prove what yours truly calls the work-mechanical-energy theorem.

a) Write down the work-energy theorem.

b) Separate the work done W in the work-energy theorem in to that done by conservative forces and that done by non-conservative forces. Nothing forbids us from doing this.

c) Use the general formula for the potential energy of conservative forces to eliminate the work done by the conservative forces.

d) The sum of kinetic and potential energy for a body is defined to be the mechanical energy. Show that change in mechanical energy energies equals the work done by the non-conservative forces. This is the work-mechanical-energy theorem.

e) If the non-conservative forces do no work—but they may be present as workless constraint forces—what is conserved? What is not necessarily conserved in this case.

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008 qfull 00210 1 3 0 easy math: gravity U to K, spanner

420. You drop a 2.0 kg spanner from rest to a friend standing on the ground which 10 m below the drop height. She will catch the spanner 1.5 m above the ground. Neglect air drag.

a) What's the work done by gravity in the drop?

b) What's the change in gravitational potential energy in the drop?

c) Using an energy conservation calculation find the speed of the spanner as it reaches your friend's hands.

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008 qfull 00220 1 5 0 easy thinking: ideal climbing

421. This gentleman of fortune Daniel Goodwin in 1981 scaled the Sears Building in Chicago using suction cups and metal clips. The building is 443 m high. Let's guess he had a mass of 70 kg. Let us assume Dan is an **IDEAL** climber: i.e., all the body chemical energy he puts out goes into his own macroscopic kinetic energy or his own gravitational potential energy and into no other forms ever even eventually.

a) How much body chemical energy did Dan expend getting to the top? He started from rest and ended at rest?

b) If he'd just climbed the stairs, how much body chemical energy again starting and ending at rest.

c) Why did he probably use a lot more body chemical energy than an ideal climber?

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008 qfull 00230 3 3 0 tough math: steel dragon

Extra keywords: CJ-177

422. The Steel Dragon in Mie, Japan is one of the world's fastest and tallest roller coasters.

a) Assuming only gravity does work on a coaster find the formula for its speed v at any height y given that its initial speed and height were, respectively, v_0 and y_0. **NOTE:** We actually have to assume that the coaster is a point mass. Otherwise, we would have to worry about the kinetic energy of its internal parts: i.e., its spinning wheels. Note we are neglecting friction and air drag.

b) What does the normal force do in the roller coaster system? **NOTE:** We will assume that the tension in the chain or cable that pulls the coaster is negligible, but this might not be the actual case.

c) Say that $v_0 = 3.0\,\mathrm{m/s}$ and $y_0 = 93.5\,\mathrm{m}$. What is the speed when $y = 0\,\mathrm{m}$?

d) Why can't we calculate the time it takes for the coaster to go from height y_0 to y in the part (c) case?

e) What is the **COMPONENT** of the force of gravity along the track direction and what is the **ACCELERATION** if only gravity is acting along the track direction? Take the angle of the track to the horizontal to be θ.

f) Assuming for the part (c) question that the motion was all downhill and the displacement in the horizontal direction was about the same as in the vertical direction, estimate the travel time between the two locations.

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008 qfull 00240 2 3 0 moderate math: bead on a wire

423. There is a bead of mass m on a rigid, unmoving wire. The wire passes fairly snugly through a hole in the bead: i.e., the bead is strung on the wire. The wire is **FRICTIONLESS** and there is **NO** air drag. The wire extends in the horizontal and vertical directions. As one moves along the horizontal direction, the wire rises and falls and even does loops. The wire has multiple maxima and minima and horizontal segments of finite length. The wire goes on and on: it has no ends: it may be closed: i.e., form a continuous loop. The whole system is located on the surface of the Earth—where the only significant gravitational force is that of the Earth. At time zero, the bead is at height y_0 and has speed v_0.

a) What is the only force (not counting insignificant wire gravity) the wire exerts on the bead? What does this force do to the bead? Does this force do any work on the bead? Why or why not?

b) Find the formula for bead speed v for any height y. **HINT:** Use conservation of mechanical energy.

c) Assuming that the wire rises to any arbitrary height (i.e., any height you care to imagine), what is the numerical value of the speed v when the bead's height is the maximum possible: i.e., when the bead is at its $y_{\max}$? Find the formula for the maximum height $y_{\max}$.

d) What must the final speed of the bead be if the wire has **SOME FRICTION**? Say the wire angle from the horizontal is θ at any point and for all points one has

$$\tan\theta < \mu_{\text{static}} \ ,$$

where μ_{static} is the coefficient of static friction. Given this condition, what can you say about where the bead will be when it reaches its final speed?

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008 qfull 00250 3 5 0 hard thinking: dropped coconut compresses spring

Extra keywords: hard, but also dull

424. A coconut of mass 2.0 kg is dropped from rest at a height y_1 above the top of an ideal (i.e., massless), **VERTICAL**, uncompressed spring with spring constant 500 N/m. The coconut sticks to the top of the spring after landing. Take the top of the uncompressed spring to be the zero level. The spring reaches a maximum compression of $y_2 = -20.0$ cm before beginning to rebound. The problem is entirely 1-dimensional.

a) To the point of maximum compression how much work was done by the spring on the coconut and by the coconut on the spring? **HINT:** Find the work done by the spring first.

b) What is y_1?

c) If the coconut was released from twice the height found in part (b) (i.e., from $y_{1,\text{new}} = 2y_{1,\text{old}}$), how much would the spring compress? **HINT:** Face it, you have to solve a quadratic for y_2.

d) What is the physical significance of the positive solution to the quadratic equation obtained in part (c)?

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008 qfull 00252 3 5 0 hard thinking: Tarquin Superbus oscillates

Extra keywords: Dull question: has no punchline.

425. Knowing it will do no harm (or good) you drop your cat Tarquin Superbus from rest from a height y_i onto a **VERTICAL** spring. Tarquin has mass m. The spring is massless and **IDEAL** (i.e., sans dissipation) with spring constant k and massless platform on the top on which Tarquin lands. The spring is initially at rest with its platform at the spring's intrinsic equilibrium position: we define the spring's intrinsic equilibrium position to be the $y = 0$ level. On landing on the spring platform, Tarquin grips it and holds on tight for the rest of the problem. The problem is entirely 1-dimensional.

a) Qualitatively what is going to happen to Tarquin after he is on the spring for the foreseeable future?

b) Using conservation of energy write down an equation relating y_i and the post-landing Tarquin parameters velocity v and height y. Tarquin and platform constitute a point particle of course.

c) Solve for y as a function of v and y_i. From a physical perspective (not just a mathematical one), why are there two solutions?

d) Give the expressions for the turning points of Tarquin's oscillations.

e) Give the expression for Tarquin's equilibrium height. **HINT:** This is where the net force on Tarquin is zero.

f) If a damping force is turned on, at what height will Tarquin come to rest.

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008 qfull 00260 2 3 0 moderate math: boll weevil and igloo 1

426. A boll weevil of mass m is sitting on top of a hemispherical igloo of radius R. An infinitesimal perturbation starts him sliding down starting from speed **ZERO**. The igloo is frictionless and there is no air drag.

a) Sketch a diagram of the system with the boll weevil at a general position on the igloo—not the top, not the bottom, a general position. Indicate the boll weevil's angle θ from the

VERTICAL as measured from the ground level at the igloo center and the forces that act on the boll weevil. Note, forces, not force components.

b) Derive an explicit formula for the boll weevil's speed on the igloo as a function of angle θ on the igloo measured from the vertical using conservation of mechanical energy.

c) At some height (measured from the ground) the boll weevil flies off the igloo. Derive explicit formulae for angle and the height at which the boll weevil flies off the igloo. **HINT:** This is purely a force analysis problem. Consider the normal force on the boll weevil and note that the radial (component of) acceleration is instantaneously given by the $a = -v^2/r$ just as for uniform circular motion even when v is not constant provided that r is constant. Recall v is just the tangential speed in the radial acceleration formula.

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008 qfull 00262 2 5 0 moderate thinking: boll weevil 2 and igloo 2

427. That egregious insect the boll weevil of mass m is back on the top of his hemispherical frictionless igloo. This time he is going to slide down hanging onto a rigid groove that runs straight down the igloo side. This time he won't fly off. The groove holds the boll weevil at surface of the igloo all the way. The groove exerts **NO** tangential force. Note at time zero, the boll weevil is exactly at the top of his igloo.

a) The radius of the igloo is again R and θ is the angular position of the boll weevil measured from the **VERTICAL**. Give an explicit formula for his velocity v as a function of θ assuming that he starts, **NOT** from rest, but with velocity $v_0 \geq 0$. The positive direction of the velocity is the direction of motion down the igloo. There are no negative velocities in the problem since boll weevil never slides up the igloo. Show all θ dependences **EXPLICITLY**. **HINT:** A diagram might help.

b) Find a formula for the force the groove exerts on the boll weevil as a function of θ. Take the radial inward direction as positive. Show all θ dependences **EXPLICITLY**. Simplify the formula as much as reasonably possible. The force could be called the groovy force.

c) Find a 1st order differential equation for angular position θ with all θ's and θ derivatives shown **EXPLICITLY**. Don't try to solve it, **JUST** find it. Is the differential equation linear? Explain why it is linear or not. **HINT:** Start from the part (a) answer.

d) An **APPROXIMATE SOLUTION** for the differential equation found in the part (c) answer is

$$t = \frac{1}{\sqrt{g/R}} \ln\left[\theta\sqrt{\frac{(g/R)}{\omega_0^2}} + \sqrt{\frac{(g/R)}{\omega_0^2}\theta^2 + 1}\right] ,$$

where θ is in **RADIANS** and $\omega_0 = v_0/R$. Note it turned out to be easier to get the slide-down time t as a function of θ than vice versa. Given that $R = 2.0\,\mathrm{m}$ and $\omega_0 = v_0/R = 1.0\,\mathrm{rad/s}$, evaluate the solution for approximate slide-down time t that the boll weevil takes to reach the ground? A single number is required.

e) If $\omega_0 = 0$, how long would it take boll weevil to slide down the igloo? Explain what the slide-down time means in this case?

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008 qfull 00270 3 5 0 tough thinking: Dingo slides on a loop falls off not

428. Dingo the daredevil dog (who has mass m), starting from **REST**, slides down a frictionless track from an initial height y_0. The track becomes level at $y = 0$, then goes into a circular loop of radius R, and then goes level at $y = 0$ again. Dingo is actually a particle dog.

a) What is Dingo's speed at any height y assuming he stays on the track? Give the speed as function of y, y_0, g, and, if necessary, m.

b) Find a general formula for the normal force on Dingo when he is on the loop as a function of the angle θ between a general radial vector and the radial vector pointing to the top of

the loop. The only variables in the formula should be y_0, R, m, g, and θ. Simplify as much as possible and take radially inward as **POSITIVE**. **HINT:** Note that the magnitude of the radial (component of) acceleration is instantaneously given by $a = v^2/r$ just as for uniform circular motion even when v is not constant provided that r is constant. Recall v is just the tangential speed in this expression and the direction of the radial acceleration is toward the center.

c) For what θ is the normal force formula smallest? What is the normal force at this angle?

d) What is the mathematical condition—sufficient, not just necessary—needed so that the normal force formula never specifies an attractive normal force (i.e., a force attracting Dingo to the track) anywhere on the loop? Explain why there is this condition. What happens to Dingo if the formula did specify an attractive normal force?

e) Say $y_0 = 50\,\text{m}$ and the loop radius $R = 10\,\text{m}$. Does Dingo stay on the loop? (A demonstration is needed, not just a yes or no answer. But it's not a long demonstration.)

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008 qfull 00380 3 5 0 tough thinking: anharmonic oscillator

Extra keywords: Because of integration probably not suitable for intro I tests.

429. You are given an oscillator of mass m obeying an anharmonic oscillator (AO) force law

$$F(x) = -ax - bx^3 \ ,$$

where a and b are positive constants.

a) What is the work W done by the AO force when the oscillator is moved from the origin to a general point x? **HINTS:** You will have to do an integration. Remember which way the force points and which way the oscillator is moving.

b) Say that the oscillator before and after it has been moved was at rest. What was the work W_{app} done by the applied force that caused the movement? Assume only the applied force and the AO force act.

c) What is the expression for the AO potential energy U if the origin is chosen to be zero potential energy?

d) Say that there are no nonconservative forces and we give the AO an initial mechanical energy of E. What is its mechanical energy at all later times?

e) What is the maximum speed that the oscillator can have given that it has mechanical energy E? Where does it reach this maximum speed?

f) What is the maximum displacement x_{max} that the AO can get from the origin (i.e., the amplitude of the oscillation) given mechanical energy E. **HINT:** Solve for $s_{\text{max}} = x_{\text{max}}^2$ first.

g) If x_{max} is so small that the 2nd term in the AO potential energy expression is negligible, approximately what kind of an oscillator do we have?

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008 qfull 00420 2 3 0 moderate math: Lance Armstrong work and power

Extra keywords: save for exams

430. Lance Armstrong has won the Tour de France bicycle race a record 7 times.

a) What is the main forward **EXTERNAL** force on the Armstrong-bike system? What are at least two of the main reverse **EXTERNAL** forces on the Armstrong-bike system? The forward forces acting alone accelerate and the reverse forces acting alone decelerate. Assume that Lance is riding on a completely horizontal road. **HINT:** We are not concerned with internal forces: e.g., forces that Lance Armstrong exerts on the bike and that the bike exerts on Lance.

b) Say Lance biked 135 km (still on the horizontal road) and started and ended at rest. What is the **NET** work done on the Armstrong-bike system by all external forces? A numerical value is expected for the answer.

c) Say Lance travels at an average speed of 12.0 m/s for the distance of 135 km. How long was the trip in seconds? How long was the trip in hours?

d) Say Lance outputs about 2.0 kW of power on average (which may be a little too high in reality) during his trip of 135 km. This power is total power: some of the energy goes into kinetic energy of his body and bike before becoming waste heat and some goes into heat directly. How much energy has has he output in total in joules?

e) Assuming Lance outputs about 0.15 kW of power for the **REST** of the of 24 hours, what is his daily average output power in **WATTS** (i.e., his sustained metabolic rate)? By "average", it is meant the kind of average that allows one to calculate the energy Lance outputs in any period of time by multiplying by that time.

Chapt. 9 Momentum

Multiple-Choice Problems

009 qmult 00100 1 1 5 easy memory: definition of vector momentum

431. Momentum (or linear momentum) is defined by the formula _____, where m is system mass and $\vec{v}$ is system center-of-mass velocity.

a) $\vec{p} = \dfrac{m}{\vec{v}}$ b) $\vec{p} = \dfrac{\vec{v}}{m}$ c) $\vec{p} = \dfrac{1}{2}mv^2$ d) $\vec{p} = \dfrac{1}{2}m\vec{v}$ e) $\vec{p} = m\vec{v}$

009 qmult 00110 1 1 5 easy memory: definition of 1-d momentum

Extra keywords: physci

432. The momentum (or linear momentum) for a 1-dimensional system is given by the formula _____, where m is system mass and v is system center-of-mass velocity for the single dimension.

a) $p = \dfrac{m}{v}$ b) $p = \dfrac{v}{m}$ c) $p = \dfrac{1}{2}mv^2$ d) $p = \dfrac{1}{2}mv$ e) $p = mv$

009 qmult 00150 1 1 3 easy memory: momentum is not energy

Extra keywords: physci KB-93-15

433. Linear momentum is **NOT**:

a) a physical quantity. b) dependent on velocity. c) a kind of energy.
d) dependent on mass. e) given by $p = mv$ for 1-dimensional cases.

009 qmult 00160 2 5 1 moderate thinking: K change and momentum change

Extra keywords: physci KB-94-13

434. If the kinetic energy of an object is doubled, the momentum magnitude changes by a factor of:

a) $\sqrt{2}$. b) 2. c) 1/2. d) $1/\sqrt{2}$. e) 1.

009 qmult 00170 1 1 5 easy memory: general form of Newton's 2nd law

435. The general form of _____ is

$$\frac{d\vec{p}}{dt} = \vec{F}_{\text{ext}} \ ,$$

where $\vec{p}$ is the total momentum of a system and $\vec{F}_{\text{ext}}$ is the net external force on the system. The net external force includes ordinary forces (field forces and contact forces), and what can be called momentum flux forces.

A momentum flux force is just the momentum added to a system by adding mass with its own momentum to the system. If all the added mass had the same velocity $\vec{v}_{\text{flux}}$, then the momentum flux force would be

$$\vec{v}_{\text{flux}}\frac{dm}{dt} \ .$$

Note dm/dt can be positive or negative: if negative, mass is actually be lost from the system. The added mass can interact with the rest of mass of the system after becoming part of it by

ordinary forces or not. Using our special case momentum flux force and the chain rule, we can specialize _____ to the formula

$$m\vec{a} = \vec{F}_{\text{ext,ordinary}} + (\vec{v}_{\text{flux}} - \vec{v})\frac{dm}{dt} \ ,$$

were $\vec{a}$ is the center-of-mass acceleration and $\vec{v}$ is the center-of-mass velocity.

If $\vec{F}_{\text{ext}} = 0$, then $\vec{p}$ is a constant: i.e., momentum is conserved. This is really the general form of Newton's 1st law: in the absence of a net external force, momentum is conserved. If $dm/dt = 0$ too (and it is hard to arrange $\vec{F}_{\text{ext}} = 0$ without having $dm/dt = 0$), then $\vec{v}$ is a constant: this is Newton's 1st law as usually stated. The 1st law is not really a law (i.e., an axiom) of classical mechanics: it is a result that is called a law because Newton called it that—maybe even for him it was a traditional law.

Since our general form of _____ is an axiom of classical mechanics, it must apply to the parts of a system treated as systems in their own right. Imagine a system broken into two parts 1 and 2. We must have

$$\frac{d\vec{p}_1}{dt} = \vec{F}'_{\text{ext},1} + \vec{F}_{21} \qquad \text{and} \qquad \frac{d\vec{p}_2}{dt} = \vec{F}'_{\text{ext},2} + \vec{F}_{12}$$

where 1 and 2 label the parts, $\vec{F}'_{\text{ext},1}$ the net external force on part 1 excluding the force part 2 exerts on part 1, $\vec{F}_{21}$ is the force part 2 exerts on part 1, $\vec{F}'_{\text{ext},2}$ the net external force on part 2 excluding the force part 1 exerts on part 2, and $\vec{F}_{12}$ is the force part 1 exerts on part 2. If we add the two part expressions and make use of the general form for the whole system itself, we find

$$0 = \vec{F}_{21} + \vec{F}_{12} \ , \qquad \text{and thus} \qquad \vec{F}_{21} = -\vec{F}_{12} \ .$$

The last result as it stands is not quite Newton's 3rd law since $\vec{F}_{12}$ and $\vec{F}_{21}$ are net forces, not particular force (e.g., gravity). However, since in special cases only particular forces exist and the last result must be true for them, the 3rd law is proven. So in reality even the 3rd law is not an axiom either: it is a consequence of _____.

Having established the 3rd law from _____, one can now see that there is no inconsistency in applications of _____. Say you divide a body into any number of parts and apply _____ to each one. You then sum up these applications and all the internal forces cancel pairwise and the sum expression is _____ applied to the body as a whole—you do not get a different formula—there is no inconsistency.

The 3rd law, in fact, is not always valid even in classical mechanics. It is violated by the magnetic force in certain cases. For these cases, one needs an even more general form of _____ in which the fields that cause forces are assigned a field momentum. But this more general form is well beyond the scope of intro physics.

a) the principle of inertia b) Newton's 4th law c) Nernst's theorem
d) Newton's zeroth law e) Newton's 2nd law

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009 qmult 00200 1 1 2 easy memory: conservation of momentum
Extra keywords: physci

436. For a system on which no net external force acts, momentum is:

a) not conserved. b) conserved. c) zero. d) never zero. e) always negative.

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009 qmult 00260 2 4 1 mod. deducto-memory: collision, explosion, momentum
Extra keywords: physci

437. A collision or explosion is an event in which relatively strong forces act between objects for a relatively short time. If one considers all the objects involved in the collision or explosion as constituting one system, then frequently in calculations it is useful to use the _____

principle provided the external forces acting on the system can be considered negligible compared to the internal forces of the system during the collision or explosion.

a) conservation of momentum b) conservation of mechanical energy
c) cosmological d) anthropic e) Peter

---

009 qmult 00270 1 1 2 easy memory: conservation of momentum, Thor
Extra keywords: physci

438. The mighty Thor is trapped in the eternal vacuum of gravity-free space with nothing to push on. But he sees Asgard glittering **YONDER** (i.e., over there). Having taken introductory physics in his young Viking days, he realizes that he will soar straight to Asgard if, with awesome strength, he throws his hammer:

a) yonder. b) anti-yonder. c) any which way. d) left. e) in a parabolic arc.

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009 qmult 00272 1 3 4 easy math: diver from boat, momentum conserved
Extra keywords: physci KB-96-25

439. A 50 kg girl (who is initially at rest) dives horizontally at 2.0 m/s in the **POSITIVE** direction from a 200 kg boat that initially is at rest. Given that the event is an ideal collision event, what is the initial recoil **VELOCITY** of the boat?

a) 0.25 m/s. b) 0.70 m/s. c) −0.70 m/s. d) −0.50 m/s. e) infinite.

---

009 qmult 00320 1 1 2 easy memory: elephant collision

440. If someone says elephant, a large, grey, tusked animal with a trunk comes into your mind. If someone says **COLLISION** in the context of a physics problem, into your mind **SHOULD** come the concept of:

a) elephant. b) conservation of momentum.
c) conservation of the sum of the kinetic energies of the colliders. d) gravity.
e) nothing.

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009 qmult 00830 1 4 4 easy deducto-memory: mass-varying acceleration

441. "Let's play *Jeopardy*! For $100, the answer is: It is the missing factor in the equation

$$\vec{a} = \frac{\vec{F}_{\text{other}}}{m} + \frac{?}{m}\frac{dm}{dt}$$

for mass varying systems."

What is _____, Alex?

a) $\vec{v}$ b) $\vec{v}_{\text{flux}}$ c) $(\vec{v} - \vec{v}_{\text{flux}})$ d) $(\vec{v}_{\text{flux}} - \vec{v})$ e) $-\vec{v}$

---

009 qmult 00910 1 1 1 easy memory: operation of rocket: I need to reconcile perspectives
Extra keywords: physci KB-92-9

442. The operation of a rocket in space is based on:

a) conservation of momentum. b) conservation of angular momentum. c) jet fuel pushing on the vacuum. d) starlight pressure. e) running an internal treadmill.

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009 qmult 00940 1 4 2 easy deducto-memory: rocket thrust force: I need to reconcile perspectives

443. The force that accelerates a rocket in the vacuum of space is the:

a) force of the vacuum. b) force exerted by the ejected fuel on the rocket: the **THRUST** force. c) force exerted by the ejected fuel on the rocket: the **THRUSH** force. d) force of light pressure. e) Force.

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009 qmult 10280 1 1 1 easy memory: You with King in space fullmult

444.* You (mass 50 kg) and your dog King (mass 90 kg) have been abducted by space aliens and have been locked in a thin spacecraft of length 50 m and mass 100 kg. The linear density of the

spacecraft is constant, and so the center of mass of the spacecraft alone is at the center (i.e., 25 m from either end). Initially, the spacecraft is **AT REST** relative to the inertial frame of the Cosmic Microwave Background (CMB). You and King have been strapped down at opposite ends of the spacecraft. But King with true dog force has broken her bonds and bounds toward you at 6.0 m/s relative to the spacecraft. We consider this King motion to be in the negative x direction. No external forces act on the you-King-spacecraft during this whole episode. What is the x coordinate **CENTER-OF-MASS VELOCITY** of the whole you-King-spacecraft system relative to the inertial frame of the CMB as King bounds along?

a) 0. b) 6.0 m/s. c) −6.0 m/s. d) 2.7 m/s. e) Indeterminate.

Full-Answer Problems

009 qfull 00100 2 3 0 moderate math: proving F=ma system

Extra keywords: For calculus-based course. Maybe for tests since it's in notes.

445. Consider a system of particles. The system is general. The particles may be rigidly bonded together. They may be non-rigidly bonded together. They may only interact through collisions. They may not interact at all. It's a general system.

a) Assume the center-of-mass version of Newton's 2nd law holds for each particle and write it down for particle i.

b) Write down the net force on particle i as an explicit sum of external and internal forces. The external forces are due to particles or fields external to the system and the internal forces are due to other particles in the system.

c) Prove that the sum of all the internal forces for the system is zero.

d) Prove that

$$\vec{F}_{\text{net}} = \sum_i m_i \vec{a}_i \ ,$$

where $\vec{F}_{\text{net}}$ is the sum of all the forces (and the sum of all the external forces) acting on the system. Note you are **NOT** allowed to prove this starting from

$$\vec{F}_{\text{net}} = m\vec{a}_{\text{cm}} \qquad \text{or} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} \ ,$$

where (m is total mass, $\vec{a}_{\text{cm}}$, and $\vec{p}$ is total momentum), since those are things proven in parts (e) and (f).

e) Define a center of mass quantity $\vec{r}_{\text{cm}}$ and show that

$$\vec{F}_{\text{net}} = m\vec{a}_{\text{cm}} \ ,$$

where m is the total mass of the system and $\vec{a}_{\text{cm}}$ is the center of mass acceleration.

f) The momentum of particle i is defined

$$\vec{p}_i = m_i \vec{v}_i \ ,$$

If $\vec{F}_{\text{net}} = 0$ and the mass of the system is conserved, show that total momentum of the system and $\vec{v}_{\text{cm}}$ (the velocity of the center of mass) are constants.

009 qfull 00270 1 5 0 easy thinking: conservation of momentum, exploding shell

Extra keywords: fragmenting shell

446. A shell of mass m is at a certain instant is exactly in horizontal flight with velocity v in the positive x-direction. At that same instant the shell explodes instantaneously (virtually instantaneously to be more realistic) into 3 fragments: two fragments have zero x velocity, one going straight up, the other straight down; the 3rd fragment, which has mass m_3, just at the instant after explosion continues in the x direction with only its x velocity nonzero.

a) What is the x-direction velocity v_3 of the 3rd fragment in terms of given variables? Derive the formula for v_3 from a basic principle.

b) Just after the explosion, what must the total y-direction momentum be of the two fragments that went straight up and down? Explain why it is this value.

c) How far does the 3rd fragment go in the x direction if it starts a height y_0 above a flat plain assuming no air drag? Derive the formula for the distance including a derivation of the the horizontal-launch projectile range formula. Give the answer in terms of given variables and g.

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009 qfull 00280 3 3 0 tough thinking: conservation of momentum in a boat with King

Extra keywords: boat-King-you system

447. Your dog King—a she, but somehow that wasn't obvious before the puppies ("Ooh, arn't they sweethearts.")—is on a long flat-bottomed boat (10 m in length to be precise) on a placid lake. King now muscles in at 90 kg—you must not have been listening when her grandsire Gross Hans the St. Bernard was being aired—"he could haul an SUV". You, also on the boat, however, are a petite 50 kg—which makes opening the door when you get home from work, King keening just inside, its own demonstration in macroscopic conservation of momentum. The boat is 60 kg: it's made of that light-weight stuff that the astronauts ate on the Moon. It is fore-aft symmetric: i.e., its center of mass is exactly **MIDSHIPS**. Assume there is no net external force acting on the boat-King-you system and that the system is **1-DIMENSIONAL** along the boat axis.

a) Act 1, Scene I: The boat is at rest (or even becalmed): King's motionless (thankfully) at the bow (bowwow?); you are at the stern likewise enjoying a breather. Give the formula in terms of **VARIABLES** (i.e., **SYMBOLS**) for the center-of-mass position of the total boat-King-you system using the stern as the **ORIGIN** and recall the system is 1-dimensional. Now what is the center-of-mass position (i.e., what is the numerical value)? **HINT:** Draw a diagram.

b) Act 1, Scene II: King in a sudden attack of nautical anxiety needs some loving and, after a brief and inconsequential acceleration phase, bounds toward you at 6.0 m/s **RELATIVE** to the boat: this, of course, should be -6.0 m/s if the positive direction is aft-to-fore. (We are really treating King as free-flying object that is not constantly pounding her paws on the deck. She's an ideal dog.) Find the formula in terms of **VARIABLES** for the boat's velocity relative to the water now. Show how you got the formula.

What is the boat's velocity relative to the water? What is your velocity relative to the water? What is King's velocity relative to the water? What is the boat-King-you system's center of mass velocity relative to the water? Take the aft-to-fore direction as positive. Numerical values are expected.

c) Act 1, Scene III: King makes a **COMPLETELY INELASTIC COLLISION** with you at the stern: i.e., she sticks to you with a deep, warm feeling—"Belay King, Belay!" Assuming you don't all tip backward into the drink, what are the velocities of you, King, the boat, and the boat-King-you system center of mass now? Briefly explain your answers.

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009 qfull 00650 2 3 0 tough math: baseball hitter, baseball batter

448. In baseball, the bigger and stronger the batter (AKA the person who hits the ball with the bat), the farther they can hit the ball with the bat. Let's create a model that may explain why this is.

For our model, let's first assume that the arm force of a batter is proportional to his/her arm mass m_1. This must be roughly true: small and large people must do about the same things with their bodies to first order. Second, let's assume that in the collision of the ball with the bat, it is really the bat and the batter's arms that are the colliding objects: the bat is held rigidly, but at the time of the collision the arms are swinging pretty freely. We neglect the bat's own mass which is probably not such a good approximation actually.

From the work-kinetic energy theorem and our assumption of batter force, the final arm kinetic energy should obey

$$\frac{1}{2}m_1 v_1^2 \propto F_{\text{arms}} \propto m_1 \ ,$$

where is v_1 the final speed of the arms in the swing just before the collision with the ball. One can see that v_1^2, and therefore v_1, is independent of mass given our assumptions. Thus all batters swing with about the same speed given our assumptions. We take v_1 to be a constant.

Note, the model may not be adequate, but we can see where it takes us.

a) Given the horizontal range formula for projectile motion

$$x = \frac{v_0^2}{g}\sin(2\theta) \ ,$$

how does one change the launch velocity v_0 to increase travel distance? What does your answer imply for hit baseballs?

b) Assume the batter-ball collision is a 1-dimensional elastic collision—which is probably not so bad. The batter is object 1 and the ball is object 2. The final direction of the ball is positive direction which means the ball's initial velocity is negative. The final velocity of object 2 is given by the elastic collision formula

$$v_{2'} = \frac{(m_2 - m_1)v_2 + 2m_1 v_1}{m_1 + m_2} \ .$$

Find the formula for the **RATE OF CHANGE** of $v_{2'}$ with respect to m_1 with all other variables constant. Show how you found the formula. Simplify the formula as much as reasonably possible.

c) Is there a mass m_1 that gives a maximum $v_{2'}$? Does $v_{2'}$ increase with m_1? Is our model consistent with the idea of bigger batter, longer travel distances? Explain your answers.

d) Given that a baseball has mass 0.145 kg, that pre-collision bat and ball speeds are of order 50 m/s, and the batter's arms mass m_1 is some reasonable value, estimate the rate of change of $v_{2'}$ with respect to m_1 for a typical batting case.

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009 qfull 00910 2 3 0 moderate math: rocket speed in space: redundant to rocket equation problem 449. Lost deep in the void of eternal space there is a rocket in an inertial frame. At time zero, it's initial velocity is v_0. Suddenly its engine fires ejecting fuel at a constant exhaust speed v_{ex} (i.e., a constant relative velocity with respect to the rocket).

a) What is the general formula for the ratio of mass to initial mass (i.e., m/m_0) in terms of velocity v, v_0, and v_{ex}? We are not asking for a derivation. Just write down the formula. Of course, if you don't know it or can't locate it somewhere, you could derive it.

b) What is the mass ratio m/m_0 when v is equal to the exhaust speed? After the time of this event what is the direction of motion of ejected fuel relative to the inertial frame?

c) What is the mass ratio when v is equal to the exhaust speed if $v_0 = 0$?

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009 qfull 00920 3 5 0 tough thinking: rocket launch

450. A rocket of initial mass 1.0×10^5 kg is about to launch vertically with constant thrust or exhaust speed $v_{\text{thrust}} = 1000\,\text{m/s}$. Assume air drag is negligible.

a) Let's assume at time zero that the rocket has just enough thrust force to cancel gravity: i.e., the thrust force entirely replaces the normal force supporting the rocket on the pad. What rate of mass change must the rocket have in this case? **HINT:** Write down the equation of motion and solve for mass change rate.

b) What rate of mass change must the rocket have to accelerate at $4g$ at time zero?

c) Write down the equation of motion again and solve for acceleration a as a function of time. Assume that mass change rate is a constant (which implies that thrust is constant) and solve for velocity as a function of time. **HINT:** Write mass as $m(t) = m_0 + (dm/dt)t$, where m_0 is initial mass.

d) Assuming the gravity is negligible, use the result of part (c) to solve for the time t it take to reach escape velocity 11.2 km/s? Use the mass rate change found in part (b). By evaluating gt assess if gravity is in fact negligible? How much mass has the rocket lost in achieving escape velocity?

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009 qfull 00800 3 5 0 tough thinking: multi-stage rocket

Extra keywords: Fr-359-9-12, but highly modified

451. Consider a fully-fueled, multi-stage rocket at rest in gravity-free space. There are I stages plus the payload which we will call stage $I+1$, but the payload has no fuel. The mass of stage i is $n_i m$, where m is the mass of the payload and n_i is a dimensionless mass ratio: $n_i \geq 1$ and the equality only holds for stage $I+1$ (i.e., the payload). The total mass of the rocket is $n_1 m$ and $n_i > n_{i+1}$ for all i.

Each stage consists of a casing and fuel. The mass of the ith casing is $r(n_i - n_{i+1})m$, where r is a dimensionless ratio that is the same for all stages.

The relative speed of ejected fuel for all the stages when in operation is v_0.

a) Before considering the situation of the preamble, we need a basic rocket velocity formula. The equation of motion for rocket of a single stage is

$$m\frac{dv}{dt} = -v_0\frac{dm}{dt} \ ,$$

where m is the rocket mass at any time t, v_0 is the relative speed of the ejected fuel of the operating rocket, and $-v_0 dm/dt$ is the rocket thrust (which is positive since the rocket mass is decreasing). Show that velocity of the rocket as a function of mass is

$$v = v_{\text{ini}} + v_0 \ln\left(\frac{m_{\text{ini}}}{m}\right) \ ,$$

where v_{ini} is the initial velocity, m_{ini} is the initial mass, and m is the mass at any time (which is not to be confused with payload mass m in the preamble).

b) Now to return to the situation of the preamble. Find an expression for the final velocity of the payload after all the stages have burned all their fuel and all the casings have been released. (The stages are burned in order: 1,2,3, etc.) Remember the rocket starts from rest. The casings are just released with zero velocity relative to the remainder of the rocket after all their fuel has been ejected. Simplify the expression as much as possible although not much simplification is possible. **HINT:** You really should be able to just write the expression down.

c) Show that the final velocity of the payload is maximized for a fixed total mass (i.e., a fixed n_1) when the following expression holds for all $i \geq 1$:

$$R = \frac{n_{i+1}}{n_i} = \frac{1}{n_1^{1/I}} \ ,$$

where R is just the label we have given to the ratio.

d) Give the expression for the maximum final velocity and give its limiting behaviors for $r = 0$ and $r = 1$. Explain the limiting behavior for $r = 1$

e) Show that the multi-stage rocket of this problem can always produce a higher final payload velocity than a single-stage rocket (with the same r) if R for the multi-stage rocket is made small enough, the multi-stage rocket stages are chosen to maximize v for a given n_1, and provided $r > 0$. What is the essential reason for this result? What is the upshot of the result in practical rocketry: e.g., in launching probes into space?

Chapt. 10 Collisions

Multiple-Choice Problems

010 qmult 00100 1 4 5 easy deducto-memory: ideal collision defined

452. "Let's play *Jeopardy*! For $100, the answer is: An interaction between two or more bodies where the interaction forces are so strong and duration so short that only the interaction forces between the bodies are significant during the interaction: other forces that arise from sources external to the interaction can be neglected during the interaction. Often the interaction can be regarded as instantaneous on the time scale of the evolution of the system of bodies before and after the interaction."

What is _____, Alex?

a) linear momentum b) uniform circular motion c) proper motion
d) a weekend in Las Vegas e) an ideal collision in physics

010 qmult 00200 2 4 2 moderate deducto-memory: impulse definition calculus-based

453. An impulse $\vec{I}$ is:

a) simply the sum of the momentum components of a momentum vector.
b) a force on an object times the time Δt it acts on the object:

$$\vec{I} = \vec{F}\Delta t \ .$$

If the force is not constant (which actually the usual case in collisions for example), then

$$\vec{I} = \int_{\Delta t} \vec{F}(t)\, dt \ ,$$

where the integral is over Δt. Net impulse yields a new version of Newton's 2nd law:

$$\vec{I}_{\rm net} = \int_{\Delta t} \vec{F}_{\rm net}(t)\, dt = \int_{\Delta t} \frac{d\vec{p}}{dt}\, dt = \Delta\vec{p} \ :$$

i.e., $\vec{I}_{\rm net}$ equals the change in momentum of the object. The concept of impulse is used mainly for strong, short-duration forces (i.e., impulsive forces) such as those that occur in collisions and explosions. In such cases, the new version of the 2nd law can be approximated by just using the impulse of the impulsive forces on the left-hand side if they give the only significant contributions to the net impulse.
c) a sudden desire.
d) the opposite of an expulse.
e) an effect used to drive an impulse engine. If your warp drive fails and you are on impulse and 30,000 light-years from Rigel 8, then it's time to get out the card games.

010 qmult 00210 2 4 2 moderate deducto-memory: impulse definition algebra-based

454. An impulse $\vec{J}$ is:

a) simply the sum of the momentum components of a momentum vector.
b) a force on an object times the time Δt it acts on the object:

$$\vec{J} = \vec{F}\Delta t \ .$$

If the force is not constant (which actually the usual case in collisions for example), then

$$\vec{J} = \vec{F}_{\rm ave}\Delta t \ ,$$

where $\vec{F}_{\rm ave}$ is the time-averaged force for time interval Δt. Net impulse yields a new version of Newton's 2nd law:

$$\vec{J}_{\rm net} = \Delta\vec{p} \ :$$

i.e., $\vec{J}_{\rm net}$ equals the change in momentum of the object. The concept of impulse is used mainly for strong, short-duration forces (i.e., impulsive forces) such as those that occur in collisions and explosions. In such cases, the new version of the 2nd law can be approximated by just using the impulse of the impulsive forces on the left-hand side if they give the only significant contributions to the net impulse.
c) a sudden desire.
d) the opposite of an expulse.
e) an effect used to drive an impulse engine. If your warp drive fails and you are on impulse and 30,000 light-years from Rigel 8, then it's time to get out the card games.

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010 qmult 00300 1 1 3 easy memory: collision conservation of momentum

455. In a collision, the impulsive forces are internal to the system of colliding objects: i.e., they do not sum to a net force on the system. Assuming external net force on the system for the short duration of the collision is insignificant compared to the internal forces, then by Newton's 2nd law there is no change in the total momentum of the system. This means that _____ conserved through the collision.

a) total kinetic energy **IS** b) total mass **IS** c) total momentum **IS**
d) total momentum is **NOT** e) total mass is **NOT**

---

010 qmult 00400 1 1 1 easy memory: collision types

456. In elementary discussions, collisions are usually divided into three types:

a) inelastic, perfectly inelastic, elastic. b) inelastic, hardly inelastic, elastic.
c) inelasticated, perfectly inelasticated, elasticated.
d) inelasticated, hardly inelasticated, elasticated. e) good, bad, ugly.

---

010 qmult 00420 1 1 4 easy memory: perfectly inelastic collision

457. In a perfectly inelastic collision:

a) total kinetic energy is conserved. b) the colliding objects bounce off each other.
c) total momentum is **NOT** conserved. d) the colliding objects stick together.
e) the colliding objects have no hair.

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010 qmult 00710 1 5 1 easy thinking: identical masses collide

458. Two identical point masses elastically collide in a 1-dimensional setup. Mass 1 had velocity v before the collision and velocity zero after. Mass 2's velocity was initially zero and after the collision it was:

a) v. b) $2v$. c) $-v$. d) $-2v$. e) still zero.

---

010 qmult 00720 2 1 5 moderate memory: relative velocity rule

459. In a 1-dimensional elastic collision (where the particles do exert forces on each other), the initial velocity = $v_{\rm rel,i}$ and final relative $v_{\rm rel,f}$ are related by:

a) $v_{\mathrm{rel,f}} = v_{\mathrm{rel,i}}^2$. b) $v_{\mathrm{rel,f}} = 2v_{\mathrm{rel,i}}$. c) $v_{\mathrm{rel,f}} = -2v_{\mathrm{rel,i}}$. d) $v_{\mathrm{rel,f}} = \frac{1}{2}v_{\mathrm{rel,i}}$.
e) $v_{\mathrm{rel,f}} = -v_{\mathrm{rel,i}}$.

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010 qmult 00850 1 1 5 easy memory: ball bounces off wall

460. A ball hits a wall with an incident angle of 30° measured from a normal to the wall. Consider the normal as a pole: angles measured from the pole are polar angles; those around the pole are azimuthal angles. The ball rebounds at polar angle:

a) 30° in a plane **PERPENDICULAR** to the one defined by the incident path and the normal.
b) 30°, but in a random azimuthal direction.
c) 60°, but in a random azimuthal direction.
d) 60° in a plane **PERPENDICULAR** to the one defined by the incident path and the normal.
e) 30° in the plane **DEFINED** by the incident path and the normal.

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010 qmult 10200 2 5 2 moderate math: goats, perfectly inelastic coll. fullmult

461.** Richthofen and Mannock are two goats with masses 80 kg and 50 kg, respectively. They charge each other head-on on a icy pond—amazingly they stay upright, but goats are sure-hoofed. Just before impact Richthofen is moving at 10.0 m/s—he's way out of control—and Mannock's moving at −4.0 m/s. On impact, they lock horns literally (i.e., stick together). Treat the goats as point masses and their collision as an ideal collision (i.e., one in which only the collision forces are non-negligible).

i) What is the center-of-mass velocity of the Richthofen-Mannock system during the collision?

ii) Given that the hoof-ice coefficient of kinetic friction is 0.05 (which is the same as steel on ice) how far does the Richthofen-Mannock system slide after the collision before coming to rest?

a) (i) 10 m/s (ii) 1.1 m. b) (i) 4.6 m/s (ii) 22 m. c) (i) 1.7 m/s (ii) 1.1 m.
d) (i) 3.6 m/s (ii) 17 m. e) (i) 3.6 m/s (ii) 14 m.

Full-Answer Problems

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010 qfull 00100 1 3 0 easy math: ideal elastic 1-dimensional 2-object collision

462. Consider the ideal elastic collision of two objects in one dimension. The objects could be point masses or finite objects, but either way their centers of mass are contrained to one dimension (i.e., to move along one line) and the sum of their kinetic energies along the line of motion is conserved. It is an ideal collision, and so external forces to the two-object system have negligible effect during the duration of the collision (i.e., the collision event), and so the total momentum of the two objects is conserved during the collision.

a) Write down the conservation of momentum and conservation of kinetic energy equations for the two objects. Label the objects 1 and 2 and use primes to indicate post-collision velocities.

b) We assume the v_1' and v_2' are our unknowns. Since we have 2 equations, we can in principle solve for them both. However, one equation is non-linear and so we need a trick which is difference of squares. First rearrange the two equations so that all object 1 quantities are on one side and all object 2 quantities are on the other. Apply difference of squares to the rearranged conservation of kinetic energy equation and divide it by the rearranged conservation of momentum equation. This gives us two equations linear in the velocities. What is the relationship between the pre-collision relative velocity of the objects $\Delta v_{21}'$ and the post-collision relative velocity of the objects Δv_{21}?

c) Solve for the formulae for v_1' and v_2' in terms of the knowns: i.e., m_1, v_1, m_2, v_2.

d) What can we say about the nature of collision forces that the objects exerted on each other?

e) Actually, there is a second solution for the post-collision velocities v_1' and v_2' that disappeared in a division by "zero". This solution is the ghost solution. What is it and can it actually be obeyed by realistic objects.

---

010 qfull 00150 2 3 0 moderate math: perfectly inelastic collisions lose tran K

463. In a perfectly inelastic collision the colliding objects stick together. Consider a set of objects i: the objects could be particles or systems of particles. The objects have masses m_i and initial velocities $\vec{v}_i$. The total mass of th objects is m and their initial center-of-mass velocity $\vec{v}_{\rm CM}$. These objects collide in some more or less complicated series of interactions and end up all stuck together in a clump: i.e., the net event is a perfectly inelastic collision of all the objects. No forces external to the system of objects act during the net event.

a) We define the deviations of initial object velocities from the center-of-mass velocity and show that their mass-weighted average value is zero.

b) Now show that the sum of the initial translational kinetic energies of the objects (i.e., their individual center-of-mass kinetic energies) is always greater than or equal to the final translational kinetic energy (i.e, center-of-mass kinetic energy) of the clump. When does the equality hold? What is the relationship between the total center-of-mass kinetic energy from before and after the collision event?

c) What has happened or may have happened to the missing translational energy.

---

010 qfull 00190 2 5 0 moderate thinking: Underdog and bullet flux

Extra keywords: Has to be renumbered and maybe reworked.

464. Few now remember Underdog, but once he was big on Saturday morning, long before Scoobie Doo—he's still Big: it's television that's gotten small. Let's assume Underdog was invulnerable, although I can't in fact remember.

a) A sub-villain—arch-villains having bigger dogs to impound—tries to mow the Canny Canine down with a hail of bullets: bullet mass $m = 3\,\mathrm{g}$, bullet length $1\,\mathrm{cm}$, bullet speed $v = 600\,\mathrm{m/s}$, bullet rate of fire $n = 500$ bullets per minute. Assuming the bullets just bounce off the immovable Heroic Hound following an elastic collision, what is the average force they exert on him. Assume a 1-dimensional setup and that the rebound speed is the same as the incident speed albeit you should know as much because he's immovable.

b) Now **CRUDELY** estimate the average force exerted by one bullet on impact during the impact time Δt itself. To do this you need **BLANK** and **BLANK**, and you already know **BLANK**. In order to get **BLANK**, you need to consider the impact process. The actual process is immensely complex and beyond your scope to consider in detail: shock waves will pass through the bullet and it may fragment. Therefore think of a crude model that should be order of magnitude good. Assume the back end of the bullet is unaffected by the impact of any parts of the bullet ahead of it and that once the impact occurs for the back end, the impact event is over. This picture certainly cannot be right in detail because shock waves will propagate through the bullet when it impacts. Nevertheless, if one imagines turning all the complicated processes off, then picture may be more or less right. Those complicated processes can't totally dominate the behavior and in any case they are likely to be determined to some degree by characteristic length and velocity scales of the problem: these include the bullet length and velocity. This replacing of complex reality by crude models that contain essential characteristics of the behavior is a powerful technique in physics and elsewhere.

c) Draw a schematic plot of bullet force as a function of time and include a line for the average force.

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010 qfull 00200 2 5 0 moderate thinking: goats, perfectly inelastic coll.

Extra keywords: goats Richthofen and Mannock

465. Richthofen and Mannock are two goats with masses 40 kg and 50 kg, respectively. They charge each other head-on on a icy pond—amazingly they stay upright, but goats are sure-hooféd. Just before impact Richthofen is moving at 10.0 m/s—he's way out of control—and Mannock's moving at −5.0 m/s. On impact, they lock horns literally (i.e., stick together). Treat the goats as point masses and their collision as an ideal collision (i.e., one in which only the collision forces are non-negligible). The problem is entirely 1-dimensional.

a) What type of collision has occurred? It is one of the basic kinds of ideal collisions.

b) What is the center-of-mass velocity of the Richthofen-Mannock system just **BEFORE** and just **AFTER** the collision? Make the collision approximation. Be explicit: which velocity is before and which is after?

c) Assuming that the coefficient of kinetic friction μ_{kin} between goat and ice is 0.13, what is the displacement of the unhappy Richthofen-Mannock system from the impact site to where it stops moving?

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010 qfull 00210 1 3 0 easy math: two objects collide elastically

466. Two objects collide elastically in a 1-dimensional setup. The first has mass $m_1 = 1.00\,\text{kg}$ and initial velocity $v_1 = 6.00\,\text{m/s}$ and the second, mass $m_2 = 8.00\,\text{kg}$ and initial velocity $v_2 = 3.00\,\text{m/s}$. Note initial velocities are given, not speeds. What are the **TWO** final velocities?

---

010 qfull 00220 2 5 0 moderate thinking: Jupiter gravity assist

467. Now you can't (to good approximation) push or pull on space. But one can take a swing about a celestial body. This maneuver in NASAese is called a gravity assist or a swingby or a slingshot encounter. Say we have a space probe on Jupiter's orbital path opposite to Jupiter's motion. The path can be regarded as straight line over the distance scale of our problem. The probe mass has $m = 1000\,\text{kg}$ and Jupiter has $m_{\text{Ju}} = 1.8988 \times 10^{27}\,\text{kg} \approx 318$ Earth masses. The probe velocity $v_{\text{probe}\,i} = -10\,\text{km/s}$ and Jupiter's velocity $v_{\text{Ju}\,i} = 13.1\,\text{km/s}$: both velocities are relative to the Sun's rest frame. The encounter is planned such that, if all goes well, the probe does 180° swing around Jupiter (i.e., a U-turn). Regarding the encounter as a 1-dimensional elastic collision what is the outgoing probe speed? Now why does NASA uses slingshot shot encounters? Give two reasons.

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010 qfull 00320 1 3 0 easy math: ballistic pendulum

468. A ballistic pendulum is a device used to measure bullet speeds. The essential idea is that a bullet is fired into a wood block suspended from a cord. The bullet embeds itself in the suspended block. The suspended block is the ballistic pendulum itself. The block is initially a rest, but picks up momentum from the bullet and rises to some height where it momentarily comes to rest. From a measurement of the height, the speed of the bullet can be calculated. Nowadays, high-speed photography has made the ballistic pendulum obsolete, except for pedagogical purposes.

a) A bullet of velocity v_1 and mass m_1 is fired into a ballistic pendulum of initial velocity zero and mass m_2. Derive a formula for the velocity v_2 of the pendulum just after the impact in terms of the formal knowns v_1, m_1, and m_2. **HINT:** Make the ideal collision approximation and draw a diagram.

b) The pendulum rises with its cord taut to height Δy about its initial height. Derive a formula for v_2 in terms of Δy and g.

c) Now derive a formula for v_1 in terms of m_1, m_2, g, and Δy only.

d) What is v_1 for $m_1 = 0.015\,\text{kg}$, $m_2 = 5.0\,\text{kg}$, and $\Delta y = 1\,\text{m}$?

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010 qfull 00400 2 5 0 moderate thinking: teenagers, symmetrical coll.

Extra keywords: This is a 2-d problem

469. Teenagers on ice—thin ice—each of mass 50 kg, slide together and cling. They are both moving at speed 3 m/s prior to the collision. An old codger of 75 kg moving at 2 m/s then collides with them and the whole conglomerate then comes to a dead stop. What is the angle between the initial velocity vectors of the two teenagers. **HINT:** A diagram might help.

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010 qfull 00500 2 5 0 moderate thinking: bullet colliding with plank

470. A 10 g bullet is shot vertically upward at 1000 m/s. Just after firing it passes straight through a 10 kg plank resting across a couple of rafters—the roof hasn't been added yet. The bullet rises to a **MAXIMUM HEIGHT** of 500 m above the rafters after going through the plank. Assume **NO** air drag and exact 1-dimensionality. **HINT:** The collision with the plank is **NOT** elastic.

a) What is the bullet's velocity immediately after the collision with the plank (i.e., immediately after it has exited the plank)?

b) What is the plank's velocity immediately after the collision (i.e., immediately after the bullet has exited the plank)? **HINT:** Make the ideal collision approximation.

c) How high does the plank rise above its initial position?

d) Approximately calculate the bullet's speed when it returns approximately to the ground level.

e) Is it dangerous to fire bullets straight up into the air? First assume no air drag and show by **SYMBOLIC CALCULATION** whether the return-to-Earth bullet velocity is dangerous or not.

Next assume there is air drag and argue whether the return-to-Earth velocity is dangerous or not. **HINT:** Raindrops reach a terminal speed of 7 m/s after a fall from rest of about 6 m. Also think of everyday experiences.

Chapt. 11 Rotational Kinematics

Multiple-Choice Problems

011 qmult 00100 1 4 4 easy deducto-memory: rotational kinematics

471. "Let's play *Jeopardy*! For $100, the answer is: It is the study of the description of rotational motion or angular motion without reference to forces or torques."

What is _____, Alex?

a) translational dynamics b) rotational dynamics c) translational kinematics
d) rotational kinematics e) rotational freight

011 qmult 00110 1 1 3 easy memory: 1-d rotational kinematics

472. The rotational kinematics analog to 1-dimensional translational (or rectilinear) motion is:

a) general rotation. b) kernal rotation. c) rotation about a single fixed axis.
d) rotation about two axes. e) rotation about three axes.

011 qmult 00120 1 1 1 easy memory: angular displacement

473. A change in angular position of a point is a/an:

a) angular displacement. b) angular velocity. c) angular acceleration.
d) translational velocity. e) translational acceleration.

011 qmult 00130 1 1 2 easy memory: angular variables

474. The usual symbols for the angular kinematic variables angular displacement, angular velocity, and angular acceleration all for rotational motion around a single fixed axis are, respectively:

a) α, β, γ. b) θ, ω, α. c) θ, α, ω. d) ω, θ, α. e) δ, ϵ, ζ.

011 qmult 00140 1 1 3 easy memory: angular velocity defined

475. About a single fixed axis,

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} \qquad \text{or, if you know calculus,} \qquad \omega = \frac{d\theta}{dt}$$

is the definition of:

a) angular displacement. b) angular acceleration. c) angular velocity.
d) angular momentum. e) torque.

011 qmult 00150 1 1 2 easy memory: angular acceleration defined

476. About a single fixed axis,

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} \qquad \text{or, if you know calculus,} \qquad \alpha = \frac{d\omega}{dt}$$

is the definition of

a) angular displacement. b) angular acceleration. c) angular velocity.
d) angular momentum. e) torque.

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011 qmult 00180 1 1 3 easy memory: rotational and tangential variables

477. The formulas

$$\Delta s = \Delta\theta r \ , \qquad v = \omega r \ , \qquad a = \alpha r$$

(where r is the radius of circular motion) relate the rotational variables to the analog:

a) units. b) transgressional variables. c) tangential variables. d) angular units. e) translational units.

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011 qmult 00200 1 4 2 easy deducto-memory: rotational kinematic equations

478. The rotational constant-angular-acceleration kinematic equations:

a) have no resemblance to the linear kinematic equations.
b) are exactly the same as the linear kinematic equations, except that the angular kinematic equations relate **ANGULAR** rather than linear variables.
c) are exactly the same as the linear kinematic equations, except that the angular kinematic equations relate **LINEAR** rather than angular variables.
d) do not allow for angular acceleration.
e) include torque terms.

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011 qmult 00202 1 1 4 easy memory: independent rot. kin. eq.

479. The number of **ALGEBRAICALLY INDEPENDENT** rotational constant-angular-acceleration equations is:

a) 5. b) 4. c) 3. d) 2. e) 1.

---

011 qmult 00232 1 1 5 easy memory: timeless equation use

480. The rotational constant-angular-acceleration equation

$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

by itself alone does **NEVER** allows you to solve for:

a) α. b) $\Delta\theta$. c) ω_0. d) ω. e) t.

---

011 qmult 00310 1 3 3 easy math: find the initial omega 1

481. A wheel spins π radians in 10.0 s with an angular acceleration of 4.00 radians/s^2. What is its final angular velocity?

a) 80.1 radians/s. b) 203 radians/s. c) 20.3 radians/s. d) 3.14 radians/s. e) 6.28 radians/s.

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011 qmult 00312 1 3 4 easy math: find the initial omega 2

482. A wheel spins $(3/2)\pi$ radians in 7.00 s with an angular acceleration of 5.00 radians/s^2. What is its final angular velocity?

a) 80.1 radians/s. b) 203 radians/s. c) 20.3 radians/s. d) 18.2, radians/s. e) 6.28 radians/s.

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011 qmult 00440 2 3 3 mod. math: discus centripetal and tangential acceleration

483. A discus thrower (in Greek, a *Diskobolos*) accelerates a discus from rest through π radians accelerating it from angular velocity 0 to angular velocity 15 rad/s. We idealize his motion as circular with radius $r = 0.81$ m. The time of the throwing motion is 0.27 s. What is the net **TRANSLATIONAL** acceleration (not tangential acceleration) of the discus at the end of the throwing motion just before release? Give the direction of acceleration relative to the inward radial direction.

a) 200 m/s^2; 10°. b) 13.9 m/s^2; 188°. c) 188 m/s^2; 13.9°. d) 150 m/s^2; 15°. e) 100 m/s^2; 50°.

011 qmult 00442 2 3 2 mod. math: discus centripetal and tangential acceleration 2

484. A discus thrower (in Greek, a *Diskobolos*) accelerates a discus from rest through π radians accelerating it from angular velocity 0 to angular velocity 12 rad/s with constant angular acceleration. We idealize her motion as circular with radius $r = 0.65\,\text{m}$. The time of the throwing motion is 0.25 s. What is the net **TRANSLATIONAL** acceleration of the discus at the end of the throwing motion just before release? Give the direction of acceleration relative to the inward radial direction.

a) $12\,\text{m/s}^2$; 15°. b) $99\,\text{m/s}^2$; 18.4°. c) $188\,\text{m/s}^2$; 13.9°. d) $144\,\text{m/s}^2$; 14°. e) $166\,\text{m/s}^2$; 32°.

011 qmult 10330 1 3 3 easy math: Waldo centrifuge, Waldo orbit fullmult

Extra keywords: just rotational kinematics

485.** Waldo's back—you know, Waldo Pepper, the Playful Pig—just accept it. This time, the Bold Boar has decided to become an astronaut and is training on NASA's giant centrifuge—the one in the film *The Right Stuff*. Let's guess it has a radius of 10.0 m. The centrifuge spins in the horizontal: i.e., the centrifuge axis is perpendicular to the ground.

i) Starting from **REST**, the centrifuge goes into a **CONSTANT** angular acceleration phase for 10.0 s. At this point Waldo—who does indeed have a mass of 150 kg—notes that the vertical weighing scale he is nauseatingly pressed on reads 2000 N. What is the centripetal force on Waldo?

ii) The Sentient Swine now does some math—some correct math. What does Waldo find for his angular velocity ω at the 10 s mark?

iii) Doing a little more correct math, the Heck-of-a-Hog now finds his angular acceleration α from time zero to the 10 s mark. What is this angular acceleration?

a) (i) 2000 N (ii) 1.33 radians/s (iii) $0.133\,\text{radians/s}^2$.
b) (i) 200 N (ii) 0.36 radians/s (iii) $0.036\,\text{radians/s}^2$.
c) (i) 2000 N (ii) 1.15 radians/s (iii) $0.115\,\text{radians/s}^2$.
d) (i) 2000 N (ii) 0.36 radians/s (iii) $0.036\,\text{radians/s}^2$.
e) (i) 200 N (ii) 0.36 radians/s (iii) $0.00\,\text{radians/s}^2$.

Full-Answer Problems

011 qfull 00330 1 3 0 easy math: Waldo centrifuge, Waldo orbit

Extra keywords: just rotational kinematics.

486. Waldo's back—you know, Waldo Pepper, the Playful Pig—just accept it. This time, the Bold Boar has decided to become an astronaut and is training on NASA's giant centrifuge—the one in the film *The Right Stuff*. Let's guess it has a radius of 10 m. The centrifuge spins in the horizontal: i.e., the centrifuge axis is perpendicular to the ground.

a) Starting from rest the centrifuge goes into a constant angular acceleration phase for 10 s. At this point Waldo—who does indeed have a mass of 150 kg—notes that the vertical weighing scale he is nauseatingly pressed on reads 3000 N. What is the centripetal force on Waldo and what kind of force is the centripetal force?

Note, Waldo is standing on the floor which provides an upward normal force which cancels the gravity force downward.

b) The Sentient Swine now does some math—some correct math. What does Waldo find for his tangential velocity at the 10 s mark? What does he find for his angular velocity at the 10 s mark?

c) Doing a little more correct math, Waldo now finds his angular acceleration from time zero to the 10 s mark. What is this angular acceleration?

d) At this point, the Shaken Bacon does a little Gedanken experiment—just like Einstein—"What if I instantaneously left NASA's big blender and found myself 1000 **KILOMETERS** from the center of Pluto (formerly the 9th planet) and moving perpendicularly to the line to Pluto in free space, but with my current velocity?" What in this case would Waldo's angular velocity be about Pluto?—for an instant since after that the Heck-of-a-Hog knows his orbital path will almost certainly not be circular.

Chapt. 12 Rotational Dynamics

Multiple-Choice Problems

012 qmult 00100 1 1 1 easy memory: rotational principles

487. Classical rotational dynamics principles are:

a) secondary principles derived from the fundamental principles of Newtonian physics (i.e., Newton's three laws, force laws, energy, etc.).
b) independent postulates completely unrelated to the fundamental principles of Newtonian physics (i.e., Newton's three laws, force laws, the energy concept, etc.).
c) secondary principles derived from quantum mechanics.
d) all gross approximations derived from the fundamental principles of Newtonian physics (i.e., Newton's three laws, force laws, the energy concept, etc.).
e) independent, but very approximate, postulates completely unrelated to the fundamental principles of Newtonian physics (i.e., Newton's three laws, force laws, the energy concept, etc.).

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012 qmult 00200 1 1 3 easy memory: cross product definition

488. The mathematical operation

$$\vec{a} \times \vec{b} = ab \sin\theta \hat{n} \ ,$$

where $\vec{a}$ and $\vec{b}$ are general vectors, θ is the angle between them (with their tails joined), and $\hat{n}$ is a unit vector normal to the plane defined by $\vec{a}$ and $\vec{b}$ and with sense determined by a right-hand rule (sweep fingers of the right hand from $\vec{a}$ to $\vec{b}$ and the thumb gives the sense), is the:

a) outer product. b) dot product. c) cross product. d) inner product. e) angry product.

---

012 qmult 00220 1 1 2 easy memory: cross product standard values

489. Behold:

$$\vec{a} \times \vec{b} = \begin{cases} ab \sin\theta \hat{n} & \text{in general;} \\ 0 & \text{for } \theta = 0^\circ \text{ or } 180^\circ; \\ ab\hat{n} & \text{for } \theta = 90^\circ; \\ \underline{\qquad\qquad} & \text{in general.} \end{cases}$$

a) $\vec{b} \times \vec{a}$. b) $-\vec{b} \times \vec{a}$. c) $-\vec{a} \times \vec{b}$. d) $-\vec{b} \cdot \vec{a}$. e) $\vec{a} \cdot \vec{b}$.

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012 qmult 00270 2 3 5 moderate math: triple vector product example

490. The quantity

$$\vec{A} \cdot (\vec{A} \times \vec{B})$$

(where θ is the angle between $\vec{A}$ and $\vec{B}$) when evaluated gives:

a) $A^2 B$. b) $A^2 B \cos(\theta)$. c) $A^2 B \cos(\theta) \sin(\theta)$. d) $A^2 B \tan(\theta)$. e) 0.

---

012 qmult 00310 1 1 5 easy memory: angular momentum and cross product

491. Angular momentum, given by the formula

$$\vec{L} = \vec{r} \times \vec{p} \ ,$$

is a quantity we define as a measure of rotational motion about a point or axis. The cross product in the definition is very useful in making angular momentum a measure of rotation. If a particle is moving directly away from the origin, then its not rotating about the origin in any common-sense sense and the cross product reflects this by giving _____ angular momentum. If a particle is moving perpendicularly to a line to the origin, then it could be rotating about the origin and even be in circular motion about the origin and the cross product reflects this by giving _____ angular momentum magnitude for the given magnitudes of displacement and momemtum.

a) zero; infinite b) the maximum; infinite c) no; zero d) the maximum; zero
e) zero; the maximum

---

012 qmult 00315 1 1 3 easy memory: v X mv=0

492. The quantity

$$\frac{d\vec{r}}{dt} \times \vec{p}$$

equals:

a) $\vec{\tau}_{\rm net}$. b) $\frac{d\vec{L}}{dt}$. c) 0 . d) $\frac{1}{2} I\vec{L}$. e) $\frac{1}{2} I\vec{\tau}$.

---

012 qmult 00320 1 1 5 easy memory: rotational 2nd law specified

493. The rotational 2nd law for a particle or a system of particles is:

a) $\vec{A} \times \vec{B}$. b) $\frac{d\vec{\tau}_{\rm net}}{dt} = \frac{1}{2} I\vec{L}$. c) $\frac{d\vec{L}}{dt} = \frac{1}{2} I\tau_{\rm net}$. d) $\frac{d\vec{\tau}_{\rm net}}{dt} = \vec{L}$.
e) $\frac{d\vec{L}}{dt} = \vec{\tau}_{\rm net}$.

---

012 qmult 00330 1 1 2 easy memory: torque definition descriptive

494. Torque is:

a) that thing (for lack of a better word) needed for an angular **VELOCITY** to exist.
b) that thing (for lack of a better word) that can cause angular **ACCELERATION**.
c) the same as angular acceleration.
d) the same as angular velocity.
e) a minor rock star of the 1960's: the fairhaired Monkee.

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012 qmult 00340 1 1 1 easy memory: vector addition
Extra keywords: new place

495. Angular momentum $\vec{L}$ and torque $\vec{\tau}$ are actually pseudovectors (AKA axial vectors), but our purposes they can be treated just like ordinary vectors—and so we will just call them vectors. In this class, we usually deal only with the z-components of angular momentum and torque which are scalars. The direction of these vectors can be determined from their definitions

$$\vec{L} = \vec{r} \times \vec{p} \qquad \text{and} \qquad \vec{\tau} = \vec{r} \times \vec{F} \ .$$

a) Vectors add like vectors. This can be done geometrically or using components in some coordinate system.
b) Vectors add just like scalars.
c) Vectors cannot be added at all. The concept of adding vectors is undefined.
d) Vectors add just like scalars **MULTIPLY**.
e) Vectors add just like scalars **DIVIDE**.

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012 qmult 00480 1 5 2 easy thinking: no net torque in equilibrium

496. There's an odd-shaped object sitting on a table. It's not rotating; it's not translating: it's just sitting for an indefinite time. The **NET** torque on this object as measured from any origin at all must be:

a) infinite. b) zero. c) incalculable. d) 0.5 N m. e) a minor rock star of the 1960's: the fairhaired Monkee.

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012 qmult 00570 2 1 2 moderate memory: conservation of ang. momentum
Extra keywords: new place

497. You are Katherina Witt (or Dorothy Hamill, Karen Magnussen, or even Barbara Ann Scott) at the Winter Olympics. After executing a flawless quad followed by a physically impossible horizontal leap, you torque yourself (using heal? toe?) into a spin. You now decide to speed up your spin (increase your ω). Remembering that without a net external torque, angular momentum ($L = I\omega$) is conserved and assuming that the ice friction torque on your blades is negligible, you do what? **HINT:** The more spread out from the axis a fixed amount of mass is, the greater the rotational inertia.

a) **PULL** your arms in to **INCREASE** your rotational inertia.
b) **PULL** your arms in to **DECREASE** your rotational inertia.
c) **FLING** your arms out to **DECREASE** your rotational inertia.
d) **FLING** your arms out to **INCREASE** your rotational inertia.
e) Belly flop.

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012 qmult 00660 1 3 4 easy math: rotational Newton's 2nd law simple example 1

498. There is a net torque of 3.0 N m on an object about a particular axis of the object. The object's rotational inertia for this axis is $10^2\,\mathrm{kg\,m^2}$. What is the angular acceleration?

a) $\alpha = 3 \times 10^2\,\mathrm{rad/s^2}$. b) $\alpha = \frac{1}{3} \times 10^2\,\mathrm{rad/s^2}$. c) $\alpha = \frac{1}{3} \times 10^{-2}\,\mathrm{rad/s^2}$.
d) $\alpha = 3 \times 10^{-2}\,\mathrm{rad/s^2}$. e) $\alpha = 10^{-2}\,\mathrm{rad/s^2}$.

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012 qmult 00662 1 3 3 easy math: rotational Newton's 2nd law simple example 2

499. There is a net torque of 30.0 N m on an object about a particular axis of the object. The object's rotational inertia for this axis is $50.0\,\mathrm{kg\,m^2}$. What is the angular acceleration?

a) $\alpha = 1.67\,\mathrm{rad/s^2}$. b) $\alpha = 0.333 \times 10^2\,\mathrm{rad/s^2}$. c) $\alpha = 0.600\,\mathrm{rad/s^2}$.
d) $\alpha = 3.00 \times 10^{-2}\,\mathrm{rad/s^2}$. e) $\alpha = 1.00 \times 10^{-2}\,\mathrm{rad/s^2}$.

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012 qmult 00664 1 3 5 easy math: rotational Newton's 2nd law simple example 3

500. There is a net torque of 20.0 N m on an object about a particular axis of the object. The object's rotational inertia for this axis is $50.0\,\mathrm{kg\,m^2}$. What is the angular acceleration?

a) $\alpha = 1.67\,\mathrm{rad/s^2}$. b) $\alpha = 0.333 \times 10^2\,\mathrm{rad/s^2}$. c) $\alpha = 0.600\,\mathrm{rad/s^2}$.
d) $\alpha = 3.00 \times 10^{-2}\,\mathrm{rad/s^2}$. e) $\alpha = 0.400\,\mathrm{rad/s^2}$.

---

012 qmult 00680 1 5 2 easy thinking: Archimedes

501. Archimedes (287?–212 BC) (the discoverer of the law of the lever) said something like "give me a lever long enough, a fulcrum (a support for a lever), and a place to stand, and I will move:

a) flea". b) the Earth". c) a horse to drink (after leading it to water)".
d) a man to drink". e) a minor rock star of the 1960's: the fairhaired Monkee."

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012 qmult 00720 1 4 5 easy deducto-memory: gravitational torque

502. "Let's play *Jeopardy*! For $100, the answer is: Its torque about any origin can be calculated as if all the object's mass were located at the center of mass."

What is _____, Alex?

a) a contact force b) friction c) a tension force d) a normal force
e) gravity near the Earth's surface

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012 qmult 00850 1 1 4 easy memory: uniform solid ball rotational inertia

503. The rotational inertia of a uniform solid ball of mass M and radius r is:

a) Mr^2. b) Mr. c) $\frac{2}{5}Mr$. d) $\frac{2}{5}Mr^2$. e) Mr^3.

---

012 qmult 00860 1 1 5 easy memory: spherical shell rotational inertia

504. The rotational inertia of an infinitely thin spherical shell of mass M and radius r is:

a) Mr^2. b) Mr. c) $\frac{2}{3}Mr$. d) Mr^3. e) $\frac{2}{3}Mr^2$.

---

012 qmult 01050 1 4 1 easy thinking: rotational analogs

505. For rigid body rotation about a fixed axis, the rotational analogs of inertial mass (m), momentum ($\vec{P} = m\vec{v}$), Newton's 2nd law ($\vec{F}_{\rm ext} = m\vec{a}$), and translational (or center-of-mass) kinetic energy ($K = (1/2)mv^2$) are, respectively:

a) rotational inertia (I), angular momentum ($L = I\omega$), the rotational Newton's 2nd law ($\tau = I\alpha$), and rotational kinetic energy ($K = (1/2)I\omega^2$).
b) rotational inertia (I), the rotational Newton's 2nd law ($\tau_{\rm ext} = I\alpha$), and angular momentum ($L = I\omega$).
c) the rotational Newton's 2nd law ($\tau_{\rm ext} = I\alpha$), angular momentum ($L = I\omega$), and rotational kinetic energy ($K = (1/2)I\omega^2$).
d) mass (m), angular velocity (ω), the static Newton's 2nd law ($0 = 0$), and rotational kinetic energy ($K = (1/2)I\omega^2$).
e) without clear definitions.

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012 qmult 01200 1 1 3 easy memory: rolling motion

506. In rolling motion (e.g., a ball rolling along the ground), there is in general both rotational and:

a) angular motion. b) centripetal motion.
c) center-of-mass or translational motion. d) slow motion. e) blocked motion.

---

012 qmult 01210 1 4 3 easy deducto-memory: no-slip condition

507. "Let's play *Jeopardy*! For $100, the answer is: The condition that is required for wheels in most ordinary circumstances: e.g., for car wheels."

What is the _____ condition, Alex?

a) no-trip b) no-rip c) no-slip d) no-grip e) no-blip

---

012 qmult 01230 1 1 1 easy memory: rolling motion relationships

508. For a roller of radius r and defining the center-of-mass or translation direction as the positive s direction and the counterclockwise as the positive rotational direction, the relationships between translational and rotational kinematic variables with the no-slip condition are

$$\Delta\theta = -\frac{\Delta s}{r}\ , \qquad \omega = -\frac{v}{r}\ ,$$

and:

a) $\alpha = -\frac{a}{r}$. b) $\alpha = -ar$. c) $\alpha = -\omega r$. d) $\alpha = \frac{a}{r}$. e) $\alpha = ar$.

---

012 qmult 01240 1 5 1 easy thinking: rolling motion and static friction

Extra keywords: new place

509. A roller (i.e., a sphere or cylinder) needs static friction to enforce the no-slip condition in general. But it doesn't need static friction to keep rolling at constant velocity as it rolls along a level surface in the limit of no rolling friction. You know this (if for no other reason) because if the roller rolls off the table, it:

a) keeps spinning. b) stops spinning. c) goes into a parabolic trajectory.
d) goes into a straight line trajectory. e) drops straight down.

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012 qmult 01270 1 5 4 moderate thinking: two rolling cylinders

Extra keywords: new place

510. Two cylinders of the same mass and radius start from rest at the top of an incline and roll down. One is just a thin cylindrical shell and the other is solid cylinder of uniform density. Which reaches the bottom **FIRST**?

a) Neither. b) The reach the bottom at the same time.
c) The shell one. d) The solid one. e) It's random which one reaches the bottom first.

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012 qmult 01280 2 5 2 moderate thinking: mechanical energy conserved

Extra keywords: new place

511. In the idealized object-rolling-down-incline system, the rolling object is perfectly round (a cylinder or sphere), the object and incline are perfectly rigid, and there is a static frictional force that causes there to be no slipping. (The frictional force is a static frictional force since ideally at the point of contact there is no relative motion between object and incline surfaces.) Mechanical energy (the sum of potential energy, center-of-mass (translational) kinetic energy, rotational kinetic energy) is conserved. And yet there is a frictional force is doing work on the object. Why doesn't the frictional force cause loss of mechanical energy to heat energy (i.e., dissipation as it is called)?

a) There is never any frictional heating with round objects.
b) The frictional force does no net work. Going down the incline, the friction force (which points uphill) does negative work lowering the object's center-of-mass kinetic energy (which being increased by gravitational potential energy), but positive work to increase its rotational kinetic energy. The friction work contributions cancel as a calculation would show. Going up the incline, the frictional force is still uphill, but now converts rotational kinetic energy into center-of mass kinetic energy (which is disappearing into gravitational potential energy). Again the friction work contributions can cancel out. The static friction, in fact, acts as channel between center-of-mass and rotational kinetic energies. Kinetic friction dissipates energy by turning it into random microscopic kinetic and potential energy. The ideal static friction force does not do this, and so does not dissipate energy. Of course, in reality there is always some slipping and some non-elastic deformation of the bodies, and so some energy gets dissipated. This slipping and deformation is often not obvious to the eye.
c) Actually heat energy is being turned into macroscopic kinetic energy by friction in this case without rejection of any heat energy to a lower temperature thermal reservoir. This violates the 2nd law of thermodynamics, but that doesn't matter in a mechanics problem.
d) The frictional force **CAUSES** a torque and forces that cause torques **NEVER** cause dissipation.
e) The frictional force **DOES NOT CAUSE** a torque and forces that **DO NOT** cause torques **NEVER** cause dissipation.

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012 qmult 11250 2 3 4 moderate math: rolling ball on incline fullmult

512.** A uniform solid ball of mass 10.0 kg starting from **REST** rolls down an incline of angle $\theta = 30°$. Note, the ball is a **ROLLER**, not a **SLIDER**. There is no slipping between the ball and incline: i.e., the no-slip condition is imposed.

i) Find the ball's center-of-mass acceleration. **HINT:** You could check an equation table.

ii) What is its center-of-mass velocity after 10 s?

iii) How far has it traveled down the incline in 10 s?

a) (i) $4.9\,\mathrm{m/s^2}$ (ii) $49\,\mathrm{m/s}$ (iii) $490\,\mathrm{m}$.
b) (i) $4.9\,\mathrm{m/s^2}$ (ii) $49\,\mathrm{m/s}$ (iii) $245\,\mathrm{m}$.
c) (i) $3.5\,\mathrm{m/s^2}$ (ii) $35\,\mathrm{m/s}$ (iii) $350\,\mathrm{m}$.
d) (i) $3.5\,\mathrm{m/s^2}$ (ii) $35\,\mathrm{m/s}$ (iii) $175\,\mathrm{m}$.
e) (i) $4.9\,\mathrm{m/s^2}$ (ii) $4.9\,\mathrm{m/s}$ (iii) $49\,\mathrm{m}$.

Full-Answer Problems

012 qfull 00300 1 3 0 easy math: deriving the rotational 2nd law

513. We define the angular momentum and torque on a particle i by, respectively, the formulae

$$\vec{L}_i = \vec{r}_i \times \vec{p}_i \qquad \text{and} \qquad \vec{\tau}_i = \vec{r}_i \times \vec{F}_i \ ,$$

where $\vec{r}_i$ is the particle position relative to a general origin, $\vec{p}_i$ is the particle momentum, and $\vec{F}_i$ is an external force on the paricle (but not necessarily the net external force on the particle). The term particle is used in the sense of a sample of matter sufficiently small that its extent in space can be neglected. Usually by particle in classical physics, one means a sample of matter that it macroscopically tiny—and so it doesn't matter exactly where you locate it's center of mass and other other properties as long as they are within the particle's actual extent—but still larger a microscopic particle (which requires quantum mechanics). Both angular momentum and torque are pseudovectors (AKA axial vectors) since they are formed by cross products of ordinary vectors. For our purposes, we can treat them just as ordinary vectors. As vectors, they have a direction in space space, but their extent is in their own abstract space. Where are they located in real space? Well nowhere really—it adds nothing to their utility to explicitly locate them. But for mental convenience, one often locates them at the origin.

From its definition, one can see that angular momentum is a sort of mass-weighted measure of rotation about an axis passing through the origin and perpendicular to $\vec{r}_i$ and $\vec{p}_i$. The particle with angular momentum does not have to be in rotation at all though: it could be going in a straight line and still have angular momentum about any origin. From its definition, one can see that torque is a sort of measure of the rotational acceleration effect of force $\vec{F}_i$ about the axis passing through the origin and perpendicular to $\vec{r}_i$ and $\vec{F}_i$. Since the definitions of angular momentum and torque are precise, they are precise measures and instead of saying "sort of something", we just say they are precisely what they are: angular momentum and torque.

Angular momentum and torque turn out to be quantitatively useful dynamical variables for analyzing rotational motion—and that's why we have defined them. In this, question we show the essentials of angular dynamics using angular momentum and torque by deriving, among other things, the rotational 2nd law. The rotational 2nd law is just called a law by convention since it is a derived result. It is called the rotational 2nd law since in rotationaly dynamics it plays a role analogous to Newton's 2nd law—which really is a law: it is a true axiom of classical mechanics.

a) What is the total angular momentum $\vec{L}$ of a system made up of particles i?

b) The dynamics of rotational systems is studied by studying the time variation of $\vec{L}$. To this end, we need formula for $d\vec{L}/dt$. Derive this formula and eliminate any zero terms. **HINT:** The product rule does apply to the cross product and the cross product of a vector with itself is zero.

c) Newton's 2nd all in general form is

$$\frac{d\vec{p}}{dt} = \vec{F}_{\text{ext}} \ ,$$

where $\vec{p}$ is the momentum of a system and $\vec{F}_{\text{ext}}$ is the net external force on the system. Use this law to replace the momentum derivative in the part (b) answer formula by the net

external force on particle i

$$\vec{F}_{\rm i}' = \vec{F}_i + \sum_j \vec{F}_{ji} \ ,$$

where $\vec{F}_i$ net force on particle i from force sources outside the system of particles, F_{ji} is the force of particle j on particle i, and F_{ii} is zero the external force of particle i on itself logically zero.

d) Now show that

$$\frac{d\vec{L}}{dt} = \sum_i \vec{r}_i \times \vec{F}_i \ ,$$

making use of Newton's 3rd law and using the assumption that all inter-particle forces are central forces. A central force is one where the force of one particle/system on another is aligned with the line joining the particles/particle centers of mass. For solid objects and many others, the assumption of central forces is valid valid.

e) Now use the definition of torque to show that

$$\frac{d\vec{L}}{dt} = \vec{\tau}_{\rm ext} \ .$$

where $\vec{\tau}_{\rm ext}$ net torque on the system due to external forces on the system. This result is the general form of the rotational 2nd law. **HINT:** There is almost nothing to show now.

f) The general rotational 2nd law is generally useful. But we can specialize it to a form which is useful for rigid body rotation about a fixed axis which conventionally is the z-axis of 3-dimensional Cartesian coordinate. As first step to this form, prove that the z component of

$$\vec{C} = \vec{A} \times \vec{B}$$

is given by

$$C_z = (\vec{A}_{xy} \times \vec{B}_{xy}) \cdot \hat{z} \ ,$$

where $\vec{A}_{xy}$ is the component of $\vec{A}$ in the xy plane $\vec{B}_{xy}$ is the component of $\vec{B}$ in the xy plane, and $\hat{z}$ is the unit vector in the z direction. The last formula show that the z component of z is independent of the z components of $\vec{A}$ and $\vec{B}$ and depends only on there xy plane componets.

g) Using the part (f) result, show that

$$L_z = \sum_i (\vec{r}_{xy,i} \times \vec{p}_{xy,i}) \cdot \hat{z} = \sum_i r_{xy,i} p_{xy,i} \sin\theta_i \ , \qquad \tau_z = \sum_i (\vec{r}_{xy,i} \times \vec{F}_{xy,i}) \cdot \hat{z} = \sum_i r_{xy,i} p_{xy,i} \sin\theta_i \ .$$

where θ_i is the angular coordinate of the second vector relative to the first measured positive in the counterclockwise direction. **HINT:** The proof is by inspection.

h) For rigid body rotation about the z axis, at least in simple analysis, you do not need consider the angular momentum or torque components for the x and y axes. In general, they will not be zero, but you just do not need them to describe the motion—in a simple analysis. For rigid body rotation about the z axis, show that

$$L_z = I\omega \ ,$$

where

$$I = \sum_i m_i r_{xy,i}^2$$

is the rotational inertia (or moment of inertai) of the body about the z axis and ω is the common angular acceleration of the body. For a rigid body, I is a constant, of course.

i) Now proof the special case rotational 2nd law

$$I\alpha = \tau_z \ .$$

where α is the angular acceleration. **HINT:** The proof is by inspection.

j) Moment arm is the name for the length of a perpendicular from the z axis of rotation to a line aligned with $\vec{F}_{xy,i}$. A signed moment arm is one where multiple the moment arm by ± 1, where the upper case is for a counterclockwise torque and the lower case for a clockwise torque. Prove that

$$r_{xy,i} F_{xy,i} \sin\theta_i = d_i F_{xy,i} \ ,$$

where d_i is the signed moment arm for $\vec{F}_{xy,i}$. What if $\theta_i = 0$? What is τ_z evaluated using signed moment arms? In many cases, using moment arms or signed moment arrms simplifies the calculation of τ_z. **HINT:** A diagram might help.

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012 qfull 00310 1 3 0 easy math: rigid-body rotation and total angular momentum

514. Let us consider some intereting features of rigid body rotation.

a) Starting from the general angular momentum definition for a system of particles

$$\vec{L} = \sum_i \vec{r}_i \times \vec{p}_i \ .$$

prove for a rigid body rotating around the z axis that

$$\vec{L} = I\omega\hat{z} - \left(\sum_i m_i \vec{r}_{xy,i} r_{z,i} \right) \omega \ ,$$

where I is the rotational inertia, $\vec{r}_{xy,i}$ is the xy component vector of $\vec{r}_i$ (which is a time-dependent quantity is the vector is rotating about the z axis with the particle), $r_{z,i}$ is the z component of $\vec{r}_i$, and ω is the angular velocity (which is not necessarily constant). The second term in the formula for $\vec{L}$ is not necessarily zero. This shows that rigid body rotators do not necessarily have the angular momentum vector aligned with the rotation z axis. However, if you are only studying the rotation, usually the studying the z component of angular momemtum suffices.

b) Give two simple cases where the second term in the part (a) result vanishes.

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012 qfull 00350 1 5 0 easy thinking: moment arm

515. The general rule for torque is

$$\vec{\tau} = \vec{r} \times \vec{F} = rF\sin(\theta)\hat{k} \ ,$$

where $\vec{r}$ is measured from the origin of the torque calculation, $\vec{F}$ is the force actually acting at position $\vec{r}$, the cross-product angle is the one between the vectors with their tails at the point, and $\hat{k}$ is unit vector perpendicular to $\vec{r}$ and $\vec{F}$ which points in a direction determined by a right-hand rule. For the special case of torque for a z axis for radius and force in the xy plane, the torque formula becomes

$$\tau_z = rF\sin(\theta) \ ,$$

where θ is measured counterclockwise from the $\vec{r}$ direction. However, there is a simple rule that provides an alternative formula for calculating the magnitude of the torque for the just aforesaid special case. Extend a line through the force vector with the force vector located at the point where the force actually acts. This line is called the line of action in old-fashioned

jargon. Consider a perpendicular from the origin to the line of action. This perpendicular is called the moment arm (HRW-230). Create a formula using the length of the moment arm and the magnitude of the force vector that equals the magnitude of torque. Let the angle betwen the line of action and radius that subtends the moment arm be γ. Prove the moment arm formula gives the magnitude of the torque. You will need the trigonometric identity $\sin(180^\circ - \theta) = \sin(\theta)$. The moment arm method for calculating the magnitude of torque gives an old-fashioned picturesque/diagrammatic way of calculating that value and the direction of torque is obvious. The moment arm is part of the lore of torque and those who are used to using moment arm probably find it an efficient way of knowing torque. **HINT:** Draw a diagram.

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012 qfull 00610 2 3 0 moderate math: LLNL door and torque
Extra keywords: (HR-243:53p)

516. The shield door of the neutron test facility at Lawrence Livermore National Laboratory (where they design the bombs) is (or was), according to HRW-243, the world's most massive hinged door: mass $44,000\,\mathrm{kg}$, rotational inertia about its hinge axis $8.7 \times 10^4\,\mathrm{kg\,m^2}$, door width 2.4 meters. Assuming zero friction, what constant force perpendicular to the door at its outer edge is needed to move the door 90° in $10\,\mathrm{s}$ starting from **REST**? Show your calculations. Could you open this door in the time given? Would you want to open this door?

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012 qfull 00612 2 3 0 moderate math: LLNL door and torque 2
Extra keywords: (HR-243:53p)

517. The shield door of the neutron test facility at Lawrence Livermore National Laboratory (where they design the bombs) has been upgraded hypothetically—though not in reality. This most massive hinged door: mass $64,000\,\mathrm{kg}$, rotational inertia about its hinge axis $1.25 \times 10^5\,\mathrm{kg\,m^2}$, door width 3.2 meters. Assuming zero friction, what constant force perpendicular to the door at its outer edge is needed to move the door 90° in $20\,\mathrm{s}$? Show your calculations.

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012 qfull 00640 3 3 0 tough math: Pippa on merry-go-round

518. Wee Pippa Passing runs up to a playground merry-go-round, initially at rest, and jumps **RADIALLY** onto the rim (i.e., all her horizongal impulse is pointed to the center of the merrry-go-round).

a) What is the torque she exerts about the rotational axis of the merry-go-round? Does the merry-go-round start to rotate? Why or why not? Does the merry-go-round move at all? Why or why not?

b) Pippa and the merry-go-round both can have angular momentum about the merry-go-round axis. She and the merry-go-round are coupled together by the static frictional force between her feet and the surface. But Pippa can directly control the relative velocity between herself and the merry-go-round by walking or running: thus she can change the coupling condition. When she is at rest on the merry-go-round, she and the merry-go-round constitute one rigid rotator. But when she moves they constitute two rigid rotators about the merry-go-round axis.

Say Pippa starts running just on the rim of the merry-go-round just after jumping on. The merry-go-round axis is frictionless: thus the total angular momentum of the system about the axis cannot change. Using conservation of angular momentum for an isolated system find an expression for the merry-go-round angular frequency ω_m in terms of Pippa's relative angular frequency $\omega_{\mathrm{p\ rel}}$ and the rotational inertias about the axis of Pippa I_p and the merry-go-round I_m. Note that $\omega_{\mathrm{p\ rel}} = \omega_\mathrm{p} - \omega_\mathrm{m}$. Show your derivation.

c) Give the expression for Pippa's final angular velocity to the ground (i.e., ω_p) using the part (b) result. What would Pippa's final angular velocity be in the limits that $I_\mathrm{m} \to \infty$ and $I_\mathrm{m} \to 0$? Show your derivation.

d) Pippa runs on the rim at $3.0\,\mathrm{m/s}$ relative to the rim. The radius of the merry-go-round is

3 m. The tangential rim velocity of the merry-go-round is $-2.0\,\mathrm{m/s}$ when Pippa is running. Pippa has a mass of 40 kg. Assuming the merry-go-round is a uniform disk, what is its mass. Show your calculation.

e) Is it at all possible with Pippa and merry-go-round starting from rest relative to the ground that both Pippa and the merry-go-round could be made to spin in the same direction relative to the ground without external torques about the merry-go-round axis? Why or why not?

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012 qfull 00840 2 5 0 moderate thinking: hollow sphere rotational inertia

519. The rotational inertia of a solid uniform sphere about an axis through its center of mass is

$$I = \frac{2}{5} m r^2 \ ,$$

where m is its mass and r is its radius.

a) What is the mass of sphere of uniform density ρ and radius r?

b) What is the rotational inertia of a uniform-density spherical shell of mass m and inner radius r_1 and outer radius r_2? Write the answer in terms of m, r_1, and r_2

c) The binomial theorem is

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^n \ .$$

Expand $(x+\Delta x)^n$ using the binomial theorem for $n = 3$ and $n = 5$ and truncate the series to 1st order in Δx. The truncated expansions are accurate to 1st order in small Δx.

d) Making use of the parts (b) and (c) results what is the rotational inertia of a differentially thin hollow shell of mass m and radius r? **HINT:** Let $r_1 = r$ and $r_2 = r + \Delta r$ in the part (b) result.

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012 qfull 00940 2 3 0 moderate math: perpendicular axis theorem, etc.

Extra keywords:

520. Do the following problems.

a) The perpendicular axis theorem applies exactly to infinitely thin planar objects and approximately to finite thin planar objects. Say you have a planar object with surface area density σ. You choose an origin and define a set of coordinates with the x and y axes in the object plane and the z axis perpendicular to the object plane. The planar axis theorem is

$$I_z = I_x + I_y \ ,$$

where I_x, I_y, and I_z are the rotational inertias about, respectively, the x, y, and z axes. Prove the theorem.

Note, the following. The general rotational inertia formula about an axis is

$$I = \sum_i m_i r_i^2 \qquad \text{or integral form} \qquad I = \int r^2 \rho \, dV \ ,$$

where r_i and r are a radii from the axis, not from an origin, and ρ is volume density, and dV is the differential volume. For a planar object, the integral rotational inertia formula for a z axis perpendicular the plane is

$$I_z = \int r^2 \sigma \, dA \ ,$$

where dA is a differential area, r is the radius from the axis in the plane, and, as aforesaid, σ is the surface area density. A radius from the x axis is y in plane and only in the plane and a radius from the y axis is x in plane and only in the plane. **HINT:** Given the preamble above, the proof is about one line.

b) The rotational inertia of an infinitely thin, uniform rod about an axis through its center and perpendicular to the rod is

$$\frac{1}{12}M\ell_{\rm rod}^2 \ ,$$

where M is the rod mass and $\ell_{\rm rod}$ is the rod length. Now consider an infinitely thin, uniform rectangular plate. The plate has mass M. Let the z axis pass through the center of the plate and be perpendicular to the plate. The x and y axes are parallel to the sides of the plate. The length of the plate perpendicular to the x axis and in the y direction is $\ell_{x\perp,y}$ and perpendicular to the y axis and in the x direction is $\ell_{y\perp,x}$. Derive the rotational inertia about the x axis? **HINT:** Just stack the rotational inertias for rods. A diagram might help

c) Now derive the rotational inertia about the y axis?

d) What is the rotational inertia of the plate about the z axis? To simplify the notation, use the replacements

$$\ell_{y\perp,x} \Rightarrow \ell_x \qquad \text{and} \qquad \ell_{x\perp,y} \Rightarrow \ell_y \ .$$

e) Now let's extend the plate in the z direction to create a rectangular box (i.e., a rectangular cuboid). A rectangular box has 6 sides all of which are rectangles and the sides all meet at right angles. The z length of our box is ℓ_z and the mass is M. Derive the rotational inertia about the z axis?

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012 qfull 01050 1 3 0 easy math: rotational energetics, car crankshaft

521. A car crankshaft rotating at 1500 rpm transfers energy from the car engine to the wheel axle at 74.6 kW (or 100 hp). What's the average torque of the crankshaft? What's the average **TANGENTIAL FORCE** of the crankshaft on the drum (or whatever one calls that thing on the axle that the crankshaft turns) assuming it has radius 0.10 m? Show your calculations.

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012 qfull 01052 1 3 0 easy math: rotational energetics, car crankshaft 2

522. A car crankshaft rotating at 1500 rpm transfers energy from the car engine to the wheel axle at 30.0 kW. What's the average torque of the crankshaft? What's the average **TANGENTIAL FORCE** of the crankshaft on the drum (or whatever one calls that thing on the axle that the crankshaft turns) assuming it has radius 0.15 m? Show your calculations.

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012 qfull 01060 2 5 0 math thinking: Demiurge spinning up the Earth
Extra keywords: demiurge Demiurge Demiourgos

523. You are the Demiurge—Plato's mythical, but not terribly responsible, artificer of the universe. Having just constructed the Earth (mass 5.97×10^{24} kg or so; radius probably 6378 km), you know you need to get it spinning—otherwise it would take a year to get through a day. You figure, offhandedly, that 86164.1 times the time of 9192631770 oscillations of light (of a specified wavelength) emitted by the cesium-133 atom ought to just about do it for a period of rotation relative to the fixed stars—you've been told off more than once about trying to set rotation relative to the Sun. (Later inhabitants of this prime real estate will call the time taken by 9192631770 oscillations of cesium light a **SECOND**.)

a) What is angular velocity in radians per second of the Earth for your chosen period of rotation?

b) Say you give yourself about one year (which you take to be $\pi \times 10^7$ s which you know is accurate enough) of constant acceleration to spin the Earth up from **REST** to your chosen

final angular velocity. What is the constant angular acceleration α you need for your given spin-up time?

c) But after a long thoughtful (i.e., vacant) pause, you realize you will need know the rotational inertia of the Earth if you are to apply the right torque to get the right angular acceleration. Not having discovered integration yet—you'll get around to it some aeon or another—you decide on an empirical method to determine the rotational inertia formula. You have a uniform density ball of mass 1 kg and radius 0.1 m. You measure rotational inertia for your ball about an axis through its center of mass to be $(3.98 \pm 0.02) \times 10^{-3}\,\mathrm{kg\,m^2}$: the uncertainty ± 0.02 shows the result is not exact. Ah, you say, the laws of nature (which weren't part of your construction contract) must give simple formulae for simple systems. So what must the **EXACT ROTATIONAL INERTIA FORMULA** be for a uniform density sphere about an axis through its center of mass? **HINT:** The units must work out correctly.

d) Assuming the Earth is a uniform density sphere, what is its rotational inertia in MKS units?

e) How much torque (in MKS units) to you need to exert for the spin-up?

f) What is the final rotational kinetic energy of the Earth (in MKS units)?

g) How much work (in MKS units) have you done spinning up the Earth?

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012 qfull 01070 3 5 0 tough thinking: falling pillar in Sparta

Extra keywords: Needs reworking: start with a finite velocity: small earthquake

524. A tall, thin Hellenic pillar (Doric style) has stood erect for 25 centuries in Sparta, now, its base eroded, it falls from rest from the vertical under the force of gravity. The base stays fixed and the pillar falls like rigid rotator about it. Take the base as being the pivot point (or origin) for all the rotational calculations. Assume the base is an ideal pivot point: i.e., the the pillar is completely free to turn about the pivot point, but the pivot point itself is fixed. The angle of the pillar from the upward pointing **VERTICAL** at any time in the fall is θ. Assume the pillar has length ℓ, its center of mass is located at ℓ_{cm} from the base, its mass is m, and its rotational inertia is I. **HINT:** Draw a good initial diagram.

a) What is its angular acceleration α of the pillar as a function of θ?

b) Using the rotational work-kinetic-energy theorem find an expression for the angular velocity ω of the pillar. **HINT:** You will have to a modest integral. An integral expression is **NOT** an acceptable full answer.

c) Assume the pillar can be treated as thin, uniform rod (rotational inertia about an axis through its center of mass is $(1/12)m\ell^2$). What is the simplified expression for the rotational inertia about the base in terms of m and ℓ only? **HINTS:** Use the parallel-axis theorem. If the pillar is uniform, what must ℓ_{cm} equal?

d) Now using the part (b) and (c) answers, write the expression for ω as a function of only g, ℓ, and θ. Since $\omega = d\theta/dt$, what kind of equation for θ is the expression just derived?

e) Alack, there is no analytic solution giving θ as a function of time for the differential equation for θ obtained in the part (d) answer. But we'd still like to find at least a crude estimate of the time for the pillar to fall down. Make the approximation $\omega = d\theta/dt \approx \Delta\theta/\Delta t$, set $\Delta\theta = \pi/2$, and calculate Δt for a judicious choice of θ in $\omega(\theta)$. The pillar is 10 m in length. The result probably has astrophysical accuracy: i.e., factor of 2ish.

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012 qfull 01080 3 5 0 tough thinking: rotational anharmonic oscillator

525. You are given an rotational oscillator of rotational inertia I obeying an anharmonic oscillator (AO) torque law

$$\tau_{\mathrm{AO}}(x) = -a\theta - b\theta^3 \ ,$$

where a and b are positive constants.

a) What is the work W_{AO} done by the AO torque when the oscillator is moved from the origin to a general point θ? **HINTS:** You will have to do an integration. Remember which way the torque points and which way the oscillator is moving.

b) Say that the oscillator before and after it has been moved was at rest. What was the work W_{app} done by the applied torque that caused the movement? Assume only the applied torque and the AO torque act.

c) What is the expression for the AO potential energy U_{AO} if the origin is chosen to be zero potential energy?

d) Say that there are no nonconservative torques and we give the AO an initial mechanical energy of E. What is its mechanical energy E_{later} at all later times?

e) What is the maximum angular speed that the oscillator can have given that it has mechanical energy E? Where does it reach this maximum angular speed?

f) What is the maximum displacement θ_{max} that the AO can get from the origin (i.e., the amplitude of the oscillation) given mechanical energy E. **HINT:** Solve for $s_{\text{max}} = \theta_{\text{max}}^2$ first.

temitemg) If θ_{max} is so small that the 2nd term in the AO potential energy expression is negligible, approximately what kind of an oscillator do we have?

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012 qfull 01220 2 3 0 moderate math: rolling ball on incline into gooey stuff

526. A uniform solid ball of mass 0.5 kg starting from rest rolls down an incline of angle $\theta = 30°$. There is no slipping between the ball and incline: i.e., the no-slip condition is imposed.

a) Calculate the ball's center-of-mass acceleration given the downhill roller acceleration formula

$$a = \frac{g \sin\theta}{1 + I/(mr^2)} \ .$$

Show your calculation.

b) What is its center-of-mass velocity after 30 s and how far has it gone down the slope? Show your calculation.

c) At the 30 s mark the ball perfectly inelastically collides with a blob of gooey stuff of mass 5.0 kg resting and stuck on a piece of wax paper of negligible mass. The ball-blob-paper then slides 20 m further down the incline and comes to a stop. What is the kinetic friction coefficient μ_{k} of the paper with the incline? Show your calculation.

Why does the ball-blob-paper stay at rest after it stops? To answer that question, ask yourself this question have the forces on it changed?

d) How far would the ball-blob-paper have gone if $\mu_{\text{k}} = 0.1$? Why does it go this far?

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012 qfull 01240 3 5 0 tough thinking: Dingo rolls on loop

527. Dingo the daredevil dog is back—just accept it. He is now on a no-slip track and he curls himself into an exactly spherically symmetric ball (but not necessarily a uniform density one) of radius r_{Dingo}. Starting from **REST**, he rolls down the track from an initial height y_0. The track becomes level at $y = 0$, does a circular loop-the-loop of radius R, and then goes level at $y = 0$ again. Note r_{Dingo} and R are different radii.

a) What is Dingo's center-of-mass speed at any height y assuming he stays on the track?

b) Find a general expression for the normal force on Dingo when he is on the loop as a function of the angle θ between a general radial vector and the radial vector pointing to the top of the loop and radius R.

c) For what θ is the normal force expression smallest? What is the normal force expression for this angle?

d) What is the condition on y_0/R for the normal force **NOT** go less than zero anywhere on the loop? What happens if it did go less than zero?

e) Say $y_0 = 50\,\text{m}$ and the radius of the loop is 20 m. Does Dingo stay on the loop?

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012 qfull 01310 3 5 0 tough thinking: double incline, rotational inertia

528. You have a triangular block which gives you two inclines (i.e., a double incline). Incline 1 is at θ_1 to the horizontal and incline 2 at θ_2. The kinetic friction coefficients are $\mu_{\text{k}\,1}$ and $\mu_{\text{k}\,2}$ for, respectively, inclines 1 and 2. You have a pulley at the apex and a ideal, massless rope connecting two blocks: block 1 (with mass m_1) on incline 1 and block 2 (with m_2 on incline 2. The rope is taut and is parallel to each incline. The rope over the pulley wheel does not slip and the pulley wheel has rotational inertia I about its axis and radius r. The axle of the pulley is frictionless. Block 1 slides up the slope and Block 2 slides down the slope. Call both these directions positive. You know all the variables except the acceleration of the blocks along the slope and the tension forces in the two regions of the rope.

a) Draw the free body diagrams for the blocks and write down their equations of motion (i.e., Newton's 2nd law applied to their special cases). Why can't you solve for acceleration now?

b) What is the rotational equation of motion for the pulley?

c) Now solve for the general acceleration formula in terms of the knowns: i.e., sans tension forces.

d) Specialize the general formula for the case where $\mu_{\text{k}\,1} = \mu_{\text{k}\,2} = 0$.

e) Specialize the general formula for the case where $\theta_1 = 0°$.

f) Specialize the general formula for the case where $\theta_1 = 0°$ and $\theta_2 = 90°$.

g) Specialize the general formula for the case where $\theta_1 = \theta_2 = 90°$.

h) Specialize the general formula for the case where $I = 0$.

i) Why is the general formula we derived wrong for the case when the blocks slide in the opposite directions? What simple demarche corrects the expression?

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012 qfull 01320 2 3 0 moderate math: Atwood's machine, rotational inertia
Extra keywords: On tests this would be hard question

529. You have an Atwood's machine: a pulley with two dangling blocks. Block 1 has $m_1 = 0.500\,\text{kg}$ and block 2 has $m_2 = 0.520\,\text{kg}$. The pulley has mass $m_{\text{pu}} = 2.0\,\text{kg}$ and effective radius $r = 0.070\,\text{m}$. The rope is massless.

a) Assume the pulley wheel magically has zero rotational inertia and a frictionless axle. What is the acceleration of the block system? Starting from rest how long does it take for the blocks to move 1 m?

b) Now assume the pulley does have rotational inertia. Estimate the rotational inertia of the pulley assuming it is a uniform disk. Answer the questions from part (a) in this case.

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012 qfull 01322 2 3 0 moderate math: Atwood's machine, rotational inertia
Extra keywords: On tests this would be hard question

530. You have an Atwood's machine: a pulley of radius r with two **DANGLING** blocks connected by an ideal massless rope that does not slip on the pulley (i.e., the no-slip constraint is imposed). Block 1 has m_1 and block 2 has m_2. The rope contrains the 1-dimensional motion of the two blocks to be identical on their respective lines of motion. Assume block 1 has upward positive and block 2 has downward positive.

a) Draw a diagram of the system with the forces on the blocks labeled.

b) Assume the pulley zero rotational inertia and a frictionless axle. Apply Newton's 2nd law to each of the two blocks. The tension T in the rope is uniform.

c) Derive the formula for acceleration a of the blocks and the most simplified formula for the tension T.

d) Now assume the pulley does have rotational inertia I. Apply Newton's 2nd law to each of the blocks and the rotational Newton's 2nd law to the pulley. **HINT:** There are now two tension forces (T_1 and T_2) and you will have to use torques.

e) Derive the formula for acceleration a of the blocks. Specialize the formula for the case of $I = 0$?

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012 qfull 01350 2 3 0 moderate math: rotational energetics with gravity

531. You have a globe of rotational inertia I_{gl} and radius r_{gl} being spun by a rope unwinding from its equator. Said rope goes out horizontally from the globe and passes over a pulley (rotational inertia I_{pu} and radius r_{pu}) and dangles down with a block of mass m_{bl} at the end. The axles of the globe and pulley are frictionless, but the rope on the globe and pulley is no-slip.

a) Why does the system conserve mechanical energy? Consider all the forces that must play a role: gravity, normal forces, tension forces, and static friction.

b) From energetics what is the expression speed of the block after it falls a height Δy from rest? The globe and pulley were at rest at time zero too.

c) Say that the globe and pulley can be treated, respectively, as a thin spherical shell (mass $m_{\text{gl}} = 5\,\text{kg}$, radius $r_{\text{gl}} = 0.3\,\text{m}$) and a uniform disk (mass $m_{\text{pu}} = 0.5\,\text{kg}$, radius $r_{\text{pu}} = 0.1\,\text{m}$), respectively. Calculate their rotational inertias.

d) Say $m_{\text{bl}} = 0.2\,\text{kg}$ and $\Delta y = 2\,\text{m}$. What is the block speed at the end of the fall?

Chapt. 13 Equilibrium and Elasticity

Multiple-Choice Problems

013 qmult 00100 2 5 3 moderate thinking: static equilibrium defined

532. In **STATIC** equilibrium for a rigid body:

a) there is no center-of-mass or rotational acceleration, but there can be **NONZERO** center-of-mass velocity and angular velocity.
b) there is no center-of-mass or rotational acceleration, and **NO** center-of-mass or rotational velocity. If static equilibrium exists in a specific reference frame, it exists in **ALL** reference frames no matter how those reference frames may be moving.
c) there is no center-of-mass or rotational acceleration and **NO** center-of-mass or rotational velocity. If static equilibrium exists in a specific reference frame, it exists **ONLY** in reference frames **NOT** moving with respect to the specific reference frame.
d) there are no forces at all.
e) there are no torques at all.

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013 qmult 00110 1 1 3 easy memory: rotational equilibrium static and non-static

533. To be in rotational equilibrium relative to some origin in an inertial frame, an object must have (relative to that origin):

a) zero angular momentum. b) non-zero angular momentum.
c) constant angular momentum. d) non-constant angular momentum. e) no hair.

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013 qmult 00200 1 4 1 easy deducto-memory: hanging and center of mass

534. "Let's play *Jeopardy*! For $100, the answer is: The net gravitational torque on an object about any origin at all is the same as if all the mass of the object were concentrated at the center of mass: i.e.,

$$\vec{\tau}_{\rm grav} = \vec{r}_{\rm cm} \times mg\hat{g} \ ,$$

where $\vec{r}_{\rm cm}$ is measured from the origin and $\hat{g}$ is a unit vector in the direction of the gravitational force. Only when $\vec{r}_{\rm cm}$ and $\hat{g}$ are aligned or when $\vec{r}_{\rm cm} = 0$ does the gravitational torque vanish. Thus for an object hanging from a frictionless pivot, equilibrium only exists for the center of mass directly above, on, or directly below the pivot. "Above" is unstable because any perturbation causes a gravitational torque away from the equilibrium. "On" is a neutral equilibrium since there is no gravitational torque whatever the orientation of the object. "Below" is stable since the gravitational torque then tries to pull the center of mass back to the equilibrium point. The gravitational torque acts as a restoring torque. With any damping to kill rotational kinetic energy (but insufficient static friction in the pivot point to oppose any gravitational torque), the object will come to a static stable equilibrium with the center of mass directly below the pivot."

a) Why does the center of mass of an object tend to come to rest directly **BELOW** a freely turning pivot from which the object is hanging, Alex?
b) Why does the center of mass of an object tend to come to rest directly **ABOVE** a freely turning pivot from which the object is hanging, Alex?
c) Why is there a center of mass, Alex?

d) Why is there a center, Alex?
e) Why is there a universe, Alex?

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013 qmult 00210 1 3 5 easy math: simple beam torque calculation

535. An object of mass 1 kg sits on a horizontal beam at 1 m from a fulcrum (i.e., the support about which a lever pivots). What is the magnitude of the torque about the fulcrum that the weight of the mass causes?

a) 1 N m. b) 2 N m. c) 3 N m. d) 4 N m. e) 9.8 N m.

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013 qmult 00222 2 3 1 moderate math: balanced masses on a beam torque calculation

536. Two objects are sitting on a horizontal beam. The beam rests on a fulrum and its center of mass is directly above the fulcrum. The beam is free to rotate about the fulcrum. Object 1 sits on the left-hand side of the pivot at a distance ℓ_1 from the fulcrum. Object 2 sits on the right-hand side at a distance ℓ_2. Given $m_1 = Nm_2$, what is ℓ_2 in terms of ℓ_1? **HINT:** Draw a diagram.

a) $\ell_2 = N\ell_1$. b) $\ell_2 = \ell_1/N$. c) $\ell_2 = \ell_1$. d) $\ell_2 = 2\ell_1$. e) $\ell_2 = 0$.

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013 qmult 00310 1 1 3 easy memory: example of an indeterminate equilibrium case

537. In a planar or 2-dimensional case of static equilibrium, can you solve for four unknown normal and static friction forces assuming perfectly rigid objects?

a) No. The system is **INDETERMINATE**: you only have **FOUR** equilibrium equations.
b) Yes. The system is **DETERMINATE** since you have **FOUR** equilibrium equations.
c) No. The system is **INDETERMINATE**: you only have **THREE** equilibrium equations.
d) Yes. The system is **DETERMINATE**: you have **THREE** equilibrium equations.
e) No. The system is **INDETERMINATE**: you only have **TWO** equilibrium equations.

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013 qmult 00410 1 4 1 easy deducto-memory: stress and strain defined

538. "Let's play *Jeopardy*! For $100, the answer is: In the field of elasticity in the linear regime, there are several formulae of the form

$$\frac{\Delta f}{f} = \frac{F/A}{\text{modulus}} \ ,$$

where $\Delta f/f$ is some kind of relative deformation, F/A is a force per unit area quantity, and modulus is an uncharacteristic value of F/A. The modulus is uncharacteristic since the formulae are only valid when $(F/A)/\text{modulus} << 1$. The generic names, respectively, for F/A and $\Delta f/f$ are keywords in the question."

What are _____, Alex?

a) stress and strain b) strain and stress c) sound and fury d) hope and glory
e) hope and faith

Full-Answer Problems

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013 qfull 00100 1 3 0 easy math: for static equilibrium torque is independent of origin

539. To be in static equilibrium relative to some inertial frame for a rigid body requires the body to be at rest in that inertial frame and

$$\vec{F}_{\text{ext}} = \sum_i \vec{F}_i = 0 \qquad \text{and} \qquad \vec{\tau}_{\text{ext}} = \sum_i \vec{\tau}_i = \sum_i \vec{r}_i \times \vec{F}_i = 0 \ ,$$

where the sums are over all external forces and torques. The zero sums mean the body center of mass is not being accelerated and its total angular momentun is not changing. So the body is not only at rest, it will stay at rest. Since the body is rigid, there are no internal motions.

a) Draw a diagram illustrating the system with the forces $\vec{F}_i$ indicated where they act at the positions given by the vectors $\vec{r}_i$ relative to torque origin O and by the vectors $\vec{r}_i'$ relative to torque origin O'. Note O' is at position $\Delta\vec{r}$ relative to O.

b) Prove mathematically that

$$\tau_{\rm ext} = \tau'_{\rm ext} \ ,$$

where the two net external torques are relative, respectively, to the two origins O and O'. Do not assume the two external torques are zero, but do assume $\vec{F}_{\rm ext} = 0$.

c) What does the part (b) result mean and what does it mean for your convenience? If the system were in static equilibrium, why non-mathetically is it rather obvious that the net torque must be zero around any origin?

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013 qfull 0112 1 3 0 easy math: origin-independent torque, wordy, rework?

540. There are some results that are useful to know in studying equilibrium for a rigid body. We will not specialize to a rigid system initially—we will say when we do so specialize. We only consider inertial frames in our derivations and discussions. Non-inertial frames can always be converted into inertial frames with inertial forces if needed.

a) The net external torque on a system about a first general origin O in an inertial frame is

$$\vec{\tau}_{\rm ext} = \sum_i \vec{r}_i \times \vec{F}_i \ .$$

The net external force on the system is

$$\vec{F}_{\rm ext} = \sum_i \vec{F}_i \ .$$

A second general origin O$'$ is located at $\Delta\vec{r}$ relative to the first origin. The vector $\Delta\vec{r}$ can in general depend linearly on time, and thus the second origin can define a second inertial frame. The displacements relative to the second origin are related to displacements relative to the first origin by

$$\vec{r}\,'_i = \vec{r}_i - \Delta\vec{r}$$

—and yes, the minus sign is right: you should draw a diagram to see this. Derive the expression for the net external torque $\vec{\tau}\,'_{\rm ext}$ about the second origin in terms of the first origin quantities. Note that forces are frame-invariant quantities in classical mechanics.

b) If $\vec{F}_{\rm ext} = 0$, what is the relationship of $\vec{\tau}\,'_{\rm ext}$ and $\vec{\tau}_{\rm ext}$. What is the relationship between the two angular momenta relative to the two origins in this case? **HINT:** You'll have to do a trivial integral to answer the second question.

c) Now we specialize to a rigid body of constant mass m and consider conditions imposed one after another. Explain what each condition implies about the system and why it does so? First, we impose $\vec{F}_{\rm ext} = 0$. Second, we impose that the system is examined in the rest frame of its center of mass. Third, we impose that $\tau_{\rm ext} = 0$ using the center of mass as the origin. Fourth, we impose that system is not rotating in anyway about the center of mass.

d) Are $\vec{F}_{\rm ext} = 0$ and $\vec{\tau}_{\rm ext} = 0$ sufficient or necessary conditions for overall equilibrium for a rigid body? Are they sufficient or necessary conditions for overall static equilibrium? Explain your answers.

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013 qfull 00200 1 3 0 easy math: general horizontal beam , rework?

541. The general formula for the net torque about a z axis is

$$\tau_z = \sum_i r_{xy,i} F_{xy,i} \sin\theta_i \ ,$$

where i indexes the applied forces, $r_{xy,i}$ is the xy component of radial vector from the origin to where a force is applied, $F_{xy,i}$ is the xy component of an applied force, and θ_i is the angular coordinate of $\vec{F}_{xy,i}$ relative to $r_{xy,i}$ measured positive in the counterclockwise direction usually.

In this problem, we will consider a general horizontal beam with a supporting fulcrum and a set of weights with masses m_i put along the beam with their center of masses at locations x_i. The number of weights is general: i.e., it's not 2 or 3; it's general. The beam itself is one of the weights. The beam is aligned with the x direction and is symmetric about the xy plane through the origin. This means there are no x and y torques. This is a usual setup for a balance scale with the rotation axis assigned to be the z axis.

a) Draw a diagram of the horizontal beam system.

b) Specialize the general formula for the net torque about a z axis to the horizontal beam system with the **ORIGIN** taken at the fulcrum point. The fulcrum is ideal and exerts no torque. **NOTE:** We are not assuming equilibrium, and so the net torque is not zero in general.

c) There are no forces in the x or z directions for the horizontal beam system. What is the general formula for the force in the y direction? **NOTE:** We are not assuming equilibrium, and so the net force is not zero in general. **HINT:** Do not forget the normal force $F_{\rm N}$ of the fulcrum.

d) What are the conditions for static equilibrium for the horizontal beam system? We assume the beam and weights act as a rigid body.

e) Usually, we can only solve for $F_{\rm N}$ from the static equilibrium equations themselves. Thus, we can only really solve the two equations of equilibrium for one other unknown, either an x_j or an m_j. Using $\sum_{i,i\neq j}$ to mean sum over i excluding j, solve for unknowns x_j and $F_{\rm N}$ and then for unknowns m_j and $F_{\rm N}$ for the horizontal beam system.

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013 qfull 00210 1 3 0 easy math: horizontal balanced beam calculation, rework

542. A horizontal beam is balanced on a point fulcrum at $x_{\rm fulcrum} = 0.20\,{\rm m}$. The beam is has mass $0.10\,{\rm kg}$ and its center of mass is at $x_{\rm beam\ cm} = 0.50\,{\rm m}$. There is also a single object of mass $2.0\,{\rm kg}$ on the beam. Where is its center of mass?

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013 qfull 00310 2 3 0 moderate math: equilibrium ladder

543. A ladder leans against a wall in static equilibrium at an angle θ from the ground. Ladder, wall, and ground are perfectly rigid. The ladder has mass m, length ℓ, and center of located at $\ell_{\rm cm}$ along its length measuring from its base. The problem is 2-dimensional: the ladder and wall are seen in the xy plane and a z axis is the only rotation axis.

a) Draw a good diagram marking on all possible forces: gravity, ground normal force $\vec{F}_{\rm N1}$, ground friction force $\vec{F}_{\rm f1}$ (assumed to point to the right) wall normal force $\vec{F}_{\rm N2}$ and. wall friction force $\vec{F}_{\rm f2}$ (assumed to point vertically). Note for clarity, we define $F_{\rm N1}$, $F_{\rm f1}$, $F_{\rm N2}$ and $F_{\rm f2}$ as magnitudes, not components. Mark the forces where they act: in the case of gravity, the center of mass is the appropriate place. Draw the ladder leaning to the **RIGHT** so that we are all consistent. Make the diagram large enough to be easily read.

b) Write out all the equations of equilibrium including all possible forces and torques. Just so we are all on the same wavelength, make to the right positive, take the origin for the torque equation to be the contact point between ladder and ground, and measure angle counterclockwise from the horizontal rightward of the ground contact point. Why is the ground contact a good choice for the torque origin? **HINT:** You will need the following trigonometric identities:

$$1)\quad \sin(-x) = -\sin(x) \qquad 2)\quad \sin(x+90^\circ) = \cos(x)$$
$$3)\quad \sin(180^\circ - x) = \sin(x) \qquad 4)\quad \sin(90^\circ - x) = \cos(x)\ .$$

Also, remember for z direction torques that

$$(\vec{a} \times \vec{b})_z = ab \sin(x) \ ,$$

where x positive/negative gives counterclockwise/clockwise torques. **HINT:** Actually, you should know the direction of the torques just by looking at the diagram. However, getting the trigonometric functions right takes some thought.

c) Given as knowns only ℓ, $\ell_{\rm cm}$, and m, can we solve for the unknowns?

Note, if you just threw a ladder against a wall, you would not necessarily know θ. Also you could know $F_{\rm N1}$ and $F_{\rm N2}$ if you used weighing scales. The friction forces could point the other way than what we have specified for this problem if somehow you wedged the ladder into place.

d) Nature has no problems solving for the unknowns. Just through let a ladder fall from the vertical to a wall. Why are the unknowns indeterminate for us? **HINT:** Think over-idealization and elasticity and stress and strain.

e) To make our problem determinate, assume the wall is frictionless (i.e., $F_{\rm f2} = 0$) and the ground static friction is at its upper limit (i.e, $F_{\rm f1} = \mu_{\rm sf} F_{\rm N1}$. Now derive formulae for the remaining unkowns: $F_{\rm N1}$, $F_{\rm N2}$, and θ.

f) What happens to results from part (e) if $\mu_{\rm sf} = 0$? Does this make sense?

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013 qfull 00320 2 3 0 moderate math: Tarquin cat on beam, breaking tension

544. Tarquin Superbus the cat is walking along a horizontal beam that is attached to a wall by a freely turning, frictionless hinge. Tarquin has mass $m_{\rm cat}$ and the beam has mass $m_{\rm b}$. The beam is not uniform, but has center of mass at $x_{\rm cmb}$, where x is the coordinate along the beam with the origin at the wall. The beam is supported by a rope reaching from the end of the beam at $x_{\rm b}$ to the wall. The angle between the rope and the horizontal is θ. The maximum tension the rope can support is $F_{\rm T\ break}$. **HINT:** Draw a good diagram.

a) Tarquin starts at the origin and walks outward. What is the explicit expression for Tarquin's position x as function of the other variables?

b) Given $m_{\rm cat} = 3.0\,{\rm kg}$, $m_{\rm b} = 20\,{\rm kg}$, $x_{\rm cmb} = 2.5\,{\rm m}$, $x_{\rm b} = 4.0\,{\rm m}$, $\theta = 30^\circ$, and $F_{\rm T\ break} = 1000\,{\rm N}$, what is position where the rope breaks: i.e., $x_{\rm break}$?

c) Assuming Tarquin falls from the beam, whether it breaks or no, how far does he fall in 3.0 s? Assume no air drag and great initial height. Don't bother worrying about how he disengages from the the beam: it's just a simple fall calculation.

d) What's Tarquin's kinetic energy at 3.0 s in the part (c) scenario?

e) Luckily cats actually reach terminal velocity after about 20 m and flex themselves around to land on their feet: they pounce on the Earth. They usually survive in fair shape—but it's not a recommended cat experiment: i.e., throwing them off skyscrapers. Which of the **THREE** kinds of collision does old Tarquin have with the Earth **AFTER** the dust has settled?

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013 qfull 00330 3 5 0 tough thinking: rolling a roller over a step

545. A roller (either a spherically symmetric ball or a cylindrically symmetric cylinder) is rolled up to a step and rests against it. The roller has radius R and mass m. The step has height y with $y \leq R$. A force $F_{\rm app}$ is applied horizontally to the roller at height R to try to push the roller over the step. A line drawn through the applied force through passes the roller's center of mass. Find the expression for $F_{\rm app}$ that just marginally lifts the roller. This is a static equilibrium situation where the normal force of the ground has just gone to zero, but the roller is still marginally touching the ground. What are the $F_{\rm app}$ values for $y = 0$ and $y = R$? **HINTS:** Draw a good diagram, use geometry, and identify the best pivot point for a torque calculation.

Chapt. 14 Gravity

Multiple-Choice Problems

014 qmult 00100 1 4 2 easy deducto-memory: Newton discovers gravitational law

546. "Let's play *Jeopardy*! For $100, the answer is: He/she discovered the gravitation law (AKA law of universal gravitation) of classical physics. This law shows that the same gravity that holds on Earth also holds throughout the space—insofar as classical physics applies."

Who is _____, Alex?

a) Galileo (1564–1642) b) Isaac Newton (1643–1727)
c) James Clark Maxwell (1831–1879) d) Albert Einstein (1879–1955)
e) Emmy Noether (1882–1935)

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014 qmult 00110 1 1 3 easy memory: Newton's Apple

547. William Stukeley (1687–1765) recorded a conversation with Newton at Kensington, 1726 April 15 (less than year before Newton's death at 84):

"when formerly, the notion of gravitation came into his mind. It was occasioned by the fall of a/an _____, as he sat in contemplative mood. Why should that _____ always descend perpendicularly to the ground, thought he to himself. Why should it not go sideways or upwards, but constantly to the earth's centre."

a) peach b) pear c) apple d) orange e) sparrow

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014 qmult 00120 1 1 1 easy memory: gravity attracts always
Extra keywords: physci

548. Gravity is the force between systems with mass and it is:

a) always attractive. b) always **REPULSIVE**, except perhaps in some cosmological applications. c) either attractive or **REPULSIVE**. d) neither attractive nor **REPULSIVE**. e) neither fish nor fowl.

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014 qmult 00130 1 1 4 easy memory: Newton's law of gravity

549. Newton's law of universal gravitation for the force exerted by point mass 1 on point mass 2 (where from 1 to 2 is indicated by subscript 12) is:

a) $\vec{F}_{12} = -Gm_1m_2r^2\hat{r}_{12}$. b) $\vec{F}_{12} = -\dfrac{Gm_1}{m_2}r^2\hat{r}_{12}$. c) $\vec{F}_{12} = -\dfrac{Gm_1}{m_2}r\hat{r}_{12}$.
d) $\vec{F}_{12} = -\dfrac{Gm_1m_2}{r^2}\hat{r}_{12}$. e) $\vec{F}_{12} = -\dfrac{Gm_1m_2}{r^3}\hat{r}_{12}$.

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014 qmult 00140 1 1 2 easy memory: 3rd law of motion and gravity

550. Newton's law of gravity is:

a) inconsistent with Newton's 3rd law of motion.
b) consistent with Newton's 3rd law of motion.
c) violates Newton's 3rd law of motion. d) Newton's 3rd law of motion.
e) Newton's 2nd law of motion.

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014 qmult 00150 1 4 5 easy deducto-memory: gravitational constant

551. "Let's play *Jeopardy*! For $100, the answer is: It is the gravitational constant with MKS units $\mathrm{N}\cdot\mathrm{m}^2/\mathrm{kg}^2$ It is actually the poorest known of the fundamental constants because gravity is such a weak force between laboratory size objects which are used to measure it."

What is _____, Alex?

a) $1.000\ldots$ b) 2.99792458×10^{-8} c) 2.99792458×10^{8} d) $6.67430(15)\times10^{11}$ e) $6.67430(15)\times10^{-11}$

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014 qmult 00160 1 1 4 easy memory: point-like gravity law: cases where it applies

552. Newton's classical gravity law is written for point masses. But this presents a paradox since point masses, at least classical point masses (and maybe not even quantum mechanical point masses, but who knows) do not really exist. The paradox is resolved by regarding the law as an ideal limiting form from which actual gravitational forces between masses can be calculated. Four cases of the application of Newton's gravity law come to mind for the situation of two objects.

1. The general case: Here one makes the classical continuum assumption for matter and integrates up over the matter of the attractor and matter of the attractee in order to find the net force of the attractor on the attractee. As an equation for attractor (object 1) and attractee (object 2), one has

$$\vec{F}_{12} = -\int\int \frac{G\rho(\vec{r}_1)\rho(\vec{r}_2)}{|\vec{r}_2-\vec{r}_1|^2}\frac{(\vec{r}_2-\vec{r}_1)}{|\vec{r}_2-\vec{r}_1|}\,dV_1\,dV_2\ ,$$

where $\vec{F}_{12}$ is the force of object 1 on object 2, $G = 6.67430(15)\times10^{-11}\,\mathrm{N\,m^2/kg^2}$ (Wikipedia: Gravitational constant 2025jun25) is the gravitational constant, ρ designates density, $\vec{r}_1$ is the position vector for object 1, $\vec{r}_2$ is the position vector for object 1, and the integrals are over the volumes V_1 and V_2 of objects 1 and 2. If one can't make the continuum approximation for some reason, a double summation over point particles can be done.

2. The case for general objects that are relatively far apart: Here the size scales of the two objects are both much smaller than the distance between them. In this case, Newton's gravity law applies directly as an approximation taking the centers of mass in the objects as being their equivalent point locations. The approximation becomes better, the greater the relative distance and and approaches being exact in the limit that the relative distance goes to infinity. We will not prove this approximation.

3. The case of spherically symmetric objects that are not overlapping: It can be proven that the gravitational field of a spherically symmetric object outside of the object is exactly the field that a point mass would give according to Newton's gravity law if the point mass had the same mass as the object and was located where the object has its center (which is necessarily the center of mass). (The gravitational field is the cause of the gravitational force. From general relativity, we know that the classical gravitational field is an approximation to the curvature of spacetime caused by mass-energy. Among other things, the gravitational field (in the classical interpretation) is force per unit mass on a point mass.) The proof is by Gauss's law for gravitation which itself is proven from Newton's law of universal gravitation (Wikipedia: Newton's law of universal gravitation). Then using logic and Newton's 3rd law, one can prove that non-overlapping spherically symmetric objects attract each other exactly as if they were point masses of the sort specified. Newton himself first proved this result—but not with modern calculus techniques and therefore with some difficulty. The result is an essential result in understanding the motions of the larger solar system bodies which are nearly spherically symmetric.

4. The case of a relatively large, spherically symmetric object and a relatively small general object outside of the spherically symmetric object. In this case, Newton's gravity law

approximately applies as if both objects were point masses: the large object's equivalent point mass is at its center and has its mass and the small object's equivalent point is located at some fiducial point inside of it (its center of mass usually being the best choice) and its mass. For planet and human size objects, the application of gravitation law is virtually exact. This case follows directly from:

a) Newton's 1st law. b) Newton's 2nd law. c) Newton's 3rd law.
d) cases 2 and 3. e) case 3.

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014 qmult 00250 2 5 3 mod. thinking: spherically symmetric grav sources to point masses

553. From the shell theorem (a corollary of the gravitational Gauss' law), it follows that a spherically symmetric system causes a gravitational field on outside systems exactly as if it were a point mass (i.e., a point mass located at the system center of mass). But does on outside general system exert a gravitational force on the first system exactly as if the first system were a point mass? The answer is yes and we now prove it.

Say we had two non-overlapping systems: system 1 which is spherically symmetric and system 2 which is general. The force of system 2 on system 1 is $\vec{F}_{2,1}$. Now the statement _____ completes the proof.

a) $\vec{F}_{2,1} = -\vec{F}_{1,2} = -\vec{F}_{1\text{point},2} = -\vec{F}_{2,1\text{point}}$ b) $\vec{F}_{2,1} = \vec{F}_{1,2} = -\vec{F}_{1\text{point},2} = \vec{F}_{2,1\text{point}}$
c) $\vec{F}_{2,1} = -\vec{F}_{1,2} = -\vec{F}_{1\text{point},2} = \vec{F}_{2,1\text{point}}$ d) $\vec{F}_{2,1} = \vec{F}_{1,2} = \vec{F}_{1\text{point},2} = \vec{F}_{2,1\text{point}}$
e) $\vec{F}_{2,1} = \vec{F}_{1,2} = \vec{F}_{1\text{point},2} = -\vec{F}_{2,1\text{point}}$

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014 qmult 00300 1 1 5 easy memory: fiducial case gravity law force calculation 1

554. A fiducial gravitational force is the force between two non-overlapping spherically symmetric objects each of mass 1 kg with the center-to-center distance one meter. The magnitude of this force is:

a) 1. b) 1/2. c) $6.67430(15) \times 10^{11}$. d) 1.67×10^{-11}.
e) $6.67430(15) \times 10^{-11}$.

---

014 qmult 00202 1 3 1 easy math: gravity law force calculation 2

555. Using Newton's gravitation law

$$\vec{F}_{12} = -\frac{Gm_1m_2}{r^2}\hat{r}_{12}$$

($G = 6.67430(15) \times 10^{-11}\,\mathrm{N\,m^2/kg^2}$: e.g., Wikipedia: Gravitational constant 2025jun25) calculate the magnitude of the force between two 1 kg objects 1 m apart. The answer is:

a) $6.67430(15) \times 10^{-11} \approx 1.5 \times 10^{-11}$ lb. b) $20.0 \times 10^{-11} \approx 4 \times 10^{-11}$ lb.
c) $4.03 \times 10^{-11}\,\mathrm{N} \approx 0.8 \times 10^{-11}$ lb. d) $2.224 \times 10^{-11}\,\mathrm{N} \approx 0.5 \times 10^{-11}$ lb.
e) $0.741 \times 10^{-11}\,\mathrm{N} \approx 0.15 \times 10^{-11}$ lb.

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014 qmult 00204 1 3 3 easy math: gravity force law calculation 3

556. Using Newton's gravitation law

$$\vec{F}_{12} = -\frac{Gm_1m_2}{r^2}\hat{r}_{12}$$

($G = 6.67430(15) \times 10^{-11}\,\mathrm{N\,m^2/kg^2}$: e.g., Wikipedia: Gravitational constant, 2025jun25) calculate the magnitude of the force between two 3 kg objects 3 m apart. The answer is:

a) $60.1 \times 10^{-11} \approx 12 \times 10^{-11}$ lb. b) $20.0 \times 10^{-11} \approx 4 \times 10^{-11}$ lb.
c) $6.67430(15) \times 10^{-11}\,\mathrm{N} \approx 1.5 \times 10^{-11}$ lb. d) $2.224 \times 10^{-11}\,\mathrm{N} \approx 0.5 \times 10^{-11}$ lb.
e) $0.741 \times 10^{-11}\,\mathrm{N} \approx 0.15 \times 10^{-11}$ lb.

---

014 qmult 00210 1 1 5 easy memory: ratio of gravity and electric forces

557. Two of the basic constituents of ordinary matter are the electron (mass $m = 9.10938215(45) \times 10^{-31}$ kg, charge $q = -1.602176487(40) \times 10^{-19}$ C) and the proton (mass $m = 1.672621636(83) \times$

10^{-27} kg, charge $q = 1.602176487(40) \times 10^{-19}$ C). The magnitude of the electric force between these particles is given by Coulomb's law

$$F = \frac{kq_1q_2}{r_{12}^2} \ ,$$

where q_1 the charge on particle 1, q_2 the charge on particle 2, r_{12} is the distance between the particles, and $k = 8.987551787\ldots \times 10^9$ in MKS units is Coulomb's constant. What is the ratio of the magnitude of the gravitational force between the particles to the Coulomb force between?

a) $1.000\ldots$. b) 2.3×10^{-39}. c) 2.3×10^{39}. d) 4.4×10^{40}. e) 4.4×10^{-40}.

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014 qmult 00212 1 4 5 easy deducto-memory: gravity and large-scale structure

Extra keywords: Sort of naturally follows the ratio of forces problem.

558. "Let's play *Jeopardy*! For \$100, the answer is: This force is essential in making the large-scale structure of objects (of the kind we know) from middling asteroids to super-clusters of galaxies and perhaps to the universe as whole."

a) What is _____, Alex?

a) a contact force b) friction c) a tension force d) a normal force
e) gravity

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014 qmult 00220 1 1 4 easy memory: ratio of gravity forces 1

559. Consider two spherically symmetric objects and the gravitational force between them. What is force for separation r relative to the force for separation r_0 (i.e., $F(r)/F(r_0)$?

a) $(r/r_0)^2$. b) r/r_0. c) r_0/r. d) $(r_0/r)^2$. e) $(r/r_0)^4$.

---

014 qmult 00222 1 1 1 easy memory: ratio of gravity forces 2

Extra keywords: physci

560. Given two spherically symmetric objects, if their separation is doubled, the force of gravity between them _____ and, alternatively, if one object's mass is doubled, the force of gravity between them _____.

a) decreases by a factor of 1/4; is doubled b) decreases by a factor of 1/2; is doubled
c) is doubled; decreases by a factor of 1/2
d) decreases by a factor of 1/4; increases by a factor of 4
e) stays the same; stays the same

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014 qmult 00230 2 1 3 mod. memory: mass and weight above Earth

Extra keywords: physci KB-59-25

561. An object has mass x and weight y on the Earth's surface. What is its mass and weight at 2 Earth radii above the Earth's surface? Note **ABOVE** the Earth's surface, not **FROM** the Earth's center.

a) x and $y/2$. b) $x/2$ and $y/2$. c) x and $y/9$. d) x and $y/4$.
e) $x/9$ and $y/9$.

---

014 qmult 00300 1 4 4 easy deducto-memory: gravitational field 1

562. "Let's play *Jeopardy*! For \$100, the answer is: It can be defined as the gravitational force per unit mass at any point in space. But more fundamentally, it is a vector field that is the cause of the gravitational force: the conventional symbol for this vector field is $\vec{g}$. In modern, physics force fields are real things that are spread through space and cause forces."

What is _____, Alex?

a) the gravitational force b) gravity c) levity d) the gravitational field
e) Gauss's law

---

014 qmult 00310 1 1 3 easy memory: gravitational field 2

563. The usual symbol for the gravitational field is $\vec{g}$ which has magnitude g. A special case of the gravitational field is the one near the Earth's surface. For this case, the gravitational field has the fiducial magnitude $g = 9.8\,\mathrm{N/kg}$ and is often called little g—big G is the gravitational constant. Context must decide if g means the general gravitational field magnitude or the special case of the gravitational field magnitude near the Earth's surface. The gravitational force on a point mass m in a general gravitational field $\vec{g}$ is given by _____. The same formula applies to an extended object provided the field is uniform over the extent of the object.

a) $\vec{F} = \vec{g}/m$ b) $\vec{F} = m/\vec{g}$ c) $\vec{F} = m\vec{g}$ d) $\vec{F} = m^2\vec{g}$ e) $\vec{F} = \vec{g}/m^2$

---

014 qmult 00320 1 1 1 easy memory: point mass grav field

564. The gravitational field

$$\vec{g} = -\frac{Gm}{r^2}\hat{r}$$

is that of a:

a) point mass at the origin. b) cubical mass centered on the origin.
c) tetrahedral mass centered on the origin.
d) dodecahedral mass centered on the origin. e) point mass **NOT** at the origin.

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014 qmult 00322 1 1 4 easy memory: spherically-symmetric mass grav field

565. The gravitational field

$$\vec{g} = -\frac{Gm}{r^2}\hat{r}$$

is that of spherically symmetric mass distribution centered on the origin where m is all the mass within the sphere of radius _____. This field result can be easily derived from Gauss's law for gravitation.

a) $2r$. b) r^2. c) $3r$. d) r. e) $1/r^2$.

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014 qmult 00324 1 1 5 easy memory: inside a gravitational hollow

566. The gravitational field

$$\vec{g} = -\frac{Gm}{r^2}\hat{r}$$

is that of spherically symmetric mass distribution centered on the origin where m is all the mass within the sphere of radius r. This field result can be easily derived from Gauss's law for gravitation. In a hollow region of spherically-symmetric shell (with inner radius r_1 and outer radius r_2), the gravitational field due to the shell is _____. So in a hollow planet one is ....

a) infinite b) a constant $-(Gm/r_2^2)\hat{r}$ c) a constant $(Gm/r_2^2)\hat{r}$
d) a constant $-(Gm/r_1^2)\hat{r}$ e) zero

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014 qmult 00330 2 3 1 mod. math: fiducial-value grav field 1 (fg1), omit since should other way

567. Given the Earth's mass $M_{\mathrm{Ea}} = 5.9736 \times 10^{24}\,\mathrm{kg}$ and mean radius $R_{\mathrm{Ea}} = 6.37101 \times 10^{6}\,\mathrm{m}$ (e.g., Wikipedia 2007sep25), compute the magnitude of the Earth's gravitational field (force per unit mass) at the mean Earth's surface assuming spherical symmetry. The formula for the magnitude of gravitational field outside of a spherically-symmetric body of mass M is

$$g = \frac{GM}{r^2}\ ,$$

where r the distance from the body center and the current favored value for the gravitational constant $G = 6.67430(15) \times 10^{-11}\,\mathrm{N\,m^2/kg^2}$ (e.g., Wikipedia: Gravitational constant, 2025jun25). By desimplifying the expression for g, one can create a fiducial-value (i.e., reference-value) expression for the magnitude of the gravitational field g in terms of the Earth's values used as fiducial values. Identify this expression.

a) $9.822\,\mathrm{N/kg} \times (m/M_{\mathrm{Ea}})(R_{\mathrm{Ea}}/r)^2$ b) $6.822\,\mathrm{N/kg} \times (m/M_{\mathrm{Ea}})(R_{\mathrm{Ea}}/r)$
c) $6.822\,\mathrm{N/kg} \times (M_{\mathrm{Ea}}/m)(R_{\mathrm{Ea}}/r)$ d) $9.822\,\mathrm{N/kg} \times (M_{\mathrm{Ea}}/m)(r/R_{\mathrm{Ea}})$
e) $9.700\,\mathrm{N/kg} \times (m/M_{\mathrm{Ea}})(r/R_{\mathrm{Ea}})$

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014 qmult 00340 1 3 4 easy math: fiducial-value grav field 2 (fg2) , boring question?
Extra keywords: Hold for tests with fg1

568. Given that the Moon's mass $M_{\mathrm{Mo}} = 0.0123 \times M_{\mathrm{Ea}} \approx (1/80)M_{\mathrm{Ea}}$ and mean radius $R_{\mathrm{Ea}} = 0.273 \times R_{\mathrm{Ea}} \approx (1/4)R_{\mathrm{Ea}}$ (e.g., Wikipedia 2007sep25) (where M_{Ea} is the Earth's mass and R_{Ea} is the Earth's mean radius), compute to **3 SIGNIFICANT FIGURES** the gravitational field at the Moon's surface and give it in MKS and in Earth g-force (i.e., in units of $g_{\mathrm{Ea}} = 9.822\,\mathrm{N/kg}$, the Earth's surface mean gravitational field here with a subscript to differentiate it from the more general meaning of g).

a) 2 N/kg; 1/5. b) 9.8225 N/kg; 1.00. c) 1.62 N/kg; 1/3. d) 1.62 N/kg; 0.165.
e) 9.8225 N/kg; 1/5.

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014 qmult 00400 2 1 2 moderate memory: solid angle definition

569. A differential definition of solid angle good Gauss' law considerations is:

a) $d\Omega = r^2\, d\vec{A} \cdot \hat{r}$. b) $d\Omega = (d\vec{A}/r^2) \cdot \hat{r}$. c) $d\Omega = (r^2 d\vec{A}/r^2) \cdot \hat{r}$. d) $d\Omega = d\vec{A} \cdot \hat{r}$.
e) $d\Omega = -(d\vec{A}r^3) \cdot \hat{r}$.

---

014 qmult 00410 1 1 5 easy memory: steradian

570. The unit of solid angle is the:

a) solang (sa). b) soliton (sn). c) solidradian (sr). d) radian (rad).
e) sterradian (sr).

---

014 qmult 00420 1 5 4 easy thinking: typical solid angles

571. The solid angles (all in steradians) subtended by a sphere, hemisphere, semi-hemisphere, and demi-semi-hemisphere (i.e., an octant) are, respectively:

a) soft solid, hard solid, harder solid, and hardest solid. b) 45°, 90°, 180°, and 360°.
c) 360°, 180°, 90°, and 45°. d) 4π, 2π, π, and $\pi/2$. e) $\pi/2$, π, 2π, and 4π.

---

014 qmult 00500 1 1 5 easy memory: gravitational Gauss law

572. Gauss's law for gravity, with $\vec{g}$ representing the gravitational field (i.e., gravitational force per unit mass) is:

a) $\oint \vec{g} \cdot d\vec{A} = GM_{\mathrm{enclosed}}$. b) $\oint \vec{g} \cdot d\vec{A} = G$. c) $\oint \vec{g} \cdot d\vec{A} = M_{\mathrm{enclosed}}$.
d) $\oint \vec{g} \cdot d\vec{A} = -4\pi GM_{\mathrm{excluded}}$. e) $\oint \vec{g} \cdot d\vec{A} = -4\pi GM_{\mathrm{enclosed}}$.

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014 qmult 00540 1 4 5 easy deducto-memory: uniform sphere gravity field

573. "Let's play *Jeopardy*! For $100, the answer is:

$$\vec{g} = GM(-\hat{r}) \begin{cases} \left(\dfrac{1}{R^2}\right)\left(\dfrac{r}{R}\right) & r \le R; \\ \dfrac{1}{r^2} & r \ge R. \end{cases}$$

a) What is Gauss's law for gravity, Alex?
b) What is Gauss's law for electrostatics, Alex?
c) What is a tension force, Alex?
d) What is a normal force, Alex?
e) What is the gravitational for per unit mass (i.e., the gravitational field) for a uniform density sphere of radius R and mass M, Alex?

---

014 qmult 00550 1 1 1 easy memory: gravity inside a hollow sphere

574. The gravitational field (gravitational force per unit mass) $\vec{g}$ inside a central cavity of an isolated spherically symmetric mass distribution of total mass M is:

a) $\vec{0}$. b) $\vec{g} = -GMr^2\hat{r}$. c) $\vec{g} = -\dfrac{GM}{r^2}\hat{r}$. d) $\vec{g} = -\dfrac{GM}{r}\hat{r}$. e) infinite.

---

014 qmult 00600 1 4 5 easy deducto-memory: Kepler's 1st law

575. According to Kepler's 1st law, the orbits of two gravitationally bound celestial bodies are:

a) ovals with the center of mass of the system at a point that is a focus for each oval.
b) squares.
c) triangles.
d) cymbals.
e) ellipses with the center of mass of the system at a point that is a focus for each ellipse.

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014 qmult 00640 2 1 2 moderate memory: orbital angular momentum

576. The angular momentum of an orbiting body in a central force system with respect to the center of force is:

a) proportional to radial distance. b) constant.
c) inversely proportional to radial distance.
d) proportional to the inverse square of radial distance. e) sinusoidal.

---

014 qmult 00680 1 5 1 easy thinking: short derivation of orbital speed

577. The centripetal force requireed for uniform circular motion is

$$\vec{F}_{\rm c} = -\frac{mv^2}{r}\hat{r} \ ,$$

where $\hat{r}$ points away from the center of force and which is really the net force of Newton's 2nd law (AKA $F = ma$) in the case of uniform circular motion. The gravitational force mass M on mass m is

$$\vec{F} = -\frac{GMm}{r^2}\hat{r} \ ,$$

where $\hat{r}$ points away from mass M. Assuming M is so much more massive than m that mass M is essentially an unaccelerated center of force, derive the speed law for a uniform circular motion orbit of mass m around mass M. The result is:

a) $v = \sqrt{\dfrac{GM}{r}}$. b) $v = \sqrt{\dfrac{GM}{r^2}}$. c) $v = \dfrac{GM}{r}$. d) $v = \left(\dfrac{GM}{r}\right)^2$.
e) $v = \left(\dfrac{GM}{r^2}\right)^2$.

---

014 qmult 00684 2 3 2 mod. math: fiducial-value orbital speed 1 (fo1)

578. Given the Sun's mass $M_\odot = 1.9891 \times 10^{30}\,\mathrm{kg}$ (where $\odot$ is the traditional astronomical symbol for the Sun) and mean radius of the Earth's orbit $R_{\rm Ea,orbit} = 1.495978875 \times 10^{11}\,\mathrm{m}$ (e.g., Wikipedia 2007sep25), compute the Earth's orbital speed in the good approximation that the Sun defines a center of force in an inertial frame. The current favored value for the gravitational constant is $G = 6.67430(15) \times 10^{-11}\,\mathrm{N\,m^2/kg^2}$ (Wikipedia: Gravitational constant, 2025jun25). One can use the above values as fiducial values to create a fiducial-value (i.e., reference-value) formula for the orbital speed $v_{\rm orbit}$ of a small-mass body around the Sun. What is this fiducial-value formula?

a) $29.790\,\mathrm{km/s} \times \sqrt{\dfrac{M_\odot}{M}\dfrac{R_{\rm Ea,orbit}}{R}}$. b) $29.790\,\mathrm{km/s} \times \sqrt{\dfrac{M}{M_\odot}\dfrac{R_{\rm Ea,orbit}}{R}}$.

c) $40.790\,\mathrm{km/s} \times \sqrt{\dfrac{M_\odot}{M}\dfrac{R}{R_{\mathrm{Ea,orbit}}}}$. d) $40.790\,\mathrm{km/s} \times \sqrt{\left(\dfrac{M_\odot}{M}\right)^2\dfrac{R}{R_{\mathrm{Ea,orbit}}}}$.

e) $29.790\,\mathrm{km/s} \times \sqrt{\left(\dfrac{M_\odot}{M}\right)^2\left(\dfrac{R}{R_{\mathrm{Ea,orbit}}}\right)^2}$.

---

014 qmult 00686 1 3 3 easy math: fiducial-value orbital speed 2 (fo2)

Extra keywords: A test follow-up to fo1 !!! It can only be given as a follow-up!!!

579. Pluto's mean radius of orbit around the Sun is 39.48168677 AU (Wikipedia: 2007sep25), where 1 AU is the mean radius of the Earth's orbit around the Sun. What approximately is Pluto's orbital speed?

a) 1200 km/s. b) 30 km/s. c) 4.7 km/s. d) 0.75 km/s. e) 200 km/s.

---

014 qmult 00720 1 1 3 easy memory: gravitational U of a point mass

580. The inter-object gravitational potential energy of two spherically symmetric masses of mass M and m with center-to-center distance r (with U for $r = \infty$ being zero as is conventional) is given by:

a) $U = -\dfrac{GMm}{r^2}$. b) $U = \dfrac{GMm}{r}$. c) $U = -\dfrac{GMm}{r}$. d) $U = \dfrac{GMm}{r^2}$.

e) $U = -\dfrac{GMm^2}{r}$.

---

014 qmult 00730 1 1 1 easy memory: grav U localizable distribution

581. If one has a localizable set of point particles or objects that can be approximated as point particles, the total potential energy of the set is

$$U = \sum_{i=1}^{I}\sum_{j=1}^{i-1}\left(-\frac{Gm_im_j}{|\vec{r}_i - \vec{r}_j|}\right) = \frac{1}{2}\sum_{i,j,i\neq j}\left(-\frac{Gm_im_j}{|\vec{r}_i - \vec{r}_j|}\right)$$

where I is the total number of particles and the $1/2$ in the second form is to prevent double counting. Localizable in this context means that the particles can all be enclosed in a finite closed surface. In the continuum approximation, the potential energy is:

a) $U = -\dfrac{1}{2}\displaystyle\int\int \frac{G\rho(\vec{r}_1)\rho(\vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|}\,dV_1\,dV_2$. b) $U = -\dfrac{1}{2}\displaystyle\int\int \frac{G\rho(\vec{r}_1)\rho(\vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|^2}\,dV_1\,dV_2$.

c) $U = \dfrac{1}{2}\displaystyle\int\int \frac{G\rho(\vec{r}_1)\rho(\vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|}\,dV_1\,dV_2$. d) $U = \dfrac{1}{2}\displaystyle\int\int \frac{G\rho(\vec{r}_1)\rho(\vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|^2}\,dV_1\,dV_2$.

e) $U = \dfrac{1}{2}\displaystyle\int\int \frac{G\rho(\vec{r}_1)\rho(\vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|^3}\,dV_1\,dV_2$.

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014 qmult 00740 1 1 4 easy memory: crude U estimate

582. Say you have an object where the characteristic (i.e., rough average) distance between parts making up the object is ℓ. The object has mass M. The crude approximation to the gravitational potential energy of the object is:

a) $U = \dfrac{GM^2}{\ell}$. b) $U = \dfrac{GM^2}{\ell^2}$. c) $U = -\dfrac{GM^2}{\ell^2}$. d) $U = -\dfrac{GM^2}{\ell}$.

e) $U = -\dfrac{GM}{\ell}$.

---

014 qmult 00760 1 1 2 easy memory: uniform sphere U

583. The gravitational potential energy of uniform density sphere of mass M and radius R can be calculated easily by imagining a symmetrical assembly process. You start with no mass at the origin and bring a differential masses dm from infinity to grow the sphere to its full mass and radius. The trick is to imagine that as you bring the differential masses to the growing

sphere radius you spread them uniformly over the surface. There is only a potential energy addition for bringing them up to the sphere surface. The spreading does not change U since the differential mass is being moved at constant radius. As the sphere is being built up the density is maintained at its final value of

$$\rho = \frac{M}{(4\pi/3)R^3} \ .$$

The potential energy itself does not depend on the assembly process. This can proven by simple analytic means. The assembly process can be represented by an integral that is easily solved to give:

a) $U = -\dfrac{GM^2}{R}$. b) $U = -\dfrac{3}{5}\dfrac{GM^2}{R}$. c) $U = -\dfrac{1}{2}\dfrac{GM^2}{R}$. d) $U = -\dfrac{1}{2}\dfrac{GM^2}{R^2}$.
e) $U = -\dfrac{1}{2}\dfrac{GM}{R}$.

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014 qmult 00770 2 3 3 mod. math: thin spherical shell U

584. The gravitational potential energy of infinitely thin spherical shell of mass M and radius R can be calculated easily by imagining a symmetrical assembly process. The shell has all mass at radius R with a uniform surface density. You start with no mass at the shell radius R and bring a differential masses dm from infinity to add to the shell to grow it to its full mass. The trick is to imagine that as you bring the differential masses to the shell radius you spread them uniformly over the shell. There is only a potential energy addition for bringing them up to the shell. The spreading does not change U since the differential mass is being moved at constant radius. The potential energy itself does not depend on the assembly process. This can proven by simple analytic means. The assembly process can be represented by an integral that is easily solved to give:

a) $U = -\dfrac{GM^2}{R}$. b) $U = -\dfrac{3}{5}\dfrac{GM^2}{R}$. c) $U = -\dfrac{1}{2}\dfrac{GM^2}{R}$. d) $U = -\dfrac{1}{2}\dfrac{GM^2}{R^2}$.
e) $U = -\dfrac{1}{2}\dfrac{GM}{R}$.

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014 qmult 00810 1 1 5 easy memory: circular orbital energy

585. The mechanical energy of a small probe in the vicinity of (but outside of) a large spherically astro-body (e.g., a planet, a moon, the Sun, a star) is

$$E = \frac{1}{2}mv^2 - \frac{GMm}{r} \ ,$$

where m is the probe mass, v is the probe speed, M is the astro-body mass, and r is the distance of the probe from the object center. If the object is in a circular orbit around the probe at radius r, the mechanical energy formula specializes to:

a) $E = \dfrac{3}{5}\dfrac{GMm}{r^2}$. b) $E = \dfrac{3}{5}\dfrac{GMm}{r}$. c) $E = -\dfrac{3}{5}\dfrac{GMm}{r}$. d) $E = \dfrac{1}{2}\dfrac{GMm}{r}$.
e) $E = -\dfrac{1}{2}\dfrac{GMm}{r}$.

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014 qmult 00820 1 1 5 easy memory: escape velocity

586. The mechanical energy of a small probe in the vicinity of (but outside of) a large spherically symmetric astro-body (e.g., a planet, a moon, the Sun, a star) is

$$E = \frac{1}{2}mv^2 - \frac{GMm}{r} \ ,$$

where m is the probe mass, v is the probe speed, M is the astro-body, and r is the distance of the probe from the object center. What minimum speed must the probe have in order to

escape infinity? This minimum speed is called the escape speed or escape velocity even though its technically a speed—escape velocity trips off the tongue.

a) $v = \sqrt{\frac{GM}{r}}$. b) $v = \sqrt{\frac{2GM}{r^2}}$. c) $v = \sqrt{\frac{GM}{r^2}}$. d) $v = \frac{GM}{r^2}$.
e) $v = \sqrt{\frac{2GM}{r}}$.

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014 qmult 00900 1 1 4 easy memory: black holes

587. A black hole is a compact celestial object with a gravity field close to the object that is so intense that general relativity must be used to describe the field: classical Newtonian gravity fails. The black hole theory is our best theory for many compact objects observed in space. But it is not certain that black holes actually exist. The defining characteristic of a black hole in the minds of most people is the _____. This is surface enclosing the black hole from which light cannot escape. Anything from below the _____ cannot escape. Anything that falls to within the _____ is lost to the outer universe forever at least as it was. The energy content of a black hole is thought to leak away as Hawking radiation. If this theory is true black holes eventually evaporate to nothing, but on a very long time scale. The bigger the black hole the longer it takes to evaporate. The estimated lifetime of a solar mass black hole (with no mass additions) is about 3×10^{67} years (Wikipedia: Hawking radiation). This time scale is much, much larger than the time since the big bang, 13.7×10^9 years.

a) Schwarzschild surface b) event radius c) Schwarzschild horizon
d) event horizon e) Schwarzschild radius

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014 qmult 01000 2 3 3 moderate math: calculate centrifugal field

588. Given the mean radius of the Earth $R_{\rm E} = 6.37123 \times 10^6\,{\rm m}$, one finds that magnitude and direction of the centrifugal force per unit mass at 45° degrees north latitude are _____ the Earth's axis:

a) $\sim 2.4 {\rm m/s}^2$ perpendicularly inward toward
b) $\sim 2.4\,{\rm m/s}^2$ perpendicularly outward from
c) $\sim 0.024\,{\rm m/s}^2$ perpendicularly outward from
d) $\sim 9.8\,{\rm m/s}^2$ perpendicularly outward from
e) $\sim 9.8\,{\rm m/s}^2$ perpendicularly inward toward

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014 qmult 10700 2 3 2 moderate math: geostationary orbit fullmult

589.** There is a special orbit about the Earth called geostationary orbit. This circular orbit is in the plane of the equator and its radius is such that the orbital period is exactly one Earth sidereal day. A sidereal day is rotation period of the Earth relative to the fixed stars. To 5 digits the sidereal day is 86164 s (Wikipedia: 2007oct29). The sidereal day is slightly shorter than the solar day of 86400 s because of the orbital motion of the Earth about the Sun. A satellite in geostationary will hang in the sky at the same point relative to the ground. For example, say you saw a geostationary satellite at your zenith (i.e., directly overhead). First of all you would be on the equator and second the satellite would stay perpetually at your zenith. Geostationary satellites are not stationary relative to the fixed stars because they circle about the Earth's axis once every sidereal day. Because geostationary satellites hold their positions relative to the ground, they are useful for communications and TV transmission.

i) Given the period P of a circular orbit of a small satellite and the mass M of the central object of the orbit (assumed to be much larger than the satellite mass), what is the formula for the radius r of the orbit?

ii) Given the Earth's mass $m = 5.9736 \times 10^{24}\,{\rm kg}$, what is the radius from the Earth's center of a geostationary orbit?

a) (i) $r = \left(\frac{P\sqrt{GM}}{2\pi}\right)^{2/3}$ (ii) $r = 6371\,\text{km}$.

b) (i) $r = \left(\frac{P\sqrt{GM}}{2\pi}\right)^{2/3}$ (ii) $r = 42170\,\text{km}$.

c) (i) $r = \left(P\sqrt{GM}\right)^{2/3}$ (ii) $r = 42170\,\text{km}$.

d) (i) $r = \left(P\sqrt{GM}\right)^{2/3}$ (ii) $r = 6371\,\text{km}$.

e) (i) $r = \left(P\sqrt{GM}\right)^{2/3}$ (ii) $r = 12742\,\text{km}$.

Full-Answer Problems

014 qfull 00100 1 3 0 easy math: calculate g gravitational acceleration, rework or superceded?

590. Calculate the gravitational acceleration g for the Earth's surface given $G = 6.67430(15) \times 10^{-11}\,\text{N}\,\text{m}^2/\text{kg}^2$ (Wikipedia: Gravitational constant, 2025jun25), Earth mass $M_\text{E} = 5.9736 \times 10^{24}\,\text{kg}$, and mean Earth radius $R_\text{E} = 6.37101 \times 10^6\,\text{m}$ (Wikipedia: 2007sep25). What is the percentage difference from the commonly used fiducial $g_\text{fid} = 9.8\,\text{m/s}^2$ (taken as being an exact value).

014 qfull 00240 1 3 0 easy math: shell theorem 5 corollaries proven

591. The shell theorem (for gravitation) is derived from Gauss' law (for gravitation) for a spherically symmetric mass distribution. The shell theorem is

$$\vec{g}(r) = -\frac{Gm(r)}{r^2}\hat{r}\ ,$$

where r is radius from the center of symmetry, $\vec{g}(r)$ is gravitation field (i.e., force per unit mass) at r, $G = 6.67430(15) \times 10^{-11}\,\text{MKS}$ units (Wikipedia: Gravitational constant, circa 2025), $m(r)$ is the mass interior to a shell of radius r (which must be distributed spherically symmetrically), and $\hat{r}$ is radially outward pointing unit vector, but the minus sign shows the gravitational field points radially inward.

There are 5 corollaries to the shell theorem that are proven almost instantly. We prove them below.

a) Given M is the total mass of a spherically symmetric distribution and R is the radius beyond which there is no mass, derive the gravitational field for $r \geq R$. What is this gravitational field like?

b) Given that the density of the spherically symmetric distribution has upper bound ρ_up near the center, prove that $\vec{g}(r = 0) = 0$. Why is this unlike a point mass gravitational field at $r = 0$?

c) Given the spherical mass distribution has no mass at all inward of R_inner, derive its gravitational field $\vec{g}(r \leq R_\text{inner})$. What is remarkable about this result.

d) Since a spherically symmetric system 1 has gravitation field outside of itself like point mass, we have

$$\vec{F}_{1,2} = \vec{F}_{1\text{point},2}\ ,$$

where $\vec{F}_{1,2}$ is the force system 1 exerts on general system 2 and $\vec{F}_{1\text{point},2}$ is force the point-mass equivalent of system 1 exerts on system 2 (which is entirely outside of system 1). But does general system 2 exert a force on system 1 exactly as if system 1 were a point mass? Yes and to prove this, prove

$$\vec{F}_{2,1} = \vec{F}_{2,1\text{point}}\ .$$

HINT: Make use of Newton's 3rd law which the gravitational force explicitly obeys.

e) What if system 2 from part (d) is also spherically symmetric? Does it exert a force on system 1 exactly as if both systems were point masses? Yes and to prove this, prove

$$\vec{F}_{2,1} = \vec{F}_{2\text{point}2,1\text{point}} \ .$$

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014 qfull 00300 1 3 0 easy math: taylor expansion of Earth's gravity and Earth's mass

592. The Earth to good approximation is a spherically symmetric astro-body, and therefore obeys (to good approximation) the shell theorem, and so outside has gravitational field (to good approximation)

$$\vec{g}(r) = -\frac{GM_\oplus}{r^2}\hat{r} = -\frac{GM_\oplus}{(R_\oplus + \Delta r)^2}\hat{r} \ ,$$

where G is the gravitational constant, r is the radial distance from the center of the Earth, $\oplus$ is the Earth symbol (one of the few astrological symbols still used by astronomers), $M_\oplus$ is the Earth's mass, $R_\oplus = 6378.137\,\text{km}$ is the Earth's radius (equatorial radius value), and $\Delta r \geq 0$ is a small positive deviation from $R_\oplus$.

a) Taylor expand the Earth's gravitational field $\vec{g}(r)$ to 1st order in small $\Delta r/R_\oplus$. You must **SHOW** the steps in doing the Taylor expansion for full marks.

b) Calculate the **RATIO** of magnitude of the gravitational field at altitude 418 km (which is about the mean altitude of the ISS: Wikipedia: International Space Station) to the magnitude of the gravitational field at ground level (i.e., where $r = R_\oplus$).

c) Given $g = 9.8\,\text{N/kg}$ (fiducial Earth's gravity field: easily measured), $R_\oplus = 6378.137\,\text{km}$ (measured by geodesy), and $G = 6.67430(15) \times 10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2$ (measured in the laboratory with human-scale objects), calculate the mass of the Earth? You must **SHOW** the calculation.

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014 qfull 00350 3 5 0 tough thinking: grav potential in spherical symmetry

593. Given that the gravitational field of a uniform density sphere of mass M and radius R is

$$\vec{g} = GM(-\hat{r}) \begin{cases} \dfrac{1}{r^2} & r \geq R; \\ \dfrac{1}{R^2}\dfrac{r}{R} & r \leq R, \end{cases}$$

what is the expression for the gravitational potential energy per unit mass V (usually called the gravitational potential)? Take the zero of the gravitational potential to be at infinity. **HINT:** Watch those constants of integration.

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014 qfull 00660 3 5 0 tough thinking: Kepler's 3rd law

Extra keywords: planetary orbital speeds

594. Obtain the following circular orbit results. Assume the more massive body (mass M) is overwhelmingly more massive than the other body. The more massive body is thus virtually at the center of mass and the other body is virtually the orbiting body or satellite.

a) The orbital speed

$$v_{\text{orb}} = \sqrt{\frac{GM}{r}} \ .$$

where r is the orbital radius. **HINT:** Use the centripetal force law for uniform circular motion and the force law for gravity about a spherically symmetric body.

b) The orbital speed for planets about the Sun using the Earth's speed as the fiducial speed:

$$v_{\text{orb}} = X\sqrt{\frac{1}{r_{\text{AU}}}}\,\text{km/s} \ ,$$

where X is the Earth's mean orbital speed in km/s—solve for it—and r_{AU} is the orbital radius in units of the astronomical unit (i.e., the mean Sun-Earth distance). Use $G = 6.67430(15) \times 10^{-11}\,\text{N}\,\text{m}^2/\text{kg}^2$ (Wikipedia: Gravitational constant, 2025jun25), $M_{\text{Sun}} = 1.9891 \times 10^{30}\,\text{kg}$, and $r_{\text{Mean Sun-Earth}} = 1\text{ AU} = 1.496 \times 10^{11}\,\text{m}$.

c) The orbital speed for satellites about the Earth using the orbital speed at the Earth's surface as the fiducial speed:

$$v_{\text{orb}} = X\sqrt{\frac{1}{r_{\text{Earth-unit}}}}\,\text{km/s}\ ,$$

where X the orbital speed at the Earth's surface in km/s—solve for it—and $r_{\text{Earth-unit}}$ is the orbital radius in units of the mean Earth radius. Use $G = 6.67430(15)\times10^{-11}\,\text{N}\,\text{m}^2/\text{kg}^2$ (Wikipedia: Gravitational constant, 2025jun25), $M_{\text{Earth}} = 5.9742 \times 10^{24}\,\text{kg}$, and $r_{\text{Earth}} = 6.37123 \times 10^6\,\text{m}$.

d) Kepler's 3rd law

$$P^2 = \left(\frac{4\pi^2}{GM}\right) r^3\ ,$$

where P is orbital period. And also derive the form I think people ought to prefer:

$$P = \left(\frac{2\pi}{\sqrt{GM}}\right) r^{3/2}\ .$$

e) Kepler's 3rd law specialized for the case of planets orbiting the Sun with period measured in years and distance measured in astronomical units: i.e.,

$$P_{\text{yr}}^2 = X r_{\text{AU}}^3$$

or

$$P_{\text{yr}} = \sqrt{X}\, r_{\text{AU}}^{3/2}\ ,$$

where X and $\sqrt{X}$ are fiducial constants you must solve for.

f) Kepler's 3rd law specialized for the case of satellites orbiting the Earth with period measured in days and distance measured in mean Earth radii: i.e.,

$$P_{\text{d}}^2 = X r_{\text{Earth-unit}}^3$$

or

$$P_{\text{d}} = \sqrt{X}\, r_{\text{Earth-unit}}^{3/2}\ ,$$

where X and $\sqrt{X}$ are fiducial constants you must solve for.

g) The Moon's period in days from Kepler's 3rd law given that Moon's mean distance from the Earth is 60.33 in mean Earth radii. Why is the value obtained significantly different from the mean lunar month of 29.531 days. **HINT:** The discrepancy is mostly not due to the circular orbit approximation nor to the perturbation effects of the Sun and other planets.

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014 qfull 00710 1 3 0 moderate math: geostationary orbit

595. There is a special orbit about the Earth called geostationary orbit. This circular orbit is in the plane of the equator and its radius is such that the orbital period is exactly one Earth sidereal day. A sidereal day is rotation period of the Earth relative to the fixed stars. To 6 digits the sidereal day is 86164.1 s (Wikipedia: Sidereal time, 2009may12). The sidereal day is slightly

shorter than the solar day (whose reference value is 86400 s) because of the orbital motion of the Earth about the Sun during the passage of day. A satellite in geostationary will hang in the sky at the same point relative to the ground. For example, say you saw a geostationary satellite at your zenith (i.e., directly overhead). First of all, you would be on the equator and second of all, the satellite would stay perpetually at your zenith. Geostationary satellites are not stationary relative to the fixed stars because they circle about the Earth's axis once every sidereal day (as view from the frame of the Earth). Because geostationary satellites hold their positions relative to the ground, they are useful for communications and TV transmission.

a) Using the universal law of gravitation and the centripetal force law for uniform circular motion **DERIVE** the formula for the orbital speed of a small satellite orbiting a large spherically symmetric astro-body of mass M in a circular orbit of radius r. The astro-body is so massive that it can be considered to be fixed in space and unaffected by the gravitational pull of satellite

b) Derive the formula for the period P of the orbit. The formula should be in terms of the astro-body of mass M and radius r.

c) Derive the formula for the radius r as a function of period P.

d) Find the radius of a geostationary satellite orbit in meters, kilometers, and Earth radii. The radius is measured from the Earth center recall. Note that $G = 6.67430(15) \times 10^{-11}\,\mathrm{N\,m^2/kg^2}$ (Wikipedia: Gravitational constant, 2025jun25), the mass of the Earth is $5.9736 \times 10^{24}\,\mathrm{kg}$, and the equatorial radius of the Earth is 6378.1 km.

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014 qfull 00720 2 5 0 moderate thinking: Ida and Dactyl

Extra keywords: (HRW-319)

596. The spacecraft Galileo was orbiting Jupiter and on its last legs circa 2002. But in 1993 on its way to Jupiter, it made a flyby of asteroid 243 Ida and discovered Ida had a small moon which has been named Dactyl. (The 243 means Ida was the 243rd asteroid discovered: asteroid mass tends to decrease as asteroid number increases. The name Ida is probably for the mountain overlooking Troy where Paris made his ill-fated choice. Dactyl was so called probably because it means finger or toe in Greek rather than for the prosodic foot dactyl: ´ ˘ ˘ .) Ida is about 55 km along its long axis and about 22 km along it's short axis as seen in projection. It is a cratered, potato-like rock. Dactyl has a scale size of 1.5 km and the radius of its orbit is of order 100 km.

a) Consider a small spherically symmetric body of mass m in circular orbit about a large spherically symmetric body of mass M. Assume m is so much smaller than M that center of the large body can be considered as being at the center of mass of the two-body system. Derive the gravitational orbital speed expression for a circular orbit

$$v_{\mathrm{orbit}} = \sqrt{\frac{GM}{r}}\ .$$

HINT: Remember the centripetal force requirement.

b) For the system of part (a), derive the expression for the orbital period for a circular orbit

$$P = \frac{2\pi r^{3/2}}{\sqrt{GM}}\ .$$

c) Given that the orbital period of Dactyl is 27 hours, what is Ida's mass? Assume as approximations that orbit is circular and that the bodies are spherically symmetric. The gravitational constant is $6.67430(15) \times 10^{-11}$ (Wikipedia: 2007sep25) in MKS units.

d) Crudely estimate Ida's volume. **HINT:** Make any reasonable assumptions you need to about it's shape.

e) Crudely estimate Ida's mean density. **HINT:** The Moon's (i.e., the Earth's moon's) mean density is $3340\,\mathrm{kg/m^3}$. Ida's won't be the same, but ...

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014 qfull 00810 3 5 0 tough thinking: building a self-gravitating shell

597. NASA has just awarded you a contract to construct in orbit a giant spherical shell of mass M and radius R. Just assume the shell is vanishingly thin and that effectively you have to bring the material to build the shell from infinity in an otherwise empty universe. Thus gravitational potential energy of the material at infinity—and I mean the differential bits are infinitely separated from each other—is zero. These are reasonable first-order approximations.

a) What's the total change in potential energy in constructing the shell? Assume less than reasonably, but to make the problem tractable, that as you bring up each differential bit of material you smear it out over the whole sphere to preserve a spherically symmetric gravitational field at all times.

b) Now what if you didn't preserve spherical symmetry in constructing the shell? Would the change in total potential energy be the same?

c) What does the potential energy get changed into? **HINTS:** There can be no definite answer without specifying the conditions exactly. But imagine you are building this sphere, where plausibly is the energy going.

d) Let $M = 10^5\,\mathrm{kg}$ and $R = 30\,\mathrm{m}$. What is the change in potential energy during construction? Is self-gravitation of the material much of a concern in this case?

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014 qfull 00820 3 5 0 tough thinking: building a self-gravitating sphere

598. O constructeur cosmique extraordinaire, once again NASA awarded you a contract to construct in space. This time it is a giant, uniform density sphere of mass M and radius R. Assume that effectively you have to bring the material to build the sphere from infinity in an otherwise empty universe. Thus gravitational potential energy of the material at infinity—and I mean the differential bits are infinitely separated from each other—is zero. This is a reasonable first-order approximation.

a) To start with what is the gravitational energy per unit mass V (i.e., the gravitational potential) just at the surface of spherically symmetric mass distribution of mass m and radius r? Make the conventional assumptions that V is the potential difference relative to infinity and infinity is set to zero potential. **HINTS:** This isn't intended to be a derivation. Just right down the correct expression from memory or a reference equation sheet. Of course, you could derive the result simply enough if you wanted to.

b) What is the differential change in gravitational potential energy dU that comes from bring a differential bit of mass dm from infinity to the surface of the spherically symmetric mass distribution considered in part (a)?

c) Now making use of the parts (a) and (b) answers, you can find the total potential energy change in assembling your giant sphere by imagining that you bring up the differential bits of mass from infinity and add them to a growing uniform density sphere and smearing them evenly over the surface when they arrive? **HINTS:** You will need to express radius as function of growing mass m in order to do the integral. Radius and mass are not constants. But density is constant.

d) Now say you didn't preserve spherical symmetry in constructing the sphere albeit the final sphere is just as specified in your contract. Is the change in total potential energy the same as found in the part (c) answer?

e) What is the potential energy of the sphere given $M = 10^{10}\,\mathrm{kg}$ and $R = 40\,\mathrm{m}$. Recall $G = 6.67430(15) \times 10^{-11}\,\mathrm{N\,m^2/kg^2}$ (Wikipedia: Gravitational constant, 2025jun25).

f) What is the magnitude of the gravitational field (which is also the gravitational acceleration) at the surface of your sphere?

Chapt. 15 Fluids

Multiple-Choice Problems

015 qmult 00100 1 1 3 easy memory: fluid defined

599. A _____ is a substance that continuously deforms under an applied shear stress. In other words a _____ can't resist a shear stress. At least ideally, it can't. In reality, some small resistance does exist for those _____s that are liquids. A shear stress is a force per unit area applied tangentially to material surface or layer. A normal stress, on the other hand, is applied perpendicularly (or normal) to a surface or layer.

a) rope b) crystal c) fluid d) solid e) ice

015 qmult 00110 1 1 3 easy memory: liquids and gases

600. Fluids are either:

a) solids or liquids. b) solids or gases. c) liquids or gases. d) crystals or gases. e) ices or metals.

015 qmult 00120 1 4 5 easy deducto-memory: solids defined

601. "Let's play *Jeopardy*! For $100, the answer is: They are materials in which the atoms or molecules are rigidly (but not perfectly rigidly) bonded together by chemical bonds. They resist shear and normal stresses. The atoms or molecules are essentially touching in these materials. Of course, atoms and molecules have no hard edges, but they do have regions of strong interaction and it is these regions that touch. Atoms and molecules strongly resist compression, and so compressing these materials from their zero-pressure state takes relatively large amounts of force for relatively small compression."

What is _____, Alex?

a) ices b) crystals c) gases d) liquids e) solids

015 qmult 00130 1 4 4 easy deducto-memory: liquids defined

602. "Let's play *Jeopardy*! For $100, the answer is: They are materials in which the atoms or molecules are bonded together by chemical bonds, but the bonds are not rigid and are interchangeable. They have very weak resistance to shear stresses. The atoms or molecules are essentially touching in these materials. Of course, atoms and molecules have no hard edges, but they do have regions of strong interaction and it is these regions that touch. Atoms and molecules strongly resist compression, and so compressing these materials from their zero-pressure state takes relatively large amounts of force for relatively small compression. But actually these materials usually do not exist in zero-pressure state since they usually rapidly evaporate if the pressure gets too low.

What is _____, Alex?

a) ices b) crystals c) gases d) liquids e) solids

015 qmult 00140 1 4 3 easy deducto-memory: liquids defined

603. "Let's play *Jeopardy*! For $100, the answer is: They are materials in which the atoms or molecules are not bonded together, but are freely moving around with their own translational

kinetic energy. They have virtually no resistance to shear stresses. The atoms or molecules only touch during collisions and are usually widely separated. Of course, atoms and molecules have no hard edges, but they do have regions of strong interaction and it is these regions that touch. Atoms and molecules resist compression, but they not relatively strongly since they are not touching. If compressed very highly, they usually change to another phase of matter.

What is _____, Alex?

a) ices b) crystals c) gases d) liquids e) solids

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015 qmult 00200 1 1 1 easy memory: density defined

Extra keywords: physci

604. Density (with standard physics symbol the Greek rho ρ) unqualified is conventionally take to be the ratio of:

a) mass over volume. b) volume over mass. c) weight over volume.
d) weight over mass. e) mass over weight.

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015 qmult 00202 1 1 2 easy memory: density symbol

605. The virtually universal symbol for density is:

a) α (the small Greek alpha). b) ρ (the small Greek rho).
c) ω (the small Greek omega). d) ζ (the small Greek zeta).
e) β (the small Greek beta).

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015 qmult 00210 1 3 3 easy math: g/cm**3 to kg/m**3

Extra keywords: physci

606. The MKS unit of density is the kg/m^3, but this unit is often inconveniently small for many ordinary terrestrial densities. Thus, these terrestrial densities in kg/m^3 are inconveniently and unmemorably large numbers. Consequently, the CGS unit of density is often used for convenience: g/cm^3 (grams per cubic centimeter). Many common terrestrial substances are of order a few g/cm^3. Now $1\,g/cm^3$ equals:

a) $1\,kg/m^3$. b) $0.001\,kg/m^3$. c) $1000\,kg/m^3$. d) $10^6\,kg/m^3$. e) $0.5\,kg/m^3$.

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015 qmult 00220 1 1 1 easy memory: density of air

607. At 20°C and 1 atm (101.325 kPa) pressure, dry air has a density of:

a) $1.2041\,kg/m^3$. b) $1204.1\,kg/m^3$. c) $1000\,kg/m^3$. d) $7.874\,kg/m^3$.
e) $7874\,kg/m^3$.

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015 qmult 00230 1 1 3 easy memory: density of water

608. For 1 atm (101.325 kPa) pressure, liquid water has about its maximum density at about 4°C. This density is

a) $1.2041\,kg/m^3$. b) $1204.1\,kg/m^3$. c) $1000\,kg/m^3$. d) $7.874\,kg/m^3$.
e) $7874\,kg/m^3$.

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015 qmult 00240 1 1 5 easy memory: density of iron

609. For about 1 atm (101.325 kPa) pressure and room temperature, the density of iron is

a) $1.2041\,kg/m^3$. b) $1204.1\,kg/m^3$. c) $1000\,kg/m^3$. d) $7.874\,kg/m^3$.
e) $7874\,kg/m^3$.

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015 qmult 00250 1 3 1 easy math: air mass in a room

Extra keywords: physci KB-132-9

610. The density of air at 20° and 1 atm pressure is $1.21\,kg/m^3$ (HRW-323). (For comparison, water density under the same conditions is $998\,kg/m^3$ (HRW-323) which is nearly the frequently

quoted 1000 kg/m^3 which is its density at 4° and 1 atm (CJ-322]).) There is a room 5 m long, 4 m wide, and 3 m high filled with air. What is the mass of air in the room?

a) 72.6 kg. b) 1.21 kg. c) 998 kg. d) 0 kg. e) −998 kg.

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015 qmult 00270 1 3 5 easy math: volume of gold
Extra keywords: physci

611. The density of gold (which as Marshal Saxe said is the father of armies) is 19.3 g/cm^3 under ordinary terrestrial conditions. What volume does 30.0 g of gold occupy?

a) infinite. b) 0 cm^3. c) 579 cm^3. d) 0.643 cm^3. e) 1.55 cm^3.

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015 qmult 00300 1 4 4 easy deducto-memory: particle density
Extra keywords: physci

612. "Let's play *Jeopardy*! For $100, the answer is: The number of particles per unit volume."

What is _____, Alex?

a) volume b) specific heat c) mass density d) particle density
e) surface density

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015 qmult 00400 1 1 2 easy memory: pressure defined
Extra keywords: physci

613. Pressure is:

a) force magnitude per unit mass. b) force magnitude per unit area.
c) force magnitude by another name. d) density by another name.
e) coercive influence.

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015 qmult 00410 1 4 1 easy deducto-memory: Earth atmosphere pressure
Extra keywords: physci KB-129-3

614. The pressure of the Earth's atmosphere at any level is caused by:

a) the weight of the overlying air mass. b) respiration by living things.
c) evaporation of sea water. d) glaciers. e) squid.

---

001 qmult 00420 1 1 2 easy memory: constant air pressure

615. Because of its low density, _____ pressure varies slowly with height and can usually can be taken as a constant over changes of height of a few meters or even hundreds of meters depending on how accurate you want to be.

a) water b) air c) mercury d) iron e) lead

---

015 qmult 00430 1 4 2 easy deducto-memory: pressure in straw
Extra keywords: physci KB-131-5

616. "Let's play *Jeopardy*! For $100, the answer is: This causes water to rise in a sucked on straw."

What is _____, Alex?

a) higher-than-room-air-pressure pressure inside the straw b) lower-than-room-air-pressure pressure inside the straw c) the electric force d) the magnetic force
e) the speed of light

---

015 qmult 00450 1 5 5 easy thinking: 2001: A Space Odyssey
Extra keywords: physci

617. In *2001: A Space Odyssey*, astronaut David Bowman finds himself trapped without his helmet in a space pod. The computer Hal has locked the direct pod-to-spaceship airlock. Bowman decides to "breathe vacuum"—to go sans helmet through space to an outside airlock—and then deal with Hal. Why doesn't Bowman explode due to his internal body pressure in the nearly zero pressure of space?

a) He is too quick to explode.
b) He holds his breath.
c) Hal has not anticipated Bowman's maneuver or at least has no contingency plan.
d) Sheer plot requirement.
e) Most of the body's internal pressure is supplied by nearly incompressible (and therefore nearly non-expandable) fluid and solid: these parts won't explode under decompression. Bowman does **NOT** hold his breathe, and so air pressure in his internal cavities drops very quickly and rupturing does not occur. He has 10 to 15 seconds before losing consciousness.

---

015 qmult 00510 2 4 2 moderate deducto-memory: pressure with depth
Extra keywords: physci?

618. The expression for pressure of an incompressible fluid (of density ρ) as a function of depth y (measured positive downward) below a surface is:

a) $P = P_{\text{surface}} + y.$ b) $P = P_{\text{surface}} + \rho g y.$ c) $P = \frac{1}{2}P_{\text{surface}} + \rho g y.$
d) $P = \rho g y / P_{\text{surface}}.$ e) $P = P_{\text{surface}}.$

---

015 qmult 00520 2 3 5 moderate math: pressure at depth 25 m
Extra keywords: physci

619. The expression for the pressure of an incompressible fluid with depth is

$$P = P_{\text{surface}} + \rho g y \ ,$$

where is P_{surface} is surface pressure, ρ is fluid density, $g = 9.8\text{N/kg}$ is the force per unit mass due to gravity near the Earth's surface, and y is depth measured downward from the surface. Air pressure and water density near the Earth's surface are to good approximation, respectively, $10^5\,\text{Pa}$ (almost 1 atm) and $1000\,\text{kg/m}^3$. The pressure in atmospheres at 25 m in depth is about:

a) $3.5 \times 10^5\,\text{atm}.$ b) $10^5\,\text{atm}.$ c) $1\,\text{atm}.$ d) $2.5\,\text{atm}.$ e) $3.5\,\text{atm}.$

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015 qmult 00700 1 1 4 easy memory: Archimedes's principle
Extra keywords: physci

620. Archimedes's principle is that buoyant force on an object surrounded by a fluid is upward and equal in magnitude to the:

a) weight of the object.
b) weight of the object submerged beneath the surface of the fluid.
c) weight of the fluid inside the object.
d) weight of the fluid displaced by the object.
e) ratio of the density of the fluid to that of the object.

---

015 qmult 00710 1 1 4 easy memory: buoyant force magnitude

621. The formula for the magnitude of the buoyant force is:

a) $F_{\text{buoy}} = m_{\text{dis}}/g.$ b) $F_{\text{buoy}} = g/m_{\text{dis}}.$ c) $F_{\text{buoy}} = 1/(m_{\text{dis}}g).$
d) $F_{\text{buoy}} = m_{\text{dis}}g.$ e) $F_{\text{buoy}} = m_{\text{dis}}^2 g.$

---

015 qmult 00720 1 1 4 easy memory: buoyant force magnitude constant density fluid

622. If the fluid density is constant, the formula for the magnitude of the buoyant force is:

a) $F_{\text{buoy}} = \rho_{\text{fluid}} V_{\text{dis}}/g.$ b) $F_{\text{buoy}} = \rho_{\text{fluid}}/(V_{\text{dis}}g).$ c) $F_{\text{buoy}} = 1/(\rho_{\text{fluid}} V_{\text{dis}}g).$
d) $F_{\text{buoy}} = \rho_{\text{fluid}} V_{\text{dis}}g.$ e) $F_{\text{buoy}} = \rho_{\text{fluid}}^2 V_{\text{dis}}g.$

---

015 qmult 00730 1 4 2 easy deducto-memory: floating formula

623. "Let's play *Jeopardy*! For \$100, the answer is: For an object floating in stable vertical equilibrium at the interface of two fluids both of constant density, the formula gives the fraction f of the object volume above the interface. Above the interface is fluid 1 with density ρ_1 and

below the interface is fluid 2 with density ρ_2: necessarily $\rho_1 < \rho_2$. The object's mean density is ρ. If $f > 1$, the object will actually float upward in fluid 1. If $f = 1$, the object actually floats in neutral equilibrium in fluid 1. If $f \in (0, 1)$, the object actually floats in stable equilibrium straddling the interface. If $f = 0$, the object actually floats in neutral equilibrium in fluid 2. If $f < 0$, the object will actually sink downward in fluid 2.

What is _____, Alex?

a) $f = \dfrac{\rho_2 - \rho}{\rho_2}$. b) $f = \dfrac{\rho_2 - \rho}{\rho_2 - \rho_1}$. c) $f = \dfrac{\rho_2}{\rho_2 - \rho_1}$. d) $f = \dfrac{\rho}{\rho_2 - \rho_1}$.
e) $f = \dfrac{\rho_2 - \rho}{\rho_1}$.

---

015 qmult 00740 1 1 5 easy memory: indeterminate unsubmerged volume

624. Say you have an object floating on the surface of fluid. Above the surface is vacuum with density zero. By Archimedes's principle, the mass m of the object is equal to the mass of the fluid displaced. Say the fluid density is ρ_2 and the displaced fluid volume is V_2. What it the volume V of the object?

a) It's V_2 since the object is floating.
b) It's $2V_2$ since the object is floating.
c) Well
$$V = \frac{m}{\rho} \qquad \text{with} \qquad \rho = \frac{\rho_1}{1 - (\rho_2 - \rho_1)(V_2/m)} \ ,$$
where ρ is the object's mean density and ρ_1 is the density of the vacuum.
d) Without further information, the volume V is indeterminate, except that it is **SMALLER** than V_2 or else the object would occupy less volume than the volume of fluid displaced which is impossible. Since there is no buoyancy force in the vacuum, the object can have any extent in the vacuum: it's not displacing any fluid in the vacuuum. So we have $V \geq V_2$ and that is all we know. Likewise the mean density ρ of the object is indeterminate. We know that $\rho V = m$, but we need some other constraint to know ρ and V individually.
e) Without further information, the volume V is indeterminate, except that it is **GREATER** than or equal to V_2 or else the object would occupy less volume than the volume of fluid displaced which is impossible. Since there is no buoyancy force in the vacuum, the object can have any extent in the vacuum: it's not displacing any fluid in the vacuuum. So we have $V \geq V_2$ and that is all we know. Likewise the mean density ρ of the object is indeterminate. We know that $\rho V = m$, but we need some other constraint to know ρ and V individually.

---

015 qmult 00750 1 3 3 easy math: buoyant force on 1 cubic meter object
Extra keywords: physci

625. What is the buoyant force on an object of volume $1\,\mathrm{m}^3$ completely submerged in water? **HINT:** The density of water is $1000\,\mathrm{kg/m}^3$.

a) 1000 N. b) 10^{-3} N. c) 9800 N. d) 4900 N. e) 9.8 N.

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015 qmult 00760 1 5 5 easy thinking: toy ship in mercury
Extra keywords: physci KB-129-5

626. Freshwater density is about $1.00\,\mathrm{g/cm}^3$, seawater density, $1.03\,\mathrm{g/cm}^3$, and mercury density, $14\,\mathrm{g/cm}^3$. A toy boat floats higher in:

a) all of them. b) none of them. c) seawater. d) freshwater. e) mercury.

---

015 qmult 00770 2 5 1 moderate thinking: ice in full cup
Extra keywords: physci KB-131-11

627. The floating equation when only one constant-density fluid needs to be considered is

$$m = \rho_{\text{fluid}} V_{\text{dis}} \ ,$$

where m is the mass of the floating object, ρ_{fluid} is the density of the fluid, and V_{dis} is the amount of fluid displaced by the floating object.

Say that you have a cup of water filled to the brim with water and an ice sample of volume V and mass m floating in the water. The volume displaced by the ice is V_{dis}. After the sample melts its mass is unchanged, of course, and its density is that of water. So after melting, sample has volume:

a) V_{dis} and the cup is filled to the brim still. b) V and the cup overflows. c) V and the cup is unfilled. d) zero and the cup is unfilled. e) zero and the cup overflows.

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015 qmult 00780 3 5 1 hard thinking: water level in pool and canoe

Extra keywords: physci KB-131-13 A hard thinking question, but memory helps

628. An empty aluminum canoe floats in swimming pool. It springs a leak and sinks to the bottom. The upshot requires a bit thinking, but one finds that the pool water level:

a) lowers. b) rises. c) stays constant. d) freezes. e) boils.

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015 qmult 00810 1 1 2 easy memory: continuity and Bernoulli equations

629. The continuity equation and the Bernoulli equation, respectively, express the conservation of _____ and the conservation of _____ in the case of fluids.

a) mass; momentum b) mass; energy c) work; energy d) work; momentum e) work; heat

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015 qmult 00830 1 5 4 easy thinking: continuity equation

630. The simple expression for the continuity equation for the flow of an **INCOMPRESSIBLE** fluid in a pipe (or a flow tube which is a pipe of the mind) is

$$\text{Constant} = vA \ ,$$

where A is the cross-sectional area of the pipe (assumed perpendicular to the flow direction) and v is the fluid speed (assumed constant over the cross-sectional area). The quantity vA can be called the volume flow rate. If A _____, then v _____. In a time Δt, the amount of volume of fluid that has flowed passed a given point is _____.

a) increases; decreases; $vA/\Delta t$ b) increases; decreases; $v/(A\Delta t)$
c) increases; increases; $vA\Delta t$ d) increases, decreases; $vA\Delta t$
e) decreases; decreases; $vA\Delta t$

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015 qmult 00840 1 3 4 easy math: continuity equation example

631. The continuity equation for an incompressible fluid in a pipe or flow tube is Constant $= vA$, where A is cross-sectional area of said pipe or flow tube and v is the fluid speed through said cross-sectional area: for simplicity v is assumed constant over the cross-sectional area. If the cross-sectional area of a water pipe doubles, the speed of the water:

a) doubles. b) goes to zero. c) increases. d) halves. e) goes to infinity.

---

015 qmult 00910 2 5 3 moderate thinking: Bernoulli equation

632. The incompressible-fluid Bernoulli equation (due to Daniel Bernoulli in 1738 [Ba-158]) for steady flow along a streamline is

$$\text{Constant} = P + \frac{1}{2}\rho v^2 + \rho g y \ ,$$

where P is pressure in the fluid, $\frac{1}{2}\rho v^2$ is the kinetic energy per unit mass, and $\rho g y$ is the gravitational potential energy per unit mass measured from some conventional zero level. The Bernoulli equation assumes no loss of energy to heat through viscous (frictional) effects. Despite its limitations the incompressible-fluid Bernoulli equation is very useful since it often applies approximately even for air which is hardly an incompressible fluid by most standards.

You have pipe with steady-state incompressible flow to which Bernoulli's equation and the continuity equation apply. (Recall, the continuity equation is Constant $= vA$, where v is fluid speed and A is cross-sectional area.) If the cross-sectional area A of pipe is **CONSTANT** and y increases, then:

a) P and v **DECREASE**.
b) P **DECREASES** and v **INCREASES**.
c) P **DECREASES** and v stays constant.
d) P and v **INCREASE**.
e) P **DECREASES** and what happens to v cannot be determined without more information.

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015 qmult 00950 1 5 5 easy easy thinking: paper and lift

633. Take this quiz and ...—no, no not that. Take this quiz—or some single sheet of paper if you arn't in a quiz *mise en scène*—in your fingers with your fingers on either side of one of narrow ends. Hold this end **JUST BELOW** your lips and blow a strong gust.

a) Nothing happens, because you've blown too hard.
b) Nothing happens, because you've blown too softly and you've never succeeded in blowing up a balloon in your life.
c) You spit.
d) The instructions are unintelligible.
e) The paper rises because you've created a high-speed, low-pressure zone above the paper. This is the Bernoulli lift which is part of aerodynamic lift by which airplanes fly. Of course, if you put the paper above your lips and blow the paper rises too. This time the rise is caused by the reaction lift which is the other part of aerodynamic lift. The blown air is deflected down by the paper, but for every force there is an equal and opposite force and so the air pushes up on the paper too.

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015 qmult 10350 1 1 2 easy memory: archimedes' principle, Tasmanian devil fullmult

634.*** Your Tasmanian devil, Gris-Gris, has mass $m = 10.0\,\text{kg}$—she's one big devil. You want to determine her density. If she would just submerge in a tank of water you could read her volume V off the water level scale and then calculate her density from

$$\rho = \frac{m}{V} \ .$$

But she doesn't submerge, but just floats indignantly. By some miracle, you measure her unsubmerged volume to be $V_2 = 0.00100\,\text{m}^3$. You know that the air density is $\rho_2 = 1.204\,\text{kg/m}^3$ (at 20°C and 1 atm) and the water density is $\rho_1 = 998.29\,\text{kg/m}^3$ (at 20°C and 1 atm). Note,

$$\frac{V_2}{V} = \frac{\rho_1 - \rho}{\rho_1 - \rho_2} \ ,$$

where ρ is the floater's density. Solve for her density.

a) $1.204\,\text{kg/m}^3$. b) $907.8\,\text{kg/m}^3$. c) $998.29\,\text{kg/m}^3$. d) $951.1\,\text{kg/m}^3$.
e) $1030.1\,\text{kg/m}^3$.

Full-Answer Problems

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015 qfull 00310 1 3 0 easy math: surface floating at interface of two fluids, buoyancy

635. This is an Archimedes' principle question. Consider a body (mass m, mean density ρ, volume V) surface floating at the interface of two fluids: lower fluid 1 (mean density ρ_1, embedded body component volume V_1; upper fluid 2 (mean density ρ_2 (with $\rho_2 \leq \rho_1$, of course), embedded body component volume V_2.

a) Write down equation that the body component volumes must satisfy. The equation has no densities in it.

b) Write down the formula for the buoyancy force $F_{\rm B}$ on the body in terms of the densities of the two fluids.

c) Write down the vertical Newton's 2nd law that the body must satisfy substituting in the buoyancy force $F_{\rm B}$ formula. The equation should be in terms of the three densities.

d) From the equations found in parts (a) and (c) solve for V_1/V as a function of the densities.

e) Write down the special cases of V_1/V formula from part (d) for:
i) $\rho_2 = 0$ or negiligible.
ii) $\rho = \rho_1$.
iii) $\rho = \rho_2$.
iv) $\rho = w\rho_1 + (1-w)\rho_2$ for $w \in [0,1]$.
v) $\rho = w\rho_1 + (1-w)\rho_2$ for $w \in [0,1]$ with $\rho_1 \to \rho_2$.
What does case $\rho = \rho_1$ imply about the position of the body and what does case $\rho = \rho_2$ imply about the position of the body?

f) Why is the case of $\rho_1 = \rho_2$ for the V_1/V formula physically meaningless if you hold fixed the value of the numerator? **HINT:** There are physical limits on V_1, V_2, and ϕ not incorporated in the equations from which the formula was derived.

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015 qfull 00350 2 5 0 mod thinking: archimedes principle, Tasmanian devil

636. We've all heard of Archimedes and King Hieron II's crown. But modern scientists and technologists are seldom called on to hallmark crowns. Let's apply Archimedes's principle to a real world situation.

a) You wish to accurately determine the density of your Tasmanian devil (Sarcophilus harrisii). You've already determined her mass to be 10.0 kg—between master and pet, what are a few scratches and one little bite to the bone? (Female Tasmanian devils have an average mass of about 6 kg, and so your devil is huge.) But now you need to determine her volume. What do you do? **HINT:** You'll need a tank of water with a water-level scale and some persuasion.

b) But Gris-Gris doesn't submerge, but just floats indignantly. By some miracle, you measure her unsubmerged volume to be $V_2 = 0.00100\,{\rm m}^3$. You know that the air density is $\rho_2 = 1.204\,{\rm kg/m}^3$ (at 20°C and 1 atm) and the water density is $\rho_1 = 998.29\,{\rm kg/m}^3$ (at 20°C and 1 atm). Note,

$$\frac{V_2}{V} = \frac{\rho_1 - \rho}{\rho_1 - \rho_2}\ ,$$

where ρ is the floater's density. Solve for her density.

c) Now you say? And Gris-Gris says?

NOTE: To find out about Tasmanian devils including what they really sound like (but not what their density is) try the Tasmanian devil page:
`https://www.youtube.com/watch?v=_Ku_Wd5CFQw`

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015 qfull 00410 2 3 0 moderate thinking: Guericke, hemispheres, horses
Extra keywords: Clydesdales. For calculus-based courses only until reevaluated.

637. Otto von Guericke (1602–1686) invented the air pump and wrote a book describing his invention and experiments that could be done with it: *Experimenta nova Magdeburgica de vacuo spatio* (1672). A famous illustration shows two teams of eight horses trying to separate a pair of joined brass hemispheres. Let the line through the horses and hemispheres be the x-axis. The hemispheres had been evacuated and external air pressure held them together. The hemispheres

were about a foot in diameter. The total horse force needed **TO JUST SEPARATE** the hemispheres was about 7500 N. **NOTE:** The diameter and total horse force values are very approximate, and so any values calculated using them will be likewise approximate. Also uncertainties tend to be large for the difference of nearly equal values.

a) Consider the x-direction forces on one hemisphere (the one that's roundy in the positive x-direction). Four forces act: the external air force, the internal air force (the vacuum's not complete), the normal force of the rim of the other hemisphere, and the pull force of the horses. For equilibrium

$$0 = F_{\rm int} + F_{\rm ext} + F_{\rm N} + F_{\rm horse} \ .$$

Note that the internal pressure force in the x-direction on a differential bit of sphere area dA is

$$dF_{\rm int} = P_{\rm int}\, dA\hat{n} \cdot \hat{x} \ ,$$

where $\hat{n}$ is the normal to the surface area. A bit of insight shows that

$$F_{\rm int} = P_{\rm int}\pi R^2 \ ,$$

where R is the radius of the hemisphere. What is the expression for $F_{\rm ext}$ in terms of external pressure? Get the sign of $F_{\rm ext}$ right.

b) Given $R = 0.5\,$ft (ft is foots), $P_{\rm ext} = 1.01 \times 10^5\,$Pa, and $F_{\rm horse} = 7500\,$N, what is $P_{\rm int}$? Assume the hemispheres have negligible thickness. **HINT:** The normal force can go to zero, but the internal pressure force cannot.

c) Would the horse force have to be any different to pull a hemisphere off a wall if the internal pressure were the same as in the two hemisphere experiment? Explain.

d) Von Guericke's horses did **NOT** separate the hemispheres. As natural-born cowgirls and cowboys what would you say: did he use Clydesdales or ponies? Explain.

NOTE, my sources for this question are Cardwell (p. 98 and 117), HRW, p. 341, and Wikipedia: Clydesdale horse.

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015 qfull 00420 3 5 0 tough thinking: parabolic surface of rotating fluid

Extra keywords: For calculus-based courses only until reevaluated.

638. You have an incompressible fluid of density ρ in a bucket. The bucket is rotating at angular speed ω. Due to viscosity the fluid has come to a steady state where it is co-rotating with the bucket: i.e., each fluid element is rotating with angular speed ω. In the rotating frame of the bucket the fluid is at rest. The bucket frame is, of course, non-inertial, but it can be treated as an inertial frame by introducing the centrifugal force (which is categorized as an inertial or fictitious force). Since fluid is at rest in the rotating frame, the Coriolis force (another inertial force) is zero (Fr-523). The external air pressure is assumed to be a constant $p_{\rm ext}$.

a) Why can we assume the external air pressure is a constant?

b) In a general sense, what do you expect the shape of the surface of the water to be. **HINT:** Try swirling water in a clear glass. In a rough qualitative way it only matters that the water is rotating something like a rigid rotator: the cup doesn't have to be.

c) What is the centrifugal force per unit mass in a rotating frame (treated as an inerital frame)? Take r to be the horizontal coordinate measured from axis of rotation. **HINT:** In the rotating frame, the centrifugal force is the force that cancels the centripetal force on any body at rest in the rotating frame. The centrifugal force is always present in the rotating frame no matter what other forces are acting.

d) Derive the expression for static equilibrium pressure in the vertical direction y taking y positive as upward. Leave the constant of integration undetermined for the moment: it will depend on r and p_{ext}.

e) Derive the expression for static equilibrium pressure in the horizontal direction r. This is done treating the rotating frame as a static reference frame with a centrifugal force. Leave the constant of integration undetermined for the moment: it will depend on y and p_{ext}. **HINT:** Consider the inner and outer horizontal pressures on a differential hollow cylinder of thickness dr and mean surface area dA centered on the rotation axis.

f) Set the zero level of y to be the height of the fluid at the $r = 0$ point. Now combine the two pressure expressions with the boundary condition $p(r = 0, y = 0) = p_{\text{ext}}$, to find pressure as a function of r and y. **HINT:** Pressure must have a unique value for each location (r, y).

g) From the general pressure expression found in the part (f) answer, determine the formula for the height y of the surface as a function of r.

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015 qfull 00460 3 5 0 tough thinking: planet pressure, Mastodon

Extra keywords: For calculus-based courses only until reevaluated.

639. Hail star voyager! After aeons of travel through deep, dark space you come a upon a strange, new world. It is a planet, that you name Mastodon, made of an absolutely incompressible fluid which you call Mastodonium. Remarkably, although the planet is fluid all the way, it is rotating like a rigid rotator: every bit has the same angular velocity ω. **NOTE:** This question could be a multi-pager in a test mise en scène.

a) The radius of Mastodon is $R = 1.0 \times 10^7\,\text{m}$; the density of Mastodonium is $\rho = 1.0 \times 10^5\,\text{kg/m}^3$. What is Mastodon's mass M?

b) Given the rotational inertia formula for a rigidly rotating uniform sphere, $I = (2/5)mr^2$, what is Mastodon's rotational inertia?

c) Mastodon rotates once every 24.0 hours which is the same as the Earth to within experimental uncertainty. What is its angular velocity ω in radians per second? **HINT:** How many radians are there in a circle?

d) Assuming the gas cloud out of which Mastodon formed was effectively isolated from all external forces, calculate that gas cloud's angular momentum about its center of mass? **HINTS:** What is Mastodon's angular momentum now? Also, "isolated from all external forces" doesn't imply the cloud had zero angular momentum.

e) Consider a differential box inside Mastodon at radius r with mass dm, height dr, and area perpendicular to the radius dA. Note r is the radial coordinate and it increases outward from 0 at the center to R at the surface. What is the differential pressure dp across dr going outward in terms of ρ and the magnitude of the gravitational field $f_g(r)$ at r? Write the gravitational field out explicitly in terms of G, the relevant mass, and r.

f) Integrate the differential expression you found in the part (e) answer to obtain the expression for the pressure at the center of Mastodon. Numerically evaluate this expression. Assume the surface pressure is zero. Note, the centrifugal force can be ignored: just as for the Earth (which Mastodon resembles in some respects) the centrifugal force is only a perturbation. Recall $G = 6.67430(15) \times 10^{-11}\,\text{N}\,\text{m}^2/\text{kg}^2$ (Wikipedia: Gravitational constant, 2025jun25).

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015 qfull 00610 1 3 0 easy math: Pascal's principle, lifting a car

Extra keywords: Doesn't use Pascal's principle really, but conventional with it.

640. You have a static water-based hydraulic system with two circular pistons. Piston 1 is 1.0 cm in **DIAMETER** and you push on it with 2.0 N of force with your finger. Piston 2 is vertical and

supporting a car of mass 2.0×10^3 kg against gravity. The piston 2 bottom is 2.0 m above the location of the piston 1 bottom. Air pressure can be considered constant with height.

a) What expression for the pressure of the 2nd piston as a function of the pressure of the 1st piston.

d) What is the **DIAMETER** of the 2nd piston?

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015 qfull 00950 2 3 0 mod math: city water tank to tap

641. Your city water tank has a water level above your taps of 100 m.

a) Assuming there's no funny business to adjust the pressure between tank and tap (which there probably is), what is the water pressure at your taps when they are closed? Assume air pressure is 1.01325×10^5 Pa (which is the standard air pressure of 1 atm [Wikipedia: 2007nov25: atmospheric pressure) and water density is 1.0×10^3 kg/m^3.

b) You open your tap and its the only open tap in the city: no toilets flushing either. Your tap DIAMETER is 1.0 cm; the city tank diameter is 20 m. What's the speed of the water from your tap? Did you really need to know the diameters? Why might this answer not be too accurate? HINT: Use Bernoulli's equation and the continuity equation. Using Bernoulli's equation implies a neglect of viscosity, and thus a neglect of energy to waste heat. Assume that the air pressure is constant and neglect the tank surface water velocity.

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015 qfull 01000 3 5 0 tough thinking: water clock, clepsydra

Extra keywords: Only for calculus-based. Really brutal.

642. The water clock or clepsydra (literally Greek for water thief) is one of the most ancient time measuring devices. The Egyptians and Babylonians used them since the 16th century BCE (Wikipedia: 2007nov25: water clock). They work at night unlike sundials and the advanced version keeps Newtonian time (the stuff we just call clock time usually). They are quite bit easier to build than hourglasses. You don't need glass and water pours, so you don't need a funnel construction. Water can easily be replaced and got rid off, so you don't to seal a water clock.

You have simple or Egyptian water clock consisting of a cylindrical primary vessel (volume V_0, surface area A_0) with a hole at the base (hole area A_h) and a catchment vessel which catches the water coming out of the primary. The volume of water V accumulated in the catchment measures time: $V_0 - V$ is the volume of water remaining in the primary.

a) Show that the differential bit of water volume sent to the catchment is

$$dV = A_\mathrm{h} v_\mathrm{h}\, dt \ ,$$

where v_h is the water velocity at the hole and dt is a time differential. **HINT:** Actually, this is one of those obviously it's ... things.

b) Using the continuity equation

$$Av = \text{Constant}$$

and Bernoulli's equation

$$P + \frac{1}{2}\rho v^2 + \rho g y = \text{Constant}$$

show that

$$A_\mathrm{h} v_\mathrm{h} = k\sqrt{V_0 - V} \ ,$$

where

$$k = \frac{A_\mathrm{h}}{\sqrt{A_0}} \sqrt{\frac{2g}{1 - (A_\mathrm{h}/A_0)^2}} \ .$$

c) Solve the differential equation

$$\frac{dV}{dt} = A_{\rm h} v_{\rm h} = k\sqrt{V_0 - V}$$

with the initial condition $V = 0$ at $t = 0$: i.e., find V explicitly as a function of time. At what time $t_{\rm max}$ does V reach a maximum? What is $V_{\rm max}$? At what time $t_{\rm empty}$ is the primary empty? If one drops the 2nd order term in the $V(t)$ solution, what would V be at time $t_{\rm empty}$?

d) Why is the simple water clock **NOT** a good measurer of Newtonian time t?

e) Does the water clock depend on air pressure or water density? Why or why not?

f) Construct a water clock and measure the relevant parameters. I suggest a fruit juice can or the like with a can opener hole at the base for the primary. A measuring cup would serve for a catchment: depending on your primary's size, you may need to empty periodically. Take a few measurements of accumulated water versus time and plot the data. Draw in the theoretical curve predicted by our theory and your parameters. Is there reasonable agreement? **NOTE:** This is just a fun experiment: no great pains required: just see how it works out: I'd like to know.

g) Ktesibios of Alexandria (circa -200) invented an improved or Ktesibian water clock. He simply had a continuous source pour into the primary with a spill-off spout at the top of the primary. Thus the primary always stayed exactly full and the flow behavior became stationary: i.e., unchanging with time. What is $V(t)$ in the catchment now? Is the Ktesibian water clock a good measurer of Newtonian time?

h) Suggest some plausible reasons why water clocks fell out of favor.

NOTE: Probably the most impressive water clock ever was engineered by Su Sung in 11th century China (Sung dynasty period): a massive fountain-water-wheel device that drove displays of lunar and other celestial cycles. A web search on Su Sung will bring up lots of information: a web search on anything brings up more than you'd ever want to know. Su Sung's clock was perhaps the real end of the road for water clock technology, but not the water clock's last hurrah—Galileo used a simple water clock in the kinematic experiments that established the constancy of gravitational acceleration. Galileo also sometimes used his pulse I believe.

Chapt. 16 Oscillation

Multiple-Choice Problems

016 qmult 00100 1 4 5 easy deducto-memory: oscillation defined

643. "Let's play *Jeopardy*! For $100, the answer is: It is a motion that repeats itself in equal time periods: i.e., a periodic motion. Perfect repetition is an ideal case that is more or less closely approached in reality. The repeated motion is sometimes called a cycle."

What is a/an _____, Alex?

a) inhalation b) exhalation c) rotation d) accleration e) oscillation

016 qmult 00110 1 1 4 easy memory: period and frequency

644. Say P is the period of an oscillation. Say you observe N oscillations (or cycles) which take time NP, of course, The frequency of the oscillation (cycles per unit time) is given by:

a) $f = NP$. b) $f = N/P$. c) $f = P/N$. d) $f = 1/P$. e) $f = N$.

016 qmult 00120 1 1 1 easy memory: unit of frequency

645. The MKS unit of frequency is the hertz (Hz). It is derived from the second:

a) $1\,\mathrm{Hz} = 1\,\mathrm{s}^{-1}$. b) $1\,\mathrm{Hz} = 1\,\mathrm{s}$. c) $1\,\mathrm{Hz} = 1\,\mathrm{s}^{-2}$. d) $1\,\mathrm{Hz} = 1\,\mathrm{s}^{2}$.
e) $1\,\mathrm{Hz} = 1\,\mathrm{s}^{-1/2}$.

016 qmult 00200 1 1 2 easy memory: linear force law

646. The linear force law (AKA linear restoring force law, spring force law, Hooke's law force, and simple harmonic oscillator force law) applies approximately to a wide variety of systems. In fact, almost any stable equilibrium system from molecules to bridges and beyond obeys the linear force law for sufficiently small perturbations of any component around the component's stable equilibrium configuration. Components moving under the linear force law alone are in simple harmonic motion. Usually some damping force affects the system. The linear force law in one-dimension with the stable equilibrium point at the origin is given by:

a) $F = -kx^{1/2}$. b) $F = -kx$. c) $F = -kx^{3/2}$. d) $F = -kx^{2}$. e) $F = kx$.

016 qmult 00210 1 1 3 easy memory: linear force law ubiquity

647. To be in stable equilibrium, an object must be subject to a restoring force. The restoring force is zero at equilibrium and for any sufficiently small displacement from equilibrium tries to push the object back toward equilibrium. If the kinetic energy built up by the force pushing an object back to equilibrium is too large, then the object can **OVERSHOOT** the equilibrium and an oscillation results. Whether an oscillation results or not depends on the system. In real systems in the absence of any driver force, a damping force will usually dissipate mechanical energy above that of rest at the equilibrium point and cause the oscillation to damp out and the object to come to rest at the equilibrium point again. If the displacement is sufficiently small from a stable equilibrium, then the restoring force will be in most cases linear in the displacement: i.e., for a one-dimensional case

$$F = -kx\ ,$$

where k is a constant and x is the displacement from the equilibrium point. The restoring force in the small-displacement limit is often linear because:

a) of no good reason. b) nature likes discontinuities.
c) nature dislikes discontinuities. d) nature is indifferent to discontinuities.
e) it's lies, all lies.

---

016 qmult 00220 1 1 3 easy memory: simple harmonic oscillator (SHO) 1

648. If the linear force alone acts on an object, the object is a simple harmonic oscillator (SHO) and undergoes simple harmonic motion (SHM). Apply Newton's 2nd law to 1-dimensional SHO with linear force $F = -kx$.

a) $m\dfrac{d^2x}{dt^2} = kx$. b) $m\dfrac{dx}{dt} = -kx$. c) $m\dfrac{d^2x}{dt^2} = -kx$. d) $m\dfrac{d^2x}{dt^2} = -\dfrac{1}{kx}$.
e) $m\dfrac{d^2x}{dt^2} = -x$.

---

016 qmult 00222 1 1 1 easy memory: simple harmonic oscillator 2

649. The simple harmonic oscillator (of mass m) in one dimension is a system in which Newton's 2nd law gives the equation:

a) $ma = -kx$. b) $ma = kx$. c) $m/a = k/x$. d) $m/a = -k/x$. e) $F = -kx$.

---

016 qmult 00230 1 1 1 easy memory: SHO solution 1

650. The general solution to the SHO differential equation

$$m\frac{d^2x}{dt^2} = -kx$$

is _____ where amplitude A and phase constant ϕ are set by initial conditions and angular frequency _____ is determined by the physics of the system itself. The motion of the solution is called simple harmonic motion (SHM).

a) $x = A\cos(\omega t + \phi)$; $\omega = \sqrt{k/m}$ b) $x = A\cos(\omega t + \phi)$; $\omega = \sqrt{m/k}$
c) $x = A(\omega t + \phi)$; $\omega = \sqrt{k/m}$ d) $x = A(\omega t + \phi)$; $\omega = \sqrt{m/k}$
e) $x = A(\omega t + \phi)$; $\omega = \sqrt{km}$

---

016 qmult 00232 1 1 2 easy memory: SHO solution 2: SHO angular frequency formula

651. For a one-dimensional simple harmonic oscillator with mass m, force law $F = -kx$, and solution obeying $a = -\omega^2 x$, the formula for the angular frequency ω is given by:

a) $\omega = m/k$. b) $\omega = \sqrt{k/m}$. c) $\omega = \sqrt{m/k}$. d) $\omega = k/m$. e) $\omega = \sqrt{km}$.

---

016 qmult 00234 1 1 4 easy memory: SHO solution 3

652. A solution to Newton's 2nd law for the simple harmonic oscillator is simple harmonic motion—which is a melodious, uncomplicated motion. The solutions for displacement, velocity, and acceleration for a case when the motion of the oscillating objects starts at time $t = 0$ at its maximum displacement A are, respectively:

a) $x = A\sin(\omega t)$; $v = \omega A\cos(\omega t)$; $x = -\omega^2 A\sin(\omega t)$.
b) $x = A$; $v = \omega A$; $a = \omega^2 A$.
c) $x = A\cos(\omega t)$; $v = -\omega A\cos(\omega t)$; $x = -\omega^2 A\cos(\omega t)$.
d) $x = A\cos(\omega t)$; $v = -\omega A\sin(\omega t)$; $x = -\omega^2 A\cos(\omega t)$.
e) $x = A\cos(\omega t)$; $v = \omega A\cos(\omega t)$; $x = \omega^2 A\cos(\omega t)$.

---

016 qmult 00240 1 1 1 easy memory: sinusoidal motion

653. In sinusoidal motion, an object's position as a function of time varies like a sine or:

a) cosine curve. b) tangent curve. c) inverse tangent curve.
d) inverse sine curve. e) straight line.

---

016 qmult 00250 1 1 3 easy memory: angular frequency

654. A sinusoid repeats its behavior every time its argument increases by 2π or 360°. An oscillation in time described by a sinusoid repeats in a time period called the period of the oscillation. The argument of the sinusoid in this case is $\omega t + phi$, where ω is the angular frequency and ϕ is a phase constant set by initial conditions. For a period P, the angular frequency is given by:

a) $\omega = 2\pi P$. b) $\omega = P/(2\pi)$. c) $\omega = 2\pi/P$. d) $\omega = 1/(2\pi P)$. e) $\omega = 2\pi P^2$.

---

016 qmult 00260 1 1 3 easy memory: SHM time formulae related

655. For simple harmonic motion,

a) $f = P = \omega$. b) $P = 1/f = 2\pi/\omega$, $f = 1/P^2 = \omega/(2\pi)^2$, $\omega = 2\pi f = 2\pi/P$.
c) $P = 1/f = 2\pi/\omega$, $f = 1/P = \omega/(2\pi)$, $\omega = 2\pi f = 2\pi/P$.
d) $P = 1/f = 2\pi/\omega$, $f = 1/P = 2\pi\omega$, $\omega = 2\pi f = 2\pi/P$.
e) $P = 1/f = 1/\omega$, $f = 1/P = \omega/(2\pi)$, $\omega = 2\pi f = 1/P$.

---

016 qmult 00270 3 4 1 hard deducto-memory: linearity of SHO equation

656. If $x_1(t)$ and $x_2(t)$ are solutions to a differential equation (DE), then the DE is linear if $c_1x_1(t)+c_2x_2(t)$ is also a solution where the c's are arbitrary constants. The differential equation for the simple harmonic oscillator

$$\frac{d^2x}{dt^2} = -\omega^2 x(t)$$

is:

a) linear. b) non-linear. c) neither linear nor non-linear.
d) both linear and non-linear. e) colinear.

---

016 qmult 00280 1 1 2 easy memory: SHO energy

657. The mechanical energy of a 1-dimensional simple harmonic oscillator of mass m and force constant k is given by:

a) $E = kx^2_{\rm max} = mv^2_{\rm max}$. b) $E = (1/2)kx^2_{\rm max} = (1/2)mv^2_{\rm max}$. c) $E = (1/2)kx^2$.
d) $E = (1/2)mv^2$. e) $E = mgy$.

---

016 qmult 00290 1 1 3 easy memory: uniform circular motion and SHM

658. The projection of uniform circular motion on a line in the plane of rotation is:

a) uniform circular motion. b) a Tusi-couple motion c) simple harmonic motion.
d) oblique motion. e) round motion.

---

016 qmult 00292 1 1 1 easy memory: SHO and uniform circular motion

659. The **COMPONENT** of a uniform-circular-motion radius vector along any axis in the plane of the motion executes:

a) simple harmonic motion. b) uniform circular motion. c) irregular motion.
d) static motion. e) no motion.

---

016 qmult 00300 1 1 4 easy memory: physical pendulum equation of motion

660. A phyiscal pendulum is any rigid object held by a free pivot axis: the axis supports the pendulum againnst gravity, but exerts no torque about itself. The equation of motion of the physical pendulum (i.e., Newton's 2nd law applied to the physical pendulum) if only gravity torques it is:

$$\frac{d^2\theta}{dt^2} = -\frac{rmg}{I}\sin\theta\ ,$$

where θ is the angle a radius from the pivot axis to the center-of-mass axis makes with a downward vertical, r is the cylindrical radius from the pivot axis to the center-of-mass axis, m is the pendulum mass, and I is the pendulum rotational inertia about the pivot axis. This equation has no simple analytical solution because of:

a) general considerations. b) the I factor. c) the r factor. d) the $\sin\theta$ factor.
e) darn good reasons.

---

016 qmult 00310 1 4 5 easy deducto-memory: small angle approximation

661. "Let's play *Jeopardy*! For $100, the answer is:

$$\sin\theta \approx \theta \ ,$$

where θ is given in radians.

What is a/an _____, Alex?

a) equality b) approximate equality c) inequality d) trigonometric function
e) small-angle approximation

---

016 qmult 00320 1 1 3 easy memory: physical pendulum in SHO approximation

662. The equation of motion of the physical pendulum (i.e., Newton's 2nd law applied to the physical pendulum) in the small-angle approximation is given by

$$\frac{d^2\theta}{dt^2} = -\frac{rmg}{I}\theta \ .$$

We recognize this equation as the differential equation of:

a) Tusi-couple motion. b) uniform circular motion. c) simple harmonic motion.
d) straight-line motion. e) uniform motion.

---

016 qmult 00330 1 1 5 easy memory: simple pendulum period

663. The period of the physical pendulum (in the small-angle approximation) is

$$P = 2\pi\sqrt{\frac{I}{rmg}} \ .$$

If the pendulum is shrunk to a point-mass bob, we have the simple pendulum. In this case, _____ and _____.

a) $I = (1/2)mr^2$; $P = 2\pi\sqrt{r/(2g)}$. b) $I = mr^2$; $P = 2\pi r/g$.
c) $I = mr$; $P = 2\pi r/g$. d) $I = mr$; $P = 2\pi\sqrt{r/g}$. e) $I = mr^2$; $P = 2\pi\sqrt{r/g}$.

---

016 qmult 00332 1 4 3 easy deducto-memory: simple pendulum period calculation

664. The period formula for the simple pendulum is

$$P = 2\pi\sqrt{\frac{r}{g}} \ .$$

If the length r is increased by a factor of 4, how does the period change?

a) Increases by a factor 4. b) Decreases by a factor 4.
c) Increases by a factor of 2. d) Decreases by a factor of 2.
e) Increases by a factor of 16.

---

016 qmult 00334 1 3 2 easy math: fiducial pendulum period

665. The period of a simple pendulum is given by

$$P = 2\pi\sqrt{\frac{r}{g}} \ ,$$

where r is the length of the pendulum and g is the gravitational fiedl. A fiducial pendulum period (i.e., period that can be used as a standard for reference or quick estimation) is obtained

for a pendulum of length exactly **ONE METER** and assuming that $g = 9.8\,\mathrm{m/s^2}$ exactly. What is this fiducial period to 2 significant figures? To 4 significant figures?

a) 1.0 s and 1.010 s. b) 2.0 s and 2.007 s. c) 2.0 s and 2.01 s. d) 1.0 s and 1.01 s. e) 1.0 s and 1.1 s.

---

016 qmult 00336 2 3 5 moderate math: pendulum period sans calculator

666. Without using a calculator (you're on your honor here), what is approximately the period of a simple pendulum of length 9 m? **HINT:** Just look at what the simple pendulum period formula and reflect on the answer to the fiducial pendulum period question.

a) 9.8 s. b) 162 s. c) 2.0 s. d) 1.0 s. e) 6.0 s.

---

016 qmult 00340 3 5 4 tough thinking: pendulum period sans mass

667. Time for deep thought. If you are clever (and not like your physics professor) before doing any elaborate dynamical analysis, why should you know that the period of the simple pendulum is independent of mass?

a) Mass is density times volume. Volume never comes into the simple pendulum problem. Ergo mass never comes into the simple pendulum problem.
b) There no mass in the kinematic equations and thus mass can never affect the motion of anything. The simple pendulum is included in the set of anything.
c) Physical intuition.
d) For any particle acted on only by gravity and workless forces of constraint (e.g., normal forces such as those of a frictionless slope or of a frictionless wire for a sliding bead), the only force that can cause acceleration is the component of gravity in the direction allowed by the constraints. This component has mass as one of its factors and is the F in $F = ma$. Thus, the mass in 2nd Newton's law applied to the system cancels out from both sides of the equation. The acceleration is thus determined by g, the direction of gravity downward, and the directions allowed by the forces of constraint. In principle, the whole kinematics of the particle can be determined from knowing the acceleration for any location and the initial conditions. In practice, it might require elaborate calculations including computer calculations to find the whole kinematics. But mass does not come into the calculation. The simple pendulum fits the case just outlined. The ideal free massless pendulum arm exerts no force in the direction the pendulum bob can move. There is no air drag. The only force that can cause acceleration is a component of gravity. Consequently, the whole kinematics of the pendulum bob is independent of mass. The pendulum period is part of the kinematics. Therefore the pendulum period is independent of mass.
e) Socrates has mass.
All humans have mass.
Ergo all humans are Socrates.

---

016 qmult 00350 1 5 5 easy thinking: pendulum clock and Galileo

668. You are Galileo Galilei (1564–1642) professor (untenured) of mathematics at the University of Pisa circa 1590. You are red-bearded and feisty (Italian word meaning ...). Tired of people making fun of your redundant name (exactly so in Latin: Galileus Galileus) as if you were Humbert Humbert or some such and bored with dropping balls off the Leaning Tower in the piazza, you seek calm in the adjacent Cathedral of Pisa. There you notice that the Cathedral lanterns oscillate in the wind with a constant period no matter what the amplitude of the oscillation provided the amplitude isn't too large and that period only varies with lantern cord length and not lantern mass as far as you can tell. At once—you are an incomparable genius after all—you realize that pendulums would make great regulators for clocks because:

a) it has **NEVER BEEN** been thought of before.
b) it has **BEEN** thought of before.

c) the hypnotic pendulum swinging motion would induce even deeper slumber in your less-amusing, clockwatching students.
d) even a bad idea can make money. All that is needed is a great advertizing campaign.
e) all clocks using pendulums as regulators for the motions of the hands and the energy input to keep the motions going would keep the periods of the hands very constant since small variations in amplitude that arise from somewhat irregular resistive and driver forces would have little effect on the pendulum period. Also the pendulum clocks could be kept synchronized to high accuracy despite varying amplitudes for the pendulum motion and masses for pendulum. Of course, the effective length of the pendulum does affect the period and has to be carefully adjusted for synchronization, but that is easy to do. It's one of those Eureka moments.

---

016 qmult 00400 1 4 2 easy deducto-memory: damped oscillations

669. "Let's play *Jeopardy*! For $100, the answer is: The motion of a system slowing down to rest at a stable equilibrium through a series of decaying oscillations about the equilibrium point."

What is _____, Alex?

a) critically motion b) underdamped motion c) overdamped motion
d) uniform circular motion e) all-wet motion

---

016 qmult 00530 1 4 4 easy deducto-memory: oscillatory resonance

670. "Let's play *Jeopardy*! For $100, the answer is: The behavior of an oscillatory system driven at its natural or resonance frequency."

What is _____, Alex?

a) underdamped oscillation b) simple harmonic motion c) uniform circular motion
d) resonance or in resonance e) loco

---

016 qmult 00540 1 4 1 easy deducto-memory: swing resonance of child

671. An oscillatory system driven at its resonance frequency will exhibit large oscillations. Every child—before they knew the name of torque—understood this when playing:

a) on a swing. b) on a ladder. c) soccer. d) hopscotch. e) with matches.

---

016 qfull 10220 2 5 5 moderate thinking: maximum force of a SHO, etc. fullmult

672.** A body of mass $m = 0.150\,\text{kg}$ is a simple harmonic oscillator (SHO) with equilibrium position $x = 0$, amplitude $x_{\max} = 10.5\,\text{cm}$, period $P = 0.250\,\text{s}$, and a linear (or spring-like) restoring force.

i) What is the force constant of the linear (or Hooke's law) force?

ii) What is the maximum absolute value of the force acting on the body?

iii) What is the maximum absolute value of the acceleration of the body?

a) (i) $94.8\,\text{N/m}$ (ii) $9.95\,\text{N}$ (iii) $85\,\text{m/s}^2$. b) (i) $120\,\text{N/m}$ (ii) $11.5\,\text{N}$ (iii) $95\,\text{m/s}^2$.
c) (i) $10.5\,\text{N/m}$ (ii) $120\,\text{N}$ (iii) $85\,\text{m/s}^2$. d) (i) $120\,\text{N/m}$ (ii) $10.5\,\text{N}$ (iii) $85\,\text{m/s}^2$.
e) (i) $94.8\,\text{N/m}$ (ii) $9.95\,\text{N}$ (iii) $66.3\,\text{m/s}^2$.

Full-Answer Problems

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016 qfull 00210 1 3 0 easy math: SHO period, frequency, ω

673. There is a simple harmonic oscillator (SHO) that takes a time $\Delta t = 0.75\,\text{s}$ before it begins to repeat. What are its (a) period P, (b) frequency f (in hertz), and (c) angular frequency ω (in radians per second)?

---

016 qfull 00220 2 5 0 moderate thinking: maximum force of a SHO, etc.

674. A body of mass $m = 0.12\,\text{kg}$ is a simple harmonic oscillator (SHO) with equilibrium position $x = 0$, amplitude $x_{\max} = 8.5\,\text{cm}$, period $P = 0.20\,\text{s}$, and a linear (spring-like) restoring force.

a) What is the force constant of the linear (or Hooke's law) force?

b) What is the maximum absolute value of the force acting on the body?

c) What is the maximum absolute value of the acceleration of the body?

---

016 qfull 00250 3 3 0 tough math: general SHO solution results

675. The general solution of the 1-dimension simple harmonic oscillator problem,

$$m\frac{d^2x}{dt^2} = -kx \qquad \text{is} \qquad x(t) = A\cos(\omega t) + B\sin(\omega t)\ ,$$

where A and B are determined by the initial conditions and can have any values in general. We will not prove that is the most general solution: that is something for a math course.

a) Verify that the general solution is a solution and determine the expression for angular frequency ω.

b) As initial conditions at $t = 0$, $x = x_0$ and $v = v_0$. Determine A and B in terms of the initial conditions.

c) Determine the ωt values for the stationary points (i.e., $\phi_{\text{st}} = (\omega t)_{\text{st}}$) within the range $\phi_{\text{st}} \in [-\pi/2, 3\pi/2]$.

Recall,

$$1)\quad \frac{y}{x} = \tan(\phi) \qquad 2)\quad \phi = \begin{cases} \arctan(x,y) + n\pi & \text{where } n = 0, \pm1, \pm2, \ldots \text{ here and below.} \\ \arctan\left(\dfrac{y}{x}\right) + n\pi & \text{for } x \geq 0 \text{ for the computational arctangent function.} \\ \arctan\left(\dfrac{y}{x}\right) + \pi + n\pi & \text{for } x < 0 \text{ for the computational arctangent function.} \end{cases}$$

For this problem, you will not know the x and y value signs.

d) Give a test determining whether a stationary point ϕ_{st} is maximum or minium. **HINT:** Recall, $d^2/dt^2 < 0$ for a maximum, $d^2/dt^2 = 0$ for an inflection point, and $d^2/dt^2 > 0$ for a minimum.

e) Verify that

$$x(t) = C\cos(\omega t + \phi)$$

is an equivalent expression for the general solution of the SHO problem: the amplitude C and phase factor ϕ are set by initial conditions and can take on any value, except of course that $\phi \in [-\pi, \pi]$. In the verification process, find A and B as functions of C and ϕ: this shows that arbitrary C and ϕ can be replaced by A and B. If the proposed equivalent solution is to be really equivalent, arbitrary A and B must be replaceable by C and ϕ. Show that they are by finding C and ϕ as functions A and B.

---

016 qfull 00260 3 5 0 tough thinking: springs in parallel, oscillations

676. You have a block of mass m sandwiched between a bunch of springs in parallel. The whole system is a 1-dimensional system. The springs are attached to opposing walls. Some springs

are from the left and some are from the right. The block sits on a level frictionless floor. The springs are ideal. Each spring has a force constant k_i and equilibrium position x_i for the center of the block: i.e., x_i is where the block center would be in equilibrium if only spring i were attached to the block.

a) What is the expression for the net force on the mass?

b) Derive the appropriate single-spring equivalent k (i.e., force constant) and $x_{\rm eq}$ (i.e., equilibrium position) expressions such that the net force expression changes to

$$F = -k(x - x_{\rm eq}) \ .$$

Why is the $x_{\rm eq}$ the equilibrium position of the total system?

c) Derive the expression for the total system ω in terms of the individual spring angular frequencies ω_i and the total system period P in terms of the individual spring periods P_i.

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016 qfull 00270 3 5 0 tough thinking: two-phase motion, collision and SHM

Extra keywords: conservation of momentum, conservation of energy.

677. There is a hunk of mass 5 kg on a frictionless horizontal surface attached to a horizontal ideal spring. The hunk is initially in the equilibrium position at rest with zero energy. A bullet on the opposite side from the spring, but traveling on the spring's axis, hits the hunk and embeds. The bullet mass is 0.050 kg and its speed is 500 m/s (which is supersonic not that it matters).

a) What is the initial speed of the hunk just after the bullet hits?

b) The spring is compressed for 2 meters before rebounding. What is its spring constant?

c) What is the **ANGULAR FREQUENCY** and the **MAGNITUDE OF THE MAXIMUM ACCELERATION** of the simple harmonic motion that results from the bullet impact?

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016 qfull 00350 3 5 0 tough thinking: physical pendulum and simple pendulum

678. Consider a physical pendulum in the small amplitude limit with a frictionless pivot axis. In this case, there is a linear restoring torque and the system exhibits simple harmonic motion. The period for the physical pendulum is

$$P = 2\pi\sqrt{\frac{I}{mgr}} \ ,$$

where I is the rotational inertia about the pivot point, m is the mass of the body that makes up the pendulum, and r is the distance from the pivot axis to the center-of-mass axis of the body.

a) Rewrite the formula substituting the parallel axis theorem formula for the rotational inertia:

$$I = mr^2 + I_{\rm cm} \ ,$$

where m is the body mass, r is the distance from pivot axis to the center-of-mass axis, and $I_{\rm cm}$ is the rotational inertia about the body center-of-mass axis. Simplify the new formula so there a term in the numerator under the square root symbol that is $I_{\rm cm}/(mr)$.

b) What is the limiting behavior of the period formula when $I_{\rm cm}/m$ goes to zero with r fixed? What is the pendulum called in this limit?

c) What is the limiting behavior of the period formula as $I_{\rm cm}/m$ goes to infinity with r fixed? What is the explication for the limiting behavior?

d) Now consider I_{cm}/m fixed. Give small r and large r limiting behaviors for the period formula. What is the large r formula? What is the limiting behavior as r goes to zero and as r goes to infinity?

e) For fixed I_{cm}/m, solve for the stationary points r_{min} and r_{max} that give the minimum and (stationary point) maximum period values and give those values. **HINT:** The r_{max} value is not finite.

f) Sketch the plot of the physical and simple pendulum periods as a function of r for $r \in [0, \infty]$. **HINT:** What must the period curve look like as it rises to the stationary maximum period. Is the simple pendulum period ever greater or equal to the physical pendulum period?

g) The physical pendulum is more realistic than the simple pendulum and its period formula gives the right behavior as r becomes small. Why, in fact, does the simple pendulum period formula give the wrong behavior as r becomes small. **HINT:** The simple pendulum assumes a massless pendulum arm and a point mass bob for all r.

Chapt. 17 Waves I: General Waves and Waves on a String

Multiple-Choice Problems

017 qmult 00100 1 4 5 easy deducto-memory: waves defined
Extra keywords: physci KB-176

679. "Let's play *Jeopardy*! For $100, the answer is: It is a extended-in-space location varying oscillation of something. Transport of energy and momentum can occur, but not in all cases."

What is _____, Alex?

a) mass b) energy c) pressure d) temperature e) a wave phenomenon

017 qmult 00110 1 4 1 easy deducto-memory: 3 kinds of waves
Extra keywords: physci

680. In one method of physically classifying waves, one has:

a) mechanical, electromagnetic, and quantum mechanical waves.
b) left, right, and middle waves.
c) S, P, and Middle Earth waves.
d) American waves, British waves, and Australian wives.
e) mechanical, financial, and emotional waves.

017 qmult 00120 1 4 5 easy deducto-memory: transverse and longitudinal waves
Extra keywords: physci

681. In one way of physically classifying waves, one has _____ waves:

a) trapeze and leotard b) terrific and lewd c) tornado and lounge
d) reverse and latitudinal e) transverse and longitudinal

017 qmult 00130 1 4 5 easy deducto-memory: linear wave equation introduced

682. "Let's play *Jeopardy*! For $100, the answer is:

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \ ,$$

where the symbol ∂ indicates partial derivatives (e.g., $\partial y/\partial x$ is the derivative of y with respect to x holding t constant), y is the displacement of some wave quantity, x is a spatial dimensional, t is time, and v is a phase velocity. This equation and multi-dimensional generalization of it hold for many wave phenomena such as small waves on a string, sound waves, and electromagnetic waves in limit of classical electromagnetism. However, not all waves obey it."

What is the _____, Alex?

a) linear medium b) phase velocity equation c) partial equation
d) phase equation e) linear wave equation

017 qmult 00132 1 4 4 easy deducto-memory: linearity of the wave equation

683. What does the linearity of the wave equation (differential wave equation) imply?

a) A linear combination of solutions is the square of a solution.

b) A linear combination of solutions is the inverse of a solution.
c) That there are no solutions.
d) A linear combination of solutions is a solution.
e) That there is only one solution.

---

017 qmult 00134 1 1 4 easy memory: superposition principle

684. The _____ principle for the wave equation—which is not the only wave equation despite its unqualified name—is not new axiom. It is is just a consequence of the linearity of the wave equation which implies that the linear combination of any two solutions is a solution. The _____ principle applies in many context which is perhaps why it is gloried with the term "principle" even though it may not be an axiom in any of them as far as yours truly knows.

a) wave b) wavelength c) symmetrization d) superposition e) werewolf

---

017 qmult 00140 1 1 3 easy memory: basic wave equation general solution

685. A very general, but not completely general, solution of the wave equation

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

is $f(x - vt)$. The proof for this solution is as follows. Let $\phi = x - vt$. Now

$$\text{LHS} = \frac{\partial^2 f}{\partial \phi^2} \times 1^2 = \frac{\partial^2 f}{\partial \phi^2} \ , \qquad \text{RHS} = \frac{1}{v^2}\frac{\partial^2 f}{\partial \phi^2} \times v^2 = \frac{\partial^2 f}{\partial \phi^2} = \text{LHS} \ ,$$

and that is QED. We have made use of the:

a) quotient rule. b) product rule. c) chain rule.
d) you-may-never-break-the-chain rule. e) right-hand rule.

---

017 qmult 00150 1 1 4 easy memory: general traveling wave solution

686. A very general, but not completely general, solution of the wave equation

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

is $f(x - vt)$. The solution is a traveling wave solution. The proof for this solution is as follows. Let $\phi = x - vt$. For a constant ϕ, one has a constant oscillation deviation $f(\phi)$. But this deviation's x position moves as _____ with velocity _____. Since ϕ is called the phase of the wave solution, v is called the phase velocity. A wave solution of the form $f(x - vt)$ is just a pattern that is traveling to the right for $v > 0$ and to the left for $v < 0$. If $v^2 = 0$, the wave equation is not defined and there is no solution. But there can be solutions that do not travel.

a) $\phi - vt$; $-v$. b) $\phi + vt$; $-v$. c) $\phi - vt$; v. d) $\phi + vt$; v. e) v; $\phi - vt$.

---

017 qmult 00160 1 1 1 easy memory: standing waves

687. A non-traveling wave solution to the wave equation

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

is

$$y = A\cos(kx + \phi_x)\cos(\omega t + \phi_t) \ ,$$

where A is amplitude, k is wavenumber t is angular frequency, and ϕ_x and ϕ_t are general phase constants. We require that $v = \omega/k$. Either or both of the cosine functions can be

changed to sine functions by defining different phase constants: e.g., $\phi_x = \phi'_x - \pi/2$ which gives $\cos(kx + \phi_x) = \sin(kx + \phi'_x)$. The proof for this solution is as follows: Now

$$\text{LHS} = \frac{\partial^2 y}{\partial x^2} = -k^2 y \ , \qquad \text{RHS} = \frac{1}{v^2}\frac{\partial^2 y}{\partial x^2} = -\frac{\omega^2}{v^2} y = -k^2 y = \text{LHS} \ ,$$

and that is QED. The solution does not travel. No shape simply glides to the right or left. At each point x, the solution is just simple harmonic motion with angular frequency ω and amplitude $A\cos(kx + \phi_x)$. Solutions of this kind are called:

a) standing waves. b) traveling waves. c) sitting waves. d) immobilized waves.
e) v; $\phi - vt$.

---

017 qmult 00170 1 4 5 easy deducto-memory: waves transport energy
Extra keywords: physci KB-212-3

688. "Let's play *Jeopardy*! For $100, the answer is: Energy."

What _____, Alex?

a) is vis viva b) is angular momentum c) is momentum d) is a vector
e) do waves transport

---

017 qmult 00200 1 1 3 easy memory: wavelength and period defined
Extra keywords: physci KB-212-1

689. The **DISTANCE** along a wave pattern in its propagation direction before the shape begins to repeat is called the _____ and the time period before the wave pattern begins to repeat itself at any point in space is called the _____.

a) frequency f; epoch e b) amplitude A; duration d c) wavelength λ; period P
d) period P; aeon a e) phase velocity v; awhile a

---

017 qmult 00210 1 1 5 easy memory: frequency-period formula

690. Say N cyles (i.e., wavelengths) of a periodic wave have passed a given point. This took time NP. The number of cycles per unit time or frequency f is given by:

a) $f = P/N$. b) $f = 1/(NP)$. c) $f = N/P$. d) $f = P$ e) $f = 1/P$.

---

017 qmult 00220 1 4 2 easy deducto-memory: frequency-wavelength formula
Extra keywords: physci

691. The time for wave cycle of wavelength λ to pass a given point is period P. The phase speed of the wave phenomena is then _____ from which one obtains the very familiar frequency-wavelength formula _____.

a) $v = \lambda P$; $f = \lambda/v$ b) $v = \lambda/P$; $f\lambda = v$ c) $v = \lambda/P$; $\lambda = vf$
d) $v = \lambda/P$; $v^2 = f\lambda$ e) $v = \lambda P$; $\lambda^2 = vf$

---

017 qmult 00222 1 1 4 easy memory: wavelength frequency relation
Extra keywords: physci KB-212-9

692. The higher the frequency of a wave, the:

a) remoter its source. b) grander its bearing. c) lower its pitch.
d) shorter its wavelength. e) closer its receiver.

---

017 qmult 00230 1 1 1 easy memory: sinusoidal waves defined

693. _____ waves in one dimesion are given by

$$y = A\cos(kx - \omega t + \phi) \qquad \text{or} \qquad y = B\cos(kx - \omega t) + C\sin(kx - \omega t) \ ,$$

where A is amplitude, k is wavenumber, x is positio, ω is angular frequency, t is time, and B are C are constants obtained by expanding $\cos(kx - \omega t + \phi)$ into cosine and sine terms.

_____ waves turn up in many contexts and, in fact, all waves can be expanded into linear combinations of them using Fourier series or Fourier transforms which gives these waves a universal use.

a) Sinusoidal b) Cosinusoidal c) Trigonometric d) Tangential
e) Cotangential

---

017 qmult 00232 1 1 4 easy memory: wavenumber wavelength, angular frequency frequency

694. A sinusoid repeat every time its argument increases by 2π. Thus, sinusoidal waves repeat as spatial coordinate alone varies when $k\Delta x = 2\pi$ and as time coordinate varies alone when $\omega\Delta t = 2\pi$ Immediately, one sees that:

a) $k = \lambda$ and $f = \omega$. b) $k = \pi\lambda$ and $f = \pi\omega$. c) $k = 2\pi\lambda$ and $f = 2\pi\omega$.
d) $k = 2\pi/\lambda$ and $f = \omega/(2\pi)$. e) $k = f$ and $f = \lambda$.

---

017 qmult 00234 1 1 3 easy memory: sinusoidal wave parameters

695. Sinusoidal waves are characterized by four common parameters wavelength λ, _____, angular frequency ω. and frequency f

a) wave arc a b) wave foam Υ c) wavenumber k d) wave lash ℓ
e) wave spit σ

---

017 qmult 00236 1 4 2 easy deducto memory: sinusoidal periodic wave

696. A sinusoidal wave is an example of a/an:

a) aperiodic wave. b) periodic wave. c) transverse wave. d) longitudinal wave.
e) trapeze wave.

---

017 qmult 00240 1 1 3 easy memory: sinusoidal waves and phase velocity

697. Sinusoidal waves, given by

$$y = A\cos(kx - \omega t + \phi) \qquad \text{or} \qquad y = B\cos(kx - \omega t) + C\sin(kx - \omega t)\ ,$$

satisfy the wave equation

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}\ .$$

This can be prove by direct subsitution into wave equation or by recognizing the sinusoidal wave function is of the form of the general traveling wave solution of the wave equation: i.e., $f(x - vt)$. Either way we find the relationship between phase velocity v and k and ω to be:

a) $v = \omega k^2 = f\lambda$. b) $v = \omega k = f\lambda$. c) $v = \omega/k = f\lambda$. d) $v = k/\omega = f\lambda$.
e) $v = \omega/k^2 = f\lambda$.

---

017 qmult 00300 1 1 2 easy memory: wave equation for string

698. Consider a string running along the x axis with tension τ. We can derive equation of motion for small transverse waves on the string. The waves are small continuously varying displacements of the string in the transverse y direction. Consider a differential segment of the string that in the x direction extends length dx. Newton 2nd law applied to this segment of string gives

$$(\mu\, dx)\frac{d^2 y}{dt^2} = (\tau\sin\theta)_2 - (\tau\sin\theta)_1\ ,$$

where μ is the linear mass density, θ is the angle of the string from the vertical, $(\tau\sin\theta)_2$ is the vertical force at the right end of the segment, and $(\tau\sin\theta)_1$ is the vertical force at the left end of the segment. Since we are considering small waves, we can make the small-angle approximation and obtain

$$\sin\theta \approx \tan\theta = \frac{dy}{dx}\ .$$

We now assume that the tension τ can be approximated as constant despite the stretching of the string by the waves. This approximation is difficult to justify a priori, but the resulting equation of motion works very well in many cases and that gives a posteriori justification. Now we have

$$(\mu\, dx)\frac{d^2y}{dt^2} = \tau\, d(\tan\theta) = \tau\, d\left(\frac{dy}{dx}\right) \ ,$$

and thus

$$\frac{d^2y}{dt^2} = \frac{\tau}{\mu}\frac{dy^2}{dx^2} \ ,$$

is our equation of motion. Note, the μ is constant if all the matter displacements are transverse. If they are not, we assume that μ can be approximated as constants. We recognize our equation of motion as the 1-dimensional wave equation with phase velocity _____. From this recognition, all the formalism developed for the wave equation applies for waves on a string in the small-wave approximation. What if the waves are not small? Then the wave equation—which is not the only wave equation despite its unqualified name–does not apply and a more complex analysis of the string waves is needed.

a) $v = \dfrac{\tau}{\mu}$ b) $v = \sqrt{\dfrac{\tau}{\mu}}$ c) $v = \dfrac{\mu}{\tau}$ d) $v = \sqrt{\dfrac{\mu}{\tau}}$ e) $v = \sqrt{\mu\tau}$

---

017 qmult 00330 1 3 5 easy math: linear density

699. A string is 8.0 m long and has mass 0.020 kg. What is its linear density μ?

a) 400 kg/m. b) 250 kg/m. c) 400 m/kg. d) 2.5×10^{-5} kg/m.
e) 2.5×10^{-3} kg/m.

---

017 qmult 00340 1 3 4 easy math: wavelength on a string

700. The wave speed for a string is

$$v = \sqrt{\frac{F_{\rm T}}{\mu}} \ ,$$

where $F_{\rm T}$ is the string tension and μ is the linear density (i.e., mass per unit length). What is wavelength as a function of $F_{\rm T}$?

a) $\lambda = f\sqrt{F_{\rm T}\mu}$. b) $\lambda = f\sqrt{F_{\rm T}/\mu}$. c) $\lambda = f^{-1}\sqrt{F_{\rm T}\mu}$. d) $\lambda = f^{-1}\sqrt{F_{\rm T}/\mu}$.
e) $\lambda = fF_{\rm T}\mu$.

---

017 qmult 00620 1 4 3 easy deducto-memory: what's an antinode?

701. What is an antinode?

a) A point of no motion in standing waves.
b) A point of minimum amplitude in standing waves.
c) A point of maximum amplitude in standing waves.
d) That which proceeds a node.
e) That which follows a node.

---

017 qmult 00630 2 3 5 moderate math: standing wave wavelength on a string

702. You have a string of length L with fixed endpoints. There are standing waves on the string. You count n antinodes. What is the wavelength of the waves?

a) $\lambda = Ln$. b) $\lambda = L/n$. c) $\lambda = L^2/n$. d) $\lambda = L$. e) $\lambda = 2L/n$.

---

017 qmult 00640 1 4 3 easy deducto-memory: antinodes and humps

703. In the 3rd harmonic of standing waves on a string fixed at both ends, how many antinodes are there:

a) Six: 2 for each of the 3 full wavelengths making up the pattern.

b) Four: the endpoints and the 2 inner points of no motion.
c) Three.
d) Two like the Bactrian camel.
e) One like the Arabian camel.

Full-Answer Problems

017 qfull 00330 1 3 0 easy math: a sinusoidal traveling wave

704. The equation of a transverse wave on a string is

$$y(x,t) = 9.0\sin(-0.01\pi x - 4.0\pi t) \ ,$$

where amplitude and x are centimeters, t is in seconds, and the argument of the sine is in radians. **HINT:** Do not forget the π values in calculing results and give results with correct units. Note, the standard form for a 1-dimensional wave solution adopted by the author is

$$y(x,t) = A\sin(kx - \omega t) \ ,$$

where ω is always taken as positive.

Find the (a) amplitude A, (b) wavelength $\lambda = 2\pi/|k|$, (c) frequency f (which is always positive), (d) direction of propagation along the x axis (which is determined by the sign of k) (e) phase velocity $v_{\rm ph} = \omega/k$ (which can be positive or negative), and (f) maximum oscillation speed of the string (i.e., its vertical maximum speed). Also (g), what is the oscillation displacement at $x = 3.5\,\text{m}$ and time $t = 0.26\,\text{s}$?

---

017 qfull 00340 2 3 0 moderate math: transverse wave speed in steel

705. Tensile stress τ is **FORCE PER UNIT AREA** and is measured in units of $\mathrm{N/m^2}$.

a) The string wave speed formula

$$v = \sqrt{\frac{T}{\mu}} \ ,$$

where T is tension force and μ is linear mass density (i.e., mass per unit length). This formula is derived in the limit of small wave amplitude and assumes the thickness of the string is negligible. Find the string wave speed formula in terms of τ and density ρ (i.e., mass per unit volume) assuming that the tensile stress is constant across the cross-sectional area of the string.

b) For safety, the maximum tensile stress to which steel wires should be subjected is $7.0 \times 10^8\,\mathrm{N/m^2}$. The density of steel is $7800\,\mathrm{kg/m^3}$. What is the fastest, small-oscillation transverse string wave that can be safely sent along a steel wire of any cross-sectional area according to our simple theory of string waves? How does this fastest string wave compare to the sound speed in steel: $v_{\mathrm{sound-in-steel}} \approx 6000\,\mathrm{m/s}$.

---

017 qfull 00360 3 5 0 tough thinking: string waves unconstant μ

706. A uniform rope hangs from a ceiling. It has length L and total mass m.

a) Show that the string speed on the rope as a function of y (measured relative to zero at the bottom end of the rope is $v = \sqrt{yg}$.

b) Show that the travel time for a wave across the whole rope is $t = 2\sqrt{L/g}$. **HINTS:** What is the travel time for the wave across a differential bit of rope dt at general y? How do you go from a differential answer to a global answer?

---

017 qfull 00640 1 5 0 easy thinking: harmonic freq. 1st order approximation

707. The formula for the harmonic resonance frequencies of string with fixed ends is:

$$f = n\frac{v}{2L} \ ,$$

where n is the harmonic number, v is the string wave speed, and L is the string length. Say one changed L by ΔL. What is the expression for Δf (the corresponding change in f) according to the 1st order approximation of calculus: i.e., $\Delta f/\Delta L \approx df/dL$.

Chapt. 18 Waves II: Sound Waves

Multiple-Choice Problems

018 qmult 00100 1 4 2 easy deducto-memory: sound waves defined

708. "Let's play *Jeopardy*! For $100, the answer is: They are presure waves in medium that can be any of gas, liquid, and solid. In gas and fluid, the waves can only be longitudinal. In solid tranverse waves can occur."

What are _____, Alex?

a) gravitational waves b) sound waves c) water waves d) seismic waves
e) quantum mechanical waves

018 qmult 00110 1 1 4 easy memory: pressure oscillates in sound
Extra keywords: physci

709. One obvious quantity that oscillates in sound waves is:

a) light. b) dark. c) frequency. d) pressure. e) gravity.

018 qmult 00120 1 1 1 easy memory: sound wave longitudinal I

710. Sound waves in fluids are always:

a) longitudinal. b) latitudinal. c) transverse. d) reverse. e) curse.

018 qmult 00130 1 1 2 easy memory: sound waves longitudinal II
Extra keywords: physci KB-212-5

711. Sound waves in fluids are always _____ waves.

a) transverse b) longitudinal c) mixed longitudinal-tranverse d) attitudinal
e) perverse

018 qmult 00200 1 4 5 easy deducto-memory: linear wave equation for sound

712. "Let's play *Jeopardy*! For $100, the answer is: For the 1-dimensional case,

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \ ,$$

where the phase velocity is given by

$$v = \sqrt{\left(\frac{\partial P}{\partial \rho}\right)_S} \ ,$$

where S means with entropy held constant: i.e., or adiabatically.

What is the _____, Alex?

a) Shrödinger equation b) linear wave equation for electromagnetic radiation
c) linear wave equation for sound d) Dirac equation
e) non-linear wave equation for sound

---

018 qmult 00210 1 4 1 moderate deducto-memory: sound speed formula for linear waves

713. The speed of sound formula for linear waves is:

a) $v = \sqrt{(\partial P/\partial \rho)_S}$, where the derivative is with constant entropy (i.e., the adiabatic condition).
b) $v = \sqrt{T/\mu}$, where T is tension and μ is linear mass density
c) $v = \sqrt{(\partial P/\partial \rho)_T}$, where the derivative is with constant temperature (i.e., the isothermal condition).
d) $v = \sqrt{T/\mu}$, where μ is tension and T is linear mass density
e) $v = \sqrt{(\partial v/\partial \rho)_S}$, where the derivative is with constant entropy (i.e., the adiabatic condition).

---

018 qmult 00340 1 3 1 easy math: sound inverse-square law

714. The intensity of sound at 10 m from a point source is $100\,\mathrm{W/m^2}$. What is it at 100 m?

a) $1\,\mathrm{W/m^2}$. b) $10\,\mathrm{W/m^2}$. c) $100\,\mathrm{W/m^2}$. d) $0.1\,\mathrm{W/m^2}$. e) $0.01\,\mathrm{W/m^2}$.

---

018 qmult 00370 1 3 4 easy math: decibel scale calculation

715. The sound level at The 'Boro (the student dive in Murfreesboro, Tennessee) has just jumped 43 dB. By approximately what factor has the sound intensity increased?

a) 43. b) 4.3 c) 430. d) 20000. e) 50000.

---

018 qmult 00700 1 4 1 easy deducto-memory: Doppler effect defined

Extra keywords: physci

716. The Doppler effect is a/an:

a) shift in wave frequency caused by the motions of sources and receivers.
b) effect of wave frequency on the motion of sources and receivers.
c) decrease of intensity as the inverse square of distance from a point source of sound or light.
d) cause of entropy.
e) cause of heat capacity.

---

018 qmult 00710 1 1 2 easy memory: Doppler effect generally

Extra keywords: physci KB-212-11

717. The Doppler effect occurs _____ waves:

a) only for sound b) for many kinds of c) only for light d) only for transverse e) for no kinds of

Full-Answer Problems

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018 qfull 00250 1 5 0 easy thinking: sound and light speeds, needs review

718. It's the 4th of July night and as usual there's half an hour of fireworks. After watching for 20 minutes—you are swift—you realize something is wrong. You hear the explosion always after you see it. In fact, the time difference is about 1.5 seconds. Explain this and tell how far you are from the rocket's red glare.

---

018 qfull 00260 1 3 0 easy math: scorpion and sound speed, need review

Extra keywords: really a kinematics and graphing problem. From HRW-438.

719. A sand scorpion can detect its prey by longitudinal and transverse sound waves the prey generates in the sand. The two kinds of sound waves arrive at different times at the scorpion

leg nearest the prey and from the arrival time difference the scorpion knows the distance to the prey.

a) The scorpion is a distance s from the prey. The scorpion detects the longitudinal waves (traveling at speed v_1) at time t_1 and the transverse waves (traveling at speed v_2) at time t_2. Both waves start at the prey at time zero. Derive the formula for s in terms of formal knowns v_1, v_2, and $\Delta t = t_2 - t_1$. Simplify as much as reasonably possible. **HINT:** There are actually three equations for three formal unknowns s, t_1, and t_2. But t_1 and t_2 are of no interest to us—nor to the scorpion. Write down the three equations and solve by eliminating the unknowns.

b) Calculate s given $v_1 = 150\,\text{m/s}$, $v_2 = 50\,\text{m/s}$, and $\Delta t = 4.0\,\text{ms}$.

c) Sketch a graph of s as a function of the $x = v_2/v_1$ taking v_1 and Δt as constants. Extend the graph from $x = 0$ to $x = \infty$. Show the asymptotic behaviors of the function curve. Actually $x > 1$ (with $\Delta t > 0$) implies s, t_1, and t_2 are all less than zero. In our setup, this cannot happen since it means sound signals existed before time zero. But extend the sketch for $x > 1$ anyway. **HINT:** Rewrite the formula from the part (a) answer eliminating v_2 in favor of x.

---

018 qfull 00550 2 3 0 moderate math: open pipe harmonics , needs review

720. The 2-open-end pipe A is 1.5 m long and is oscillating in its 3rd harmonic. The 1-closed-end pipe B is oscillating in its second lowest harmonic. The frequencies of the two pipes happen to match. The air speed is 343 m/s (appropriate for air at 20° C and 1 atm).

a) What is the fundamental frequency of pipe A (which is not the frequency it is oscillating at in this problem).

b) How long is pipe B?

c) If an x-axis extends along pipe A with a zero at one end, where along the pipe are the displacement nodes?

---

018 qfull 00650 2 3 0 moderate math: lines of destructive interference, needs review

721. There are two identical coherent, in-phase sources of sound separated by 2.5 m. (Coherence means having a definite [usually constant] phase relation. In-phase, means that the crests, troughs, and nodes are all aligned at the same time.) The sound wavelength is 1.0 m. Far from the sources, what is the angle of the first line destructive interference measured from the line bisecting the sources.

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018 qfull 00700 1 3 0 easy math: deriving the Doppler effect

722. The Doppler effect is a shift in wave frequency depending on the motion of source and/or observer. It is common to all wave phenomena, but the actual formulae vary from case to case. A prominent difference is the one between the sound Doppler effect (which is in the classical limit) and the light Doppler effect (which is relativistic). Here we derive the formulae for the sound Doppler effect.

a) Draw a schematic diagram of transverse wave cycle traveling in the positive x direction with wavelength λ and phase velocity v'. The wave cycle (i.e., a complete up-and-down cycle) takes time P to pass a given x position in the reference frame in which v' is observed. Derive, the formula

$$f\lambda = v' \ ,$$

where f' is the frequency of the wave.

Note, this is the formula relating frequency, wavelength, and phase velocity that everyone remembers, but without the prime symbol that we use to distinguish observer frame phase velocity from the phase velocity v observed in the rest in medium's frame

(i.e., the frame in which the medium is at rest which we assume to be an inertial frame). The medium-frame phase velocity is derived from the medium properties and is just called the phase velocity without qualifications unless context says otherwise. So the formula everyone really remembers is

$$f\lambda = v \ .$$

b) The formula $f\lambda = v'$ derived in part (a) is obviously valid for a general frame of reference moving in the x direction. We will now derive the basic (classical) Doppler effect formula in the opinion of the author.

First note that the phase velocity in a general observer frame moving at velocity $\vec{v}_{\rm obs}$ relative to the medium is

$$v' = v - \vec{v}_{\rm obs} \cdot \hat{v} \ ,$$

where $\hat{v}$ is unit vector in the direction of wave propagation and $\vec{v}_{\rm obs} \cdot \hat{v}$ is the component of the observer velocity in that direction. Second note that the wavelength λ of the wave is invariant for all observers (i.e., the particular set of waves being observed by all observers including one moving with the source of the waves). The invariance of the wavelength is just because length is a frame invariant quantity in classical physics. Recall length is a position difference that can be measured at one instant in time. In relativistic physics where time flows at different rates in different frames of reference, length becomes a frame dependent quantity (which effect is called FitzGerald contraction), but this frame-dependence is unobservably small in all common sound applications of the Dopper effect. The FitzGerald contraction has is needed for high relative velocities, but we are not going to consider those cases in this question.

As the author likes to write the basic Doppler effect formula, $1/\lambda$ appears on the right-hand side of the formula. Making use of this convention for $1/\lambda$, $v' = v - \vec{v}_{\rm obs} \cdot \hat{v}$, and part (a), derive the basic sound Doppler shift formula.

c) Give the formula for frequency of observer 2 (moving with $\vec{v}_2$) in terms the frequency of observer 1 (moving with $\vec{v}_1$). For $\vec{v}_1 = \vec{0}$, give and explicate briefly the special cases for:

i) in general.
ii) $\cos\theta_2 = 1$, $v_2 < v$.
iii) $\cos\theta_2 = 1$, $v_2 = v$.
iv) $\cos\theta_2 = 1$, $v_2 > v$.
v) $\cos\theta_2 = -1$, $v_2 < v$.
vi) $\cos\theta_2 = -1$, $v_2 = v$.
vi) $\cos\theta_2 = -1$, $v_2 >> v$.
vii) $\cos\theta_2 = -1$, $v_2 = \infty$.

Note, v_2 is interpreted as a magnitude (and therefore $v_2 \geq 0$ always) and not as component of a velocity vector along some axis.

d) Taylor expand the first result of part (d) to 1st order in small $(\vec{v}_1 \cdot \hat{v})/v$ and $(\vec{v}_2 \cdot \hat{v})/v$, and derive the 1st order formula for

$$\frac{f_2 - f_1}{f_1} \ .$$

What is the qualitative difference between the general result and the 1st order result? **HINT:** The Taylor expansion can be speeded up by remembering the geometric series:

$$1) \quad \frac{1}{1-x} = \sum_{k=0}^{\infty} x^k \qquad 2) \quad \frac{1}{1+x} = \sum_{k=0}^{\infty} (-x)^k \qquad 3) \quad |x| < 1 \quad \text{for absolute convergence}$$

(Arf-279). Note there is no conditional convergence at $x = 1$ for the alternating series (Arf-279,293).

e) A source can also be considered an observer. The source frequency in its own frame of rest is fixed at f_s where the subsript Roman s stands for source. The fixedness is because source observer always measures f_s ideally since the source observer can just count the pulses emitted. However, the emitted wavelength does depend on source velocity $\vec{v}_s$ and wave propagation direction $\hat{v}$. Note, $\vec{v}_s \cdot \hat{v} = v_s \cos\theta$, where θ is the angle between wave propagation direction and source velocity. Many sources emit in all directions, often isotropically or nearly. Using the basic Doppler effect formula, derive the formula for $\lambda(\vec{v}_s \cdot \hat{v}/v)$ (i.e., the formula for λ of emission as a function of $\vec{v}_s \cdot \hat{v}/v$). What happens when $v - \vec{v}_s \cdot \hat{v} = 0$? What does it mean when $v - \vec{v}_s \cdot \hat{v} < 0$?

---

018 qfull 00730 3 3 0 tough math: submarine Doppler effect, needs review

723. Two submarines cruising the Arctic Ocean approach each other with **SPEEDS** 50 km/h (sub 1) and 100 km/h (sub 2). Sub 1 emits a sonar signal of frequency $f_1 = 1000$ Hz. The sonar sound speed in sea water is 5470 km/h.

a) What is the signal frequency f_2 detected by sub 2?

b) What is the reflected signal frequency f_1' detected by sub 1?

---

018 qfull 00740 3 3 0 tough math: TGV trains, hurricane wind, Doppler shift, needs review

724. Two trains are traveling toward each other on the same straight track—but don't worry, the situation is just hypothetical—each at 60 m/s **RELATIVE** to the ground (i.e., 216 km/h: they are TGVs). The speed of sound in air (i.e, the speed of sound **RELATIVE** to the air) is 343 m/s. Train A blows its whistle at 500 Hz (in its own frame). **NOTE:** The 1st order formulae for the Doppler effect are not accurate enough for this problem.

a) What is the frequency heard on train B if the air is still?

b) What frequency is heard on train B if the air is blowing at 60 m/s (it's hurricane force, but very steady and nonturbulent) in the direction from train A to train B?

c) What frequency is heard on train B if the wind direction is reversed?

---

018 qfull 00780 2 5 0 hard thinking: sweet Polly Purebred, needs review

725. The foul Batbird has kidnapped sweet Polly Purebred and secreted her in his lair in an unkown reach of the Carlsbad Caverns—the enormous Batbird Cave! This looks like a job for Underdog.

a) Having penetrated the pitch-dark Cave, the Sober Sealyham gives gives a mighty Woof to which the Sultry Saluki upon hearing it instantly replies with a plaintive whimper. The time between Woof and the reply is 2.20 s and the speed of sound is 340 m/s. What is distance from Underdog to Polly?

b) Just then the batholithic Batbird swoops at Underdog having detected his location by bat sonar. But two can play that game: the Musical Mutt, still at rest himself, emits a concert A growl (440 Hz) and detects the echo exactly 1.00 s later at frequency of 660 Hz. How far is Batbird at the time the sound echoes off of him? What is his velocity toward Underdog then? What is Batbird's distance at the time Underdog hears the echo of the growl assuming Batbird's velocity is constant?

c) The Soldier Setter launches himself straight at Batbird with sufficient velocity to send Batbird reeling straight back at twice his initial speed following a 1-dimensional **ELASTIC** collision. Underdog has mass 70 kg and Batbird, 300 kg. What is Underdog's pre-collision speed? (Note: The analytic solution given as explicitly as possible is worth full marks even if you weren't able to answer part (b). By analytic I mean don't substitute in any numbers. By explicit I mean Underdog's velocity should appear isolated on one side of the equation and any analytic simplifications that can be made should be made.)

Denouement: Leaving the imprecating Batbird behind, tucking the Fair Fox under a foreleg, the Dogged Dog careens out of the Batbird Cave into the star-crossed night.

Chapt. 19 Thermodynamics I: Temperature, Heat, 1st Law

Multiple-Choice Problems

019 qmult 00100 1 4 5 easy deducto-memory: thermodynamics defined 1

726. "Let's play *Jeopardy*! For $100, the answer is: The short definition is that it is the science of heat and temperature. The words heat and temperature require definitions of course."

What is _____, Alex?

a) classical mechanics b) classical physics c) quantum mechanics
d) thermostatics e) thermodynamics

---

019 qmult 00110 1 1 3 easy memory: thermodynamics defined 2

727. In modern understanding, _____ is the science of how microscopic structures control or affect certain macroscopic properties of the materials. These properties are described/measured by thermodynamic variables (also called state functions) which include internal energy, temperature, pressure, volume, density, and entropy. There are other variables. If none of its thermodynamic variables is changing, a system is in thermodynamic equilibrium (TE). At the microscopic level, atoms and molecules are changing state all the time in TE and out of TE. TE _____ is much easier than non-TE _____. Fortunately, many non-TE systems may be treated approximately as TE systems or can be understood as passing through a sequence of TE states. This extends the utility of TE _____ into the non-TE realm. In this extension use is made of quasistatic processes. A quasistatic process is change in a system sufficient slow that the system can always be regarded as being in TE. To always exactly in TE, the quasistatic process would have to be infinitely slow. Fortunately, many not-so slow changes can be regarded as quasistatic process. Also imagining the infinitely slow quasistatic processes allows one to study limiting behaviors.

a) classical mechanics b) classical physics c) thermodynamics
d) quantum mechanics e) thermostatics

---

019 qmult 00112 1 1 3 easy memory: quasistatic process

728. A quasistatic process in thermodynamics is one in which the system is changed so:

a) **QUICKLY** that thermodynamic **EQUILIBRIUM** is virtually maintained throughout the process.
b) **SLOWLY** that thermodynamic **NON-EQUILIBRIUM** is virtually maintained throughout the process.
c) **SLOWLY** that thermodynamic **EQUILIBRIUM** is virtually maintained throughout the process.
d) **QUICKLY** that thermodynamic **NON-EQUILIBRIUM** is virtually maintained throughout the process.
e) **SLOWLY** that thermodynamic **EQUILIBRIUM** cannot be reached by the **END** of the process.

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019 qmult 00120 1 1 3 easy memory: internal energy defined

729. _____ is the energy of microscopic structures that participate in thermodynamics. _____ includes the kinetic energy of atoms, molecules and electrons relative to the rest

frame of the bulk material. It also includes field energies for fields that vary on the atomic scale. The field energies can often be described as potential energies or as the electromagnetic radiation field energy. _____ since it is an energy can be converted to or from macroscopic energies in various processes.

a) in-to-it energy b) inner energy c) internal energy d) infernal energy
e) gas energy

---

019 qmult 00122 1 1 1 easy memory: heat and internal energy

730. Formally, _____ (given the symbol Q) is internal energy transferred—but not transformed—by microscopic processes. In fact, almost everyone uses _____ or _____ energy as synonyms for internal energy. Frequently, one says this is the _____ transferred by a process and not as formally correct the _____ of the process.

a) heat b) warmth c) calor d) frisson e) temperature

---

019 qmult 00124 1 5 1 easy thinking: spontaneous heat flow
Extra keywords: physci

731. In the macroscopic world, heat spontaneously always flows from _____. This result is common observation. In thermodynamics, it is understood as a consequence of the 2nd law of thermodynamics. We can make heat go the other way, but that takes work: it doesn't happen spontaneously.

a) hot to cold b) cold to hot c) hot to hotter d) hot to hottest
e) objects at 0 K

---

019 qmult 00130 1 1 2 easy memory: zeroth law of thermodynamics

732. If two systems each in TE are put into thermal contact (i.e., contact whereby internal energy can flow between) and there are no thermodynamic changes in either, then the two systems are said to be in TE with respect to each other whether they are in contact or not. If two systems are both in TE with respect to a third system, then it is an observational fact that they are in TE with respect to each other. This last fact is called the _____ of thermodynamics. Why that name? Well it was decided that it was logically needed for thermodynamics after the first 3 laws had been well established. It could have been the called the 4th law, but it seemed more elementary than the other 3 laws. So the _____ seems a happy choice.

a) 5th law b) zeroth law c) half law d) 1.1th law e) the old 100th law

---

019 qmult 00140 1 4 4 easy deducto-memory: zeroth law and temperature

733. The existence of the zeroth law of thermodynamics suggests that there exists a thermodynamic variable that controls/describes the TE state of a system. It also suggests a way of measuring this variable for a system by changes in a small non-perturbing system as it comes into TE with the first system. The thermodynamic variable is an intensive variable: i.e., one that is independent of the size of a system. This must be since the condition of system being in TE with respect to each other is independent of their sizes. The TE state controlling/describing variable is called:

a) pressure. b) respiration. c) expiration. d) temperature. e) entropy.

---

019 qmult 00150 1 4 2 easy deducto-memory: temperature defined

734. "Let's play *Jeopardy*! For $100, the answer is: Well it's not the easiest thing to define. In modern understanding, it is thermodynamic variable that controls/describes the distribution of internal energy among the microscopic states of a system. Its control is such that two systems with the same value of the parameter are in TE with respect to each other. The variable is an intensive variable since it is independent of the size of a system: it controls distribution, not amount, of energy. Unlike most intensive variables, the variable in question is not ratio of extensive quantities (those that do depend on the size of the sample) at least in any ordinary

way of thinking of it. In a usual sense, we do cannot measure the variable in question directly. We measure some other quantity that is correlated with it over some range of behavior. For example, we it by measuring fluid volume in a fluid thermometer, gas pressure in a constant-volume gas thermometer, and electrical potential (i.e., voltage) with a thermocouple."

What is _____, Alex?

a) heat b) temperature c) hotness d) coldness e) lukewarmness

---

019 qmult 00160 1 1 4 easy memory: 1st law of thermodynamics

735. The 1st law of thermodynamics is energy conservation in the field of the thermodynamics. There are various ways of writing it depending on circumstances. However, a conventional form for beginners is

$$dE = dQ - dW \ ,$$

where E is the internal energy of a system, Q is heat added to the system by microscopic heat transfer processes, and W is work (i.e., macroscopic work) done by the system. This formula for the 1st law is written in terms of differentials since that is convenient in developing the formalism of thermodynamics and in problem-solving as it turns out. All the terms in the formula can be positive or negative. The minus sign in formula is annoying, but by convention one counts work done by a system (which takes away energy from it) rather than work done on the system (which adds energy to it). We can see that 1st law embodies energy conservation: a system's energy changes only by the sum of contribuitons to that energy and is constant if there are no contributions.

The formula for differential work is _____, where the p stands for system pressure and the V for system volume. This kind of work is given the name that comes from the vocalization of the letters in order followed by the word work.

a) $dW = p/V$ b) $dW = pV$ c) $dW = V\,dp$ d) $dW = p\,dV$ e) $dW = dp/V$

---

019 qmult 00162 1 1 2 easy memory: 1st law of thermodynamics

736. A standard form for the 1st law of thermodynamics (prior to introducing entropy anyway) is:

a) $dE = dQ + p\,dV$. b) $dE = dQ - p\,dV$. c) $dE = -dQ + p\,dV$.
d) $dE = dQ + V\,dp$. e) $dE = -dQ - V\,dp$.

---

019 qmult 00170 1 4 3 easy deducto-memory: pressure defined

737. "Let's play *Jeopardy*! For $100, the answer is: It is the thermodynamic variable that is the magnitude of the force per unit area that a material exerts on any internal or external surface. We usually think of it as being isotropic: i.e., having the same value for all directions at a point in the material. It can be anisotropic in some case."

What is _____, Alex?

a) temperature b) internal energy c) pressure d) density e) entropy

---

019 qmult 00180 1 1 5 easy memory: derivation of pdV work

738. Actually, the formula for $p\,dV$ work requires a derivation. Consider a vectorized differential surface element $d\vec{A}$ at the surface of a system. The vector points in the perpendicular outward direction from the differential surface element itself and is well defined since a differential surface element is planar. The system pressure does work on the differential surface element if the surface element moves. We find

$$dW = p\,d\vec{A}\cdot d\vec{s} = p\,(dA\,ds\cos\theta) \ ,$$

where $d\vec{s}$ is the displacement of the surface element. A little thought shows that $(dA\,ds\cos\theta) = dV$ the change in volume with the displacement. Reflecting on the area of a rectangle and drawing a diagram might help. Thus, we have

$$dW = p\,dV$$

for the work done by the system. We note that $p\,dV$ work can be:

a) positive only. b) negative only. c) zero only. d) a square root.
e) positive or negative.

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019 qmult 00190 1 1 2 easy memory: isothermal and adiabatic

739. A thermodynamic process at constant temperature is called _____ and that with no heat flow (i.e., $dQ = 0$) is called _____.

a) adiabatic; isothermal b) isothermal; adiabatic c) isobaric; isothermal
d) adiabatic; isobaric e) isobaric; adiabatic

---

019 qmult 00192 1 4 1 easy deducto-memory: ideal thermal processes

740. Isothermal, adiabatic, engine cycle, and free expansion processes: briefly and in order these processes can be characterized by the phrases:

a) (i) constant temperature, (ii) no heat flows (no entropy change), (iii) final state is the initial state and **NET** heat and work flow, and (iv) no heat flow and no work and not quasistatic.
b) (i) constant temperature, (ii) final state is the initial state and **ZERO NET** heat and work flow, (iii) no heat flows (no entropy change), and (iv) no heat flow and no work and not quasistatic.
c) (i) constant temperature, (ii) no heat flow and no work and not quasistatic, (iii) final state is the initial state and **ZERO NET** heat and work flow, and (iv) no heat flows (no entropy change).
d) (i) constant pressure, (ii) constant volume, (iii) constant entropy, and (iv) constant free energy.
e) (i) resonance frequency, (ii) first harmonic, (iii) Doppler effect, and (iv) Manifest Destiny.

---

019 qmult 00200 1 1 1 easy memory: 3 temperature scales
Extra keywords: physci

741. The three common temperature scales are:

a) Fahrenheit, Celsius, and Kelvin. b) Fahrenheit, Celsius, and Newton.
c) Fahrenheit, Vesuvius, and Kelvin. d) Fahrenheit, Celsius, and Calvin.
e) Gesundheit, Vesuvius, and Calvin.

---

019 qmult 00202 1 4 2 easy deducto-memory: kelvin/absolute temperature
Extra keywords: physci

742. "Let's play *Jeopardy*! For $100, the answer is: This temperature scale is considered to be the absolute temperature scale and its zero-point is absolute zero."

What is the _____ scale, Alex?

a) Fahrenheit b) Kelvin c) Celsius d) thermometer e) Hobbes

---

019 qmult 00210 1 4 4 easy deducto-memory: absolute zero defined
Extra keywords: physci KB-129-9

743. "Let's play *Jeopardy*! For $100, the answer is: At this Kelvin temperature, the internal energy of a system is as low as quantum mechanics allows."

What is _____, Alex?

a) zero celsius b) the boiling point of water c) the freezing temperature of water
d) absolute zero e) relative zero

---

019 qmult 00220 1 4 3 easy deducto-memory: triple point of water

744. "Let's play *Jeopardy*! For $100, the answer is: The name for the state in which water coexists in its three phases (vapor, liquid, and solid) at Kelvin temperature 273.16 K (by definition) and pressure 610 Pa."

What is _____, Alex?

a) the unique point of water b) the water plasma phase c) the triple point of water d) the state where the Celsius and Fahrenheit temperature scales agree e) is hockey weather

---

019 qmult 00311 1 1 1 easy memory: temperature a ratio

Extra keywords: physci I've finally concluded intensive quantities are ratios

745. Absolute or Kelvin temperature in modern physical understanding is, in fact, a ratio of quantities: these quantities can be determined by addition. There are a variety of ratios that can be used to determine temperature. The simplest ratio to understand is set by the ideal gas law. Because temperature is a ratio, a sample of any size above the microscopic level can have

a) any temperature at all, except absolute zero (or so it seems) and infinite temperature. b) only negative temperatures. c) only temperatures proportional to its volume. d) only temperatures proportional to its mass. e) only infinite temperatures.

---

019 qmult 00312 1 4 5 easy deducto-memory: temperature: heat per unit mass

Extra keywords: physci

746. "Let's play *Jeopardy*! For $100, the answer is: This **RATIO** is does not give absolute temperature, but it increases with absolute temperature and gives a concrete, though not definitive, sense of what temperature is."

What is _____, Alex?

a) pressure b) density c) mass d) heat energy e) heat energy per unit mass

---

019 qmult 00240 1 1 1 easy memory: temperature conversion rules

747. The three main temperature scales (Kelvin, Celsius, and Fahrenheit) are related by following conversion rules

$$T_{\mathrm{K}} = T_{\mathrm{C}} + 273.15\,\mathrm{K}\ , \qquad T_{\mathrm{F}} = 1.8 \times T_{\mathrm{C}} + 32^{\circ}\mathrm{F}\ .$$

What are 290 K, 300 K, and 310 K in Fahrenheit?

a) 62, 80, 98. b) 59, 73, 96. c) 57, 71, 95. d) 56, 69, 94. e) 49, 64, 91.

---

019 qmult 00242 1 3 2 easy math: temperature conversion example

Extra keywords: physci

748. What are 10° C, 20° C, and 30° C in Fahrenheit? **HINT:** I always multiply a Celsius temperature by 1.8 and add 32° to get Fahrenheit. Going the other way subtract 32° and divide by 1.8.

a) −273.15° F, 273.15° F, 373.15° F. b) 50° F, 68° F, 86° F. c) 40° F, 48° F, 56° F. d) 48° F, 68° F, 90° F. e) 0° F, 100° F, 212° F.

---

019 qmult 00244 1 3 1 easy math: boiling ethyl alcohol

Extra keywords: physci KB-130-23

749. Ethyl alcohol boils at about 172°F (at 1 atmosphere pressure one assumes). The conversion formula from Fahrenheit to Celsius is

$$T_{\mathrm{C}} = \frac{T_{\mathrm{F}} - 32}{9/5}\ .$$

The boiling point on the Celsius scale is:

a) 77.8°C. b) 32°C. c) 0°C. d) 100°C. e) 172°C.

---

019 qmult 00250 3 3 3 hard math: higher/lower Tfahren Tcelsius

Extra keywords: physci KB-129-1 a stupid, but tricky question actually

750. For the same physical temperature, a Fahrenheit numerical reading is _____ than a Celsius numerical reading. **HINT:** Recall there are negative temperatures for both these scales.

a) always higher b) always lower c) higher, lower, or equal depending on the physical temperature d) both higher and lower e) neither higher or lower

---

019 qmult 00252 1 1 4 easy memory: -40 same in celsius and fahrenheit

751. There is one temperature where the Celsius and Fahrenheit scales give the same number. What is that number?

a) 40. b) 0. c) 32. d) −40. e) −32.

---

019 qmult 00400 1 4 1 easy deducto-memory: state variable

752. "Let's play *Jeopardy*! For $100, the answer is: A thermodynamic quantity for a sample of material (which is in thermodynamic equilibrium) that is entirely a function of the thermodynamic state—not a function of past history—nor future history for that matter. The thermodynamic state for a fixed mass of material is completely determined by specifying any two such quantities: e.g., temperature and pressure or temperature and volume."

What is _____, Alex?

a) a state variable or function of state b) temperature c) pressure d) caloric e) no such thing

---

019 qmult 00500 1 1 1 easy thinking: linear expansion in fluids

Extra keywords: This question is too wordy.

753. Volume expansion coefficients are usually given for liquids (which together with gases constitute fluids) instead of linear expansion coefficients. Linear expansion coefficients (although they can be specified) arn't very meaningful theoretically or practically. This is because:

a) fluids (simple fluids at least) **CANNOT** withstand the tendency of any applied force to deform them in a way that leaves volume unchanged. Thus, if a fluid expands thermally, it's shape will usually change unless, for example, the fluid container scales up **ISOTROPICALLY** with the same volume expansion coefficient as the water. Therefore, point-to-point distances between specified fluid elements will usually **NOT SCALE UP** uniformly.

b) fluids (simple fluids at least) **CAN** withstand the tendency of any applied force to deform them in a way that leaves volume unchanged. Thus, if a fluid expands thermally, it's shape will usually change unless, for example, the fluid container scales up **ISOTROPICALLY** with the same volume expansion coefficient as the water. Therefore, point-to-point distances between specified fluid elements will usually **NOT SCALE UP** uniformly.

c) fluids (simple fluids at least) **CANNOT** withstand the tendency of any applied force to deform them in a way that leaves volume unchanged. Thus, if a fluid expands thermally, it's shape will usually change unless, for example, the fluid container scales up **ISOTROPICALLY** with the same volume expansion coefficient as the water. Therefore, point-to-point distances between specified fluid elements will usually **SCALE UP** uniformly.

d) you cannot draw a line in a fluid.

e) fluids never expand thermally.

---

019 qmult 00510 1 5 1 easy thinking: linear expansion of invar

754. Typical common metals (steel, aluminum, copper, brass) have linear expansion coefficients in the range $\sim$ 10–25 in units of $10^{-6}\,\mathrm{K}^{-1}$ at about room temperature. (**NOTE:** Expansion coefficients are themselves temperature dependent in general, but that extra effect is beyond the scope of this class.) Glass is more rigid with a coefficient of $\sim 3\times10^{-6}\,\mathrm{K}^{-1}$. A special alloy *invar* (probably meaning invariable) made of 64 % iron and 36 % nickel has a coefficient of only

$0.9 \times 10^{-6}\,\mathrm{K}^{-1}$. What is the fractional linear expansion of invar $\Delta\ell/ell$ when the temperature rises 1° C (or 1 K)?

HINT: Remember

$$\Delta\ell = \ell\alpha\Delta T \ ,$$

where ℓ is any length in the material, $\Delta\ell$ is the change in that length, α is the linear expansion coefficient, and ΔT the change in temperature.

a) 0.9×10^{-6}. b) 0.9×10^{-5}. c) 3×10^{-5}. d) 25×10^{-5}. e) 9×10^{-5}.

---

019 qmult 00600 1 4 4 easy deducto-memory: phase change
Extra keywords: physci

755. During a phase change of the common kind, the temperature of a substance in self-equilibrium (so all the material has the same temperature) is:

a) 273.15 K. b) 32°F. c) changed by 10 % or more. d) constant. e) infinite.

---

019 qmult 01100 1 1 1 easy memory: specific heat
Extra keywords: physci

756. Specific heat of a material is its:

a) rate of change of heat energy content (sometimes called internal energy) per unit mass per unit temperature.
b) rate of change of temperature per unit mass per unit heat energy content (sometimes called internal energy).
c) temperature in a pressure of 1 atmosphere.
d) temperature at its melting point.
e) heat energy content (usually called internal energy) at its melting point.

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019 qmult 01300 1 4 3 easy deducto-memory: 3 modes of heat transfer I
Extra keywords: physci

757. The three common heat transfer processes are:

a) conduction, invection, and radiative transfer. b) induction, convection, and radiative equilibrium. c) conduction, convection, and radiative transfer. d) conduction, invection, and radiative equilibrium. e) introduction, insurrection, and radiative hibernation.

---

019 qmult 01400 1 4 5 easy deducto-memory: 3 modes of heat transfer II
Extra keywords: physci

758. The three commonly known means of heat transfer are:

a) conduction, convection, hyperventilation. b) conduction, contrition, contemplation. c) conduction, neutrinos, gravitational radiation. d) crenelation, navigation, mutation. e) conduction, convection, radiative transfer.

Full-Answer Problems

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019 qfull 00100 2 5 0 moderate thinking: thermal linear expansion, invar
Extra keywords: see WP, p. 447 for the expansion coefficients

759. You are trying to fit an invar bolt of diameter 1.001 cm into a steel nut of diameter 1.000 cm. The temperature is 20°C. The linear expansion coefficients of invar and steel at about room temperature are $\alpha_{\mathrm{invar}} = 0.9 \times 10^{-6}$ and $\alpha_{\mathrm{steel}} = 12 \times 10^{-6}$, respectively. To what temperature

must raise the objects so that the bolt fits in the nut? Why could this calculation be a poorish prediction?

---

019 qfull 00200 2 3 0 moderate math: equilibration calculation

760. You have objects of copper ($m_1 = 0.30\,\text{kg}$, $T_{i,1} = 20°\text{C}$, and $c_1 = 386$ mks units), aluminum ($m_2 = 0.60\,\text{kg}$, $T_{i,2} = 100°\text{C}$, and $c_2 = 900$ mks units), and silver ($m_3 =?$, $T_{i,3} = 50°\text{C}$, and $c_3 = 236$ mks units). The m_k's are masses, the $T_{i,k}$'s are initial temperatures, and the c_k's are specific heats. You put the objects in thermal contact with each other, but in isolation from the rest of the world. The equilibrium temperature is found to be $T_f = 80°$. What is the mass of silver?

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019 qfull 00300 2 3 0 moderate math: work done on a straight line PV diagram

761. In a quasistatic process, you take a system from a state specified by the function of state pair (V_1, p_1) to a state specified by the pair (V_2, p_2). The path on a p-V diagram is a straight line. Obtain an explicit formula (i.e., not just an integral expression) for the work done by the system. **HINT:** Draw a diagram.

---

019 qfull 00400 2 5 0 moderate thinking: solar luminosity, solar constant

762. The temperature of the Sun's photosphere (the layer from which comes most of the light we see) is about 5800 K and has a radius of about 6.96×10^8 m.

a) Assuming the photosphere radiates like a blackbody (which isn't so bad an approximation), what is the total luminosity of the sun (i.e., it's total power)? (The Stefan-Boltzmann constant $\sigma = 5.67 \times 10^{-8}$ in mks units.)

b) What is the intensity (i.e., power per square meter) at the distance of the Earth (1.496×10^{11} m)? This intensity is often called the Solar Constant and it better had be for life on Earth. **HINT:** Remember the inverse-square law.

c) What is total power received by the Earth from the Sun? (The Earth's mean radius is 6378 km.)

d) The Earth collects sunlight at one time only on one side and because of the curvature of the Earth's surface the power absorbed varies with the tilt of the surface to the incoming beams. However, rotation and air circulation very crudely can be assumed to evenly spread the power over the whole Earth. Obviously this is not exactly true: the further from the equator the less power is received on average. Let's assume though that the power received from the Sun is spread uniformly over the Earth's surface. Now the Earth is more or less in an energy steady state. It re-radiates back to space in the infrared virtually all of the energy it gets from the Sun which mostly comes in the optical. Assuming the Earth is a spherical black-body absorber/radiator in an exact energy-steady state, what is its surface temperature? Can the Earth be exactly a black-body absorber/radiator?

Chapt. 20 Thermodynamics II: Microscopic Treatment

Multiple-Choice Problems

020 qmult 00110 1 1 2 easy memory: classical thermodynamics and the micro realm

763. Classical thermodynamics as it existed in the 19th century avoided reference to microscopic entities like atoms and _____ since their existence was then considered problematic. If you make theories based on wrong premises, your theories are likely to be wrong. However, by the end of the 19th century evidence for atoms and _____ as real things was increasing and increasingly classical thermodynamics could be explained and extended by using the statistical behavior of the atoms and _____. The study of this statistical behavior is now called statistical mechanics. Although there is still some value in considering thermodynamics from a purely macroscopic point of view, it quickly becomes tedious to avoid referencing the microscopic realm since we know it is there and provides the basis for macroscopic thermodynamics. So it is physically right and educationally sound to start referencing the microscopic realm right from the start in studying thermodynamics.

a) atoms b) molecules c) monads d) imperceptables
e) little itty bitty things

020 qmult 00150 1 5 1 easy thinking: mole defined and used

764. The total mass of a sample of atomic or molecular species i is

$$m_{\text{total}} = Nm_i = NA_i u = N(A_i \times 1\,\text{g})\left(\frac{u}{1\,\text{g}}\right) = \left(\frac{N}{u/1\,\text{g}}\right)(A_i \times 1\,\text{g}) \ ,$$

where A_i is the atomic mass of the species, $u = (1/12)m_{\text{C}-12} = 1.66053906892(52) \times 10^{-24}\,\text{g}$ is the dalton (AKA atomic mass unit) which is defined to be exactly the mass of an unperturbed carbon-12 atom, the commonest isotope of carbon. Note, all basic microscope particles are absolutely identical in principle as dictated by quantum mechanics: they cannot, in principle, be individually identified. We define the mole or Avogadro's number to be

$$N_{\text{A}} = \frac{1}{u/1\,\text{g}} = 6.02214076^{23} \ ,$$

which is now defined as an exact number even though the dalton in grams isn't. We now have

$$m_{\text{total}} = \left(\frac{N}{N_{\text{A}}}\right)(A_i \times 1\,\text{g}) = n_{\text{mol}}(A_i \times 1\,\text{g}) \ ,$$

where n_{mol} is the number of moles in the sample. If you have 1 mole of species i, the mass in grams is

a) A_i. b) $1/Ai$. c) 1. d) N_{A}. e) $1/N_{\text{A}}$.

020 qmult 00160 1 1 4 easy memory: simple calculation with moles

765. You have 32 grams of molecular oxygen, O_2 (each molecule having containing 16 protons and 16 neutrons altogether). How many moles of O_2 do you have?

a) 32 moles. b) 2 moles. c) 16 moles. d) 1 mole. e) None.

---

020 qmult 00162 1 3 5 easy math: Avogadro's number

766. Say you have A grams of some element whose elemental atomic mass is A: mass in macroscopic units is Am_a, where $m_a = 1.6605 \times 10^{-24}\,\mathrm{g}$ is the atomic mass unit. How many atoms do you have?

a) 6.02×10^{23} or 2 moles. b) 1.6605×10^{-24} or 2 moles. c) 6.02×10^{23} or 3 moles.
d) 1.6605×10^{-24} or 1 mole. e) 6.02×10^{23} or 1 mole.

---

020 qmult 00164 1 4 5 easy deducto-memory: moles keyword

767. "Let's play *Jeopardy*! For \$100, the answer is: 10 g of HF, where hydrogen has atomic mass 1 and and fluorine has atomic mass 19."

a) What is hydrochloric acid, Alex?
b) What is one mole of HF molecules, Alex?
c) What is a small burrowing mammal with shovel-like front paws, Alex?
d) What is half a small burrowing mammal with shovel-like front paws, Alex?
e) What is half a mole of HF molecules, Alex?

---

020 qmult 00230 2 3 5 moderate math: ideal gas and pressure, volume, and temperature
Extra keywords: physci KB-129-6

768. The ideal gas law obeyed by all sufficiently dilute gases is

$$PV = NkT\ ,$$

where P is pressure, V is volume, N is the number of molecules, $k = 1.380649 \times 10^{-23}\,\mathrm{J/K}$ is Boltzmann's constant (exact by definition), and T is temperature on the Kelvin scale. Say we keep some gas at constant volume. The pressure is then _____:

a) 100 Pa. b) infinite. c) proportional to molecule size. d) zero.
e) proportional to temperature.

---

020 qmult 00250 1 3 4 easy math: ideal gas law and gas constant
Extra keywords: physci if moles are introduced

769. The ideal gas law is obeyed by ordinary gases to some degree:

$$PV = nRT\ ,$$

where P is pressure, V is volume, n is the number of moles, $R = 8.31446261815324\,\mathrm{J \cdot K^{-1} \cdot mol^{-1}}$ (exact by definition) is the gas constant, and T is temperature on the Kelvin scale. In the limit of vanishing density all ordinary gases are obey the ideal gas law exactly. Say we keep some gas at constant temperature (isothermal) and constant number of moles (iso-something) and double its volume. The pressure:

a) doubles. b) goes to zero. c) increases. d) halves. e) goes to infinity.

---

020 qmult 00270 1 1 3 easy memory: real gases condense at low enough temperature
Extra keywords: physci KB-131-17

770. At absolute zero, a true ideal gas would be a gas at zero pressure or zero volume or both. This is not true for a real gases as long as their density was high enough since they would _____ to a liquid or directly to a solid when temperature went too low—except arguably in some exotic cases.

a) boil b) evaporate c) condense d) duplicate e) sigh

020 qmult 00450 1 1 1 easy memory: number degrees of freedom of typical gas

771. How many degrees of freedom at room temperature does a typical molecule have that is monatomic, diatomic, and polyatomic, respectively?

a) 3, 5, and 6. b) 3/2, 5/2, and 3. c) 1, 2, and 3. d) 1, 2, and indeterminate. e) 0, 2, and poly.

020 qmult 00500 2 1 2 moderate memory: gas molecule K

Extra keywords: physci KB-129-11

772. At a given temperature, the different molecule types of a mixed gas have the same _____.

a) kinetic energy. b) average kinetic energy. c) mass. d) average speed. e) fate.

Full-Answer Problems

020 qfull 00210 2 3 0 moderate math: ideal gas and work

773. You have 7 moles of ideal gas at 300 K at a pressure of 50,000 Pa.

a) What is the volume?

b) If you compress the volume quasistatically and isothermally by a factor of 3, how much work have you done and how much work has the gas done?

c) How can the gas's internal energy not change?

020 qfull 00220 1 3 0 easy math: pdV work done under different conditions

774. Let's derive the explicit formulae for $p\,dV$ work done by a system on the environment under various conditions.

a) Derive the formula for the work done expanding an **IDEAL GAS** from volume V_1 to volume V_2 under the quasistatic isothermal condition.

b) Derive the formula for the work done expanding a general sample under the condition of constant volume. Briefly explain the result.

c) Derive the formula for the work done expanding a general sample from volume V_1 to volume V_2 under the condition of constant pressure p.

d) Derive the formula for the work done by a gas in free expansion into a vacuum from volume V_1 to volume V_2. Briefly explain the result.

e) Derive the formula for the work done by an **IDEAL GAS** in expansion from volume V_1 to volume V_2 under the quasistatic adiabatic condition.

020 qfull 00320 2 5 0 moderate thinking: adiabatic ideal gas stiffness

775. The formula relating pressure and volume for an ideal gas in an adiabatic state is $PV^\gamma =$Constant.

a) Which gas is "stiffer" (i.e., resists compression more): a monatomic gas or diatomic gas?

b) From a microscopic point of view, why is the less stiff gas less stiff?

020 qfull 00830 1 3 0 easy math: classical gas and photon gas energy ratio

776. The thermodynamic equilibrium translationl kinetic energy density of a classical gas (which is what causes pressure) and the thermodynamic equilibrium energy density of a photon gas

(AKA blacbody radiation) are, respectively,

$$1) \quad \epsilon_{\rm cl} = \frac{3}{2} nkT \qquad 2) \quad \epsilon_{\rm ph} = aT^4 \ ,$$

where n is particle density, $k = 1.380649 \times 10^{-23}\,{\rm J/K}$ is Boltzmann's constant (exact by definition), $a = (7.56573325028\ldots) \times 10^{-16}\,{\rm J \cdot K^{-4} \cdot m^{-3}}$ is the radiation density constant (exact by definition), and T is temperature on the Kelvin scale. If matter and radiation are in thermodynamic equilbrium with respect to each other, they have the same temperatue T.

a) Determine the formula for ratio $R = \epsilon_{\rm cl}/\epsilon_{\rm ph}$ assuming the two gases have different temperatures.

b) The warm-hot intergalactic medium may make up $\sim 50\,\%$ of the baryonic matter (i.e., ordinary matter consisting of protons, neutrons, and electrons) in the observable universe. It is mostly an ionized hydrogen-helium gas with density of order $1\,{\rm particle/m^3}$ and temperature of order $10^6\,{\rm K}$ (Wikipedia: Warm-Shot intergalactic medium). On the other hand, the dominant radiation in space is the cosmic microwave background (CMB, $T = 2.72548(57)\,{\rm K}$ (Fixsen 2009)) arguably the most perfect blackbody spectrum in nature. What is ratio R for these two cosmic components? Does one or the other dominate the pressure-causing energy?

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020 qfull 01650 1 3 0 easy math: adiabatic process and adiabatic index for an ideal gas

777. In this problem, we determine the adiabatic process curve (i.e., the adiabat) and the abiabatic index for an ideal gas.

a) An ideal gas (assuming fixed degrees of freedom and energy equipartition) has internal energy given by

$$E = \frac{d}{2} n_{\rm mol} RT \ ,$$

where d is number of degrees of freedom, (d=3: monatomic gas; d=5: diatomic gas; d=7: diatomic gas with vibration; d=6: polyatomic gas with no vibration), $n_{\rm mol}$ is the number of moles of the gas, $R = 8.31446261815324\,{\rm J \cdot K^{-1} \cdot mol^{-1}}$ (exact) is the gas constant, and T is temperature on the Kelvin scale. Remarkably, the ideal gas internal energy is independent of volume and only depends on $n_{\rm mol}$ and T. Derive $(\partial E/\partial T)_{n_{\rm mol}}/n_{\rm mol}$: the rate of change of internal with temperature for constant $n_{\rm mol}$.

b) The molar heat capacity C_V at constant volume (and implicitly a constant number of moles) is the amount of heat energy needed to raise the temperature of 1 mole of a species by 1 K at constant volume. Given the rather general version of the 1st law of thermodynamics

$$dE = \delta Q - \delta W = \delta Q - p\,dV \ ,$$

derive C_V. What is relationship of C_V and $(\partial E/\partial T)_{n_{\rm mol}}/n_{\rm mol}$? Note, the symbol δ is used in this context to mean inexact differential: i.e., a differential whose integral is path dependent (Wikipedia: Inexact differential).

c) Because, in fact, the ideal gas internal energy E is independent of volume any change in E due to a change in temperature can be written in terms of C_V whether volume is constant or not. Given this fact, determine the formula for dT in terms of C_V and $n_{\rm mol}$ moles in the case where $\delta Q = 0$ (i.e., the adiabatic case) and $p\,dV \neq 0$.

Note, the formula can be understood as a relationship between $dT/d\ell$ and $dp/d\ell$ where ℓ is a path parameter parameterizing an adiabatic path $(p(\ell), V(\ell), T(\ell))$ through pVT space. The formula must be obeyed along any adiabatic path (i.e., along any adiabat). By a simplifying convention of thermodynamics, one usually does not explicitly talk of paths and path parameters. They are taken as understood.

d) Given the ideal gas law:

$$PV = n_{\rm mol}RT \ ,$$

where P is pressure and V is volume, take the differentials of both sides and replace dT by the part (c) result which has the effect of constraining the resulting expression to adiabatic changes since the right-hand side forces the left-hand side to be consistent only with changes in temperature that result from $p\,dV$ work only. Collect all differentials of the same kind and simplify the expression recalling the molar heat capacity at constant volume $C_v = (d/2)R$ and the molar heat capacity at constant pressure for an idea gas is $C_p = C_V + R$.

Note, C_V and C_p turn up in the derivation just as parameters since neither V nor p are constant along an adiabat for an ideal gas.

e) The ideal gas adiabatic index is defined by

$$\gamma = \frac{C_p}{C_V} = \frac{d+2}{d} \ .$$

Substitute γ into the result from part (d). The resulting expression is, in fact, a diffential equation for an adiabatic curve (i.e., an adiabat). Solve the differential equation by integration to get the formula for an adiabat and simplify the formula in such way that left-hand side equals the constant $p_{\rm fid}V_{\rm fid}^{\gamma}$, where $p_{\rm fid}$ and $V_{\rm fid}^{\gamma}$ fiducial or reference values along the adiabat.

Chapt. 21 Thermodynamics III: Entropy and 2nd Law

Multiple-Choice Problems

021 qmult 00100 1 4 5 easy deducto-memory: heat engine definition
Extra keywords: physci

778. "Let's play *Jeopardy*! For $100, the answer is: A device that operates in a cycle. It extracts heat from a hot reservoir and converts some fraction of this heat into macroscopic work and rejects the rest of the heat to a cold reservoir. The hot reservoir is at higher temperature than the cold resevoir as the names suggest."

What is a _____, Alex?

a) working fluid b) piston c) cylinder d) refrigerator e) heat engine

---

021 qmult 00110 1 1 3 easy memory: working fluid

779. The three main components of a heat engine are a hot reservoir, a cold reservoir, and a _____. The _____ is a substance sample (usually a gas at least part of the time during an engine cycle) that absorbs heat from a hot reservior, rejects heat to a cold and performs the macroscopic work of the engine. At the end of an engine cycle, the _____ has returned to its original thermodynamic state.

a) valve b) idling stuff c) working fluid d) engine fluid e) radiator

---

021 qmult 00120 1 4 2 easy deducto-memory: example heat engine 1

780. An example of a heat engine is a/an:

a) fire. b) internal combustion engine. c) electric motor.
d) common household refrigerator. e) bicycle.

---

021 qmult 00150 1 4 4 easy deducto-memory: refrigerator definition
Extra keywords: physci

781. "Let's play *Jeopardy*! For $100, the answer is: A device that operates in a cycle. It extracts heat from a cold reservoir and rejects the rest of the heat hot reservoir. This process requires an input of work. The work energy is rejected to the hot reservoir too. The device needs a working fluid that absorbs and rejects the heat and has work done on it. At the end of a cycle, the working fluid has returned to its orginal state. The device is a heat engine run in reverse. However, there are very few heat engines that are designed to run in reverse."

What is a _____, Alex?

a) working fluid b) piston c) cylinder d) refrigerator e) heat engine

---

021 qmult 00160 1 1 2 easy memory: refrigerator gives off more heat
Extra keywords: physci KB-130-20

782. The heat a refrigerator absorbs from its cold bath (or cold reservoir) is:

a) more than the heat is reject to its hot bath. b) less than the heat it rejects to its hot bath. c) zero. d) infinite. e) the same as it rejects to its hot bath.

---

021 qmult 00200 1 1 3 easy memory: heat energy refrigerator efficiency

783. The performance of heat engines and refrigerators need to be evaluated in many ways since they are used in many ways. However, a generally useful performance measure for a heat engine is the efficiency defined by

$$\varepsilon = \frac{W}{Q_{\rm H}} = \frac{Q_{\rm H} - Q_{\rm C}}{Q_{\rm H}} = 1 - \frac{Q_{\rm C}}{Q_{\rm H}} \ ,$$

where W is the work output, $Q_{\rm H}$ is the heat absorbed from the hot bath, and $Q_{\rm C}$ is the heat rejected to the cold bath. Two generally useful performance measures for a refrigerator are the heating and cooling coefficients of performance:

$$\eta_{\rm heating} = \frac{Q_{\rm H}}{W} = \frac{Q_{\rm H}}{Q_{\rm H} - Q_{\rm C}} = \frac{1}{1 - Q_{\rm C}/Q_{\rm H}} = \frac{1}{\varepsilon} \ ,$$

and

$$\eta_{\rm cooling} = \frac{Q_{\rm C}}{W} = \frac{Q_{\rm H} - W}{W} = \frac{1}{\varepsilon} - 1 = \eta_{\rm heating} - 1 \ ,$$

where W is the work input, $Q_{\rm H}$ is the heat rejected to the hot bath, $Q_{\rm C}$ is the heat absorbed from the cold bath, and ε is just a parameter in these equations not an efficiency since they are refrigerators, not heat engines. Given $Q_{\rm C}/Q_{\rm H} = 1/2$, what are the values of ε, $\eta_{\rm heating}$, and $\eta_{\rm cooling}$?

a) 1/2, 2, 1/2. b) 2, 1/2, 2. c) 1/2, 2, 1. d) 2, 2, 1. e) 1, 1, 0.

---

021 qmult 00220 1 1 3 easy memory: reversible heat engine highest efficiency

784. The argument that a reversible heat engine—if it exists—is the most efficient heat engine and highest performance refrigerator (when run in reverse) is as follows. Imagine two identical reversibles: one runs forward as a heat engine and one runs in reverse as a fridge. For the forward/reverse engine let W be the work output/input, $Q_{\rm H}$ be the heat absorbed from/rejected to the hot bath, and $Q_{\rm C}$ be the heat rejected to/absorbed from the cold bath. We use the work output of the forward engine to drive the reverse engine. From an outside perspective, nothing happens averaged over an engine cycle. No net work is done and no net heat leaves or enters the baths.

Now imagine a more efficient heat engine than the reversible and use it to replace the forward reversible. Scale it to reject $Q_{\rm C}$. Since it is more efficient, it absorbs $Q'_{\rm H} > Q_{\rm H}$ and does work $W' > W$. Now the net effect over a cycle is that net work $W' - W > 0$ is done and net heat $Q'_{\rm H} - Q_{\rm H}$ is removed from the hot bath and there is no net change to the cold bath. Thus, an amount of heat $Q'_{\rm H} - Q_{\rm H}$ is turned entirely into work with no other effect. This process is just never observed in nature and no one has ever been able to construct anything like it. It is a reasonable, but not incontrovertible, conclusion that the more efficient heat engine is impossible and the reversible—if it exists—must be the highest efficiency engine.

Now imagine a higher performance refrigerator than the reversible and use it to replace the reverse reversible. Scale it to use work W. Since it is higher performance, it absorbs $Q'_{\rm C} > Q_{\rm C}$ from the cold bath and rejects heat $Q'_{\rm H} > Q_{\rm H}$ to the hot bath. Now the net effect over a cycle is that net work is zero and net heat $Q'_{\rm H} - Q_{\rm H} = Q'_{\rm C} > Q_{\rm C}$ is removed from the cold bath and transferred to the hot bath. This process is just never observed in nature and no one has ever been able to construct anything like it. Heat does not spontaneously flow from hot to cold. It is a reasonable, but not incontrovertible, conclusion that the higher performance refrigerator is impossible and the reversible—if it exists—must be the highest performance refrigerator.

As a matter of fact, the _____ of thermodynamics (which is an axiom of thermodynamics) dictates the non-existence of the more efficient engine and the higher performance refrigerator, and therefore leads to the conclusion the reversible—if it exists—must be the most efficient heat engine and highest performance refrigerator.

a) zeroth law b) 1st law c) 2nd law d) 3rd law e) 4th law

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021 qmult 00222 1 4 5 easy deducto-memory: Carnot engine exists

785. "Let's play *Jeopardy*! For $100, the answer is: It is the reversible engine: i.e., it can be run as a heat engine or in reverse as a refrigerator. Because it is the reversible engine, it has the highest possible efficiency. The formula for this efficiency is

$$\varepsilon = 1 - \frac{T_{\rm C}}{T_{\rm H}} \ ,$$

where $T_{\rm C}$ is the Kelvin temperature of the engine cold bath and $T_{\rm H}$ is the Kelvin temperature of the engine hot bath. An exactly true reversible engine cannot actually exist, but it can be approached arbitrarily closely. Thus, the behavior of the reversible engine provides a useful limiting case for the study of real engines. Approximate revesible engines can be built, but have extremely low power, and so despite their high efficiency have very limited use. They have some special scientific uses like measuring temperatures $T_{\rm C}$ and $T_{\rm H}$ in certain cases."

What is the _____, Alex?

a) carnivore engine b) new-comer engine c) Newcomen engine
d) Stirling engine e) Carnot engine

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021 qmult 00230 1 1 2 easy memory: adiabatic expansion forever

786. All though it is not possible to convert internal energy into

a) b) c) d) e)

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021 qmult 00300 1 1 2 easy memory: reversible engine efficiency

787. What do the two classical statements of the 2nd law of thermodynamics imply about a reversible engine?

a) It is the least efficient engine possible.
b) It is the most efficient engine possible.
c) It is a fairly efficient engine.
d) It can be used as refrigerator.
e) It can be used as a vacuum cleaner.

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021 qmult 00320 2 4 2 moderate deducto-memory: if Carnot engine wrong

788. If you could build an engine/refrigerator more efficient than an Carnot engine, what are the immediate contrafactual implications.

a) You could then build a machine that would turn **WORK** entirely into heat and that would transfer heat from a **HOT BATH** to a cold bath without work being done.
b) You could then build a machine that would turn **HEAT** entirely into **WORK** and that would transfer heat from a **COLD BATH** to a hot bath without work being done.
c) You could then build a machine that would turn **HEAT** entirely into **QUARKS** and that would give you a **HOT BATH**.
d) Entropy would always increase to a maximum in a closed system.
e) Entropy would always increase to a maximum in an open system.

---

021 qmult 00330 1 4 3 easy deducto-memory: Carnot heat flows

789. What is a key reason why the ideal Carnot engine is a reversible engine?

a) Heat flows occur across non-zero temperature gradients.
b) Heat flows occur across large temperature gradients.
c) Heat flows occur across zero temperature gradients.
d) Heat flows occur from cold to hot baths.
e) There are no heat flows at all.

---

021 qmult 00410 1 1 5 easy memory: Carnot efficiency
Extra keywords: physci

790. The efficiency ε of the ideal Carnot engine (which is the most efficient of all possible heat engines) working between a hot bath of temperature T_h and a cold bath of temperature T_c is:

a) $\varepsilon = T_\mathrm{c}/T_\mathrm{h}$. b) $\varepsilon = T_\mathrm{h}/T_\mathrm{c}$. c) $\varepsilon = 2 - T_\mathrm{c}/T_\mathrm{h}$. d) $\varepsilon = T_\mathrm{c}/T_\mathrm{h} - 1$. e) $\varepsilon = 1 - T_\mathrm{c}/T_\mathrm{h}$.

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021 qmult 00420 1 4 2 easy deducto-memory: Carnot engines in practice
Extra keywords: physci

791. A Carnot engine that achieves the ideal maximum efficiency may be impossible in practice, but Carnot engines that can achieve almost this ideal maximum probably can be built and there Carnot engines are built for certain very special purposes. Why do Carnot engines not have wide use? They

a) have dangerously high power. b) tend to have very low power. c) have zero power. d) have toxic fumes. e) are old-fashioned.

---

021 qmult 00430 1 1 4 easy memory: limit on engine efficiency
Extra keywords: physci KB-130-17

792. The **UPPER LIMIT** on the efficiency of a heat engine:

a) depends on the amount of friction present.
b) is 100 %.
c) is 10 % always.
d) depends on the intake and exhaust temperatures (or hot and cold bath temperatures) of the engine.
e) is zero.

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021 qmult 00440 1 3 2 easy math: maximum efficiency calculation 1
Extra keywords: physci KB-133-33

793. An engine operates between baths of 2000 K and 700 K. Its actual efficiency is 40 %. What is its ideal maximum efficiency?

a) 35 %. b) 65 %. c) 100 %. d) 40 %. e) 0 %.

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021 qmult 00450 2 3 1 moderate math: maximum efficiency calculation 2
Extra keywords: physci KB-133-31

794. An enterprising designer plans an engine which will operate between baths at 200°C and 50°C with an efficiency of 50 %. Will the engine reach its specified efficiency and what is the theoretical upper limit on the efficiency of an engine working between the two baths?

a) No and 32 %. b) No and 25 %. c) Yes and 68 %. d) Maybe and 68 %. e) Maybe and 75 %.

---

021 qmult 00500 1 1 4 easy memory: state function implied by integration

795. If you integrate a change in a quantity around any closed path on a p-V diagram, and the result is always zero, this suggests that the quantity is:

a) heat. b) work. c) a state fair i.e., an equation of state with variable as fair result. d) a state function (i.e., an equation of state with variable as function result). e) a mare's-nest.

---

021 qmult 00510 1 1 1 easy memory: entropy defined
Extra keywords: physci

796. The thermodynamic quantity that is a measure of microscopic disorder is:

a) entropy. b) temperature. c) pressure. d) density. e) volume.

---

021 qmult 00520 1 4 3 easy deducto-memory: entropy law emergent principle

Extra keywords: physci

797. The 2nd law of thermodynamics (which the instructor at least calls the entropy law) says that the entropy of a closed, macroscopic system will increase until it reaches a maximum at which point the system will be in thermodynamic equilibrium and will be unchanging at the macroscopic level. Entropy in thermodynamics is a well-defined measure of microscopic disorder. The essential reason for the entropy law is that random processes will continue to make a closed system more disorderly until a maximum disorder has been reached. The entropy law is called the "arrow of time" because it dictates that certain sequences of events will never happen in reverse in practice: one never in practice sees the entropy of a closed, macroscopic system decrease noticeably even though that is energetically possible and must happen sometimes after immensely long time periods.

Physicists, in fact, believe that the entropy law would apply in worlds with different laws of motion than our own because it is based on the concept of randomizing processes which is more general than any particular law of motion. It is also reasonable to say that a generalized and perhaps not quantifiable version of the entropy law applies in realms other than that of microscopic particles. For example, a household will tend to some maximum of disorder by random dropping, kicking around, and dog processes unless someone (e.g., Mom) picks things up and re-orders them.

The entropy law can be regarded as an example of:

a) intelligent design. b) negligent design. c) an emergent principle. d) a submergent principle. e) an unprinciple.

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021 qmult 00620 1 5 1 easy thinking: entropy and spontaneous heat flow 2

Extra keywords: physci

798. In the macroscopic world (i.e., on scales much larger than atomic), heat energy flows spontaneously (i.e., without any outside cause) from **COLD** bodies to **HOT** bodies:

a) never.
b) whenever the temperature is greater than 300 K.
c) whenever the temperature is greater than 27°C.
d) always.
e) whenever the two bodies are both in the gas state.

---

021 qmult 00710 1 1 1 easy memory: Santa Claus the fate of the universe

Extra keywords: physci

799. Santa Claus musing correctly on the physics of the universe says to himself "If the universe is a closed system and the 2nd law of thermodynamics is universally valid, then the universe as a whole will ultimately approach a state of:

a) timeless, lifeless thermodynamic equilibrium."
b) thermodynamic disequilibrium."
c) varying, life-sustaining thermodynamic equilibrium."
d) elves in charge."
e) peace on Earth and goodwill to humankind."

Full-Answer Problems

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021 qfull 00100 3 5 0 tough thinking: reversible engine/frig I

800. Using words, diagrams, or equations clearly show that a refrigerator more efficient than a reversible engine leads to a contradiction of the classical statements of the second law and all observations too.

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021 qfull 00200 3 5 0 tough thinking: reversible engine/frig II

801. Let's consider an engine and frig operating between a hot and a cold bath. The engine is used to drive the frig. The two baths are in self-thermodynamic equilibrium. Use primes to distinguish frig from engine variables: i.e., for the frig
$Q'_{\rm c}$ (heat absorbed from the cold bath),
$Q'_{\rm h}$ (heat rejected to the hot bath),
and W' (work input);
and for the engine
$Q_{\rm c}$ (heat rejected to the cold bath),
$Q_{\rm h}$ (heat absorbed from the hot bath),
and W (work output).
HINT: This question just has to be stepped through very carefully.

a) Given that the engine and frig work in cycles, what is the change in the internal energy ΔE for the working fluid (or substance) of either over a cycle? Given the answer to the first question and the 1st law of thermodynamics $dE = dQ - dW$, what are W and W' in terms of $Q'_{\rm c}$ and $Q'_{\rm h}$ and of $Q_{\rm c}$ and $Q_{\rm h}$, respectively.

b) What are the expressions for the efficiencies of the engine (ε) and frig (η')? Note we are not asking for ideal Carnot expressions, just the general expressions. **HINT:** Since it is perhaps idiosyncratic to the author of this question, we will just tell you that the frig efficiency is not the standard coefficient of performance (i.e., $Q'_{\rm c}/W'$: e.g., WP-494), but $\eta' = Q'_{\rm c}/Q'_{\rm h}$.

c) For the engine solve for $Q_{\rm c}$ and W as functions of $Q_{\rm h}$ and ε. Then for the frig solve for $Q'_{\rm c}$ and W' as functions of $Q'_{\rm h}$ and η'.

d) Write down the expressions analogous to $Q_{\rm c\,net} = Q_{\rm c} - Q'_{\rm c}$ for $Q_{\rm h\,net}$ and $W_{\rm net}$. These are just the parameters of the net engine-frig system assuming it acts like an engine: i.e., assuming that the net parameters are all positive.

e) If we assume the engine and frig are identical and reversible, what are $Q_{\rm h\,net}$, $Q_{\rm c\,net}$, and $W_{\rm net}$? What is the relation between ε and η' in this special case?

f) Now replace the reversible engine of the reversible engine-frig pair of part (e) by a more efficient engine with $\varepsilon_* > \varepsilon$. Assume the more efficient engine is scaled to reject the same $Q_{\rm c}$ to the cold bath as the reversible engine. Prove that the net system violates the 2nd law of thermodynamics and hence is impossible.

g) Now replace the reversible frig of the reversible engine-frig pair of part (e) by a more efficient frig with $\eta_* > \eta'$. Assume the more efficient frig is scaled to absorb the same $Q'_{\rm c}$ form the cold bath as the reversible frig. Prove that the net system violates the 2nd law of thermodynamics and hence is impossible.

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021 qfull 00300 2 3 0 moderate math: efficiency and waste heat

802. A power plant extracts energy from steam at 250°C and delivers 800 MW (megawatts) of electric power. It rejects waste heat to a river at 30°C. The overall efficiency is 0.28.

a) What is the ideal Carnot efficiency for a heat engine operating between the two heat baths, steam and river? **HINT:** Remember to use Kelvin temperatures.

b) Using the **ACTUAL** efficiency and the general expression $\varepsilon = W/Q_{\text{hot bath}}$, what is the total power extracted from the steam: i.e., delivered power plus waste-heat power?

c) What is the actual waste-heat power discharged to the river?

d) How many houses requiring $10^4\,\text{W}$ each to heat could be heated by the waste heat if it were somehow accessible.

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021 qfull 00420 1 3 0 easy math: carnot engine efficiency calculation, carnot efficiency

Extra keywords: carnot engine efficiency calculation, Carnot efficiency

803. Sadi Carnot (1796–1832) argued that a reversible engine (i.e., a reversible heat engine) must the most efficient engine possible and all reversible engines would have equal efficiency based on his understanding of the 2nd law of thermodynamics (which would not have been called that in his day).

Efficiency ε in the heat engine meaning is defined by

$$\varepsilon = \frac{W}{Q_{\text{H}}} = \frac{Q_{\text{H}} - Q_{\text{C}}}{Q_{\text{H}}} = 1 - \frac{Q_{\text{C}}}{Q_{\text{H}}} \ .$$

where W is work done by the engine in one engine cycle, Q_{H} is heat energy absorbed from a hot bath in one engine cycle, and Q_{C} is heat energy rejected to a cold bath in one engine cycle.

By reversible engine, one means an engine that can be run in reverse as a refrigerator (which just a heat engine run in reverse) and whose cycles are reversible processes. A reversible process is an ideal limit in which a process run in reverse restores both system and environment to their original state. In a reversible process, any heat flows must be at zero temperature gradient, and therefore can be done forward and in reverse. Any heat flow over a temperature gradient is irreversible since the 2nd law forbids spontaneous heat flow from cold to hot. A reversible heat flow is in reality impossible since any heat flow requires some temperature gradient, but is the limit of what can be done by using small temperature gradients. A reversible heat flow must be quasistatic.

Sadi Carnot argued that a heat engine (i.e., a heat engine cycle: i.e., closed path on pressure-volume diagram) consisting of (1) an isotherm (for flow from a hot bath), (2) an adiabat (for positive work done), (3) an isotherm (for flow to a cold bath), and (4) adiabat (for negative work done) would be a reversible engine and calculated its efficiency using an ideal gas as a working fluid.

a) Recall, for an ideal gas (assuming fixed degrees of freedom and energy equipartition) has internal energy given by

$$E = \frac{d}{2} n_{\text{mol}} RT \ ,$$

where d is number of degrees of freedom, (d=3: monatomic gas; d=5: diatomic gas; d=7: diatomic gas with vibration; d=6: polyatomic gas), n_{mol} is the number of moles of the gas, $R = 8.31446261815324\,\text{J}\cdot\text{K}^{-1}\cdot\text{mol}^{-1}$ (exact) is the gas constant, and T is temperature on the Kelvin scale. Remarkably, the ideal gas internal energy is independent of volume and only depends on n_{mol} and T. This means that along an isotherm (which means T is constant) the internal energy cannot change, and thus heat absorbed and work done must cancel as per the 1st law version

$$dE = \delta Q - \delta W \ .$$

Determine the formula for Q_{H} for the 1st isotherm of the Carnot cycle (i.e., point (P_1, V_1) to point point (P_2, V_2)) from the ideal gas law:

$$P = \frac{n_{\text{mol}} RT}{V} \ ,$$

where P is pressure, V is volume, and $T = T_{\rm H}$ for the hot bath temperature

b) Now determine the formula for $Q_{\rm C}$ for the 2nd isotherm (3rd step) of the Carnot cycle (i.e., point (P_3, V_3) to point point (P_4, V_4)) from the ideal gas law with $T = T_{\rm C}$ for the cold bath temperature. **HINT:** Note that $Q_{\rm C} = -\delta Q$, where δQ is the heat absorbed by the ideal gas working fluid.

c) Determine the formula for the Carnot engine efficiency without eliminating the volume factors.

d) Show how the volume factors are eliminated from the Carnot engine efficiency using the adiabatic constraints. The result is the Carnot engine efficiency formula which gives the in-principle upper limit on heat engine efficiency. **HINT:** Note, the adiabatic constaints for the Carnot cycle obey

$$1) \quad P_2 V_2^\gamma = P_3 V_3^\gamma \qquad 2) \quad P_1 V_1^\gamma = P_4 V_4^\gamma \ ,$$

where the adiabatic index

$$\gamma = \frac{d+2}{d} \ .$$

You will need to do a division and use the ideal gas law.

---

021 qfull 00520 3 5 0 tough thinking: entropy always increases

Extra keywords: I hope this question is OK now.

804. You have two systems in thermal contact, but isolated from the rest of the world. Initially system 1 has temperature T_1 and system 2, temperature T_2. Without loss of generality say that $T_1 \leq T_2$.

a) Since the two systems are in thermal contact what can you say about their final temperatures?

b) Assume that the systems have heat capacities $C_1(T)$ and $C_2(T)$ that are appropriate for the conditions under which you let them equilibrate: e.g., constant volume, constant pressure, or some strange other condition. Give the integral expressions for the change in heat each systems experience in the equilibration process: i.e., the expressions for ΔQ_1 and ΔQ_2. What can one say about $\Delta Q_1 + \Delta Q_2$ and why can one say it?

c) Recall the differential expression for a change in entropy is

$$dS = \frac{dQ}{T} \ .$$

Find the integral expressions for the changes in entropy of the two systems in the equilibration process in terms of the heat capacities: i.e., the expressions for ΔS_1 and ΔS_2.

d) Find inequalities relating ΔS_1 and ΔS_2 to $\Delta Q_1 / T_f$. Then find the inequality relating ΔS_1 and $|\Delta S_2|$.

e) What is the sign of the total change in entropy. Is this result consistent with the entropy statement of the 2nd law?

---

021 qfull 00550 1 3 0 easy math: entropy change in an ideal gas

805. A constant volume you have increased the temperature of an ideal diatomic gas by a factor of 10. You have 6 moles. What is the change in entropy?

Chapt. 22 Electric Charge

Multiple-Choice Problems

022 qmult 00100 1 1 1 easy memory: electric charge definition, incomplete
Extra keywords: physci
806. Electric charge is:

a) a fundamental property of matter: it comes in invariant quantized amounts of size $\pm e$.
b) a derived property of matter: it comes in somewhat variable quantized amounts of size $\pm e$ more or less.
c) a fundamental property of matter which comes in a continuum of quantities: there is no smallest bit of charge.
d) a derived property of matter which comes in a continuum of quantities: there is no smallest bit of charge.
e) amber.

022 qmult 00110 1 1 5 easy memory: Ben Franklin named charges
Extra keywords: physci KB-137
807. Who named positive and negative electric charge?

a) George Washington (1732–1799). b) John Adams (1735–1826).
c) Benedict Arnold (1741–1801). d) Aaron Burr (1756–1836).
e) Benjamin Franklin (1706–1790).

022 qmult 00120 1 1 1 easy memory: elementary charge 1
Extra keywords: physci KB-169-1
808. The absolute value of the smallest nonzero physical amount of electric charge (not considering quarks) is:

a) e or $1.602\ldots \times 10^{-19}\,\mathrm{C}$. b) $1\,\mathrm{C}$. c) indefinitely small. d) indefinitely large.
e) indeterminate.

022 qmult 00130 1 4 5 easy deducto-memory: elementary charge 2
Extra keywords: physci KB-169-1
809. The electric charge comes naturally quantized in amounts of:

a) 1 coulomb. b) $e = 2.71828\ldots$. c) $\varepsilon_0 = 8.854\ldots \times 10^{-12}\,\mathrm{F/m}$.
d) $k = 8.987\ldots \times 10^{9}\,\mathrm{N\text{-}m^2/C^2}$. e) $e = 1.602\ldots \times 10^{-19}\,\mathrm{C}$.

022 qmult 00200 1 4 5 easy deducto-memory: ordinary matter neutral
Extra keywords: physci
810. "Let's play *Jeopardy*! For $100, the answer is: It is usually approximately electrically neutral."

What is _____, Alex?

a) an electron b) a proton c) highly charged matter
d) highly negatively charged matter
e) ordinary, terrestrial matter above the atomic scale

022 qmult 00210 1 4 5 easy memory: no microscopic charge neutralization

Extra keywords: physci

811. Why don't positive and negative charge largely neutralize each other at the microscopic level just like they mostly do at the macroscopic level? The short answer is it is forbidden in most circumstances by _____.

a) classical physics b) James Clerk Maxwell c) Michael Faraday
d) Ben Franklin e) quantum mechanics

---

022 qmult 00220 1 1 3 easy memory: positively charged matter
Extra keywords: physci KB-169-9

812. Ordinary matter is positively charged whenever protons:

a) outnumber neutrons. b) outvote electrons. c) outnumber electrons.
d) are fewer than electrons. e) are absent.

---

022 qmult 00310 1 1 2 easy memory: electrons not permanently bound
Extra keywords: physci KB-169-7

813. Electrons in an atom are:

a) permanently bound to the atom. b) not permanently bound to the atom.
c) confined to the nucleus. d) more massive than the nuclues. e) green.

---

022 qmult 00320 1 1 5 easy memory: electron or proton removal
Extra keywords: physci KB-171-3

814. Which of electrons, protons, neutrons are easier to remove from an atom?

a) Protons. b) They are all irremovable. c) They are equally easily removable.
d) Neutrons. e) Electrons.

---

022 qmult 00330 2 3 3 moderate math: charge on 1 g of protons
Extra keywords: physci KB-173-1

815. Given the mass of a proton is 1.67×10^{-24} g and the elementary charge is 1.602×10^{-19} C, what approximately is the total charge of 1 g of protons?

a) 1.67×10^{-24} C. b) 1.602×10^{-19} C. c) 10^{5} C. d) 2.5×10^{-43} C. e) zero.

---

022 qmult 00400 1 1 4 easy memory: conduction classes
Extra keywords: physci

816. There are four main charge conduction categories for materials:

a) insulator, conductor, demiconductor, orchestra conductor.
b) insulator, conductor, semiconductor, infraconductor.
c) insulator, conductor, semiconductor, Superman-conductor.
d) insulator, conductor, semiconductor, superconductor.
e) insulator, conductor, semiconductor, demisemiconductor.

---

022 qmult 00410 1 4 2 easy deducto-memory: conductors defined
Extra keywords: physci

817. "Let's play *Jeopardy*! For $100, the answer is: These materials allow an electric current to flow through them easily."

What are _____, Alex?

a) insulators b) conductors c) vacuum states d) solids e) crystals

---

022 qmult 00420 1 4 1 easy deducto-memory: insulators defined
Extra keywords: physci

818. "Let's play *Jeopardy*! For $100, the answer is: These materials do **NOT** allow an electric current to flow through them easily."

What are _____, Alex?

a) insulators b) conductors c) vacuum states d) solids e) crystals

---

022 qmult 00510 1 1 4 easy memory: electromagnetism
Extra keywords: physci

819. The electricity and magnetism are now understood to be a united set of phenomena which we call:

a) magnoelectrism. b) magmaelectrism. c) electromagi-ism.
d) electromagnetism. e) electromagnanimousism.

---

022 qmult 00520 1 4 1 easy deducto-memory: contact forces

820. What are contact forces?

a) Forces so short range that macroscopically they require bodies to physically **TOUCH** in order to act. All everyday contact forces are the **ELECTROMAGNETIC** force manifested by macroscopic matter.
b) Forces so short range that macroscopically they require bodies to physically **BE REMOTE** in order to act. All everyday contact forces are the **ELECTROMAGNETIC** force manifested by macroscopic matter.
c) Forces so short range that macroscopically they require bodies to physically **TOUCH** in order to act. All everyday contact forces are the **GRAVITATIONAL** force manifested by macroscopic matter.
d) Forces so short range that macroscopically they require bodies to physically **BE REMOTE** in order to act. All everyday contact forces are the **GRAVITATIONAL** force manifested by macroscopic matter.
e) The forces holding contact lenses in place. They're not for the squeamish.

---

022 qmult 00610 1 1 1 easy memory: Coulomb's law vector form

821. The formula

$$\vec{F}_{12} = \frac{kq_1q_2}{r_{12}^2}\hat{r}_{12} \ ,$$

where $k = 8.987\ldots \times 10^9\,\mathrm{N\,m^2/C^2}$, is _____ law.

a) Coulomb's b) Faraday's c) Ampère's d) Biot-Savart's e) Gauss's

---

022 qmult 00620 1 1 1 easy memory: Coulomb's law scalar form
Extra keywords: physci

822. The formula

$$F_{12} = \frac{kq_1q_2}{r_{12}^2} \ ,$$

where $k = 8.987\ldots \times 10^9\,\mathrm{N\,m^2/C^2}$, is _____ law in scalar form.

a) Coulomb's b) Faraday's c) Ampère's d) Biot-Savart's e) Gauss's

---

022 qmult 00622 1 4 5 easy deducto-memory: Coulomb's constant

823. "Let's play *Jeopardy*! For $100, the answer is:

$$k = \frac{1}{4\pi\varepsilon_0} = \frac{\mu_0 \times 10^{-7}}{4\pi} = \frac{c}{10^7} = 8.9875517873681764\times10^9\,\mathrm{N\,m/C^2} \approx 8.99\times10^9\,\mathrm{N\,m/C^2} \approx 9\times10^9\,\mathrm{N\,m/C^2} \approx 10^{10}\,\mathrm{N\,m/C^2}$$

where ε_0 is the vacuum permittivity, μ_0 is the vacuum permeability, and c is the vacuum light speed. The k constant is an exact rational number because of the way the basic units are defined.

What is _____, Alex?

a) Boltzmann's constant b) Planck's constant c) Avogadro's number
d) Fred's constant e) Coulomb's constant

---

022 qmult 00630 1 1 5 easy memory: category of Coulomb's law
Extra keywords: physci KB-169-5

824. Coulomb's law is a:

a) societal law. b) law of thermodynamics. c) law of motion.
d) conservation law. e) force law.

---

022 qmult 00640 1 1 2 easy memory: electric and gravity force inverse-square
Extra keywords: physci KB-140 sort of

825. The electrostatic force and gravity are both _____ force laws.

a) inverse-linear b) inverse-square c) inverse-cube d) linear e) quadratic

---

022 qmult 00650 1 1 2 easy memory: net charge and force
Extra keywords: physci KB-172-9 This is a simplified version.

826. For two **POINT** masses to exert a Coulomb force on each other:

a) both must be uncharged. b) both must be charged. c) one must charged and the other uncharged. d) their charges must both be greater than 1 C. e) their charges must both be less than 1 C.

---

022 qmult 00660 3 5 4 hard thinking: neutral finite body attract-repel
Extra keywords: physci-172-9 This is the hard version

827. Can two finite bodies each electrically neutral overall ever attract or repel each other?

a) No.
b) Always.
c) Always and no.
d) **YES**. For example, consider two small, non-conducting balls attached by a non-conducting bar: give one ball a positive charge (uniformly spread) and the other ball a negative charge (uniformly spread) of the same magnitude. The structure is an electric dipole that is overall neutral. Now consider a second identical dipole. Align the two with unlike ends closest and then with like ends closest. The distance between the balls is a fixed distance a in both cases. The force between the two unlike ends is attractive and between the like ends is repulsive. What of the other forces between the balls? We can make the bars as long as we like. The other forces between the balls get smaller and smaller as we make the bars longer and longer. Eventually the other forces become negligible and the closest ball forces dominate.
e) They can **REPEL**, but never **ATTRACT**. That is the valid conclusion of answer (d).

---

022 qmult 00710 2 3 1 moderate math: Coulomb's law in action
Extra keywords: physci KB-171-29

828. If the distance between charges is changed by a **MULTIPLICATIVE** factor of 1/4, the magnitude of electric force between the charges changes by a **MULTIPLICATIVE** factor of:

a) 16. b) 4. c) 1. d) 1/4. e) 1/16.

---

022 qmult 00720 3 3 3 hard math: Coulomb and gravitational force
Extra keywords: physci KB-171-5

829. Coulomb's law and the point-mass gravitational formula in scalar form are, respectively:

$$F_{\mathrm{C}} = \frac{kq_1q_2}{r^2} \qquad \text{and} \qquad F_{\mathrm{G}} = \frac{Gm_1m_2}{r^2} \ ,$$

where $k = 8.98755179 \times 10^9\,\mathrm{N\,m^2/C^2}$, $G = 6.67430(15) \times 10^{-11}\,\mathrm{N\,m^2/kg^2}$ (Wikipedia: Gravitational constant, 2025jun25), q stands for charge, m for mass, 1 and 2 for particles 1 and 2, and r is the distance between the particles. The mass of electrons is $9.1093826 \times 10^{-31}\,\mathrm{kg}$ and their electric charge is $1.60217653 \times 10^{-19}\,\mathrm{C}$. Approximately, what is the ratio of the magnitude of gravitational force to the magnitude of the electrical force between two electrons (i.e., $F_\mathrm{G}/F_\mathrm{C}$) for any distance r?

a) 1. b) 10. c) 3×10^{-43}. d) 3×10^{43}. e) 10^{-19}.

---

022 qmult 00800 1 1 3 easy memory: polarization

830. Charge separation in a object is called:

a) rasterization. b) pasteurization. c) polarization. d) north polarization. e) miniaturization.

---

022 qmult 00910 1 1 3 easy memory: two-ball induction charging 1

831. Two uncharged conducting balls A and B are mounted on insulating stands. The balls are touching. A positively charged rod is brought from infinity to near B, but not near A. The balls are then separated and the rod is put back at infinity. The charges on A and B are, respectively:

a) positive and positive. b) negative and negative. c) negative and positive. d) positive and negative. e) zero and zero.

---

022 qmult 00912 1 1 3 easy memory: two-ball induction charging 2

832. Two uncharged conducting balls A and B are mounted on insulating stands. The balls are touching. A positively charged conducting rod is brought from infinity to near B, but not near A. The rod touches B. Then the rod is then removed and the balls are separated. The charges on A and B are _____.

a) both positive b) both negative c) negative and positive d) positive and negative e) both zero.

---

022 qmult 10300 2 3 4 moderate math: elementary charge disparity mult

833.** A modern American **DIME** has a mass of 2.268 g and is actually 91.67 % copper and 8.33 % nickel. We assume that it is entirely copper for this question.

i) Given that copper's atomic mass is 63.546 in atomic mass units (i.e., in units of $1.66 \times 10^{-27}\,\mathrm{kg}$), how many copper atoms are there in a dime? **HINT:** If you have 10 kg of eggs and each egg weighs 50 g, then you how many eggs?

ii) Given that a copper atom has 29 protons (i.e., atomic number Z=29), how much gross positive charge in Coulombs does a dime have?

a) (i) 2.37×10^{22} (ii) $1.10 \times 10^5\,\mathrm{C}$. b) (i) 2.15×10^{22} (ii) $1.66 \times 10^5\,\mathrm{C}$. c) (i) 63.546 (ii) $1.66 \times 10^{-27}\,\mathrm{C}$. d) (i) 2.15×10^{22} (ii) $9.99 \times 10^4\,\mathrm{C}$. e) (i) 63.546 (ii) 1.66 C.

Full-Answer Problems

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022 qfull 00100 1 3 0 easy math: elementary charges in a coulomb

834. A typical current is 1 A (A for ampere) or 1 Coulomb per second. Given that the elementary charge is $e = 1.602 \times 10^{-19}\,\mathrm{C}$, how many elementary charges flow past a point in 1 second if the current is 1 A?

---

022 qfull 00110 2 3 0 moderate math: elementary charge disparity

835. A copper penny weighs about 2.5 g (by actual, but not terribly accurate, measurement with a mail weighing scale). It is a pure copper penny, not one of those new-fangled things with zinc cores.

a) Given that copper's atomic mass is 63.5 in atomic mass units (i.e., in units of 1.66×10^{-27} kg), how many copper atoms are there in the penny? **HINT:** If you have 10 kg of eggs and each egg weighs 50 g, then you how many eggs?

b) Given that a copper atom has 29 protons (i.e., atomic number Z=29), how much gross positive charge in Coulombs does the penny have?

c) Imagine (contrafactually as they say in Washington) that the absolute value of the elementary proton charge was 1.000001 times larger than the absolute value of the elementary electron charge (i.e., there was a fractional charge difference of $(\delta e/e) = 10^{-6}$), what would be the force between two pennies a distance 1 meter apart assuming that the pennies would be neutral if there were no fractional charge difference.

d) Is a charge difference between proton and electron of the order suggested in the part (c) question likely? Explain.

---

022 qfull 00200 3 3 0 tough math: a vector calculation with charges

836. Four point charges are set at the corners of a square 2 meters on a side. Going clockwise from the upper left hand corner, the charges are labeled A (1 C), B (2 C), C (−3 C), and D (4 C). What is the net force exerted on charge B? Remember to specify a direction. **HINTS:** Pick a convenient set of axes and compute the force components along those axes. The components are just scalars. Recall Coulomb's law for the force due to particle 1 on particle 2 is:

$$\vec{F}_{12} = k\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12} \ ,$$

where $k = 8.987\ldots \times 10^9\,\mathrm{N\,m^2/C^2}$ and $\hat{r}_{12}$ points from particle 1 to particle 2. Draw a diagram.

---

022 qfull 00210 3 3 0 easy math: 3 charges on a line and equilibrium
Extra keywords: Serway-667-8, suitable for group work

837. Imagine three point charges on a line. Charge $q_1 \neq 0$ is at $x = 0$. Charge $q_2 \neq 0$ is at $x = a > 0$. The third charge is q and is located **BETWEEN** the two other charges.

a) Is there an equilibrium position for charge q if q_1 and q_2 have different signs? Why or why not?

b) Find the equilibrium position solutions x for charge q. Mathematically what happens to these solutions if q_1 and q_2 have different signs? Mathematically what happens to these solutions if $q_1 = q_2$?

c) Which solution from part (b) is physically allowed by the setup conditions. To save tedium just consider the cases where charges q_1 and q_2 are both greater than 0.

d) When is the equilibrium position stable and when is it unstable. **HINT:** A word argument gives the answer easily.

---

022 qfull 00212 3 3 0 tough math: 3 charges on a line and equilibrium 2
Extra keywords: too tough for students? Needs reworking?

838. Image three point charges on a line. Charge $q_1 \neq 0$ is at $x = 0$. Charge $q_2 \neq 0$ is at $x = a > 0$. The third charge is q and can be located anywhere except at $x = 0$ and $x = a$. This question concerns itself with the equilibrium positions for q along the line. We don't bother with the trivial equilibrium positions are $x = \pm\infty$ in the discussion.

a) Qualitatively where is the (Coulomb force) equilibrium position for charge q if q_1 and q_2 have the same sign? Can there be more than one equilibrium? Is the equilibrium stable or not? Explain your answers.

b) Qualitatively where is the equilibrium position for charge q if q_1 and q_2 have different signs and $|q_1| > |q_2|$? Explain your answer.

c) Qualitatively where is the equilibrium position for charge q if q_1 and q_2 have different signs and $|q_1| < |q_2|$? Explain your answer.

d) Qualitatively where is the equilibrium position for charge q if q_1 and q_2 have different signs and $|q_1| = |q_2|$? Explain your answer.

e) The trouble with find the equilibrium positions in general for the three-charges on a line case, is that the direction of the force changes when one passes a charge. If it wasn't for that, the equation would be a simple quadratic. After a lot of thought, I think a good way to formulate the equation to solve as

$$F = k\left[\frac{d_1 q_1}{x^2} - \frac{d_2 q_2}{(a-x)^2}\right] q = 0 \ ,$$

where $d_1 = -1$ for q to the negative of q_1 and positive otherwise and $d_2 = 1$ for q to the negative of q_2 and positive otherwise. This simplifies to

$$\frac{1}{y^2} - \frac{Q}{(1-y)^2} = 0 \ ,$$

where $y = x/a$ and $Q = (d_2 q_2)/(d_1 q_1)$. Find the general solutions for y. What is a necessary condition on Q for a physically real solutions? What happens if $q_1 = q_2$ or $q_2 = 0$ (which actually violates are givens for the problem)?

f) Given that q_1 and q_2 have the same signs, what are the actual physical solutions for y for $|q_1| > |q_2|$ and for $|q_1| < |q_2|$.

f) Given that q_1 and q_2 have different signs, what are the actual physical solutions for y for $|q_1| > |q_2|$ and for $|q_1| < |q_2|$.

---

022 qfull 00910 1 3 0 easy math: induction charging of a ball

839. A person, who is acting as a grounded conductor, touches a metal ball as a positively charged rod is brought near the ball. Then the person is removed and then the rod is removed. Is the ball charged? Explain.

Chapt. 23 The Electric Field

Multiple-Choice Problems

023 qmult 00100 1 4 1 easy deducto-memory: electric field definition
Extra keywords: physci

840. The electric field is a:

a) vector field that emanates from charge (but not only charge) and causes the electric force.
b) scalar field that emanates from charge (but not only charge) and causes the electric force.
c) scalar field that emanates from current (but not only current) and causes the electric force.
d) vector field that emanates from current (but not only current) and causes the magnetic force.
e) scalar field that emanates from current (but not only current) and causes the magnetic **FARCE**.

023 qmult 00150 1 1 3 easy memory: electric field line
Extra keywords: physci

841. A curve through space that is tangent to the electric field vector at each point is a/an:

a) streamline. b) stream curve. c) electric field line. d) magnetic field line. e) straight line always.

023 qmult 00200 1 4 5 easy deducto-memory: E-force on charge

842. The force on a point charge q caused by an electric field $\vec{E}$ is given by:

a) $\vec{F} = m\vec{a}$. b) $\vec{F} = \dfrac{\vec{E}}{q}$. c) $\vec{\tau} = \vec{p} \times \vec{E}$. d) $\vec{F} = \dfrac{\left(\vec{E}\right)^2}{q}$. e) $\vec{F} = q\vec{E}$.

023 qmult 00202 1 1 3 easy memory: electric force and a charge distribution

843. Say that you have charge distribution of point charges with net charge q in an external electric field $\vec{E}$. Since it is external, $\vec{E}$ does not include contributions from the charge distribution. Also $\vec{E}$ is **NOT** necessarily uniform. The electric force on the distribution due to $\vec{E}$ is

$$\vec{F} = \sum_i q_i \vec{E}_i \ ,$$

where q_i is the charge of one member of the distribution and $\vec{E}_i$ is the external electric at the location of charge q_i. The force $\vec{F}$ is _____ on the distribution and in the absence of any other net external force dictates _____ of the distribution.

a) the net electric force; individual charge accelerations
b) not the net electric force; center-of-mass acceleration
c) the net electric force; center-of-mass acceleration
d) not the net electric force; individual charge accelerations
e) half the net electric force; individual charge accelerations

023 qmult 00210 1 1 4 easy memory: electromagnetic field

844. A self-propagating electromagnetic field is:

a) a static electric field. b) a static magnetic field. c) a grass field. d) light. e) sound.

---

023 qmult 00220 1 1 1 easy memory: point charge electric field

845. The electric field about a point charge or any spherically symmetric charge distribution is given by

$$\vec{E} = \frac{kq}{r^2}\hat{r} \ ,$$

where $k = 8.987\ldots \times 10^9\,\mathrm{N\,m^2/C^2}$, r is the radius from the center of system, q is the charge inside of a shell of size r, and $\hat{r}$ is a/an:

a) radially outward pointing unit vector. b) a radially inward pointing unit vector.
c) an azimuthal angle unit vector. d) a polar angle unit vector.
e) an r with tilde on it.

---

023 qmult 00230 1 4 4 easy deducto-memory: E-field for arbitrary charge distribution

846. "Let's play *Jeopardy*! For $100, the answer is: The formula for a discrete set of charges is

$$E(\vec{r}) = \sum_i \frac{kq_i}{|\vec{r} - \vec{r}_i|^3}(\vec{r} - \vec{r}_i) \ ,$$

where the sum is over all charges q_i, $\vec{r}_i$ is the charge location, $\vec{r}$ is the point of evaluation, and $\vec{r}$ is the point of evaluation."

What is the discrete charge electric field formula for _____, Alex?

a) an overall neutral charge distribution b) a point charge
c) a dipole charge distribution. d) an arbitrary charge distribution
e) a huge charge distribution

---

023 qmult 00232 1 1 3 easy memory: changing charge signs

847. Say you had a static distribution of charge that gave rise to an electric field $\vec{E}$ and a test charge q (which not counted in the distribution). Say you magically changed all the charge signs in the distribution, but not q sign. The force on the charge is now _____. Now say you magically changed all the charge signs including changing q to $-q$. The force on the charge is now _____. Note we do **NOT** redefine the values assigned to the symbols $\vec{E}$ and q.

a) $q\vec{E}$; $q\vec{E}$ b) $-q\vec{E}$; $-q\vec{E}$ c) $-q\vec{E}$; $q\vec{E}$ d) $q\vec{E}$; $-q\vec{E}$ e) $-\vec{E}$; $\vec{E}$

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023 qmult 00240 1 1 2 easy memory: far field limit

848. The electric field around a localized distribution of charge (with a non-zero net charge) in the far-field limit looks like the electric field of a/an:

a) dipole. b) point charge. c) infinite plane of charge. d) infinite cylinder of charge. e) infinite line of charge.

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023 qmult 00300 1 1 1 easy memory: electric dipole moment definition

849. The quantity

$$\vec{p} = \int \vec{r}\rho(\vec{r})\,dV$$

for a continuous distribution (and where the integral is over the whole distribution) or

$$\vec{p} = \sum_i \vec{r}_i q_i$$

for a discrete distribution (and where the sum is over the whole distribution) is called:

a) an electric dipole moment. b) an electric monopole moment.
c) a magnetic dipole moment. d) a magnetic monopole moment.
e) an electric field moment.

---

023 qmult 00310 1 1 3 easy memory: dipole definition

850. All finite charge distributions have a dipole moment (which may be zero), but usually only one with an non-zero dipole moment and that is overall neutral (i.e., has zero monopole moment) is called a

a) tripole. b) monopole. c) dipole. d) monocle. e) maypole.

---

023 qmult 00320 1 4 2 easy deducto-memory: dipole moment electric field

851. "Let's play *Jeopardy*! For \$100, the answer is:

$$E(\vec{r}) = k\left[\frac{3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}}{r^3}\right] .$$"

What is the _____ moment electric field in the far-field limit, Alex?

a) monopole b) dipole c) quadrupole d) octupole e) hexacontatetrapole

---

023 qmult 00330 1 1 3 easy memory: dipole far-field

852. The far-field electric field of an electric dipole falls off as:

a) $1/r$. b) $1/r^2$. c) $1/r^3$. d) $1/r^4$. e) $1/r^5$.

---

023 qmult 00340 1 1 1 easy memory: dipole electric field description

853. The pattern of the electric field lines about a dipole viewed in a cross section plane in which the dipole axis lies is sort of:

a) butterfly-like. b) ladybug-like. c) dragonfly-like. d) aphid-like.
e) praying-mantis-like.

---

023 qmult 00350 1 1 2 easy memory: dipoles in nature

854. The permanent or induced dipoles at the microscopic level are:

a) uncommon. b) common. c) never found. d) never found on Earth.
e) never found in space.

---

023 qmult 00360 1 1 3 easy memory: electric dipole torque

855. The torque on an electric dipole of dipole moment $\vec{p}$ in an electric field $\vec{E}$ is:

a) $U = \vec{p}\cdot\vec{E}$. b) $U = -\vec{p}\cdot\vec{E}$. c) $\tau = \vec{p}\times\vec{E}$. d) $\tau = -\vec{p}\times\vec{E}$. e) $\tau = -\vec{p}\cdot\vec{E}$.

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023 qmult 00370 1 1 2 easy memory: electric dipole potential energy

856. An electric dipole (with moment $\vec{p}$) in an electric field $\vec{E}$ has a potential energy of alignment:

a) $U = \vec{p}\cdot\vec{E}$. b) $U = -\vec{p}\cdot\vec{E}$. c) $\tau = \vec{p}\times\vec{E}$. d) $\tau = -\vec{p}\times\vec{E}$. e) $\tau = -\vec{p}\cdot\vec{E}$.

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023 qmult 00380 1 1 1 easy memory: electric dipole simple harmonic motion

857. The equation of angular motion for a rigid dipole (of dipole moment $\vec{p}$ and rotational inertia I) in a uniform electric field $\vec{E}$ is

$$I\alpha = I\frac{d^2\theta}{dt^2} = -pE\sin\theta ,$$

where θ is the angle of the dipole moment from the electric field direction. This innocent-looking differential equation (DE) has no exact analytic solution—that gosh-darn transcendental function of the solution θ is pretty intractable. But one can make the small angle approximation

$$\sin\theta \approx \theta$$

(with θ in radians of course) which is exact to 2nd order in small θ since the Taylor's expansion for $\sin\theta$ is

$$\sin\theta = \sum_{\ell=0}^{\infty} (-1)^{\ell} \frac{\theta^{2\ell+1}}{(2\ell+1)!}$$

(Ar-264). The angle θ in radians is larger than $\sin(\theta)$ at 10° by 0.51 %, 20° by 2.1 %, 30° by 4.7 %, 45° by 11 %, and 60° by 21 %. So the small angle approximation is pretty good even for not-so-small angles. With the small angle approximation, the DE becomes

$$I\frac{d^2\theta}{dt^2} = -pE\theta$$

which is the simple harmonic oscillator DE. The general solution of this linear 2nd order DE (linear because the solution function and its derivatives appear linearly and 2nd order because the highest derivative is 2n order) is

$$\theta = A\cos(\omega t) + B\sin(\omega t) \ ,$$

where ω here is not the angular velocity (not $d\theta/dt$), but the angular frequency and is given by

$$\omega = \sqrt{\frac{pE}{I}} \ .$$

We see that the dipole oscillation is simple harmonic oscillation in the small angle approximation. The period of the solution is

$$P = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{pE}} \ .$$

The period increases as I increases (increasing resistance to angular acceleration) and decreases as pE increases (increasing torque magnitude per angular displacement from equilibrium). The constants A and B are independent of ω and P and set by the initial conditions. Say at time $t = 0$, the dipole is at $\theta = 0$ and has angular velocity $\omega\theta_0$. The solution for this special case is:

a) $\theta_0\sin(\omega t)$. b) $\theta_0\cos(\omega t)$. c) $\omega\theta_0\sin(\omega t)$. d) $(\theta_0/\omega)\sin(\omega t)$. e) ωt.

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023 qmult 10600 3 3 2 tough math: classical hydrogen atom mult

858.*** The hydrogen atom cannot really be understood in classical physics: quantum mechanics is required. Nevertheless, one can get some idea of the size of effects from classical estimates.

i) Given that the separation of the electron and proton in the hydrogen atom is 0.529×10^{-10} m, what is the magnitude of the electric force between these particles: i.e., the electric force each exerts on the other? (The distance 0.529×10^{-10} m is the characteristic separation of proton and electron as determined by quantum mechanics.)

ii) Given that the electrostatic force is the only force between electron and proton (which is not quite true) and that the proton can be regarded as at rest (which is almost true because of its much greater mass), what is the electron's acceleration magnitude?

iii) Assuming the motion of the electron is uniform circular motion, what is the electron speed v?

a) (i) 1.776×10^{3} N (ii) 9.05×10^{22} m/s^2 (iii) 2.19×10^{6} m/s.
b) (i) 8.24×10^{-8} N (ii) 9.05×10^{22} m/s^2 (iii) 2.19×10^{6} m/s.
c) (i) 8.24×10^{-8} N (ii) 9.05×10^{22} m/s^2 (iii) 2.19×10^{-6} m/s.
d) (i) 8.24×10^{-8} N (ii) 9.05×10^{2} m/s^2 (iii) 2.19×10^{-6} m/s.
e) (i) 0.529×10^{-10} N (ii) 8.99×10^{9} m/s^2 (iii) 8.85×10^{-12} m/s.

Full-Answer Problems

023 qfull 00240 2 5 0 moderate thinking: charge rod, integration

859. There is a thin uniformly charged rod of length L. The total charge is q. The rod's thickness is negligible: it can be treated as a line of charge.

a) What is the formula for the electric field a distance x away from one end of the rod and along the axis of the rod? **HINTS:** Find the expression for charge per unit length and then divide and conquer by integration. Draw a diagram.

b) What is the far-field limit (but not infinite-distance limit) approximation to the formula for part (a).

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023 qfull 00250 3 3 0 tough math: infinite line charge

860. What is the electric field (size and direction) around an infinite line of charge with linear charge density λ. **HINTS:** Divide and conquer: what is the electric field at an arbitrary point in space due to an arbitrary differential bit of the line? Also: by symmetry what must be the direction of the electric field? And lastly, a conversion to angular integration saves nine (of what? cat lives?): $x = r \tan\theta$ where x is the coordinate along the line of charge reaching from $-\infty$ to $+\infty$ and r is the perpendicular distance from your arbitrary point to the line at the $x = 0$ location.

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023 qfull 00280 1 3 0 tough math: E-field of a circular disk

861. The goal of this problem is to find the electric field on the symmetry axis of an infinitely thin circular disk of charge. The disk has radius R and uniform surface charge density σ. Let the symmetry axis be the z axis with the origin at the disk center.

a) Draw a diagram of the system.

b) Find the formula for the electric field on the axis at a general z due to a differential ring of charge that is part of the disk. The ring is at general r from the disk center with $r \leq R$. Don't forget the electric field is a vector.

c) Integrate the part (a) result over the whole disk. Write the formula with the vacuum permittivity ε_0 (instead of Coulomb's constant k) and as a function of z/R.

d) What is the value of the formula from part (c) in the limit that $z \to 0$ for finite R or $R \to \infty$ for finite z or $z/R \to 0$.

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023 qfull 00360 1 3 0 hard math: classical hydrogen atom

862. The hydrogen atom cannot really be understood in classical physics: quantum mechanics is required. Nevertheless, one can get some idea of the size of effects from classical estimates.
NOTE: There are parts a,b,c,d.

a) Given that the separation of the electron and proton in the hydrogen atom is $a_0 = 0.52917721067(12) \times 10^{-10}\,\mathrm{m}$, what is the magnitude of the electric force between these particles: i.e., the electric force each exerts on the other? Is the force attractive or repulsive?
The distance a_0 is the Bohr radius and it is the characteristic separation of proton and electron in a hydrogen atom as determined by quantum mechanics.
HINT: You will need the Coulomb constant $k = 8.9875517873681764 \times 10^9\,\mathrm{N\,m^2/C^2}$ and the elementary charge $e = 1.6021766208(98) \times 10^{-19}\,\mathrm{C}$.

b) Given that the electrostatic force is the only force between electron and proton (which is not quite true) and that the proton can be regarded as at rest (which is almost true because of its much greater mass), what is the electron's acceleration magnitude and which way does the acceleration point? **HINT:** The electron mass is $9.10938356(11) \times 10^{-31}$.

c) Given that the answer to part (b), what can you suggest as the classical reason why the electron doesn't crash into the proton? **HINT:** Why doesn't the Moon crash into the Earth?

d) Assuming the motion of the electron is uniform circular motion, what is the electron speed v? What is the ratio v/c, where c is the speed of light? Recall v/c is conventionally designated β in relativity.

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023 qfull 00530 2 3 0 moderate math: simple electric dipole electric field

863. A simple electric dipole consists of two point charges equal magnitude and opposite in sign (i.e., q and $-q$).

a) Draw a diagram of a simple electric dipole and sketch in its electric field lines.

b) Give the general formula of the electric field of the simple dipole in polar coordinates where the radius r is measured from the midpoint between the two charges (which is the conventional origin) and the polar angle θ is measured from the positive side of the axis through the two charges. Give the formula in terms of $\vec{r}$, r (the magnitude of $\vec{r}$), θ, $\vec{a}$ (the vector from the origin to the positive charge q), $-\vec{a}$ (the vector from the origin to the positive charge $-q$), and a (the magnitude of $\vec{a}$ and $-\vec{a}$). **HINT:** You will need to use the law of cosines.

c) Taylor expand the formula of the electric field of the simple dipole to 1st order in a/r in order to obtain the far-field limit formula for the electric dipole field. Write this formula in terms of the dipole moment

$$\vec{p} = 2q\vec{a}$$

for the simple dipole.

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023 qfull 00580 3 5 0 tough thinking: SHO electric dipole

864. The torque on a dipole moment $\vec{p}$ in an electric field $\vec{E}$ is

$$\vec{\tau} = \vec{p} \times \vec{E} \ .$$

If we write this formula in scalar form measuring θ from the electric field direction (which is conventional), then

$$\tau = -pE \sin\theta \ ,$$

where p and E are, respectively, the magnitudes of $\vec{p}$ and $\vec{E}$ and the negative sign shows that the torque always tries to align the dipole moment with the field: if $\theta > 0$ the torque points in the negative θ direction; if $\theta < 0$ the torque points in the positive θ direction. The torque of the electric field is the only torque acting on the dipole.

a) From the rotational 2nd law for rigid-body rotation around a fixed axis, find the differential equation for the motion of the dipole in the electric field. Recall this form of the second law is

$$\tau = I\alpha \ ,$$

where τ is **NET** external torque about the fixed axis, I is rotational inertia, and $\alpha = d^2\theta/dt^2$ is angular acceleration.

b) If you make the small angle approximation, what differential equation for a standard kind of motion do you have?

c) For the solution to the approximate equation in the part (b) answer, what is the expression for the angular frequency and period of the motion?

d) Now let's be concrete and consider a water molecule (H_2O) in an applied uniform electric field of strength of 10^3 N/C (like near a charged comb: HRW-521). The dipole moment

of a water molecule is about 6.2×10^{-30} C-m (HRW-537). Assume the water molecule is a uniform solid sphere of radius 1 Å (i.e., 10^{-10} m) to calculate its rotational inertia: a pretty so-so approximation. What is the frequency f of the motion? Note frequency f, not angular frequency ω.

---

023 qfull 00700 3 3 0 tough math: far-field limit of a charge system

865. Let us consider the electric field in the far-field limit for a localizable system of point charges. In this context localizable means that all the charges can be enclosed in finite closed surface: e.g., a sphere or a box.

a) Draw a diagram indicating the charges and in particular a general charge q_i. Put an origin someplace within the system of charges. Draw vector $\vec{r}_i$ from the origin to q_i and also draw a general vector $\vec{r}$ to a point well outside of the system in the far field. The angle between $\vec{r}_i$ and $\vec{r}$ is θ_i.

b) Give the formula for the electric field at $\vec{r}$ due to the whole system. Give it in a form that makes explicit use of the law of cosines.

c) Expand in a Taylor's series about $r_i/r = 0$, the electric field formula to 2nd order in r_i/r. Truncate the series to 1st order and then zeroth order in r_i/r. **HINT:** Actually, it's slightly simpler to Taylor expand about

$$y = -2(r_i/r)\cos\theta_i + (r_i/r)^2 = 0$$

to second order in y and then collect like powers of r_i/r to second order r_i/r. The result will be the same by the uniqueness of power series theorem (Arf-268).

d) Under what simple condition does the 1st order term (the dipole term) in the Taylor's expansion vanish? By simple condition, we mean that it is easily understood why the 1st order term vanishes when the condition holds. The condition is sufficient for making the dipole term vanish, but not necessary: i.e., the dipole term can vanish even the condition does not hold.

e) Under what necessary condition does the zeroth order term (the monopole term) in the Taylor's expansion vanish?

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023 qfull 01010 1 3 0 easy math: electrical breakdown and sparks in air

866. Consider the following systems.

a) A rubber rod is charged to a very high negative voltage (i.e., it a very high negative potential). Why will a spark jump if the rod is brought near a sufficiently highly positively charged rod?

b) Will a spark occur if the negative rod is brought near a neutral object? Explain.

Chapt. 24 Gauss's Law

Multiple-Choice Problems

024 qmult 00100 1 1 1 easy memory: Gauss law, high symmetry

867. Gauss's law gives a way to easily obtain a formula for the electric field of a charge distribution in cases of:

a) high symmetry: principally, planar, cylindrical and spherical symmetry.
b) low symmetry: e.g., irregular objects, charged atmospheric clouds.
c) indifferent symmetry.
d) strange symmetry.
e) charm symmetry.

024 qmult 00200 1 4 1 easy deducto-memory: equivalent physical laws

868. To say that two physical laws or postulates are equivalent means:

a) that **EACH** implies the other. Thus **EVERYTHING** that can be deduced from one can also be deduced from the other. Coulomb's law and Gauss's law in electrostatics **ARE** an example of equivalent laws.
b) that **ONE** implies the other, but not vice versa. Thus **EVERYTHING** that can be deduced from one can also be deduced from the other. Coulomb's law and Gauss's law in electrostatics **ARE NOT** an example of equivalent laws.
c) that **NEITHER** implies the other. Thus **NOTHING** that can be deduced from one can also be deduced from the other. Coulomb's law and Gauss's law in electrostatics **ARE** an example of equivalent laws.
d) that **EACH** implies the other. Thus **NOTHING** that can be deduced from one can also be deduced from the other. Coulomb's law and Gauss's law in electrostatics **ARE NOT** an example of equivalent laws.
e) that they **REFUTE** each other.

024 qmult 00300 1 1 3 easy memory: electric flux

869. The quantity

$$\int_{\text{surface}} \vec{E} \cdot d\vec{A}$$

is the:

a) electric potential. b) electric field. c) electric flux through a surface. d) magnetic flux through a surface. e) magnetic potential.

024 qmult 00400 1 1 2 easy memory: Gauss law formula

870. The formula

$$\oint_{\text{surface}} \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0}$$

is:

a) the differential equation form of Gauss's law.
b) the integral equation form of Gauss's law.

c) Coulomb's law.
d) Faraday's law of induction.
e) the Einstein equation.

---

024 qmult 00500 1 1 1 easy memory: Gauss law spherical

871. For a spherically symmetric charge distribution, Gauss's law gives the magnitude of the electric field to be

a) $E = \dfrac{q_{\text{encl}}}{4\pi\varepsilon_0 r^2}$. b) $E = \dfrac{\lambda_{\text{encl}}}{2\pi\varepsilon_0 r}$, where λ_{encl} is linear charge density.
c) $E = \dfrac{\sigma_{\text{encl}}}{2\varepsilon_0}$, where σ_{encl} is area charge density. d) $E = \dfrac{q_{\text{encl}}}{4\pi\varepsilon_0 r}$. e) $V = \dfrac{q_{\text{encl}}}{4\pi\varepsilon_0 r}$.

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024 qmult 00510 2 5 1 moderate thinking memory: force on a spherically sym. distribution

872. From a Gauss's law analysis, we know that a spherically symmetric charge distribution of total charge q creates an electric field outside of the distribution which is the same electric field that a point charge q located at the center of symmetry would create. Say we had a symmetrical charge distribution 1 and general charge distribution 2. The net force of distribution 1 on distribution 2 is $\vec{F}_{12}$. The net force of distribution 2 on distribution 1 is $\vec{F}_{21}$. From the discussion above, a point charge of the same net charge as distribution 1 located at the center of distribution 1 would also exert a force $\vec{F}_{12}$ on distribution 2. Is the net force that distribution 2 exerts the point charge the same as would exert on distribution 1?

a) Yes. b) No. c) Sometimes. d) There is not enough information to say.
e) Yes and no.

---

024 qmult 00600 1 1 2 easy memory: Gauss law cylindrical

873. For a cylindrically symmetric charge distribution, Gauss's law gives the magnitude of the electric field to be

a) $E = \dfrac{q_{\text{encl}}}{4\pi\varepsilon_0 r^2}$. b) $E = \dfrac{\lambda_{\text{encl}}}{2\pi\varepsilon_0 r}$, where λ_{encl} is linear charge density.
c) $E = \dfrac{\sigma_{\text{encl}}}{2\varepsilon_0}$, where σ_{encl} is area charge density. d) $E = \dfrac{q_{\text{encl}}}{4\pi\varepsilon_0 r}$. e) $V = \dfrac{q_{\text{encl}}}{4\pi\varepsilon_0 r}$.

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024 qmult 00700 1 1 3 easy memory: Gauss law planar

874. For a planar symmetric charge distribution, Gauss's law gives the magnitude of the electric field to be

a) $E = \dfrac{q_{\text{encl}}}{4\pi\varepsilon_0 r^2}$. b) $E = \dfrac{\lambda_{\text{encl}}}{2\pi\varepsilon_0 r}$, where λ_{encl} is linear charge density.
c) $E = \dfrac{\sigma_{\text{encl}}}{2\varepsilon_0}$, where σ_{encl} is area charge density. d) $E = \dfrac{q_{\text{encl}}}{4\pi\varepsilon_0 r}$. e) $V = \dfrac{q_{\text{encl}}}{4\pi\varepsilon_0 r}$.

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024 qmult 00800 1 1 3 easy memory: Gauss law surface of conductor

875. Gauss's law applied to **JUST** outside of an electrostatic conductor gives the magnitude of electric field to be:

a) $E = \dfrac{q_{\text{encl}}}{4\pi\varepsilon_0 r^2}$. b) $E = \dfrac{\lambda_{\text{encl}}}{2\pi\varepsilon_0 r}$, where λ_{encl} is linear charge density.
c) $E = \dfrac{\sigma_{\text{encl}}}{\varepsilon_0}$, where σ_{encl} is surface area charge density. d) $E = \dfrac{q_{\text{encl}}}{4\pi\varepsilon_0 r}$.
e) $V = \dfrac{q_{\text{encl}}}{4\pi\varepsilon_0 r}$.

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024 qmult 00900 1 1 3 easy memory: net charge on conductor

Extra keywords: Some day check the long answer for rightness.

876. Where is all the (net) charge on a conductor in an electrostatic situation?

a) Clumped in the middle.
b) Spread uniformly throughout.

c) Spread (in general non-uniformly) over the surface.
d) Clumped into two orbiting mass in the inside.
e) All at the bottom where gravity pulls it.

---

024 qmult 00910 1 4 5 easy deducto-memory: Faraday cage

877. "Let's play *Jeopardy*! For $100, the answer is: It is a conducting container used to shield the interior cavity from external electric fields. For perfect shielding the container must completely enclose the cavity and the external field must be static, but in practice a non-complete enclosure will shield against even fairly rapidly varying fields to some degree of adequacy."

What is a/an _____, Alex?

a) Franklin stove b) Whitney cotton gin c) Arkwright spinning frame
d) Carnot engine e) Faraday cage

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024 qmult 00915 1 1 3 easy memory: 3 ball charge exchange

878. Say you had 3 **IDENTICAL** conducting balls. Call them A, B, and C. Initially, A has charge $8q$, B has charge $4q$, and C has charge $-2q$. All other matter and entities are at infinity relative to the balls. Step 1, you bring A and B into (electrical) contact while putting C at infinity and let electrostatic equilibrium be reached between A and B. Step 2, you move A to infinity, bring C into contact with B, and let electrostatic equilibrium be reached between B and C. Step 3, you move B to infinity, bring A again into contact with C, and let electrostatic equilibrium be reached between C and A. What is the final charge on A?

a) 0. b) $-4q$. c) $4q$. d) $10q$. e) $6q$.

---

024 qmult 00920 1 1 5 easy memory: Ben Franklin inverse-square

879. Ben Franklin (1706–1790: the ideal Ben, not the crafty operator of history) on noticing that bits of paper floating in a charged metal tankard were not attracted to the tankard (as they would be if they were outside the tankard due to polarization of the paper molecules [SJ-645] though Ben would not have known about polarized molecules) deduced that

a) the tankard was half empty.
b) the tankard was half full.
c) Sam Adams's (1722-1803) ale lacked body.
d) the electric **POTENTIAL** in the tankard was apparently constant, and thus the electrostatic force obeyed an **INVERSE-CUBE** law.
e) the electric **FORCE** on the bits in the tankard was apparently zero, and thus the electrostatic force obeyed an **INVERSE-SQUARE** law.

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024 qmult 10610 2 5 2 mod. thinking: Guass's law sphere with hollow fullmult

880.** You have a sphere of radius R with a uniform charge density ρ. The electric field at any radius $r \leq R$ is _____. Now say that we magically remove a spherical hollow of radius a from entirely inside the sphere: the hollow is entirely enclosed by the sphere. The hollow contains no charge. The electric field at the hollow center at radius r of the sphere is _____.

HINT: Use Gauss's law for the first part and then model the sphere and hollow with a sphere and superimposed charge distribution for the second.

a) $\vec{E} = \dfrac{4\pi R^3}{3}\dfrac{k\rho}{r^2}\hat{r}$; $\vec{E} = \dfrac{4\pi R^3}{3}\dfrac{k\rho}{r^2}\hat{r}$ b) $\vec{E} = \dfrac{4\pi}{3}k\rho r\hat{r}$; $\vec{E} = \dfrac{4\pi}{3}k\rho r\hat{r}$
c) $\vec{E} = \dfrac{4\pi}{3}k\rho r\hat{r}$; 0 d) $\vec{E} = \dfrac{4\pi R^3}{3}\dfrac{k\rho}{r^2}\hat{r}$; 0
e) $\vec{E} = \dfrac{4\pi R^3}{3}\dfrac{k\rho}{r^2}\hat{r}$; $\vec{E} = \dfrac{4\pi}{3}k\rho r\hat{r}$

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024 qmult 10620 3 5 2 tough thinking: Gauss's law spherical shell with hole fullmult

881.*** You have an infinitely thin spherical shell with surface charge density σ (i.e., charge per unit area σ). It is **NON-CONDUCTING**, and so the charge cannot move. The electric field

just outside of the shell is _____ and everywhere inside is _____. Now say you remove a differential bit of surface area dA from the shell to create a hole. Now the electric field just at the hole is _____.

HINT: Use Gauss's law for the first part and then use Gauss's law for the differential area dA exploiting the planar symmetry at the dA location for a differentially small box enclosing dA.

a) $\frac{\sigma}{\varepsilon_0}\hat{r}$; $\frac{\sigma}{\varepsilon_0}\hat{r}$; $\frac{\sigma}{2\varepsilon_0}\hat{r}$ b) $\frac{\sigma}{\varepsilon_0}\hat{r}$; 0 ; $\frac{\sigma}{2\varepsilon_0}\hat{r}$ c) $\frac{\sigma}{2\varepsilon_0}\hat{r}$; 0 ; $\frac{\sigma}{2\varepsilon_0}\hat{r}$
d) $\frac{\sigma}{2\varepsilon_0}\hat{r}$; 0 ; $\frac{\sigma}{\varepsilon_0}\hat{r}$ e) $\frac{\sigma}{2\varepsilon_0}\hat{r}$; $\frac{\sigma}{\varepsilon_0}\hat{r}$; $\frac{\sigma}{2\varepsilon_0}\hat{r}$

Full-Answer Problems

024 qfull 00100 2 5 0 moderate thinking: Gauss law and a cube

882. Consider a charged conducting cube. The cube has net charge $q \neq 0$. The cube is isolated from any electric fields not arising from itself. The system is electrostatic.

a) Can Gauss's law be used to derive a simple exact expression for the electric field around the cube? Why or why not?

b) What about in the far-field limit? If so, why and what is the expression?

c) Derive the expression for the electric field at the surface in terms the surface charge density σ_{surface} (which is not constant).

024 qfull 00200 1 3 0 easy math: charged spacecraft

Extra keywords: but there is a tough thinking discussion bit

883. Spacecraft traveling in the radiation belts can accumulate a net charge by intercepting electrons. Say you have a spherically symmetric, conducting satellite of radius 1.5 m that acquires a net charge of 2.3 μC in one orbit.

a) What is the surface charge density?

b) Calculate the magnitude of surface electric field in two ways? How does it compare to the breakdown field of air? Of course, there isn't much air in orbit.

c) Electronic components on the spacecraft can be damaged by the electric fields caused by this charge build-up. How can they be protected? **HINTS:** This is a discussion question. The instructor is almost completely ignorant of real electrical and electronic devices.

024 qfull 00300 1 3 0 easy math: artificial electric field of Earth

884. If the solid Earth (which is a fair conductor) had an average excess of charge of 1.0 μC per square meter (which is not the case actually), what would be its average electric field just at the surface? How does this compare to the breakdown electric field for air (3×10^6 N/C: HRW-521)?

024 qfull 00400 2 3 0 moderate math: electron crossing between plates

885. There are two large parallel plates (plate 1 and plate 2) 0.050 m apart with area charge densities $\sigma_1 > 0$ and $\sigma_2 < 0$: note $\sigma_2 \neq -\sigma_1$ necessarily. There is vacuum in the gap between the plates. We treat the system as having exact planar as an approximation: this means among other things that the plates are treated as infinite. Just so we all agree on the coordinate conventions: plate 1's inner edge is at $x = 0$, plate 2's inner edge is at $x = d = 0.05$ m, and the unit vector in the positive x direction is $\hat{x}$. The inner edges are the edges facing the gap. **HINT:** Draw a diagram.

NOTE: The charge densities have planar symmetry, but we don't know or need to know how the charge is distributed with depth through the plates. But we do know that each differential

planar slab with charge density $d\sigma$ in a plate gives rise to a differential electric field

$$d\vec{E} = \frac{d\sigma}{2\varepsilon_0}\hat{n} \ ,$$

where $\hat{n}$ points away from the slab on either side of the slab, ε_0 is the vacuum permittivity, $d\vec{E}$ is constant outside of the differential slab, except that it direction changes from one side to the other of the slab as determined by $\hat{n}$. This result follows from Gauss's law. We can integrate up all these slabs immediately to get

$$\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{n} \ ,$$

where $\vec{E}$ is the electric field outside of the plate and $\hat{n}$ points away from the plate on either side of the plate.

a) In the exact planar symmetry approximation, what is the electric field between the plates? Give both magnitude and direction. Only a symbolic answer is possible.

b) At time zero, a proton (charge e) is released from the positive plate and an electron (charge $-e$) from the negative plate. Both particles are at rest at the time of release. They don't interact with each other and they only experience forces due to the plate electric field. At what time are they at the same x coordinate? Solve using symbols only.

c) What is their distance from plate 1 in actual numbers at this time of passing? **HINT:** The ratio of proton to electron mass is 1836.15.

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024 qfull 00500 2 5 0 moderate thinking: parallel plates

886. You have two infinite conducting plates in parallel. The system has exact planar symmetry and is electrostatic. On the left plate you have an area charge density $\sigma > 0$ and on the right plate you have area charge density $-\sigma < 0$. The system is in fact an ideal parallel-plate capacitor. Some of interesting properties of this capacitor are revealed in the answers to the questions.

a) Where on the two plates must all the charge be?

b) What is the electric field in the five regions: left of the plates, inside the positive plate, between the plates, inside the negative plate, right of the plates. **HINT:** Recall that a single infinite planar slab of charge with area charge density σ gives a field

$$\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{n} \ .$$

where $\hat{n}$ points outward from either side of the slab.

c) What is the charge density on each of the four conductor surfaces?

d) Does the answer to part (c) make sense? Why does it make sense?

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024 qfull 001010 1 3 0 easy math: electrical shielding explained

887. Explain electrical shielding. Uses diagrams as needed.

Chapt. 25 Electric Potential

Multiple-Choice Problems

025 qmult 00100 1 1 3 easy memory: electric potential defined 1
888. The electric potential energy per unit point charge is the electric:

a) power. b) fulfillment. c) potential. d) shock. e) college.

---

025 qmult 00110 1 4 5 easy deducto-memory: electric potential defined 2
889. What is electric potential? It is the electric potential energy:

a) **PER** unit point charge. It is a **VECTOR** field like the electric field.
b) **OF** a charge. It is a **VECTOR** field like the electric field.
c) **PER** unit point charge. It is a **TENSOR** field unlike electric charge.
d) **OF** of a charge. It is a **TENSOR** field unlike electric charge.
e) **PER** per unit point charge. It is a **SCALAR** field unlike the electric field.

---

025 qmult 00120 1 1 4 easy memory: electric force conservative force
890. The electric potential can be defined since electrostatic charge gives rise to a:

a) non-field force. b) field force. c) non-conservative force.
d) conservative force. e) liberal force.

---

025 qmult 00130 1 1 2 easy memory: path-independence of potential
891. The change in electric potential between any two points is _____ the path between them.

a) dependent on b) independent of c) 0 V. d) 1 V. e) 10 V.

---

025 qmult 00140 1 4 1 easy deducto-memory: closed paths and potential
892. If you go around any closed path in an electric field (caused by an static distribution of charge), your change in electric potential is:

a) 0 V. b) ± 100 V. c) always greater than zero. d) always less than zero.
e) shocking.

---

025 qmult 00150 1 1 2 easy memory: electric potential scalar addition
893. Since electric potential is a scalar it adds:

a) like a vector. b) like a scalar. c) like a tensor. d) to zero always.
e) not at all.

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025 qmult 00160 1 1 1 easy memory: potential unit volt
Extra keywords: physci
894. The units of electric potential are:

a) volts (V=J/C). b) joules. c) newtons per coulomb (N/C). d) webers (Wb).
e) oersteds (Oe).

---

025 qmult 00200 1 4 2 easy deducto-memory: electronvolt defined

895. "Let's play *Jeopardy*! For $100, the answer is: It is the energy an elementary charge acquires if accelerated through a 1 V potential."

What is a/an _____, Alex?

a) joule b) electronvolt or eV c) electron-joule or eJ
d) kilo-electronvolt or KeV e) mega-electronvolt or MeV

---

025 qmult 00220 1 4 1 easy deducto-memory: electronvolt calculation

896. If an electron is accelerated through a potential difference of 100 V, what kinetic energy does it acquire in eVs (electronvolts)?

a) 100 eV. b) 0.01 eV. c) 1.6×10^{-17} eV. d) 1.6×10^{-17} J. e) 1.6×10^{17} eV.

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025 qmult 00230 1 3 4 easy math: fiducial electronvolt speed

897. Say an electron is accelerated from rest through a 1 V potential. What is it's final speed?

a) 1×10^5 m/s. b) 3×10^5 m/s. c) 3×10^8 m/s. d) 6×10^5 m/s. e) 6 m/s.

---

025 qmult 00300 1 1 4 easy memory: differential potential change

898. The differential change dV in electric potential along a differential bit of path $d\vec{s}$ in the electric field $\vec{E}$ of a point charge (located at the origin with spherical components (E_r, E_θ, E_ϕ)) is given by:

a) $dV = -\vec{E} \cdot d\vec{s} = -E_\theta \, d\theta$. b) $dV = \vec{E} \cdot d\vec{s} = E_\theta \, d\theta$.
c) $dV = \vec{E} \cdot d\vec{s} = E_r \hat{r} \cdot d\vec{s} = E_r \, dr$. d) $dV = -\vec{E} \cdot d\vec{s} = -E_r \hat{r} \cdot d\vec{s} = -E_r \, dr$.
e) $dV = d\vec{E}$.

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025 qmult 00310 1 1 3 easy memory: point charge potential

899. The potential of a point charge (relative to infinity) is:

a) $V = \dfrac{kq}{r^3}$. b) $V = \dfrac{kq}{r^2}$. c) $V = \dfrac{kq}{r}$. d) $V = kqr$. e) $V = kqr^2$.

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025 qmult 00400 1 1 5 easy thinking: potential relative to infinity

900. For a localized charge distribution (i.e., one that can be contained in finite closed surface), the potential change from infinity (i.e., infinitely far from the charge distribution) to some region a finite distance from the distribution is a finite number. When one specifies the potential at some point for such a charge distribution without qualification, one means that it is potential difference of that point from a point at infinity which assigned to be:

a) infinite potential. b) negative potential. c) positive potential.
d) potential 1 V. e) zero potential.

---

025 qmult 00410 1 2 3 moderate memory: discrete point charge

901. The general formula to calculate the potential of a localized set of point charges (with infinity set to zero potential as is conventional) is:

a) $V(\vec{r}) = \displaystyle\sum_i \frac{kq_i}{|\vec{r} - \vec{r}_i|^3}$. b) $V(\vec{r}) = \displaystyle\sum_i \frac{kq_i}{|\vec{r} - \vec{r}_i|^2}$. c) $V(\vec{r}) = \displaystyle\sum_i \frac{kq_i}{|\vec{r} - \vec{r}_i|}$.
d) $V(\vec{r}) = \sum_i kq_i |\vec{r} - \vec{r}_i|$. e) $V(\vec{r}) = \sum_i kq_i |\vec{r} - \vec{r}_i|^2$.

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025 qmult 00420 1 2 3 moderate memory: continuous charge

902. The general integral to calculate the potential due to a localized continuous distribution of charge ρ (with infinity set to zero potential as is conventional) is:

a) $V(\vec{r}) = \displaystyle\int \frac{k\rho(r')}{|\vec{r}' - \vec{r}|^3} \, d\vec{r}'$. b) $V(\vec{r}) = \displaystyle\int \frac{k\rho(r')}{|\vec{r}' - \vec{r}|^2} \, d\vec{r}'$. c) $V(\vec{r}) = \displaystyle\int \frac{k\rho(r')}{|\vec{r}' - \vec{r}|} \, d\vec{r}'$.
d) $V(\vec{r}) = \displaystyle\int k\rho(r') \, d\vec{r}'$. e) $V(r) = \int k\rho(r') |\vec{r}' - \vec{r}| \, d\vec{r}'$.

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025 qmult 00430 1 2 1 moderate memory: potential enery for continuous charge

903. The general formulae for the potential energy for a localized set of point charges and a localized continuous distribution of charge are, respectively,

$$U = \frac{1}{2}\sum_i q_i V_i \qquad \text{and} \qquad U = \frac{1}{2}\int \rho(\vec{r})V(\vec{r})\,d\vec{r}\,.$$

The 1/2 factor prevents:

a) double counting. b) undercounting. c) random counting. d) triple counting. e) rash counting.

---

025 qmult 00440 1 2 1 moderate memory: potential energy for continuous charge

904. Say you had localized continuous charge distribution with total charge Q and characteristic length ℓ. The potential energy of the distribution is approximately:

a) $\frac{kQ^2}{\ell}$. b) $\frac{kQ}{\ell}$. c) $\frac{kQ^2}{\ell^2}$. d) $\frac{kQ}{\ell^2}$. e) $\frac{k^2Q^2}{\ell}$.

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025 qmult 00500 1 1 4 easy memory: dipole potential

905. The far-field electric potential of an electric dipole is

$$V = \frac{k\vec{p}\cdot\hat{r}}{r^2}\,,$$

where $\vec{p}$ is the _____, $\hat{r}$ is a unit vector pointing the direction to the position where the potential is being evaluated, and r is the distance from the origin (which is best located near the dipole center as somehow defined) to the point of evaluation. Consider a simple two-point charge dipole where the positive charge is q and the separation on one of the charges is d. The dipole moment magnitude is _____.

a) dipole moment; d/q b) dipole second; d/q c) dipole second; dq
d) dipole moment; dq e) momentum; d/q.

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025 qmult 00610 1 1 1 easy memory: gradient of potential

906. The electric field is given by

$$\vec{E} = -\nabla V\,,$$

where in Cartesian coordinates

$$\nabla V = \left(\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z}\right)$$

is the _____ of the potential.

a) gradient b) Laplacian c) divergence d) D'Alembertian e) Einsteinium

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025 qmult 00700 1 1 5 easy memory: conductor equipotential

907. A conductor in an electrostatic case (and in some practical non-electrostatic cases at least approximately) is a/an _____ object.

a) non-equipotential b) pseudo-equipotential c) faux-equipotential
d) equiporpoise e) equipotential

---

025 qmult 00720 1 1 3 easy memory: set of charged conductors

908. Say that you had a localized set of charge conductors with charges $\{q_i\}$ and potentials $\{V_i\}$—and remember each conductor has only one potential since it is an equipotential object. It follows from

$$U = \frac{1}{2}\int \rho(\vec{r})V(\vec{r})\,d\vec{r}$$

that the potential energy of the set is given by

a) $\frac{1}{2}\sum_i V_i$. b) $\sum_i q_i V_i$. c) $\frac{1}{2}\sum_i q_i V_i$. d) $\frac{1}{2}\sum_i q_i$. e) $\sum_i q_i$.

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025 qmult 00740 1 1 1 easy memory: conductor sharp points

909. At the sharp edges and points of a charged conductor, the surface charge density and electric field tend to be:

a) relatively high. b) relatively low. c) mediocre. d) less than you might think. e) relatively relative.

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025 qmult 10150 1 1 3 tough math: work-energy theorem, el. potential fullmult

910.*** A particle of charge q_1 is held at the origin. A second particle of charge q_2 and mass m_2 is held a distance r_{initial} away and then released from rest. Both charges are positive. The only force around is the Coulomb force between the particles. What is the expression for particle 2's speed when it reaches $r = \infty$?

a) $v(r=\infty) = \sqrt{\frac{2kq_1q_2m_2}{r_{\text{initial}}}}$. b) $v(r=\infty) = \sqrt{2kq_1q_2m_2r_{\text{initial}}}$.

c) $v(r=\infty) = \sqrt{\frac{2kq_1q_2}{m_2r_{\text{initial}}}}$. d) $v(r=\infty) = \sqrt{\frac{2k}{q_1q_2m_2r_{\text{initial}}}}$.

e) $v(r=\infty) = \frac{1}{\sqrt{2kq_1q_2m_2r_{\text{initial}}}}$.

Full-Answer Problems

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025 qfull 00150 2 5 0 moderate thinking: work-energy theorem, el. potential

911. A particle of charge q_1 is held at the origin. A second particle of charge q_2 and mass m_2 is held a distance r_{initial} away and then released from rest. Both charges are positive. The only force around is the Coulomb force between the particles.

a) What is the general expression for the speed of particle 2 as a function of radius r from charge q_1? **HINT:** Recall the work-energy theorem.

b) What is the expression for particle 2's speed when it reaches $r = \infty$?

c) Assume the charges the both have $2\,\mu$C of charge, $r_{\text{initial}} = 3\,$m, and the mass of the particle 2 is $0.02\,$g. What is particle 2's speed at infinity?

---

025 qfull 00170 2 5 0 moderate thinking: ring of charge

912. You have ring of vanishingly small thickness. The radius of the ring is b. The total charge on the ring is $q_{\text{ring}} > 0$. This charge is uniformly spread around the ring.

a) Draw a diagram of the ring and an x-axis through the center of the ring. The x-axis is perpendicular to the plane of the ring and extends to infinity in both directions. The ring center is at $x = 0$.

b) Consider a bit of arc length of the ring Δs. The charge on Δs is $q_{\text{ring}}\Delta s/(2\pi b)$. What is the formula for potential due to the arc length on the x-axis at the general point x in terms of q_{ring}, Δs, b, and x? Assume that the bit of arc can be approximated as a point charge.

c) Why is the formula obtained in part (b) exact even though the bit of arc length is not a point charge? **HINT:** An argument, not a rigorous proof is expected.

d) What is the formula for the potential at general x due to the whole ring in terms of q_{ring}, b, and x?

e) If a point charge of charge $q > 0$ is put on the x-axis exactly at $x = 0$, what is the net force on it? What would happen to the point charge if it were slightly displaced along the x-axis from $x = 0$? Assume that the point charge is constrained to move along the x-axis: for instance it may be thought of as a bead on a frictionless wire. Other than the force of constraint assume that the only the Coulomb force due to the ring acts on the bead.

f) Assume the point charge from part (e) is displaced by a vanishingly small amount from $x = 0$ along the x-axis and initially has zero velocity. The point charge has mass m. Derive the point charge's **VELOCITY** at $x = \pm\infty$. Express the velocity in terms of $q_{\rm ring}$, q, b, and m?

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025 qfull 00250 2 3 0 moderate math: 2 charges on a line

Extra keywords: This needs fixing up in light of the similar question in ch.23.

913. Consider two charges q_1 and q_2 on the x-axis at, respectively, points x_1 and x_2: $x_2 > x_1$. The zero of potential is at infinity.

a) Write out the expression for the total potential on the x-axis.

b) Sketch the potential as a function of x for the case of $q_1 > 0$ and $q_2 > 0$ and the case of $q_1 > 0$ and $q_2 < 0$ with $q_1 + q_2 > 0$. Mark the places where the electric field is zero.

c) If q_1 and q_2 have the same sign, then the potential is only zero at $\pm\infty$. Show that if q_1 and q_2 have the different signs that the location of the zeros of the potential satisfy

$$0 = \frac{|q_1|}{|x - x_1|} - \frac{|q_2|}{|x - x_2|} \ .$$

d) Given that q_1 and q_2 have the different signs, there is a zero of the potential $x \in (x_1, x_2)$ (i.e., x is in between x_1 and x_2). Find the expression for x.

e) Given that q_1 and q_2 have the different signs, there is a zero of the potential $x \notin (x_1, x_2)$ (i.e., x is in not between x_1 and x_2) that is not at $\pm\infty$. Find the expression for x.

f) Given $x_1 = 1.0\,\text{m}$, $x_2 = 2.0\,\text{m}$, $q_1 = 3.0\,\mu\text{C}$, and $q_2 = -1.0\,\mu\text{C}$, find the locations of zero potential.

g) Given $x_1 = 1.0\,\text{m}$, $x_2 = 2.0\,\text{m}$, $q_1 = 3.0\,\mu\text{C}$, and $q_2 = -1.0\,\mu\text{C}$, calculate the potential energy of the assembly of charges.

---

025 qfull 00290 2 3 0 moderate math: potentials for high symmetry

914. Derive the potential formulae required in the following parts. **HINT:** Draw diagrams.

a) The potential for a point charge at $\vec{r}$ relative to infinity which we take as the zero point for potential energy. Recall the electric field for a point charge is

$$\vec{E} = \frac{kq}{r^2}\hat{r} \ ,$$

where the charge is at the origin and spherical polar coordinates are used. What happens if $\vec{r}$ goes to zero. How is the resulting problem avoided in practice.

b) The potential difference for an infinite line of charge between the general point $\vec{r}$ and $\vec{r}_0$ which is chosen as the zero point for the potential. Note small $\vec{r}$'s denote displacement from an arbitrary origin somewhere on the line of charge. Let the capital R's denote the cylindrical radial component of the vectors $\vec{r}$. Recall the electric field for an infinite line of charge is

$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 R}\hat{R} \ ,$$

where λ is the linear charge density. What happens if either of R goes to infinity or zero? How is the resulting problem avoided in practice?

c) The potential difference for an infinite plane of charge between points $\vec{r}$ and $\vec{r}_0$ which is a vector in the plane is chosen as the zero point for the potential. The z direction is perpendicular to the plane. Recall the electric field for an infinite plane of charge is

$$\vec{E} = \pm\frac{\sigma}{2\varepsilon_0}\hat{z} \ ,$$

where σ is the area charge density and upper case for the positive z side of space and lower case is for the negative z side of space. What happens if z (the z component of $\vec{r}$) goes to infinity? How is the resulting problem avoided in practice?

---

025 qfull 00300 2 3 0 moderate math: U of uniform charged ball

915. You have uniformly charged sphere of charge Q and radius R. We're going to find the potential **ENERGY** of this charge assembly (relative to zero when all the charge bits are at infinity relative to each other).

a) First, what is the differential bit of potential energy dU of a differential bit of charge dq in a potential V?

b) Now what is the differential bit of potential energy dU of a differential bit of charge dq at the surface of uniformly charged sphere of charge q and radius r?

c) You can think of bringing up bits of charge dq from infinity and adding them to the sphere of charge Q that you are assembling. Thus you can integrate the expression you got for the answer to part (b) from $q = 0$ to $q = Q$ if you can express the integrand in terms of a single variable: i.e., q or r. So express q in terms of r or r in terms of q. In the first case you will also need the expression for dq in terms of dr.

d) Substitute the expressions from the part (c) answer that you have chosen to work with into the part (b) answer and integrate from 0 to R if you are using r as the integration variable or from 0 to Q if you are using q as the integration variable. For neophytes to integration,

$$\int_a^b f(x)\,dx = F(b) - F(a) \ .$$

where F is the antiderivative of f: i.e., $dF/dx = f$.

e) *Mutatis mutandis* what is the gravitational potential energy of a uniform density sphere of mass M and radius R? Why is this formula the right one.

---

025 qfull 00400 2 3 0 moderate math: potential of disk of charge

916. You have a thin disk of charge q with the charge spread uniformly over the disk.

a) What is the area charge density σ of the disk given that it has radius R?

b) What is the differential charge on a ring in the disk of radius ρ and width $d\rho$, where ρ is the radius coordinate of the disk.

c) What the potential dV at a height z above the disk along the axis of the disk due to a ring of charge of width $d\rho$. Remember potential is a scalar and adds like a scalar therefore. Assume the potential is zero at infinity as is conventional for a localizable charge distribution.

d) Find the total potential at height z due to the whole ring. Eliminate σ in favor of q from the expression.

e) Using Taylor's series to 1st order, find the far-field expression for the potential of the disk on the axis.

f) Using Taylor's series, find the first order in small z/R form of the solution for the potential: i.e., the approximate form the solution for being very close to the disk. Eliminate q in favor of σ and k in favor of ε_0. Recall

$$k = \frac{1}{4\pi\varepsilon_0} \ .$$

g) What is the z-component of the electric field E_z for the small z/R case?

---

025 qfull 00500 3 5 0 tough thinking: potential of disk of charge 2

917. You have a disk of radius R. The surface charge density is given by $\sigma(r) = \sigma_0/r$ where r is the radial coordinate of the disk.

a) What is the total charge q on the disk?

b) What is the electric potential at distance z above the disk on its axis. **HINT:** You will probably need a table of integrals.

c) What is the potential at z if $R \to \infty$?

d) What is far-field limiting form for the potential at z (i.e., when $z >> R$)?

---

001 qfull 00600 1 3 0 easy math: electrostatic conductor potential energy

Extra keywords: Needs a bit more work.

918. Electrostatic systems of conductors have some interesting features.

a) The potential energy for an electrostatic distribution of point charges is

$$U = \frac{1}{2}\sum_i q_i V_i \ ,$$

where q_i is the charge on particle i and V_i is the potential energy at the location of particle i. For many theoretical calculations, one can just use the discrete particle formula for continuous charge distribution cases too since all the operation are analogous and one can always take the continuum limit when needed. Say one had a electrostatic system conductors. So that the potential energy of the system is

$$U = \frac{1}{2}\sum_\ell q_\ell V_\ell \ ,$$

where q_ℓ is the charge on conductor ℓ and V_ℓ is the potential of conductor ℓ. Remember conductors are equipotentials in electrostatic cases. **HINT:** This isn't very hard.

b) Say one had an isolated conductor (i.e., one isolated from all external electric fields) in an electrostatic situation. Show that its potential energy is given by

$$U = \frac{1}{2}\alpha q^2 \ ,$$

where q is the conductor charge and α is a constant that depends on the geometry of the conductor. Assume the conductor is an ideal conductor with an impenetrable surface. **HINT:** Remember Newton's 2nd law.

c) Determine α for a spherically symmetric isolated conductor.

---

025 qfull 00610 2 5 0 moderate thinking: van de graaf generator

919. The Van de Graaf generator was invented in 1929 by Robert J. Van de Graaf (1901–1967). Without going into great detail, one builds up net charge on a nearly spherically symmetric conducting shell (called a dome) by literally conveying charge there on a nonconducting conveyor belt that enters the dome through holes. A conducting comb (that just brushes the belt) allows

the charge to flow to the dome. The charge can be positive (absence of electrons) or negative (excess of electrons). We assume positive for definiteness in this problem.

a) Say the belt conveys a constant net current (charge per unit time) of I to the dome. What is the formula for time t for charge on the dome to reach q when it started at zero?

b) Say the charge q ends up uniformly spread over the outer surface of the dome which we can approximate to high accuracy as spherical shell of radius R. Let the radial coordinate be r. Assuming exact spherical symmetry, what are the formulae for the **ELECTRIC FIELD** and **ELECTRICAL POTENTIAL** for $r \geq R$? **HINT:** General formulae are not acceptable answers. What are the formulae for this specific case.

c) The Van de Graaff generator is a neat trick to build up net charge. But in building up net charge, one also builds up potential energy in the distribution of charge on the dome's surface. But you don't get things for free. By what process done by what component of the generator does the energy flow into the potential energy form.

d) Assuming exact spherical symmetry for the charge distribution on the dome's outer surface and that the charge is brought to the dome from infinity, show that the potential energy stored in building the dome charge to q is

$$U = \frac{1}{2}\frac{kq^2}{R} \ .$$

e) The breakdown electric field magnitude for dry air is about $3 \times 10^6 \, \mathrm{V/m}$. For what potential at the dome's surface will air break down if the dome radius is $4 \, \mathrm{m}$?

f) The electric charge collected by the comb mostly ends up on the surface of the dome. Explain why the comb collects charge and why the charge mostly ends up on the dome's outer surface and why the dome's outer surface charge density is nearly uniform. **HINTS:** Remember that like charges repel. The system is in some sense close to electrostatic as charge build-up becomes large despite a slow inflow of charge. The holes can be assumed to be a negligible perturbation on a nearly completely enclosing conducting dome. The dome system (i.e., dome and inside parts) is nearly spherically symmetric.

Chapt. 26 Capacitors and Capacitance

Multiple-Choice Problems

026 qmult 00100 1 1 5 easy deducto-memory: capacitor defined

920. "Let's play *Jeopardy*! For $100, the answer is: An electrical device consisting of two conducting elements (called plates) separated by vacuum or a dielectric. The plates carry equal and opposite charges. The device is used to store electrical potential energy and charge."

What is a/an _____, Alex?

a) resistor b) inductor c) transistor d) and-gate
e) condenser—no, no a capacitor

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026 qmult 00200 1 1 3 easy memory: capacitance geometry

921. The capacitance of a capacitor ideally depends only on:

a) its dielectric environment. b) its geometry. c) its geometry and its dielectric environment. d) the potential put on the capacitor. e) the charge put on the capacitor.

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026 qmult 00300 1 1 4 easy memory: C=Q/V

922. The standard formula for capacitance with standard symbols is:

a) $C = V/Q$. b) $C = V^2/Q$. c) $C = Q^2/V$. d) $C = Q/V$. e) $C = Q - V$.

---

026 qmult 00400 1 1 2 easy memory: farad size

923. The MKS unit of capacitance is the farad (F=C/V). In fact, 1 F is such a huge capacitance—you must hold a Coulomb of charge with a single volt for a 1 F capacitor—that standard capacitors are generally rated in:

a) kF, MF, and GF. b) pF, nF, and μF. c) TF, PF, and EF. d) aF and fF.
e) ferrets, stoats, and weasels.

---

026 qmult 00500 1 1 4 easy memory: capacitor symbol

924. The standard circuit diagram symbol for a capacitor is:

a) $\overset{\bigcirc}{+}$. b) $\overset{\uparrow}{\odot}$. c) $\odot$. d) $\dashv\vdash$. e) $\oplus$.

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026 qmult 00600 1 1 3 easy memory: planar capacitor

925. The simplest of all capacitor capacitance formulae to remember is that for an ideal planar capacitor with vacuum between the plates. (Ideal means that it can be treated as a segment of an infinite planar capacitor.) The formula involves only A (the area of the plates), d the separation of the plates, and the vacuum permittivity ε_0 (which has units of F/m). By dimensional analysis (i.e., analysis of the units), the formula must be:

a) $C = A + d + \varepsilon_0$. b) $C = \dfrac{\varepsilon_0 d}{A}$. c) $C = \dfrac{\varepsilon_0 A}{d}$. d) $C = \dfrac{A}{d\varepsilon_0}$.
e) $C = \dfrac{(A-d)}{\varepsilon_0}$.

---

026 qmult 00700 1 4 3 easy deducto-memory: capacitor formulae

926. The capacity formulae for the three ideal, high-symmetry, vacuum-separated capacitors—planar, cylindrical, and spherical—in order are:

a)
$$C = \frac{q}{V} \qquad C_{\text{eq}} = \sum_i C_i \qquad \frac{1}{C_{\text{eq}}} = \sum_i \frac{1}{C_i} \ .$$

b)
$$u = \frac{1}{2}\varepsilon_0 E^2 \qquad \varepsilon = \kappa\varepsilon_0 \qquad \vec{E} = \frac{q}{4\pi\kappa\varepsilon_0 r^2}\hat{r} \ .$$

c)
$$C = \frac{\varepsilon A}{d} \qquad C = \frac{2\pi\varepsilon L}{\ln(r_{\text{outer}}/r_{\text{inner}})} \qquad C = 4\pi\varepsilon\frac{r_{\text{inner}}}{1 - r_{\text{inner}}/r_{\text{outer}}} \ .$$

d)
$$C = \frac{2\pi\varepsilon L}{\ln(r_{\text{outer}}/r_{\text{inner}})} \qquad C = \frac{\varepsilon A}{d} \qquad C = 4\pi\varepsilon\frac{r_{\text{inner}}}{1 - r_{\text{inner}}/r_{\text{outer}}} \ .$$

e)
$$C = 4\pi\varepsilon\frac{r_{\text{inner}}}{1 - r_{\text{inner}}/r_{\text{outer}}} \qquad C = \frac{\varepsilon A}{d} \qquad C = \frac{2\pi\varepsilon L}{\ln(r_{\text{outer}}/r_{\text{inner}})} \ .$$

---

026 qmult 00800 1 1 5 easy memory: capacitors in parallel

927. If you have a set of $\{C_i\}$ capacitances in parallel, then the equivalent capacitance C_{equ} is given by:

a) $1/C_{\text{equ}} = \sum_i 1/C_i$. b) $1/C_{\text{equ}} = \sum_i C_i$. c) $C_{\text{equ}} = \sum_i 1/C_i$.
d) $1/C_{\text{equ}} = \sum_i 1/C_i^2$. e) $C_{\text{equ}} = \sum_i C_i$.

---

026 qmult 00900 1 1 1 easy memory: capacitors in series

928. If you have a set of $\{C_i\}$ capacitances in series, then the equivalent capacitance C_{equ} is given by:

a) $1/C_{\text{equ}} = \sum_i 1/C_i$. b) $1/C_{\text{equ}} = \sum_i C_i$. c) $C_{\text{equ}} = \sum_i 1/C_i$.
d) $1/C_{\text{equ}} = \sum_i 1/C_i^2$. e) $C_{\text{equ}} = \sum_i C_i$.

---

026 qmult 01000 1 4 1 easy deducto-memory: E-field energy density

929. "Let's play *Jeopardy*! For $100, the answer is: The energy density of the electric field in vacuum."

What is _____, Alex?

a) $u = \frac{1}{2}\varepsilon_0 E^2$ b) $u = \frac{B^2}{2\mu_0}$ c) $u = -kx^2$ d) $u = \frac{1}{2}\varepsilon_0 E$ e) $u = \frac{1}{2}\varepsilon_0 E^3$

---

026 qmult 01100 1 1 1 easy memory: dielectric

930. By the word dielectric, one usually means a/an:

a) insulator considered as electrically active. b) conductor. c) semi-conductor.
d) superconductor. e) two-electric-field material.

---

026 qmult 10150 3 3 1 tough thinking: infinite ladder capacitor mult

931.*** Consider an infinite ladder capacitor. There are two terminals A and B at the left: A is the upper terminal; B is the lower terminal. From each terminal, a wire extends to infinity in the rightward direction. There are vertical wires at equal intervals connecting the A wire and the B wire: these are the rungs of the ladder. On each vertical wire (each rung) is a capacitor of capacitance C. On the A wire before each node (going rightward) is also a capacitor of capacitance C. Draw a diagram.

What is the capacitance C_∞ of the infinite ladder (i.e., what is the capacitance of the infinite ladder viewing A and B as the leads to capacitor plates)?

As Yoda would say, **HINT** you want? If you draw a vertical line just to the right of leftmost rung, the capacitance of the ladder to the right of that line must be C_∞ also since the ladder is infinite.

a) $C_\infty = C\left(\frac{\sqrt{5}-1}{2}\right)$. b) $C_\infty = C\left(\frac{\sqrt{5}+1}{2}\right)$. c) $C_\infty = C\left(\frac{\sqrt{3}+1}{2}\right)$.
d) $C_\infty = C\left(\frac{\sqrt{3}-1}{2}\right)$. e) $C_\infty = \frac{C}{2}$.

---

026 qmult 11000 3 5 1 tough thinking: classical electron radius fullmult

Extra keywords: Uses electric field energy, and so is capacitor lecture problem.

932.*** People once tried to devise a semi-classical model of an electron in which the rest mass energy given by the Einstein equation $E = mc^2$ (E for energy here) was entirely stored as electric field energy. Remember this energy can be easily calculated from the potential energy of the assembled charge distribution provided you don't have to calculate the potential energy of a point charge.

The idea was that the electron charge ($q = 1.602176487 \times 10^{-19}\,\mathrm{C}$) was spread uniformly over a sphere of radius R. (Note this distribution avoids any point charges). They took the potential energy formula for this sphere and equated it to mc^2 for the electron and solved for R. The value they would have gotten using modern input values is:

a) $1.40897\ldots \times 10^{-15}\,\mathrm{m}$. b) $2.81794\ldots \times 10^{-15}\,\mathrm{m}$. c) $1.99792458 \times 10^{8}\,\mathrm{m}$.
d) $9.8\,\mathrm{m}$. e) $1.602176487 \times 10^{-19}\,\mathrm{m}$.

Full-Answer Problems

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026 qfull 00100 2 3 0 moderate math: capacitance formulae

933. The capacitance formulae of high-symmetry capacitors are easy to calculate starting from Gauss's law:

$$\oint_{\text{surface}} \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0} \ .$$

One uses Gauss's law to calculate the electric field between the plates of the capacitor in terms of geometry and charge q on the positive plate (the negative plate has charge $-q$) and then determines the potential difference going from the negative plate to the positive plate using

$$V = -\int_{-\text{ve}}^{+\text{ve}} \vec{E} \cdot d\vec{s} \ .$$

The capacitance is given by

$$C = \frac{q}{V}$$

which should turn out to be independent of q and V.

a) What is the capacitance formula for an ideal parallel plate capacitor? Assume plate area A and plate separation d. We have to assume the characteristic length of A is very large compared d for the infinite plate approximation to be valid.

b) What is the capacitance formula for an ideal cylindrical plate capacitor? Assume cylinder length L, inner and outer radii a and b, and the positive charge on the inner plate. We have to assume $L >> a$ and $L >> b$ for the high symmetry arguments to apply approximately.

c) What is the capacitance formula for an ideal spherical plate capacitor? Assume inner and outer radii a and b, and the positive charge on the inner plate. In this case the high-symmetry condition applies exactly.

d) What limiting case formula do you get from the part (c) answer if $b \to \infty$? In this case you put the outer plate at infinity. It's shape so far away has no effect since the electric field where it is important obeys spherical symmetry. Remember the universe is overall neutral (it seems). So if you isolate a spherical plate of positive charge, somewhere off in the universe is negative charge to balance it.

---

026 qfull 00200 2 3 0 moderate math: capacitor and switch

934. Let's analyze the static states of a very simple circuit consisting of an ideal battery with emf $\mathcal{E}$, a capacitor of capacitance C_c, switch of capacitance C_s when open, and connecting wires. An ideal switch would have zero capacitance, but our switch is not ideal and has $C_s > 0$. However, $C_s << C_c$. The assembled circuit is a single loop running battery, capacitor, switch in a clockwise direction: wires connect the components. The wires have no potential drops across them whether current is flowing or no. The potential change around the complete circuit should be zero at all times: this just follows from the conservative nature of the electrostatic force. You should draw a circuit diagram.

a) Initially, the capacitor and switch are uncharged and connected to the battery. The switch is open. Ideally, no current should flow at all when the circuit is constructed. But, in fact, there will be a tiny transient current that will put a charge q_1 on both the capacitor and switch: i.e., both are capacitors charged to q_1. The current stops when potential drop across the capacitor and switch equals the potential rise across the battery (which equals the emf): $\mathcal{E} = V_c + V_s$. Determine V_c and V_s in terms of $\mathcal{E}$, C_c, and C_s. How do the sizes of V_c and V_s compare to $\mathcal{E}$? What would V_c and V_s be if the switch were ideal (i.e., had $C_s = 0$). **HINTS:** Remember $q = CV$. This is an algebra problem: two unknowns for which two equations are needed for solution.

b) Now you close the switch which essentially converts it into a piece of wire with no potential drop across it. There is some transient current for awhile, but the system settles down to a state with charge q_2 on the capacitor. What are V_c and q_2?

c) Now you open the switch again. What happens? **HINT:** There is no potential difference across the switch.

d) Now you take the battery out of the circuit leaving the wires that connected to the battery terminals free. What happens?

e) Now you simply connect the terminal wires. There will be transient current. The end result will be as if the capacitor and open circuit were capacitors in parallel in a static case. What is the charge on each in terms of q_2 and the capacitances? **HINTS:** Use charge conservation and the fact that the potential differences across the capacitor and switch must be equal. You are solving for two unknowns again.

---

026 qfull 00300 2 3 0 moderate math: energy stored in a capacitor

935. The potential energy stored in a capacitor is a function of the charge and its distribution only. It does not depend on how the distribution was assembled. The work done to assemble the distribution, however, equals this potential energy. So you can imagine any simple way of assembly you like as means of calculating the potential energy. Say that you just took positive charge from one plate little bit at a time and moved it to the other plate. You started from uncharged state and built up to charge q on the capacitor. The change in potential energy of the system due the transfer of dq' is

$$dU = V'\, dq' \ ,$$

where V' is the potential difference between the plates at the time of transferring dq'. What is the total potential energy of charging the capacitor to q? **HINT:** You need to do an integration.

---

026 qfull 01000 1 3 0 easy math: classical electron radius

Extra keywords: Uses electric field energy, and so is capacitor lecture problem.

936. People once tried to devise a semi-classical model of an electron in which the rest mass energy given by the Einstein equation $E = mc^2$ (E for energy here) was entirely stored as electric field energy. Remember this energy can be easily calculated from the potential energy of the assembled charge distribution provided you don't have to calculate the potential energy of a point charge.

a) What they did was assume that all the electron charge was uniformly spread on the surface of a sphere of radius R. (Note this distribution avoids any point charges). What is the potential energy of a sphere of radius R with uniform surface density Q?

b) What formula and value for R did those long-ago folks arrive at from their model?

c) As a matter of fact, the proton has a measurable radius of about 0.9×10^{-15} m: this is the radius of the sphere over which charge seems to be spread: i.e., the charge radius. Given that charge radii of this size are measurable and the electron has no measured radius, what can one say about the old electron model of the electron radius discussed in this problem?

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026 qfull 01200 3 3 0 tough math: capacitance with pyrex dielectric

937. Boy or girl wizard scientist that you are, you decide to construct your own capacitor (little recking how dangerous they can be.) You use a tall pyrex drinking glass as dielectric and line the inside and outside of the curved surfaces with tin foil. The glass is 15 cm high and has inner and outer radii of 3.6 cm and 3.8 cm, respectively. What is the capacitance and breakdown potential of your capacitor? To find the breakdown potential exactly requires finding the linear charge density of the inner foil when the dielectric breaks down.

NOTE: By the by, this device is called a Leyden Jar and was one of, if not the first, consciously constructed capacitors. Eighteenth century investigators first learnt how dangerous non-lightning electricity could be fooling around with these things—or so I seem to recall.

Chapt. 27 Currents and Resistance

Multiple-Choice Problems

027 qmult 00100 1 1 1 easy memory: electrical current partially defined
Extra keywords: physci

938. An electrical current, partially defined, is:

a) moving charge. b) static charge. c) electrical potential. d) moving water. e) static water.

027 qmult 00110 1 4 5 easy deducto-memory: electrical current defined

939. "Let's play *Jeopardy*! For $100, the answer is: It is the rate of charge per unit time passing or flowing through a specified surface in a specified sense and we'll call this quantity flow for the nonce (i.e., for the rest of this answer). By 'charge' without qualification, one means the net charge (i.e., positive plus negative charge or positive charge minus the absolute value of negative charge). By 'specified sense', one means that one defines one side of the surface as the positive side where all charge flows to that side are added and all charge flows to the other side are subtracted. This means that added positive charge or subtracted negative charge both increase flow and added negative charge and positive subtracted charge both decrease flow. The above specifications for adding up the contributions to the flow make perfect sense if one considers the net charge in a box into which charge is flowing. For example, positive inflows and negative outflows increase charge in the box and positive outflows and negative inflows decrease charge in the box. Integrated flow as we have defined it gives the change in the box charge.

Positive and negative flows can occur. Usually one chooses one's conventions so that the flows are positive, but obviously in cases where the flow can reverse or where you don't know the direction of flow initially, negative flows can arise."

What is electrical _____, Alex?

a) swarm b) tide c) flux d) tirade e) current

027 qmult 00120 1 1 2 easy memory: ampere named
Extra keywords: physci

940. The MKS unit of current is the:

a) coulomb (C). b) ampere (A). c) volt (V). d) watt (W). e) joule (J).

027 qmult 00130 1 4 5 easy deducto-memory: electrical units
Extra keywords: physci KB-170-11

941. "Let's play *Jeopardy*! For $100, the answer is: They are, respectively, the units of power, current, potential, resistance."

What are, respectively, _____, Alex?

a) joule, watt, newton, pascal b) ohm, watt, volt, ampere
c) ambig, volte-face, omen, watkin d) ampere, volt, ohm watt
e) watt, ampere, volt, ohm

027 qmult 00140 1 1 3 easy memory: current density defined

942. _____ for a single kind of charge carrier of charge q is defined by

$$\vec{J} = nq\vec{v} \ ,$$

where n is the density of charge carriers at that point and $\vec{v}$ is the velocity of the charge carriers at some point. _____ is a vector and note that it points opposite to $\vec{v}$ when $q < 0$. Imagine a horizontal differential bit of surface area dA. The net charge flowing through this area in a direction an angle θ from the vertical in time Δt is

$$\Delta Q = nq(v\Delta t)\, dA \cos\theta = (nq\vec{v} \cdot d\vec{A})\Delta t = (\vec{J} \cdot d\vec{A})\Delta t \ ,$$

where $(v\Delta t)\, dA \cos\theta$ is the volume of a parallelepiped that sweeps through dA in Δt, $v\Delta t$ is the length of one side of the parallelepiped, and $d\vec{A}$ is the vectorized differential area. The differential current through $d\vec{A}$ is

$$dI = \vec{J} \cdot d\vec{A} \ .$$

Integrating over a finite surface gives a current:

$$I = \int_{\text{surface}} \vec{J} \cdot d\vec{A} \ .$$

a) Current b) Current mass c) Current density d) Current velocity
e) Current acceleration

---

027 qmult 00150 1 1 1 easy memory: units of current density

943. The MKS units of current density are:

a) amperes per square meter (A/m^2). b) amperes per meter per second (A/(m s)).
c) coulombs per square meter (C/m^2). d) coulombs per meter per second (C/(m s)).
e) coulombs per second squared (C/s^2).

---

027 qmult 00160 2 3 3 moderate math: electrons in a current
Extra keywords: physci KB-173-9

944. A current of 2 A flows past a point in a wire. How many electrons flow past per second? (More exactly what is the net flow of electrons per second?)

a) 3.2×10^{-19} electrons/s. b) 2 electrons/s. c) 1.25×10^{19} electrons/s.
d) 2×10^{19} electrons/s. e) none.

---

027 qmult 00200 1 1 4 easy memory: metal charge carriers

945. The charge carriers in metals are:

a) tightly bound electrons. b) bound electrons. c) positrons. d) free electrons.
e) holes.

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027 qmult 00210 1 1 4 easy memory: Fermi gas

946. The free electrons (more strictly quasi-free electrons) in metals approximately constitute a:

a) solid. b) molten mess. c) classical ideal gas. d) Fermi gas.
e) semiconductor.

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027 qmult 00240 1 2 2 moderate memory: effective speed of free electrons

947. The effective speed of random thermal motion of electrons in metals at room temperature is of order:

a) 3×10^8 m/s. b) 10^6 m/s. c) 10^{-6} m/s. d) 3×10^{10} m/s. e) 1 m/s.

---

027 qmult 00250 1 1 5 easy memory: metal conduction

948. In the classical Drude model of conduction of electrons in metals, a macroscopic applied electric field $\vec{E}$ in the metal causes electrons to accelerate with acceleration _____ for a mean time τ between collisions with ions, impurities, and defects in the metal. This acceleration imposes a non-random velocity on the electrons which causes net current flow. Collisions effectively cancel the non-random velocity by scattering the electron isotropically. We will call the non-random velocity just the velocity hereafter for brevity. The mean final velocity between collisions is $\vec{a}$ is $\vec{a}\tau$ and the average velocity is _____. This average velocity is in fact the drift velocity. Given the current density formula _____, the current density in the metal is _____. We call _____ the conductivity of the metal. If the conductivity is independent of $\vec{E}$, the metal is ohmic.

a) $\vec{a} = \vec{E}$; $\vec{v}_\mathrm{d} = (1/2)\vec{a}\tau$; $\vec{J} = nq\vec{v}_\mathrm{d}$; $\vec{J} = -\dfrac{ne\tau}{2}\vec{E}$; $\sigma = -\dfrac{ne\tau}{2}$.

b) $\vec{a} = e\vec{E}$; $\vec{v}_\mathrm{d} = (1/2)\vec{a}\tau$; $\vec{J} = nq\vec{v}_\mathrm{d}$; $\vec{J} = -\dfrac{ne^2\tau}{2}\vec{E}$; $\sigma = -\dfrac{ne^2\tau}{2}$.

c) $\vec{a} = -e\vec{E}$; $\vec{v}_\mathrm{d} = (1/2)\vec{a}\tau$; $\vec{J} = nq\vec{v}_\mathrm{d}$; $\vec{J} = \dfrac{ne^2\tau}{2}\vec{E}$; $\sigma = \dfrac{ne^2\tau}{2}$.

d) $\vec{a} = -e\vec{E}/m_e$; $\vec{v}_\mathrm{d} = (1/2)\vec{a}\tau$; $\vec{J} = nq\vec{v}_\mathrm{d}$; $\vec{J} = \dfrac{ne^2\tau}{2m_e}\vec{E}$; $\sigma = \dfrac{ne^2\tau}{2m_e}$.

e) $\vec{a} = -e\vec{E}/m_e$; $\vec{v}_\mathrm{d} = \vec{a}\tau$; $\vec{J} = nq\vec{v}_\mathrm{d}$; $\vec{J} = \dfrac{ne^2\tau}{m_e}\vec{E}$; $\sigma = \dfrac{ne^2\tau}{m_e}$.

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001 qmult 00260 1 1 1 easy memory: microscopic Ohm's law

949. The formula

$$\vec{J} = \sigma\vec{E}$$

where σ is independent of $\vec{E}$ is the microscopic form of:

a) Ohm's law. b) Kirchhoff's voltage law. c) Kirchhoff's current law.
d) Capt. Kirchhoff's current law. e) Faraday's law of induction.

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027 qmult 00270 1 5 1 easy thinking: unit of conductivity

950. The MKS unit of conductivity (formula $\sigma = nq^2\tau/m$) must be:

a) $\mathrm{C^2\,s\,m^{-3}\,kg^{-1}}$. b) $\mathrm{C^3\,s\,m^{-3}\,kg^{-1}}$. c) $\mathrm{C^2\,s\,m^{-2}\,kg^{-1}}$. d) $\mathrm{C^2\,s\,m^{-3}\,kg^{-4}}$.
e) $\mathrm{C^2\,s\,m^{-4}\,kg^{-4}}$.

---

027 qmult 00280 1 1 1 easy memory: current direction

951. In the wires of electrical circuits, current (speaking more carefully, current density) flows in the direction of the electric field since positive charge will be pushed in that direction and negative charge pushed in the opposite direction. However, most of the time the electrical force only gives rise to an average velocity (i.e., the drift velocity). Electrical resistance (which is caused by collisions) prevents continual acceleration. But it is _____ that current always flows in the direction of the electric field. Current is just a net flow of charge. A net flow can happen for beam of charged particles flying in a vacuum with no external forces acting on the particles. Or charged particles forming a current may be carried along on a conveyor belt as in a Van der Graaf generator without out any net force on them or in such a way that the current flows opposite to the electric field.

a) not true b) true c) correct d) corresponding e) the objective correlative

---

027 qmult 00300 2 4 4 moderate deducto-memory: Ohm's law
Extra keywords: physci KB-149

952. Ohm's law is:

a) $P = VI$. b) $C = q/V$. c) $V = I/R$. d) $V = IR$. e) $\displaystyle\oint \vec{E}\cdot d\vec{A} = \frac{q_\mathrm{enc}}{\varepsilon_0}$.

027 qmult 00310 2 4 4 moderate deducto-memory: Ohm's law and micro form

953. Ohm's law _____ can be derived from the microscopic Ohm's law _____ which assumes _____.

a) $I = VR$: $\vec{J} = (1/2)\varepsilon E^2$: constant density.
b) $V = IR$: $\vec{J} = (1/2)\varepsilon E^2$: constant capacitance.
c) $V = IR$: $\vec{J} = \sigma\vec{E}$: constant electric field.
d) $V = IR$: $\vec{J} = \sigma\vec{E}$: conductivity independent of E.
e) $V = IR$: $\vec{J} = \sigma\vec{E}$: conductivity strongly dependent on E.

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027 qmult 00320 1 1 4 easy math: current in toaster

Extra keywords: physci KB-171-32

954. What is the current in a toaster of $12\,\Omega$ with a DC potential (or voltage) drop across it of 120 V?

a) 0 A. b) 1440 A. c) 120 A. d) 10 A. e) 12 A.

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027 qmult 00330 1 1 3 easy memory: semi-conductors

955. Semiconductors

a) under all conditions obey Ohm's law which makes them so useless.
b) under all conditions obey Ohm's law which makes them so useful.
c) obey Ohm's law for some range of conditions, but their conductivity can be modified by applied electric fields which allows semiconductor devices to behave non-ohmically. This is one of the features that makes semiconductors so useful.
d) under all conditions obey Ohm's law which makes them so unusual.
e) semi-obey Ohm's law which makes them so mediocre.

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027 qmult 00400 1 1 4 easy memory: power of potential rise/drop

Extra keywords: physci KB-151

956. The expression for DC power input/output by a current I through a potential rise/drop V is:

a) $P = I/V$. b) $P = V/I$. c) $P = V - I$. d) $P = IV$. e) $P = I^2V$.

---

027 qmult 00410 1 1 2 easy memory: energy lost in resistance

Extra keywords: physci KB-170-13

957. The electrical energy lost in a resistance is converted into:

a) electric field energy. b) heat energy. c) magnetic field energy.
d) macroscopic mechanical energy. e) gravitational potential energy.

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027 qmult 00420 1 1 4 easy memory: power in resistance

958. The electrical power extracted in a resistance R with a potential drop V and a current I is:

a) $P = VIR = V/R = I/R$. b) $P = VIR = VR^2 = I^2R$.
c) $P = VI = VR^2 = I^2/R$. d) $P = VI = V^2/R = I^2R$. e) $P = VI = V/R = IR$.

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027 qmult 00430 1 3 1 easy math: power calculation 1

Extra keywords: physci

959. A current of 10 A (an ampere is a coulomb per second) goes through a device with voltage difference of 10 V (a volt is joule per coulomb). What is the power (energy per unit time with MKS units of watts [symbol W]: i.e., joules per second) delivered to the device?

HINT: Do the units come out right from the math?

a) 100 W. b) 100 J. c) 10 W. d) 10 J. e) 1 W.

---

027 qmult 00450 1 1 3 easy memory: power in X-ray tube

960. An X-ray tube operates at a current of 7.0 **mA** and a potential difference of 80 **kV** (note kilovolts). The energy is going out of electrical potential energy into electron kinetic energy and then into X-ray energy. What is the X-ray tube's power in watts?

a) 0.56 W. b) 5.6×10^5 W. c) 560 W. d) 11.4 W. e) 1.14×10^7 W.

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027 qmult 10200 2 3 4 mod. math: current from current density fullmult

961.** The current density in a very large cylindrical wire, for whatever reason, obeys the expression

$$J = J_0 \left[1 + \frac{r}{r_0} + \left(\frac{r}{r_0} \right)^2 \right] ,$$

where J_0 is the central current, r is the cylindrical radial coordinate in the wire, and r_0 is a radius parameter. The outer radius is R. What is the expression for the total current in the wire?

a) $2\pi J_0 \left[\frac{R}{r_0} + \frac{1}{2} \left(\frac{R}{r_0} \right)^2 + \frac{1}{3} \left(\frac{R}{r_0} \right)^3 \right]$. b) $2\pi J_0 \left[\frac{1}{2} \left(\frac{R}{r_0} \right)^2 + \frac{1}{3} \left(\frac{R}{r_0} \right)^3 + \frac{1}{4} \left(\frac{R}{r_0} \right)^4 \right]$.

c) $2\pi r_0^2 J_0 \left[\frac{R}{r_0} + \frac{1}{2} \left(\frac{R}{r_0} \right)^2 + \frac{1}{3} \left(\frac{R}{r_0} \right)^3 \right]$.

d) $2\pi r_0^2 J_0 \left[\frac{1}{2} \left(\frac{R}{r_0} \right)^2 + \frac{1}{3} \left(\frac{R}{r_0} \right)^3 + \frac{1}{4} \left(\frac{R}{r_0} \right)^4 \right]$.

e) $2\pi r_0^2 J_0 \left[\frac{1}{3} \left(\frac{R}{r_0} \right)^3 + \frac{1}{4} \left(\frac{R}{r_0} \right)^4 + \frac{1}{5} \left(\frac{R}{r_0} \right)^5 \right]$.

Full-Answer Problems

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027 qfull 00110 1 3 0 easy math: fuse melting

962. A fuse is a wire in a circuit designed to melt at a prescribed current. It thereby opens the circuit and stops the flow. The point is to open the circuit before anything important melts. Say a wire material melts at a current density

$$J_{\text{melt}} = 440 \, \text{A/cm}^2 .$$

Assuming that current density is uniform over a wire (i.e., that $I = JA$) what is the formula for the area for melting for general current I_{melt}? Assuming a circular wire, what is the formula for the diameter for melting for general current I? What is the diameter of a wire that melts with $I_{\text{melt}} = 0.50$ A?

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027 qfull 00120 1 3 0 easy math: current from current density

963. The current density in a very large cylindrical wire, for whatever reason, obeys the expression

$$J = J_0 e^{-r/r_e} ,$$

where J_0 is the central current, r is the cylindrical radial coordinate in the wire, and r_e is the e-folding radius of the current density distribution. Assuming that the radius of the wire is so large it can be set to infinity, what is the expression for the total current in the wire? **HINT:** You might find the factorial integral useful:

$$n! = \int_0^\infty e^{-x} x^n \, dx .$$

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027 qfull 00250 3 3 0 tough math: mean speed between collisions

Extra keywords: I think the question is all right, but I'm still thoughtful.

964. The probability of electron collision (with an ion, impurity, or defect) in a metal happening in the time span from last collision to time t is

$$P(t) = 1 - e^{-t/\tau}$$

where τ is the e-folding time for collisions.

a) Take the derivative of $P(t)$ with respect to time.

b) The derivative from part (a) is a probability density ρ for collisions: i.e., $\rho\,dt$ is the probability for a collision happening in the differential time interval dt. The time interval over which the density applies is from time 0 (immediately after the last collision) to time infinity. If you have N electron flights between collisions, $N\rho\,dt$ is the mean number of them that end in dt. Say you have a quantity Q associated with the electron flights: in general Q depends on t. The mean total value of Q for flights that end in dt is $QN\rho(t)\,dt$. The mean total value for all N flights is

$$N\int_0^\infty Q\rho\,dt\ .$$

The average value of Q per flight (i.e., $\langle Q\rangle$) given by

$$\langle Q\rangle = \int_0^\infty Q\rho\,dt\ .$$

Say $Q = t^n$, where n is a positive integer. What is mean of t^n (i.e, what is $\langle t^n\rangle$). The quantity $\langle t^n\rangle$ is called the nth moment of the probability distribution. What are the 0th and 1st moments and what are they called? **HINT:** You might find the factorial integral useful:

$$n! = \int_0^\infty e^{-x}x^n\,dx\ .$$

c) In metal conductors with applied electric field of typical electrical equipment size, there is an electrical force on the electrons that causes a current. The force accelerates the electrons in a particular direction for the time between collisions. This acceleration only causes a minute extra velocity to be added to the random thermal velocity of the electrons. The random velocity is taking the average electron nowhere. The minute extra velocity gives the electrons a net flow direction. When a collision occurs the electron direction is randomized and the extra velocity is lost: the energy gained from the electric force is dissipated to heat in the conductor. The extra velocity is so minute that it does not affect the probability distribution of the time between collisions. Determine the expression for the mean velocity of the electrons due to the electric field given that the magnitude of the acceleration due to the field is a constant $a = eE/m_e$? This velocity is called the drift velocity. It is the average velocity of the electrons: the random velocities of individual electrons average to zero. There is a subtlety though. There are different kinds of means. The mean velocity we want here is the one that given a time Δt tells us how far the mean electron has gone in that time Δt.

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027 qfull 00270 2 3 0 moderate math: atmospheric current

Extra keywords: HRW-631-26

965. The Earth's lower atmosphere contains positively and negatively charged ions produced by radioactive elements in the soil and cosmic rays. Assume their densities are $620\,\mathrm{cm}^{-3}$ and $550\,\mathrm{cm}^{-3}$, respectively, and that they are **SINGLY** charged. (Lower atmospheric molecule density is about $2.5\times10^{19}\,\mathrm{cm}^{-3}$ for comparison: the molecules are mostly neutral.) Assume also an atmospheric electric field of $120\,\mathrm{V/m}$ pointed downward.

a) In which direction does each kind of ion move?

b) Given that the conductivity of the air is given by $\sigma = 2.70 \times 10^{-14}\,\Omega^{-1}\,\mathrm{m}^{-1}$, what is the current density?

c) Assume that the ions have the same drift speed which is plausible since they are in the same electric field, have about the same mass on average, the same absolute value of charge, and about the same collision time on average. Recall in that

$$\vec{J} = nq\vec{v}_\mathrm{d}\ ,$$

where n is carrier density, q is carrier charge, and $\vec{v}_\mathrm{d}$ is the drift velocity.

What is this drift speed in our case? (Note that current is net flux of charge, not flux of one kind or another of charge. Be careful about signs.)

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027 qfull 00300 1 3 0 tough math: deriving Ohm's law

966. We will now derive Ohm's law in its usual form (which is what one means when one just says Ohm's law without qualification) from the microscopic form of Ohm's law. The microscopic form of Ohm's law is

$$\vec{J} = \sigma\vec{E}$$

where $\vec{J}$ is the current density and σ is the conductivity which is independent of $\vec{E}$ for an ohmic material. We have to make some assumptions that we can't justify here.

a) Draw a diagram of sum conductor of arbitrary shape with two surface segments A and B. The surface segments do not touch each other, and for diagramatic clarity should be on opposite sides of the conductor. One segment is held at potential V_A and the other at V_B. The potential drop across the conductor is $V = V_\mathrm{A} - V_\mathrm{B}$. The conductivity is uniform in the conductor.

b) Imagine a steady state situation in which a current I flows into the conductor at A and out at B. Charge doesn't flow out or in anywhere else. Assume that the electric field in the conductor surface on the inner side, except at segments A and B, is parallel to the conductor surface. Guiding charge on the conductor surfaces arranges this. We don't have to worry about the E-fields outside of the conductor which might have a complex nature and be discontinuous at the conductor surface. The E-fields in this case are conservative: i.e., there is a defined potential at every point in space. What is the direction of the electric field at segments A and B?

c) Draw schematic electric field lines and equipotentials on your diagram.

d) Now say we scale the conductor potential drop up by a factor f. In the simplest imaginable steady-state solution to the new situation, how should the E-field change at every point? What would this E-field change imply for changes in the guide guide charge and all potential differences in the conductor?

e) Given the microscopic form of Ohm's law, what would happen to $\vec{J}$ and I everywhere with the scale up of the potential drop in the simplest possible scaled up solution?

f) Making the assumption of unique steady state solutions for our conductor system does the ratio of $R = V/I$ depend V or I? What does R depend on? What is the formula

$$V = IR\ ?$$

What is R called?

g) To get a formula for R first integrate the microscopic Ohm's law over a general equipotential surface i and second integrate the E-field along a general electric field line j from surface

A to B. Using the two results find the formula for ratio R. What does one actually need to evaluate the formula?

h) Say one had the special case of exact cross sectional symmetry along an axis: i.e., the conductor cross section was constant along the axis and the E-field was uniform everywhere in the conductor and its direction was parallel to the axis of symmetry. In this case, there is no guide charge. The cross sectional area is A and the conductor length is ℓ.

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027 qfull 00410 2 3 0 moderate math: power integrated for heater

967. A 1250 W radiant heater operates with a potential difference across it of 115 V.

a) What is the current in the heater?

b) What is the resistance R_0 of the heater coil?

c) How much energy (in MKS units) does the heater absorb and emit (it's the same amount) in a time $t_0 = 1$ hour?

d) Now say that due to a defect the resistor starts to increase in resistance from its proper value R_0 calculated in part (b). The resistance increases according to the expression

$$R(t) = R_0(t/t_0)^2 \ ,$$

where $t_0 = 1$ hour again and time t starts at the one hour mark: i.e., at t_0. Given that the **POTENTIAL** is held constant, how much thermal energy in principle is generated from the one hour mark to time equals infinity? **HINTS:** You'll have to integrate power over time. Identify what's constant and what's not.

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027 qfull 00420 2 3 0 moderate math: incandescent light bulbs

Extra keywords: SJ-771-40

968. You are given two incandescent light bulbs. We will assume counterfactually that the bulbs are ohmic: i.e., their resistances are independent of the potential drop across them and current through them. One is rated at 25 W and the other at 100 W. These are the powers that the bulbs require for normal operation with a potential of 120 V across them: the potential is constant for DC and the root-mean-square (RMS) average for AC. Actually, only a few percent of the power goes into visible light for conventional incandescent light bulbs: the rest ends up as waste heat without going through a visible light stage: it mostly goes through an infrared light stage. Let us not worry about efficiency and assume DC operation for simplicity. Both bulbs are put in circuits with 120 V across each of them and the circuits are in **STEADY-STATE** operation.

a) What are the resistances of the bulbs?

b) How long does it take for 1 C of charge to flow into each bulb? How much charge passes out of the bulbs in these time periods?

c) How long does it take for 1 J of energy to be converted in each bulb?

d) What is the cost of running each bulb for 30 days assuming the power company charges \$0.070 per kWh?

Chapt. 28 Circuits

Multiple-Choice Problems

028 qmult 00100 1 4 4 easy deducto-memory: circuit definition

969. "Let's play *Jeopardy*! For $100, the answer is: A closed loop or loops of conductors in which currents can flow."

What are _____, Alex?

a) magnets b) Faraday's and Lenz's laws c) capacitors d) circuits
e) bread and circuits

028 qmult 00110 1 1 3 easy memory: Kirchhoff's current law

970. Charge conservation applied to steady state currents in circuits leads to:

a) Kirchhoff's voltage law. b) Ohm's law. c) Kirchhoff's current law.
d) Newton's 2nd law. e) energy conservation.

028 qmult 00120 1 4 5 easy deducto-memory: Kirchhoff's voltage law 1

971. The potential change around any closed path is zero, and thus in the special case of an electrical circuit the potential changes summed over all elements in the circuit must be:

a) infinite: this is Mozart's 3rd law. b) positive: this is Krankheit's 1st law.
c) negative: this is Wilson's 14th point. d) indefinite: this is Hemingway's conjecture.
e) zero: this is Kirchhoff's voltage law.

028 qmult 00130 1 4 5 easy deducto-memory: Kirchhoff's voltage law 2
Extra keywords: physci

972. The circuit law that states that the sum of voltages rises (potential rises) equals the sum of the voltages drops (potential drops) going around a closed path is:

a) Mozart's 3rd law. b) Krankheit's 1st law. c) Wilson's 14th point.
d) Hemingway's conjecture. e) Kirchhoff's voltage law.

028 qmult 00200 1 4 2 easy deducto-memory: emf defined

973. "Let's play *Jeopardy*! For $100, the answer is: An abbreviation for the misnomer electromotive force. An electromotive force is not a force. It is a work done along some path per unit charge: i.e.,

$$\mathcal{E} = \int \vec{f} \cdot d\vec{s} \, ,$$

where $\mathcal{E}$ is the electromotive force, f is the force per unit charge (which could be an electric field, in fact, but there are many other possibilities), and the integral is along some path."

What is _____, Alex?

a) elforce b) emf c) ef d) elfor e) elf

028 qmult 00210 1 4 5 easy deducto-memory: emf device

974. "Let's play *Jeopardy*! For $100, the answer is: It is a device that sets up a separated charge structure inside itself such that there is a potential difference between a positive terminal and

a negative terminal. Ideally, the device maintains the potential difference at a constant value and the separated charge structure no matter what current flows through the device from the negative terminal to the positive terminal and then from the positive terminal to the negative terminal through an external circuit. In real cases, the potential usually decreases when current flows. Since electrical potential energy is lost as current flows from high to low potential in the external circuit, the device must convert energy from some form into electrical potential energy by forcing the current to flow from negative to positive terminals internally.

What is a/an ____ device, Alex?

a) fme b) mfe c) mef d) efm e) emf

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028 qmult 00220 1 1 3 easy memory: internal resistance

975. Real emf devices are not perfect because of:

a) magnetic fields. b) electric fields. c) internal resistance.
d) external resistance. e) la Résistance.

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028 qmult 00300 1 1 1 easy memory: resistors series/parallel

976. The equivalent resistor formulae for resistors in series and parallel are, respectively:

a) $R_{\text{eq}} = \sum_i R_i$ and $\frac{1}{R_{\text{eq}}} = \sum_i \frac{1}{R_i}$.

b) $\frac{1}{R_{\text{eq}}} = \sum_i \frac{1}{R_i}$ and $R_{\text{eq}} = \sum_i R_i$. c) $R_{\text{eq}} = \prod_i R_i$ and $\frac{1}{R_{\text{eq}}} = \prod_i \frac{1}{R_i}$.

d) $R_{\text{eq}} = \sqrt{\sum_i R_i^2}$ and $R_{\text{eq}} = \sum_i R_i$.

e) $R_{\text{eq}} = \sqrt{\sum_i R_i^2}$ and $R_{\text{eq}} = \sum_i \frac{1}{R_i}$.

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028 qmult 00320 1 1 3 easy memory: parallel resistor calculation

977. What is the equivalent resistance of $10\,\Omega$ and $1\,\Omega$ resistors in parallel?

a) $11\,\Omega$. b) $1.1\,\Omega$. c) $0.91\,\Omega$. d) $1.0\,\Omega$. e) $10\,\Omega$.

---

028 qmult 00400 1 1 2 easy memory: solving circuits

978. Using Kirchhoff's voltage and current laws, simple circuits involving only emfs and resistors can be solved by:

a) differential calculus. b) algebra. c) variational calculus. d) integration.
e) guesswork.

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028 qmult 00450 1 1 1 easy memory: electric eel

Extra keywords: HRW-646

979. You are swimming with your pet electric eel Fred—who may not be a terribly realistic electrophorus electricus—and playfully grab his tail and just below his head: the potential difference between these regions is $\sim 750\,\text{V}$. Wet the typical resistance between human body parts is of order $1000\,\Omega$ and let's assume contrafactually that Fred's internal resistance is negligible. Human resistance is much higher if you are dry: of order $10^5\,\Omega$. Given that death is pretty certain if you carry a current of $0.2\,\text{A}$, what are your chances of living?

a) Close to nil. b) So-so. c) Great. d) You have never been safer.
e) You are under 25, you are immortal.

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028 qmult 00600 1 4 5 easy deducto-memory: ammeter voltmeter

980. "Let's play *Jeopardy*! For $100, the answer is: One measuring device is put in series and has a very low resistance; the other is put in parallel and has a very high resistance."

What are _____, Alex?

a) odometer and speedometer b) taximeter and altimeter
c) multimeter and ohmmeter d) kilometer and Demeter
e) ammeter and voltmeter

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028 qmult 00710 1 4 4 easy deducto-memory: RC circuit time constant

981. The e-folding time or time constant of a simple RC (resistor-capacitor) circuit is given by:

a) $\tau = C/R$. b) $\tau = 1/(RC)$. c) $\tau = R/C$. d) $\tau = RC$. e) $\tau = R^2/C$.

---

028 qmult 10300 3 3 2 tough thinking: infinite ladder resistor mult

982.*** Consider an infinite ladder resistor. There are two terminals A and B at the left: A is the upper terminal; B is the lower terminal. From each terminal, a wire extends to infinity in the rightward direction. There are vertical wires at equal intervals connecting the A wire and the B wire: these are the rungs of the ladder. On each vertical wire (each rung) is a resistor of resistance R. On the A wire before each node (going rightward) is also a resistor of resistance R. Draw a diagram.

What is the resistance R_∞ of the infinite ladder (i.e., what is the resistance of the infinite ladder viewing A and B as the leads to an effective resistor)?

As Yoda would say, **HINT** you want? If you draw a vertical line just to the right of leftmost rung, the resistance of the ladder to the right of that line must be R_∞ also since the ladder is infinite.

a) $R_\infty = R\left(\dfrac{\sqrt{5}-1}{2}\right)$. b) $R_\infty = R\left(\dfrac{\sqrt{5}+1}{2}\right)$. c) $R_\infty = R\left(\dfrac{\sqrt{3}+1}{2}\right)$.
d) $R_\infty = R\left(\dfrac{\sqrt{3}-1}{2}\right)$. e) $R_\infty = 2R$.

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028 qmult 10510 2 3 4 moderate math: load matching fullmult

983.** You have a 1-loop circuit containing an emf source with emf value $\mathcal{E}$ and internal resistance r and one resistor with resistance R. What is the formula for the power delivered to the resistor? What is the formula for R that maximizes the power delivered to the resistor? What is the formula for maximum power delivered to the resistor?

a) $P = \dfrac{\mathcal{E}^2}{r+R}$, $R = \dfrac{r}{2}$, $P = \dfrac{2\mathcal{E}^2}{3r}$.
b) $P = \dfrac{\mathcal{E}^2}{r+R}$, $R = 2r$, $P = \dfrac{\mathcal{E}^2}{3r}$.
c) $P = \dfrac{\mathcal{E}^2 R}{(r+R)^2}$, $R = 2r$, $P = \dfrac{\mathcal{E}^2}{9R}$.
d) $P = \dfrac{\mathcal{E}^2 R}{(r+R)^2}$, $R = r$, $P = \dfrac{\mathcal{E}^2}{4r}$.
e) $P = \dfrac{\mathcal{E}^2}{r+R}$, $R = r$, $P = \dfrac{\mathcal{E}^2}{2r}$.

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028 qmult 10520 2 1 4 moderate math: 1 loop circuit fullmult

984.** There is a single loop circuit with two reversible emfs ($\mathcal{E}_1 = 10\,\mathrm{V}$ and $\mathcal{E}_2 = 5\,\mathrm{V}$) and two resistors ($R_1 = 5.0\,\Omega$ and $R_2 = 10.0\,\Omega$). (Reversible in this context means they can supply or absorb energy.) Consider the loop to be rectangular as usual: R_1 is on the top segment; R_2 is on the right segment; $\mathcal{E}_1$ is on the bottom segment with positive terminal aimed **RIGHT**; $\mathcal{E}_2$ is on the left segment with positive terminal aimed **UP**. **HINT:** Draw a schematic circuit diagram.

i) What is the current in the circuit with direction specified?

ii) What is the power dissipated in each resistor?

a) (i) 0.33 A counterclockwise (ii) $P_1 = 0.66\,W, P_2 = 0.99\,\mathrm{W}$.
b) (i) 0.67 A clockwise (ii) $P_1 = 0.56\,W, P_2 = 1.1\,\mathrm{W}$.
c) (i) 0.33 A clockwise (ii) $P_1 = 0.56\,W, P_2 = 1.1\,\mathrm{W}$.
d) (i) 0.33 A counterclockwise (ii) $P_1 = 0.56\,W, P_2 = 1.1\,\mathrm{W}$.
e) (i) 0.33 A clockwise (ii) $P_1 = 0.28\,W, P_2 = 0.56\,\mathrm{W}$.

Full-Answer Problems

028 qfull 00310 1 5 0 easy thinking: resistor maze
Extra keywords: (HRW-653:6)

985. HRW, p. 653, Question 6. There is a maze of resistors and ideal batteries. All the batteries have $\mathcal{E} = 4\,\mathrm{V}$. There is a specified resistor with resistance $R_{\mathrm{sp}} = 4\,\Omega$. What is the current through it? **HINT:** Of course, there's a trick.

028 qfull 00320 1 5 0 easy thinking: capacitor maze
Extra keywords: (HRW-653:8)

986. HRW, p. 653, Question 8. There is a maze of capacitors and ideal batteries. The system is static: i.e., all the capacitors are charged up and no current is flowing. All the batteries have $V = 10\,\mathrm{V}$. The is a specified capacitor with capacitance $C_{\mathrm{sp}} = 6\,\mu\mathrm{F}$. What is the charge on C_{sp}? **HINT:** Think like a mouse.

028 qfull 00430 3 3 0 tough math: 2 loop circuit

987. Draw rectangle. Put battery 1 of potential V_1 on the left-hand side. Put battery 2 of potential V_2 on the right-hand side. Both batteries have their positive terminals above their negative terminals. Put resistor R_1 on the top of the rectangle nearest battery 1. Put resistor R_2 on the top of the rectangle nearest battery 2. Draw a line bisecting the rectangle and put resistor R_3 on this line.

a) Using Kirchhoff's laws write down 3 equations that allow you to solve for the 3 currents I_1 (which flows through battery 1 and resistor R_1 to the right), I_2 (which flows through battery 2 and resistor R_2 to the left), and I_3 (which flows through resistor R_3 downward).

b) Solve the three equations in part (a) for the three currents in terms of the knowns V_1, V_2, R_1, R_2, and R_3. **HINT:** This is easy, but tedious, and there are lots of ways of proceeding. Exploit symmetry wherever possible.

c) Find the limiting forms for the current expressions when $R_3 = 0$. Are these expressions what you would expect from a direct solution of the problem with $R_3 = 0$?

d) Find the limiting forms for the current expressions when $R_3 = \infty$. Are these expressions what you would expect from a direct solution of the problem with $R_3 = \infty$?

028 qfull 00510 1 3 0 easy math: power absorbed in segment
Extra keywords: (HR-654:9E)

988. There is a circuit **SEGMENT** (note segment, not a complete circuit loop) containing a resistor $R = 2.0\,\Omega$ and an emf of unknown potential and unknown orientation with respect to the current. The current flow is left to right and is given by $I = 1.0\,\mathrm{A}$. The segment absorbs $P = 50\,\mathrm{W}$.

a) Draw a diagram of the circuit segment. Since the orientation of the emf is not known, you will have to finish the problem to put that component in.

b) What is the potential difference across the segment?

c) Assuming the emf has no internal resistance what is its $\mathcal{E}$?

d) What is the polarity of the emf: i.e., what is the orientation of its terminals relative to the current direction?

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028 qfull 00520 1 3 0 easy math: 1 loop circuit
Extra keywords: (HR-654:5E)

989. There is a single loop circuit with two reversible emfs ($\mathcal{E}_1 = 12\,\text{V}$ and $\mathcal{E}_2 = 6\,\text{V}$) and two resistors ($R_1 = 4\,\Omega$ and $R_2 = 8.0\,\Omega$). (Reversible in this context means they can supply or absorb energy.) Consider the loop to be rectangular as usual: R_1 is on the top segment; R_2 is on the right segment; $\mathcal{E}_1$ is on the bottom segment with positive terminal aimed right; $\mathcal{E}_2$ is on the left segment with positive terminal aimed up.

a) Draw a schematic circuit diagram with correct symbols for the components.

b) What is the current in the circuit with direction specified?

c) What is the power dissipated in each resistor?

d) What is the power of each battery and is it emitted or absorbed?

---

028 qfull 00530 2 5 0 moderate thinking: power in simple loop
Extra keywords: with a variable resistor

990. You have a single loop circuit with a battery of fixed emf $\mathcal{E}$ and two resistors (in series of course) with resistances R_1 and R. The resistor R is variable.

a) Draw a circuit diagram.

b) Give the analytic expression for the current in the circuit.

c) Give the analytic expression for the power P output from resistor R in terms of $\mathcal{E}$, R_1, and R. What is the power in the limits that $R = 0$ and $R = \infty$? Sketch a graph of P as a function of R.

d) Derive the analytic expression for R that maximizes the power P generated in resistor R in terms of $\mathcal{E}$, R_1, and R. What is the expression for P_{max}?

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028 qfull 00550 3 5 0 tough thinking: power in variable resistor
Extra keywords: (HRW-655:34)

991. HRW, p. 655, Fig. 28-36. The figure shows a two-loop circuit with an emf $\mathcal{E}$, two fixed resistors which we take to have resistance R_1 and R_2, and a variable resistor R.

a) What is the power generated in resistor R if $R = 0$ or $R = \infty$?

b) What are lower and upper bounds on the power generated in resistor R? The upper bound is tricky since there are several ways of thinking about the upper bound. But you want a real absolute upper bound that doesn't depend on the value of R at all. Recall R_1, R_2, and $\mathcal{E}$ are fixed and finite. The upper bound doesn't have to be a least upper bound.

c) Should there be a maximum power generated in R? Give an argument.

d) Find the analytic expression for I through resistor R in terms of R, R_1, R_2 and $\mathcal{E}$.

e) Find the analytic expression for P for resistor R in terms of R, R_1, R_2 and $\mathcal{E}$.

f) Find the analytic expression for R that maximizes P. What is the P_{max} expression?

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028 qfull 00610 2 3 0 moderate math: disguised parallel resistors
Extra keywords: (HRW-656:40)

992. HRW, p. 656, Pr. 40.

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028 qfull 00620 2 5 0 moderate thinking: disguised series resistors

993. HRW, p. 656, Pr. 41.

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028 qfull 00630 2 3 0 moderate math: tedious circuit problem

994. HRW, p. 656, Pr. 42. **HINT:** Do part (b) first.

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028 qfull 00710 2 3 0 moderate math: charging an RC circuit

995. There is 15.0 kΩ resistor and a capacitor in series. The capacitor is initially uncharged. At time zero a constant potential of $V_0 = 12\,\mathrm{V}$ is suddenly applied across the resistor and capacitor. The potential difference across the capacitor rises to 5.0 V in 1.30 μs.

a) What is the e-folding time or time constant of the circuit?

b) What is the capacitance of the capacitor?

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028 qfull 00712 2 3 0 tough : charging an RC circuit

996. There is 15.0 kΩ resistor and a capacitor in series. The capacitor is initially uncharged. At time zero a constant potential of $V_0 = 12\,\mathrm{V}$ is suddenly applied across the resistor and capacitor. The potential difference across the capacitor rises to 5.0 V in 1.30 μs.

There are parts a,b,c. The parts (b) and (c) can be done independently of part (a) if you you recall the solution to the differential equation.

a) From Kirchhoff's voltage law, the differential equation for an RC circuit is

$$R\frac{dQ}{dt} + \frac{Q}{C} = V_0 \ ,$$

where R is resistance, C is capacitance, V_0 is a constant applied potential, and Q is the charge on the capacitor. Solve for the $V(t)$ the potential drop across the capacitor with the initial condition $Q = 0$.

b) Solve for numerical value of e-folding time or RC time constant of the circuit.

c) What is the capacitance C?

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028 qfull 00750 2 3 0 moderate math: RC circuit charging disguised

997. HRW, p. 657, Pr. 52. **HINT:** This problem is easy once you see past the camouflage.

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028 qfull 00760 3 3 0 tough math: capacitor bridgeoid

998. Here is a circuit description. There is an emf $\mathcal{E}$. From $\mathcal{E}$'s positive terminal is a wire connecting $\mathcal{E}$ to two vertical parallel circuit segments each with two resistors in series: the left segment going downward has resistors R_1 and R_2 and the right segment going downward has resistors R_3 and R_4. The segments reunite in a wire that connects to the negative terminal of $\mathcal{E}$. There is a connection between the two segments below resistors R_1 and R_3 and above resistors R_2 and R_4. The connection has capacitor C on it. The circuit has 6 current branches in all. The assumed current direction in $\mathcal{E}$ branch is out of the positive terminal and into the negative terminal. In the resistor branches, the assumed current directions are all downward. In the capacitor branch, the assumed current direction is to the right.

a) Draw the circuit diagram. Indicate the current directions. **HINT:** This isn't meant to be a hard part.

b) What happens to the calculated current value for a current branch if the assumed current direction is not correct?

c) Since there are 6 current branches, one requires 6 equations to solve for the currents. We can obtain these equations from what circuit laws?

d) There are, in fact, only 6 independent equations because there are only 6 current branches. There is some freedom in choosing the 6 independent ones though. A good choice is

$$I_{\mathrm{emf}} = I_1 + I_2 = I_3 + I_4 \ , \tag{1}$$

$$I = I_1 - I_2 = I_4 - I_3 \ , \tag{2}$$

$$\mathcal{E} = I_1 R_1 + I_2 R_2 \ , \tag{3}$$

$$\mathcal{E} = I_3 R_3 + I_4 R_4 \ , \tag{4}$$

$$\mathcal{E} = I_1 R_1 + \frac{q}{C} + I_4 R_4 \ , \tag{5}$$

$$\mathcal{E} = I_3 R_3 - \frac{q}{C} + I_2 R_2 \ , \tag{6}$$

where $I_{\rm emf}$ is the current through the emf branch, I is the current through capacitor branch, q is the charge on the capacitor, and the number-indexed currents are for the correspondingly numbered resistors. Change the 5th equation into an ordinary differential equation for I and q with coefficients in terms of the given resistances and capacitances. Note $q = dI/dt$.

No **UNKNOWNS**, except I and q, should appear in the differential equation. Make any reasonable simplifications. **HINT:** You have to do some algebra. Don't make use of the 6th equation: it turns out to be redundant for this part. The rest of the problem does **NOT** depend on getting part (d) right.

e) In the part (d) answer, you should have obtained a differential equation of the form

$$f\mathcal{E} = gI + \frac{q}{C} \ .$$

Say we have just connected the circuit with the capacitor initially uncharged. What is the formula for time-zero (i.e., initial) current in terms quantities appearing in the formula just given? At time infinity, what is the formula for capacitor charge in terms quantities appearing in the formula just given?

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028 qfull 00840 3 5 0 tough thinking: non-ohmic devices

999. Consider a generic device where current I through in the direction of the potential drop V across the device is a strictly increasing function of the potential drop V. For $V = 0$, $I = 0$. We assume the device operates symmetrically whichever way the potential drop is placed across it.

a) The I-V curve (current-versus-voltage curve AKA current-voltage or I-V characteristic) for a device is a plot of current through versus voltage drop across. Since our generic device behaves symmetrically, is the I-V curve an even or odd function of V?

b) Sketch the I-V curve (current-versus-voltage curve AKA current-versus-voltage characteristic) for the device if it is ohmic (i.e., if it obeys Ohm's law.) What is the slope of the curve?

c) Resistance can be defined for non-ohmic devices as long as current is a function of potential as it is for our generic devices. The definition is just

$$R = \frac{V}{I(V)} \ ,$$

where we have explicitly shown current as a function of V. If I strictly increases with V, then one can also write the inverse function $V(I)$ which gives V as a strictly increasing function of I.. For many non-ohmic devices, resistance increases with V (considering only $V \geq$) This is because electrical energy in the device is converted into heat which increases the temperature of the device and thus increases the resistivity of its material. Recall power delivered to the device is

$$P = IV \ .$$

If current is a strictly increasing function of V as for our generic device, power is a strictly increasing function of V too. Not all of the power is necessarily turned into heat, but it is in resistors and in incandescent light bulbs. Resistors are officially ohmic, but if heated too much that ceases to be true. Incandescent light bulbs are non-ohmic in general use.

A non-ohmic devices will be ohmic for small enough V usually. Sketch an I-V curve for a non-ohmic device (which is otherwise our generic device) where resistance increases with V. Sketch a plot of R versus V for the device. If V is halved, what happens to current? If V is doubled, what happens to current?

d) Say you have one battery driving currents through circuit of consisting of a single loop with generic devices in series. The battery voltage is $V_{\rm ba}$. If one device is removed from the circuit, prove that the loop current must rise. The proof should be complete: i.e., no loopholes.

e) Say you have one battery driving currents through circuit of consisting of a single loop with two generic devices in series: device 1 and device 2. Now a generic device 3 is added to the circuit in paralle to device 2, but series with device 1. Prove that the current through device 1 rises. The proof should be complete: i.e., no loopholes. **HINT:** Consider the two cases: (a) the current increases; (b) the current decreases or stays constant

f) Say you have one battery driving currents through circuit containing devices. The battery voltage is $V_{\rm ba}$. The circuit can be complex with many branches and with multiple devices in series or none possible on any branch. A branch is any wire between nodes with no node on it. The circuit is finite. The devices are all of our generic type, but are not in general identical. Give a complete proof that the current through the battery is the maximum possible if and only if the circuit has all the devices in parallel with the battery. The proof should be complete: i.e., no loopholes.

There is one bit that is tricky to the complete proof, and so you can take it as a given: the potential drop across any branch is less than or equal to $V_{\rm ba}$. **HINT:** The proof isn't very long. You just have to think of things in the right way.

Chapt. 29 Magnetic Fields Causing Forces

Multiple-Choice Problems

029 qmult 00100 1 1 4 easy memory: magnetic field incomplete definition
Extra keywords: magnetic field definition

1000. The magnetic field is a:

a) vector field that emanates from charge (but not only charge) and causes the electric force.
b) scalar field that emanates from charge (but not only charge) and causes the electric force.
c) scalar field that emanates from current (but not only current) and causes the electric force.
d) vector field that emanates from current (but not only current) and causes the magnetic force.
e) scalar field that emanates from current (but not only current and causes the magnetic **FARCE**.

029 qmult 00110 1 4 5 easy deducto-memory: magnetic field lines
Extra keywords: mathematical physics

1001. "Let's play *Jeopardy*! For $100, the answer is: They have no ends except at points of zero magnetic field(unless magnetic monopoles exist after all) and a every point along them the magnetic field vector is tangent to them and points in their direction."

What are _____, Alex?

a) transmission lines b) telephone lines c) streamlines d) electric field lines
e) magnetic field lines

029 qmult 00120 1 1 2 easy memory: magnetic field lines loopy

1002. Magnetic field lines tend to be:

a) droopy. b) loopy. c) Snoopy. d) croupy. e) loup garouy.

029 qmult 00130 1 4 2 easy deducto-memory: electromagnetic fields are real

1003. The main reason for regarding electric and magnetic fields as actual things and not just descriptive devices for calculating electric and magnetic forces is (in the instructor's view anyway) that:

a) they are so useful as descriptive devices.
b) self-propagating electromagnetic radiation (consisting of coupled electric and magnetic fields) has no connection with its source (after its emission) and none with its receiver until it is received. Such radiation can propagate across billions of light years of space long after its source has been wiped from existence and long before its receiver has come into existence. Thus electric and magnetic fields are as real as—well—as lots of other things.
c) they are both vector fields
d) they are both scalar fields.
e) they are so useless as descriptive devices.

029 qmult 00140 1 1 5 easy memory: north and south pole

1004. In the view of the instructor, a region where magnetic field lines diverge can be called a/an _____ and a region where they converge can be called a/an _____.

a) up pole; down pole b) left pole; right pole c) magnet; magnetite
d) south pole; north pole e) north pole; south pole

---

029 qmult 00150 1 1 4 easy memory: pole mnemonic

1005. The magnetic pole mnemonic is:

a) likes attract, unlikes attract. b) likes contract, unlikes detract.
c) likes attract, unlikes repel. d) likes repel, unlikes attract.
e) likes repel, unlikes rebell.

---

029 qmult 00160 1 4 5 easy deducto-memory: fairly strong magnetic field

1006. On the Earth's surface, a fairly large magnetic field (including laboratory systems) would be:

a) 10^8 T. b) 10^{-4} T. c) 10^{-4} gauss. d) 1 gauss. e) 1 T.

---

029 qmult 00200 1 1 4 easy memory: magnetite

1007. Magnetic effects are everywhere in nature, but are mostly not perceivable as such by casual observation. The major exception is the attractive and repulsive forces due naturally occurring magnetic materials. But these materials occur in geopraphically restricted areas. In pre-history, magnetism was probably known only to peoples who lived in such areas or traded for magnetic materials. Magnetism was scientifically described by Thales as early as circa 600 BCE and by the Chinese since circa 400 BCE. The strongest common naturally occurring magnetic material is:

a) electromagnetium. b) magnesium. c) Cartesianism.
d) magnetite or lodestone (Fe_3O_4). e) refrigeratite (Fe_3O_4).

---

029 qmult 00210 1 1 1 easy memory: compass

1008. The first useful device exploiting magnetism (and it was known since the 11th century or earlier in China and the 13th century in Europe) is the:

a) compass. b) electromagnet. c) MRI machine. d) maglev train.
e) refrigerator magnet.

---

029 qmult 00212 1 1 2 easy memory: compass and north magnetic pole

1009. The north pole of a compass needle points approximately toward the north magnetic pole. This means that the north magnetic pole is:

a) a magnetic north pole. b) a magnetic south pole. c) at the north pole (i.e., where the Earth's rotational axis sticks out in the north). d) at the south pole (i.e., where the Earth's rotational axis sticks out in the south). e) a magnetic dipole.

---

029 qmult 00220 1 1 3 easy memory: William Gilbert

1010. The person who first suggested that the Earth itself was magnetic was:

a) the Devil Valley Master (circa 4th century BCE). b) Omar Khayyam (1048–1131).
c) William Gilbert (1544–1603). d) Hans Christian Ørsted (1777-1851).
e) James Clerk Maxwell (1831–1879).

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029 qmult 00230 1 1 4 easy memory: Hans Christian Oersted

1011. The effective discoverer that macroscopic current causes magnetic fields was

a) the Devil Valley Master (circa 4th century BCE). b) Omar Khayyam (1048–1131).
c) William Gilbert (1544–1603). d) Hans Christian Ørsted (1777-1851).
e) James Clerk Maxwell (1831–1879).

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029 qmult 00400 1 4 3 easy deducto-memory: magnetic force law

Extra keywords: magnetic force law.

1012. The magnetic force law for a point charge is:

a) $\vec{F} = q\vec{E}$. b) $\vec{F} = q\vec{v} \times \vec{E}$. c) $\vec{F} = q\vec{v} \times \vec{B}$. d) $\vec{F} = \vec{v} \times \vec{B}$. e) $\vec{F} = \vec{v} \times \vec{E}$.

---

029 qmult 00410 1 1 5 easy memory: magnetic force does no work

1013. The point-charge magnetic force law _____ can _____ work on a point charge since _____. Thus, a useful potential energy _____ be defined for the magnetic force for the case of single point charge.

a) $\vec{F} = \vec{v} \times \vec{B}$; do; $dW = \vec{F} \cdot d\vec{s} = q\vec{v} \times \vec{B} \cdot \vec{v}dt \neq 0$; can
b) $\vec{F} = q\vec{v} \times \vec{B}$; do; $dW = \vec{F} \cdot d\vec{s} = q\vec{v} \times \vec{B} \cdot \vec{v}dt \neq 0$; can
c) $\vec{F} = q\vec{v} \times \vec{B}$; do no; $dW = \vec{F} \cdot d\vec{s} = q\vec{v} \times \vec{B} \cdot \vec{v}dt \neq 0$; can
d) $\vec{F} = q\vec{v} \times \vec{B}$; do no; $dW = \vec{F} \cdot d\vec{s} = q\vec{v} \times \vec{B} \cdot \vec{v}dt = 0$; can
e) $\vec{F} = q\vec{v} \times \vec{B}$; do no; $dW = \vec{F} \cdot d\vec{s} = q\vec{v} \times \vec{B} \cdot \vec{v}dt = 0$; cannot

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029 qmult 00420 1 1 2 easy memory: Lorentz force transformation

1014. A striking fact about the magnetic force on a point charge is that it is velocity dependent. But that frame of reference is the velocity with respect to? Any inertial frame. This means that the magnetic force is inertial-frame dependent and in the special case of moving in the frame of the particle (which at least at any instant can be regarded as in an inertial frame) no magnetic force is exerted on the particle. But what happens then to Newtonian physics where force and acceleration are inertial frame independent? Well, the resolution of the paradox is that only net force actually has to be inertial frame independent in order for the acceleration to be inertial frame independent: remember the 2nd law $\vec{F}_{\text{net}} = m\vec{a}$. What happens is that net electromagnetic force (which is called the Lorentz force)

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

is inertial frame independent in the classical limit of velocities much less than the speed of light. The electric and magnetic fields are inertial-frame dependent quantities and transform between inertial frames in such a way that the Lorentz force is inertial-frame independent in the classical limit. Velocity transforms too, of course. The classical limit transformations needed an unprimed frame S to a primed frame S' are

$$\begin{aligned} \vec{v}\,' &= \vec{v} - c\vec{\beta}\ , \\ \vec{E}' &= \vec{E} + \vec{\beta} \times c\vec{B}\ , \\ \vec{B}' &= \vec{B} - \vec{\beta} \times \vec{E}/c\ , \end{aligned}$$

where the velocity parameter of the transformation

$$\vec{\beta} = \frac{\vec{v}_{SS'}}{c}$$

with $\vec{v}_{SS'}$ being velocity of frame S' relative to frame S. One can show explicitly in the classical limit that

$$\vec{F}' = q(\vec{E}' + \vec{v}\,' \times \vec{B}') = \vec{F}$$

where in the proof one has dropped the classically negligible term _____. Terms are usually classically negligible in transformations if they have two velocity-over-c factors by definition of the classical limit where velocities are small relative to c. Note that electric charge is inertial frame invariant.

Note that if one transforms to an inertial frame in which the point charge is at rest at least momentarily, only the electric field in that frame acts on the particle at that moment anyway. But that does **NOT** mean there is no magnetic field in that frame.

Note also that the Lorentz force formula is actually the same in any inertial frame, but if you are not in the classical limit, it's evaluated value is **NOT** the same in any inertial frame.

But this is OK, since we don't require Newton physics to hold if we are not in classical limit. One must use relativistic dynamics to understand motion when not in the classical limit.

a) $(\vec{v}/c - \vec{\beta}) \times (\vec{\beta} \times \vec{E})$. b) $-(\vec{v}/c - \vec{\beta}) \times (\vec{\beta} \times \vec{E})$. c) $-(\vec{v}/c + \vec{\beta}) \times (\vec{\beta} \times \vec{E})$.
d) $(\vec{v}/c + \vec{\beta}) \times (\vec{\beta} \times \vec{E})$. e) $-\vec{v} \times (\vec{\beta} \times \vec{E}/c)$.

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029 qmult 00430 1 4 5 easy deducto-memory: circling charges or helixing charges
Extra keywords: circling charges.

1015. Charged particles moving in magnetic fields tend to:

a) travel in straight lines. b) travel in sinusoidal curves. c) move in zigzags.
d) stop and start constantly. e) move in circles or helixes.

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029 qmult 00440 1 1 3 easy memory: cyclotron quantities

1016. The cyclotron quantities for a free charge of charge q, mass m, and velocity in the plane perpendicular to the magnetic field $v_{\rm per}$ moving in a uniform magnetic field $\vec{B}$ and acted on by the magnetic force alone are

$$r = \frac{m v_{\rm per}}{|q|B} , \qquad \omega = \frac{|q|B}{m} , \qquad f = \frac{|q|B}{2\pi m} , \qquad P = \frac{2\pi m}{|q|B} .$$

It may be hard to memorize these quantities, but one can always rederive them at the drop of a hat by equating the magnitudes of the centripetal force and the magnetic force on the charge: i.e.,

a) $m v_{\rm per}^2 r = |q| v_{\rm per} B$. b) $m \dfrac{v_{\rm per}^2}{r} = |q| B$. c) $m \dfrac{v_{\rm per}^2}{r} = |q| v_{\rm per} B$.
d) $\dfrac{v_{\rm per}^2}{r} = |q| v_{\rm per} B$. e) $\dfrac{v_{\rm per}^2}{mr} = |q| v_{\rm per} B$.

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029 qmult 00450 1 1 1 easy memory: cyclotron radius

1017. A charged particle (charge q and mass m) in a uniform magnetic field $\vec{B}$ with velocity $\vec{v}$ perpendicular to the field will move in a circle with the cyclotron radius given by:

a) $\dfrac{mv}{|q|B}$. b) $\dfrac{2\pi m}{|q|B}$. c) $\dfrac{|q|B}{2\pi m}$. d) $q\vec{v} \times \vec{B}$. e) $\dfrac{\mu_0 I}{2\pi r}$.

---

029 qmult 00460 1 4 1 easy deducto-memory: aurora and helixing charged particles
Extra keywords: aurora

1018. The aurora:

a) is essentially caused by **CHARGED** particles helixing under the influence of the Earth's magnetic field into the atmosphere mainly at high latitudes and causing massive atmospheric fluorescence.
b) is essentially caused by **NEUTRAL** particles helixing under the influence of the Earth's magnetic field into the atmosphere mainly at high latitudes and causing massive atmospheric fluorescence.
c) is caused by magnetic fields swishing the trees in the Tiaga.
d) is caused by magnetic fields swishing the caribou.
e) is caused by electric fields swishing the caribou.

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029 qmult 00470 1 4 2 easy deducto-memory: crossed fields
Extra keywords: crossed fields, crossed electromagnetic fields

1019. Crossed fields is an expression that customarily means:

a) two electric fields that are perpendicular to each other.
b) perpendicular electric and magnetic fields.
c) two magnetic fields that are perpendicular to each other.
d) parallel electric and magnetic fields.
e) oblique electric and magnetic fields.

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029 qmult 0480 1 4 4 easy deducto-memory: electron discovery
Extra keywords: electron discovery, J. J. Thomson

1020. The discovery of the electron is usually credited to:

a) Irishman George Johnstone Stoney in the 1874.
b) American Ben Franklin in the 1760's.
c) Englishman J.J. Thomson in 1997.
d) Englishman J.J. Thomson in 1897.
e) New Zealander Ernest Rutherford in more or less 1911.

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029 qmult 00500 1 4 5 easy deducto-memory: magnetic force on a wire 1
Extra keywords: magnetic force law.

1021. The magnetic force law for a current I on an infinitely thin differential current path $d\vec{s}$ is:

a) $d\vec{F} = dq\vec{E}$. b) $d\vec{F} = q\vec{v} \times \vec{E}$. c) $d\vec{F} = q\vec{v} \cdot (d\vec{s} \times \vec{B})$. d) $d\vec{F} = (\vec{v} \cdot d\vec{s}) \times \vec{B}$.
e) $d\vec{F} = Id\vec{s} \times \vec{B}$.

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029 qmult 00510 1 4 4 easy deducto-memory: magnetic force on a wire 2
Extra keywords: Suitable for calculus-based only. SJ-820.

1022. The magnetic force law for an infinitely thin differential current I segment of path $d\vec{s}$ is:

a) $\vec{F} = q\vec{E}$. b) $\vec{F} = q\vec{v} \times \vec{E}$. c) $\vec{F} = q\vec{v} \times \vec{B}$. d) $d\vec{F} = I\, d\vec{s} \times \vec{B}$.
e) $d\vec{F} = I\, d\vec{s} \times \vec{E}$.

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029 qmult 00520 1 3 3 easy math: magnetic force on a wire 3

1023. An thin wire of length 2.0 m carries a current of 5.0 A. There is a uniform magnetic field of 0.20 T at an angle of 30° to the wire. What is the magnitude of the magnetic force on the wire?

a) 5.0 N. b) 0.20 N. c) 1.0 N. d) 30.0 N. e) 2.0 N.

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029 qmult 00630 1 4 2 easy deducto-memory: magnetic moment
Extra keywords: magnetic moment

1024. For a magnetic moment:

a) $\vec{\tau} = \vec{\mu} \times \vec{E}$ and $U = -\vec{\mu} \cdot \vec{E}$. b) $\vec{\tau} = \vec{\mu} \times \vec{B}$ and $U = -\vec{\mu} \cdot \vec{B}$.
c) $\vec{\tau} = -\vec{\mu} \cdot \vec{B}$ and $U = \vec{\mu} \times \vec{B}$. d) $\vec{\tau} = -\vec{\mu} \cdot \vec{E}$ and $U = \vec{\mu} \times \vec{E}$.
e) $\vec{\tau} = U$ and $U = \vec{\mu} \times \vec{E}$.

029 qmult 00730 1 4 1 easy deducto-memory: Hall effect
Extra keywords: Hall effect

1025. The Hall effect showed that:

a) the charge carriers in metals had negative charge.
b) the charge carriers in metals had positive charge.
c) the charge carriers in metals had neutral charge.
d) the charge carriers in metals had both negative and positive charge.
e) metals had no charge carriers.

Full-Answer Problems

029 qfull 00400 2 3 0 mod math: electron in crossed E and B fields

1026. A free electron is in a region of uniform electric and magnetic fields. (Recall $e = 1.602 \times 10^{-19}\,\mathrm{C}$ and $m_e = 9.109 \times 10^{-31}\,\mathrm{kg}$.) The system is described by Cartesian coordinates. The electron's initial velocity is $(0.0, 5.0, 5.0) \times 10^3\,\mathrm{m/s}$ and it has a constant acceleration in the x-direction of $2.0 \times 10^{12}\,\mathrm{m/s^2}$. In the y-z plane it is merely circling. The magnetic field is $(400, 0, 0)\,\mu\mathrm{T}$: note the given units are **MICROTESLAS**.

a) Draw a diagram of the system.

b) What is the electric field, magnitude and direction? **HINT:** If any component of the electric field were in the y-z plane, could the electron just be circling there?

c) What is the speed in the y-z plane at any time?

d) What is the radius of the circular motion and what is the direction of circling?

029 qfull 00410 2 3 0 mod math: mass spectrometer, analytic and numerical
Extra keywords: mass spectrometer, analytic and numerical

1027. A particle of charge q is accelerated through an electric potential V.

a) What is the expression for its final speed assuming it started from rest?

b) The particle is then injected into a mass spectrometer which is a chamber with a uniform magnetic field $\vec{B}$. The field is perpendicular to the particle motion. The particle travels for half a circle and strikes a wall. The radius of the circle is measured. What is the formula (with velocity eliminated) for the mass of the particle?

c) Assume the particle is a doubly charged positive ion (i.e., positive charge $2e$), the potential was $10^5\,\mathrm{V}$, the magnetic field strength was $0.1\,\mathrm{T}$, and the **DIAMETER** of the circle was $1.5\,\mathrm{m}$. What is the mass of the particle?

029 qfull 00430 2 5 0 moderate thinking: beam of electrons

1028. A beam of electrons is aligned in a plane perpendicular to the local direction of the Earth's magnetic field. This magnetic field is constant over the size of the beam system to very high accuracy. The beam was originally created to fly in a straight line. You place yourself so that you are looking down on the beam and **IN** the direction of the magnetic field.

NOTE: There are parts a,b,c. The some of the parts can be done independently. So don't stop if you can't do a part.

a) The kinetic energy of each electron is $12.0\,\mathrm{keV}$. What is their speed? **NOTE:** $q_e = -e = -1.6021766208(98) \times 10^{-19}\,\mathrm{C}$ and $m_e = 9.10938356(11) \times 10^{-31}\,\mathrm{kg}$.

b) Qualitatively what will happen to the beam as it travels in the magnetic field? In what direction does this happening happen?

c) Given that the magnetic field magnitude is 55.0 μT, what is the magnitude of the maximum displacement of the electron from its starting point and how long does the electron take to return to its starting point?

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029 qfull 00600 1 3 0 easy math: Earth current, Earth magnetic moment

1029. The magnetic moment of the Earth is 8.0×10^{22} J/T. As a simple model of the Earth's current assume that it flows in a circular path of radius 3500 km in the Earth's core. Calculate this current.

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029 qfull 00630 3 5 0 super tough think: arbitrary loop magnetic torque

1030. Prove (to general satisfaction if not with complete rigor) that the magnetic torque about an axis through the center of mass on an arbitrary planar loop of current in a uniform magnetic field is

$$\vec{\tau} = IA\hat{n} \times \vec{B} \ ,$$

where I is the current in the loop, A is the area of the loop, and $\hat{n}$ the normal to the loop. The sense of the normal is fixed by a right-hand rule: curl your right-hand fingers with the current direction and your thumb points in the direction of the normal. **HINTS:** Imagine replacing the loop by long, thin rectangular loops of current I, where the long side is in the plane defined by the B-field and the normal and the short side is perpendicular to this plane. The currents of the rectangles all have the same sense as the current of the original loop. ("Sense" in mathematical jargon is direction when there are only two possibilities.) The rectangles tile the original loop with their short sides and end bits of long sides tracing the original loop as closely as possible. These rectangles are bound rigidly together by some unspecified force. We only have to be concerned with torque component along the axis perpendicular to the magnetic field and the normal to the loop: let us call this the pivot axis for brevity. All the other torque components cancel to zero though this isn't so easy to see without a diagram. A diagram of the replacement loop would help.

Chapt. 30 Currents Causing Magnetic Fields

Multiple-Choice Problems

030 qmult 00100 1 1 5 easy memory: electro-magnetic connection
Extra keywords: physci if magnetism is covered

1031. Hans Christian Ørsted (1777–1851) in 1820 found that a current (of electric charge) generated:

a) a Bessel function. b) a Hermite polynomial. c) Michael Faraday.
d) a Hooke's law force. e) a magnetic force.

030 qmult 00110 1 1 4 easy memory: cause of magnetic fields

1032. Magnetic fields are caused by time-varying electric fields and by:

a) magnetic monopoles. b) magnetic torques. c) static charge distributions.
d) currents and elementary particle magnetic dipoles. e) streams.

030 qmult 00120 1 1 1 easy memory: Biot-Savart law

1033. The Biot-Savart law is:

a) $d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{I\, d\vec{s}\times\hat{r}}{r^2}$. b) $\vec{E}(\vec{r}) = \dfrac{kq}{r^2}\hat{r}$. c) $\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{\rm enc}$.
d) $\oint \vec{E}\cdot d\vec{A} = \dfrac{q_{\rm enc}}{\varepsilon_0}$. e) $E = mc^2$.

030 qmult 00130 1 4 3 easy deducto-memory: Ampere's law
Extra keywords: Ampère's law

1034. Ampère's law is:

a) $d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{I\, d\vec{s}\times\hat{r}}{r^2}$. b) $\vec{E}(\vec{r}) = \dfrac{kq}{r^2}\hat{r}$. c) $\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{\rm enc}$.
d) $\oint \vec{E}\cdot d\vec{A} = \dfrac{q_{\rm enc}}{\varepsilon_0}$. e) $E = mc^2$.

030 qmult 00140 1 1 1 easy memory: Ampere's law and generality

1035. Introductory physics derivations of _____ from _____ (a law which actually applies exactly only for time independent or steady currents) demand that the line integral of the magnetic field be around a planar closed contour and that the current density distribution be steady and not vary in the direction perpendicular to the plane of the contour. In fact, the general version of blank the first is the more fundamental law from which blank the second is derived in higher level physics courses.

a) Ampère's law; the Biot-Savart law b) the Biot-Savart law; Ampère's law
c) Gauss's law; Ampère's law d) Ampère's law; Gauss'law
e) French's 1st law; French's 2nd law

030 qmult 00150 1 4 5 easy deducto-memory: Ampere's law usage

1036. "Let's play *Jeopardy*! For $100, the answer is: It is an integral formula that allows one to directly calculate the magnetic field for a few current systems of very high symmetry."

What is the _____, Alex?

a) Jude law b) differential equation form of the Jude law
c) integral equation form of the Jude law
d) differential equation form of Ampère's law
e) integral equation form of Ampère's law

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030 qmult 00200 2 1 1 moderate memory: straight wire B-field 1

1037. The magnitude of the magnetic field outside of an axially-symmetric, infinite straight wire (with current I, coordinate r perpendicular to the wire, and coordinate z along the wire) which can be found directly found easily by an application of Ampère's law is:

a) $B = \dfrac{\mu_0 I}{2\pi r}$. b) $B = \dfrac{\mu_0 I\theta}{4\pi r}$. c) $B = \mu_0 nI$. d) $B = \dfrac{\mu_0 NI}{2\pi r}$. e) $\vec{B} = \dfrac{\mu_0 \vec{\mu}}{2\pi z^3}$.

---

030 qmult 00210 1 1 2 easy memory: straight wire B-field 2

1038. The magnitude of the magnetic field of an axially-symmetric, infinite straight wire (with current I, coordinate r perpendicular to the wire, coordinate z along the wire) which can be found directly found easily by an application of Ampère's law is:

a) $B = N\dfrac{\mu_0 I}{2z}$. b) $B = \dfrac{\mu_0 I}{2\pi r}$. c) $B = \mu_0 nI$. d) $B = \mu_0 I$. e) $B = I$.

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030 qmult 00220 1 1 2 easy memory: magnetic field lines about wire

1039. The magnetic field lines about an circularly-symmetrc infinite wire are circles whose sense is determined by the directions of the fingers used with:

a) a left-hand rule where the thumb aligns with the current.
b) a right-hand rule where the thumb aligns with the current.
c) a right-hand rule where the thumb aligns opposite to the current direction.
d) wind direction.
e) guesswork.

---

030 qmult 00230 1 1 4 easy memory: wire attraction

1040. Parallel current-carrying wires will attract if the currents are parallel and repel if the currents are antiparallel. It's easy to remember this since if you imagine deforming the wires into loops, you will qualitatively have the same attraction-repulsion situation and the poles of the loops are identified by:

a) geometry. b) symmetry. c) a left-hand rule. d) a right-hand rule.
e) an up-down rule.

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030 qmult 00240 1 4 5 easy deducto-memory: ampere defining force
Extra keywords: ampere defining force

1041. What is the magnitude of the force per unit length between two wires one meter apart each with a current of 1 A in the same direction?

a) 1 N. b) 1 lb. c) 1 kg. d) 2×10^7 N/m. e) 2×10^{-7} N/m.

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030 qmult 00320 1 4 2 easy deducto-memory: solenoid defined
Extra keywords: mathematical physics

1042. "Let's play *Jeopardy*! For $100, the answer is: It is a long coil of wire with a current running through it and a nearly constant magnetic field inside."

What is a/an _____, Alex?

a) commutator b) solenoid c) diode d) triode e) electric motor

---

030 qmult 00330 2 1 3 moderate memory: magnetic field in ideal solenoid

1043. The magnitude of the magnetic field inside an ideal solenoid (with current I, N turns, n turns per unit length, r the coordinate perpendicular to the central axis, and z the coordinate along the central axis) is given by:

a) $B = \dfrac{\mu_0 I}{2\pi r}$. b) $B = \dfrac{\mu_0 I\theta}{4\pi r}$. c) $B = \mu_0 nI$. d) $B = \dfrac{\mu_0 NI}{2\pi r}$. e) $\vec{B} = \dfrac{\mu_0 \vec{\mu}}{2\pi z^3}$.

---

030 qmult 00340 1 1 1 easy memory: right-rule for solenoids

1044. The magnetic field lines running through the middle region of a solenoid have their sense determined the direction of the thumb used with:

a) a right-hand rule in which the fingers curl in the direction of current flow.
b) a left-hand rule in which the fingers curl in the direction of current flow.
c) a right-hand rule in which the fingers curl in the opposite to the direction of current flow.
d) wind speed.
e) guesswork.

---

030 qmult 00420 1 4 5 easy deducto-memory: magnetic material
Extra keywords: mathematical physics

1045. "Let's play *Jeopardy*! For $100, the answer is: A material in which the atomic-scale magnetic dipoles can (but not necessarily do) align to create a strong net macroscopic magnetic field."

What is a/an _____, Alex?

a) electric material b) rock c) current d) copper e) ferromagnetic material

---

030 qmult 00430 1 1 3 easy memory: soft magnetic material

1046. A magnetic material that tends **NOT** to remain magnetized in the absence of an applied field is called:

a) mushy. b) hard. c) soft. d) bold. e) hardy.

---

030 qmult 00800 1 1 3 easy memory: railgun

1047. A railgun is a magnetic:

a) pistol. b) water pistol. c) launching device. d) lunching device.
e) train robber in the old West: i.e., that old charmer Butch Cassidy.

---

030 qmult 10200 2 3 4 moderate math: four wires magnetic force per unit length mult

1048.** There are **FOUR** infinitely long, straight, vanishingly thin wires each carrying current I running parallel to each other: the currents all flow in the same direction. The projection of the wires unto a plane perpendicular to the wires gives four dots at the corners of a **SQUARE** of side a. **HINT:** Draw a diagram.

i) Which way must the net force on any one of the wires point? **HINT:** Exploit symmetry.

ii) What is the magnitude of the force per unit length f on any one of the wires?

a) (i) Away from the square center. (ii) $f = \dfrac{\mu_0 I^2}{2\pi a}\dfrac{3}{\sqrt{2}}$.

b) (i) Away from the square center. (ii) $f = \dfrac{\mu_0 I^2}{2\pi a}\dfrac{(2\sqrt{2}+1)}{\sqrt{2}}$.

c) (i) To the square center. (ii) $f = \dfrac{\mu_0 I^2}{2\pi a}\dfrac{(2\sqrt{2}+1)}{\sqrt{2}}$.

d) (i) To the square center. (ii) $f = \dfrac{\mu_0 I^2}{2\pi a}\dfrac{3}{\sqrt{2}}$.

e) (i) Parallel to the current (ii) $f = \dfrac{\mu_0 I^2}{2\pi a}\dfrac{(2\sqrt{2}+1)}{\sqrt{2}}$.

---

030 qmult 10202 2 3 3 moderate math: three wires magnetic force per unit length mult

1049.** There are **THREE** infinitely long, straight, vanishingly thin wires each carrying current I running parallel to each other: the currents all flow in the same direction. The projection of the wires unto a plane perpendicular to the wires gives three dots at the corners of an

EQUILATERAL TRIANGLE of side a. The angles at the corners of the triangle are all 60°, of course. **HINT:** Draw a diagram.

i) Which way must the net force on any one of the wires point? **HINT:** Exploit symmetry.

ii) What is the magnitude of the force per unit length f on any one of the wires?

a) (i) Away from the triangle center. (ii) $f = \dfrac{\mu_0 I^2}{2\pi a} \times \sqrt{3}$.

b) (i) Away from the triangle center. (ii) $f = \dfrac{\mu_0 I^2}{2\pi a} \times 2$.

c) (i) To the triangle center. (ii) $f = \dfrac{\mu_0 I^2}{2\pi a} \times \sqrt{3}$.

d) (i) To the triangle center. (ii) $f = \dfrac{\mu_0 I^2}{2\pi a} \times \dfrac{3}{\sqrt{2}}$.

e) (i) To the triangle center. (ii) $f = \dfrac{\mu_0 I^2}{2\pi a}$.

Full-Answer Problems

030 qfull 00100 3 3 0 tough math: finite straight wire magnetic field

1050. A thin, straight wire of length L carries current I. The current comes out of nowhere and goes into nowhere which is really unrealistic.

a) Draw a diagram of the system. Describe the magnetic field lines everywhere.

b) Find the expression for the magnetic field at a point a distance R away from the **CENTER** of the wire along a line perpendicular to the wire. **HINTS:** Ampère's law can't be used because the current is discontinuous. So divide and conquer. You might need the table integral

$$\int \frac{dx}{(ax^2+b)^{3/2}} = \frac{x}{b\sqrt{ax^2+b}} \ .$$

c) If $L \to \infty$, what does the magnetic field become and where along the wire does it apply?

030 qfull 00200 2 3 0 moderate math: four wires magnetic force per unit length

1051. There are four infinitely long, straight, vanishingly thin wires each carrying current I running parallel to each other: the currents all flow in the same direction. The projection of the wires unto a plane perpendicular to the wires gives four dots at the corners of a square of side a.

a) Draw a diagram with the wires running straight into the page and indicate this straight-into-the-page condition with appropriate symbol.

b) Which way must the net force on any one of the wires point? **HINT:** Exploit symmetry.

c) What is the magnitude of the force per unit length f on any one of the wires.

d) Given $I = 10.0\,\mathrm{A}$ and $a = 0.01\,\mathrm{m}$, what is f?

030 qfull 00300 2 3 0 moderate thinking: two cylindrical currents

Extra keywords: two cylindrical currents, superposition, Ampère's law

1052. Say that you have an infinitely long cylindrical conductor with uniform current density $\vec{J}$ with direction aligned with the symmetry axis and radius a.

a) What is the magnetic field at a distance r from center inside and outside the conductor expressed using J? Justify the use of Ampère's law and remember to give the direction of the field.

b) Imagine that you superimpose another cylindrical conductor of radius b on the first. The two cylinders have aligned symmetry axes. The second cylinder has current $-\vec{J}$ with direction aligned with the symmetry axis. The second cylinder is smaller than the first cylinder and entirely inside it, but off-center by an amount c: i.e., c is the distance from the center of the first cylinder to the center of the superimposed one. What is the magnetic field at the center of second cylinder due to its own current only? What is the net magnetic field at the center of second cylinder? What is net current density inside the second cylinder? **HINT:** Draw a diagram.

c) What is the net magnetic field inside the second cylinder at any point along the line from the center of the first cylinder and passing through the second cylinder's center? Be very careful about getting the signs of the components correct.

d) What is the net current of the two cylinders?

---

030 qfull 00400 3 3 0 tough math: magnetic field along solenoid axis

1053. You have a solenoid of length L, N turns, current I in each turn, and radius R.

a) What is the current per unit length i? (I mean the current flowing around the solenoid if that helps.)

b) In what direction does the magnetic field point along the axis of the solenoid?

c) Let z be the coordinate along the axis. It is positive in the direction of the field and $z = 0$ is at the center of the solenoid. What is the magnetic field strength at a point z due to a differential bit of solenoid at z'? Recall that for single coil the magnetic field strength on-axis at a distance ρ from the center is

$$B(\rho) = \frac{\mu_0 I R^2}{2\left(R^2 + \rho^2\right)^{3/2}} \ .$$

The direction of the field is along the z-axis.

d) By integration find the total field strength at z due to the solenoid. **HINTS:** Divide and conquer. You might need the table integral

$$\int \frac{dx}{(ax^2 + b)^{3/2}} = \frac{x}{b\sqrt{ax^2 + b}} \ .$$

e) What is the limiting form if $L \to \infty$ for fixed z.

f) For extra credit (in insight, not marks) just guess what the far-field limit of the field strength is. Actually the answer depends on whether $R \gtrsim L$ or $R \lesssim L$. Don't bother trying to work out the limiting expressions: it's too tedious to do it right.

Chapt. 31 Faraday's Law of Induction and Inductors

Multiple-Choice Problems

031 qmult 00100 1 1 2 easy memory: Faraday's law of induction 1

1054. The general form of _____ is

$$\mathcal{E} = \oint (\vec{E} + \vec{v} \times \vec{B}) \cdot d\vec{s} = -\frac{d\Phi}{dt} \ ,$$

where $\mathcal{E}$ is the induced emf around a closed contour, $\vec{E}$ is the Maxwell-Faraday law induced electric field, $\vec{v}$ is the velocity of the contour in the observer's inertial frame of reference, $\vec{B}$ is the magnetic field, Φ is the magnetic flux linked by the contour, the time derivative includes variations in flux (as seen in the observer's frame) due to changing magnetic field (giving the Maxwell-Faraday law of induction) and due the contour motion (giving the motional emf Faraday law of induction), and the minus sign along with other conventions gives the direction of the emf in the contour.

a) Ampère's law b) Faraday's law of induction c) Gauss's law
d) Gauss's law for magnetism e) Lenz's law

---

031 qmult 00110 1 1 3 easy memory: Faraday's law of induction 2

1055. It follows from Faraday's law of induction that a changing magnetic field will cause an:

a) deduced emf. b) reduced emf. c) induced emf. d) adduced emf.
e) seduced emf.

---

031 qmult 00120 1 1 2 easy memory: Faraday's law of induction 3

1056. Faraday's law of induction (in its Maxwell-Faraday equation version) implies that time-varying _____ induce _____.

a) electric fields; magnetic fields b) magnetic fields; electric fields
c) magnetic fields; capacitor fields d) magnetic fields; inductor fields
e) electric fields; capacitor fields

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031 qmult 00124 1 1 2 easy memory: Faraday's law of induction 4

Extra keywords: Faraday's law of induction formula for algebra-based

1057. Faraday's law of induction is:

a) $B = \dfrac{\mu_0 I}{2\pi r}$. b) $\mathcal{E} = -\lim\limits_{\Delta t \to 0} \dfrac{\Delta \Phi}{\Delta t}$. c) $\mathcal{E} = \dfrac{\mu_0 I}{2\pi r}$. d) $\mathcal{E} = -\lim\limits_{\Delta t \to 0} \dfrac{\Delta B}{\Delta t}$.
e) $E = mc^2$.

---

031 qmult 00130 1 1 2 easy memory: Maxwell-Faraday equation

Extra keywords: Maxwell-Faraday law of induction formula

1058. The Maxwell-Faraday equation (which is one version of Maxwell's law of induction) is:

a) $d\vec{B} = \dfrac{\mu_0}{4\pi} \dfrac{I\, d\vec{s} \times \hat{r}}{r^2}$. b) $\oint \vec{E} \cdot d\vec{s} = -\dfrac{d}{dt} \int B \cdot d\vec{A}$. c) $\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{enc}}$.
d) $\oint \vec{E} \cdot d\vec{A} = \dfrac{q_{\text{enc}}}{\varepsilon_0}$. e) $E = mc^2$.

---

031 qmult 00132 1 1 1 easy memory: induced electric field

1059. The induced electric field in _____ is not caused by charge, but by a time-varying magnetic field in the inertial frame of the observer. But it is just like a charge-caused electric field in fundamental respects. It cases an electric force on a point charge according to the formula $q\vec{E}$. It has the same vacuum energy density $(1/2)\varepsilon E^2$. It source is different. But induced electric fields has a different distribution in space. Charge-caused electric fields from static charge have a distribution that allows potential (and thereby potential energy) to be defined which are actually convenient accounting mechanisms for electric field energy. The condition for allowing potential to be defined is that path integral

$$\int_{\rm A}^{\rm B} \vec{E} \cdot d\vec{s}$$

be independent of the path between between general points A and B or equivalently that

$$\oint \vec{E} \cdot d\vec{s} = 0$$

for any closed contour. Induced electric fields do not obey these equations and a potential cannot be defined for them in general although it can in restricted ways that we won't go into here. The electric field lines of induced electric fields formed closed loops or perhaps in some cases extend to infinity.

a) the Maxwell-Faraday equation b) Gauss's law c) Gauss's law for magnetism
d) Lenz's law e) Boltzmann's equation

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031 qmult 00134 1 4 5 easy deducto-memory: Maxwell-Faraday equation status

1060. "Let's play *Jeopardy*! For $100, the answer is: In classical electrodynamics, it is an exact fundamental law or axiom that cannot be derived. It is one of Maxwell's four equations of classical electromagnetism. These equations account for all of classical electromagnetism although one must take the electrical and magnetic properties of materials as givens. These properties cannot be understood at a fundamental level in classical physics."

What is _____, Alex?

a) Zipf's law b) the Bernoulli equation c) Gauss's law for gravity
d) Lenz's law e) the Maxwell-Faraday equation

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031 qmult 00140 1 1 2 easy memory: Faraday's law of induction formula
Extra keywords: Faraday's law of induction formula for algebra-based

1061. Faraday's law of induction is:

a) $B = \dfrac{\mu_0 I}{2\pi r}$. b) $\mathcal{E} = -\lim\limits_{\Delta t \to 0} \dfrac{\Delta \Phi}{\Delta t}$. c) $\mathcal{E} = \dfrac{\mu_0 I}{2\pi r}$. d) $\mathcal{E} = -\lim\limits_{\Delta t \to 0} \dfrac{\Delta B}{\Delta t}$.
e) $E = mc^2$.

---

031 qmult 00150 1 1 4 easy memory: magnetic flux defined
Extra keywords: physci

1062. Magnetic flux is defined by the formula:

a) $d\vec{\Phi} = \vec{E} \times d\vec{A}$. b) $\oint \vec{E} \cdot d\vec{s} = -\dfrac{d}{dt} \int B \cdot d\vec{A}$. c) $d\Phi = \vec{E} \cdot d\vec{A}$.
d) $d\Phi = \vec{B} \cdot d\vec{A}$. e) $d\vec{\Phi} = \vec{B} \times d\vec{A}$.

---

031 qmult 00152 1 1 4 easy memory: magnetic flux unit
Extra keywords: physci

1063. The units of magnetic flux are:

a) volts (V=J/C). b) joules (J). c) newtons per coulomb (N/C).
d) webers (Wb). e) oersteds (Oe).

---

031 qmult 00154 1 1 1 easy memory: magnetic flux algebra-based
Extra keywords: algebra-based

1064. The magnetic flux of uniform magnetic field $\vec{B}$ through a planar area segment of vector area $\vec{A}$ (which has magnitude equal to the area and direction normal to the plane in whichever sense is convenient) is given by:

a) $\Phi = \vec{A} \cdot \vec{B}$. b) $\Phi = A/B$. c) $\Phi = \vec{A} \times \vec{B}$. d) $\Phi = B/A$. e) $\Phi = \vec{A}\vec{B}$.

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031 qmult 00160 1 1 2 easy memory: unique magnetic flux linked

1065. The verb _____ used with the subject contour or loop in the context of magnetic flux means that contour bounds a surface area through which magnetic flux is being evaluated. For example, "This wire _____s the magnetic flux". This usage of _____ is common, but rarely defined it seems, except by implication.

a) baffle b) link c) snark d) chain e) snerf

---

031 qmult 00170 1 3 3 easy math: magnetic flux calculation

1066. A uniform magnetic field has magnitude 0.10 T and there is a planar area segment of area $0.040\,\mathrm{m}^2$ embedded in this field. The normal of the area segment is at an angle of 60° to the magnetic field direction. What is the magnetic flux through the segment?

a) 3.5×10^{-3} Wb. b) 4.0×10^{-3} Wb. c) 2.0×10^{-3} Wb. d) 0.4 Wb.
e) 2.5 Wb.

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031 qmult 00200 1 4 4 easy deducto-memory: Lenz's law 1
Extra keywords: Lenz's law

1067. "Let's play *Jeopardy*! For $100, the answer is: The direction of an induced emf is such that the induced current it would cause (if there were a conducting closed loop present on the contour where the Faraday's law was being applied) would create an induced magnetic field to oppose the change in the linked magnetic flux giving rise to the induced emf."

a) What is Bernoulli principle, Alex?
b) What is Fermat's last theorem, Alex?
c) What is Schubert's unfinished last symphony, Alex?
d) Lenz's—excuse me, what is Lenz's law, Alex?
e) What is Faraday's 2nd law of induction, Alex?

---

031 qmult 00210 1 1 5 easy memory: Lenz's law 2

1068. If one strictly followed all the sign conventions that go with Faraday's law of induction (which is a tricky thing to do), one could do without _____ which is really just a consequence of Faraday's law and not an independent law.

a) Bell's theorem b) Hund's rules c) Fermi's golden rule d) Ampère's law
e) Lenz's law

---

031 qmult 00220 1 5 4 easy thinking: Lenz's law application 1

1069. A uniform magnetic field is increasing in magnitude and it points in the positive x direction. The sense or polarity of the induced emf around any closed path in matterless space in the y-z plane is:

a) counterclockwise looking in either the positive or negative x direction.
b) counterclockwise looking the negative x direction.
c) clockwise looking the positive x direction.
d) counterclockwise looking the positive x direction.
e) undefined since there is no emf in matterless space.

---

031 qmult 00300 1 1 3 easy memory: motional emf Faraday's law

1070. The _____ is

$$\mathcal{E} = \oint (\vec{v} \times \vec{B}) \cdot d\vec{s} = -\frac{d\Phi}{dt} \ ,$$

where $\mathcal{E}$ is the induced emf around a closed contour, $\vec{v}$ is the velocity of the contour in the observer's inertial frame of reference, $\vec{B}$ is the magnetic field which is constant in time in the observer's inertial frame, Φ is the magnetic flux linked by the contour, the time derivative is for variations in the contour with time. and the minus sign along with other conventions gives the direction of the emf in the contour. Unlike the Maxwell-Faraday equation Faraday's law, the _____ is not a fundamental law of classical electromagnetism. It a derived result from the fundamental laws of classical electromagnetism (i.e., Maxwell's equations). If the conditions of the derivation are not met, _____ can fail. This is not a failure of classical electromagnetism which is exact in the classical limit, but just a case where a particular result (the motional emf Faraday's law does not a apply). It is a remarkable fact that both versions Faraday's law have the same appearance,

$$\mathcal{E} = -\frac{d\Phi}{dt} \ ,$$

and can be used in similar ways. Lenz's law works for the both Faraday's law versions.

a) Gauss law b) Maxwell-Faraday equation Faraday's law
c) motional emf Faraday's law d) Gauss law for magnetism e) Ampère law

---

031 qmult 00310 1 1 4 easy memory: motional emf gone into

1071. Emfs are rather strange quantities. They are path integrals of a force per unit point charge where the integral is done at one instant in time. They are useful for calculating energy input and outputs to the form of electrical kinetic and potential energy in steady or approximately steady current flow situations. The motional emf of the motional emf Faraday's law is particularly tricky. It is a magnetic force per unit charge. But the magnetic force is velocity dependent. What velocity does one use? If one prescribes a path and a velocity at each point along that path, one can calculate the magnetic emf for those conditions. But in general that will not be a useful emf. The useful magnetic emf is one that follows that path that charge carriers actually will flow along. Say charge carriers following a path that have velocity

$$\vec{v}_{\rm car} = \vec{v}_{\rm rel} + \vec{v} \ ,$$

where $\vec{v}_{\rm rel}$ is the carrier relative to the path and $\vec{v}$ is the velocity of the path. The differential magnetic emf along the path (which is at one instant in time recall) is

$$(\vec{v}_{\rm rel} + \vec{v}) \times \vec{B} \cdot d\vec{s} \ ,$$

where $d\vec{s}$ is differential bit of path vector But since carriers are following the path

$$\vec{v}_{\rm rel} \times \vec{B} \cdot d\vec{s} = 0$$

since $\vec{v}_{\rm rel} \times \vec{B}$ is _____ to $d\vec{s}$. So for a useful magnetic emf which we call the motional emf, the differential emf is

$$\vec{v} \times \vec{B} \cdot d\vec{s} \ ,$$

where $\vec{v}$ is just the velocity of the path. This emf can inject or remove electric kinetic and potential energy. We calculate the motional emf for paths the particles take which in practical cases are usually wires or other conductors. Note that the magnetic force due to the relative velocity of the carriers can do things. It can cause a charge separation (i.e., a Hall effect charge separation) or for free carriers a helixing motion, but it can't contribute to the motional emf. If

the path is unmoving in the inertial frame of observation, then $\vec{v} = 0$, and there is no motional emf.

a) close b) antiparallel c) parallel d) perpendicular e) infinitesimally close

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031 qmult 00312 2 5 1 moderate thinking: induced emf and magnetic force

1072. Consider a wire loop held rigidly in place. The wire loop is idealized as infinitely narrow and completely confines the free charges inside of it so that they can only move along the wire. A changing magnetic field creates an induced emf in the wire. The charges have some motion caused by the induced emf. The magnetic field also causes a magnetic force on the moving charges. But in this case the charge motion **CANNOT** be affected by the magnetic force. This is because magnetic force is:

a) always **PERPENDICULAR** to the direction of motion of a charge and is always canceled by the confining force of the **WIRE**.
b) always **PARALLEL** to the direction of motion of a charge and is always canceled by the confining force of the **WIRE**.
c) always **PERPENDICULAR** to the direction of motion of a charge and is always canceled by the confining force of the **EMF FIELD**.
d) always **PARALLEL** to the direction of motion of a charge and is always canceled by the confining force of the **EMF FIELD**.
e) zero.

---

031 qmult 00320 2 3 2 moderate math: motional emf Faraday's law example 1

Extra keywords: Can be used for an algebra-based course

1073. Say you have two infinite parallel wires running in the x direction and a distance ℓ apart and two conducting bars. You lay the bars across the wires: they are perpendicular to the wires. There is a uniform magnetic field perpendicular to the bars and wires. You move the **LEFTMOST** bar to the **LEFT** at constant **SPEED** v_1 and the **RIGHTMOST** bar to the **RIGHT** at constant **SPEED** v_2. What is the **MAGNITUDE** of the induced emf around the loop formed by the wires and bars?

a) $\mathcal{E} = \ell(-v_1 + v_2)B$. b) $\mathcal{E} = \ell(v_1 + v_2)B$. c) $\mathcal{E} = \ell v_1 B$. d) $\mathcal{E} = (v_1 + v_2)B$.
e) $\mathcal{E} = \ell(v_1 + v_2)$.

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031 qmult 00330 2 3 4 moderate math: motional emf Faraday's law example 2

Extra keywords: Can be used for an algebra-based course.

1074. Say you have two infinite parallel wires a distance ℓ apart and two conducting bars. You lay the bars across the wires: they are perpendicular to the wires. There is a uniform magnetic field perpendicular to the bars and wires. You move the **LEFTMOST** bar to the **RIGHT** at constant **SPEED** v_1 and the **RIGHTMOST** bar to the **LEFT** at constant **SPEED** v_2. Assume the bars do **NOT** touch each other in the time frame of interest. What is the **MAGNITUDE** of the induced emf around the loop formed by the wires and bars?

a) $\mathcal{E} = \ell(-v_1 + v_2)B$. b) $\mathcal{E} = -\ell(v_1 + v_2)B$. c) $\mathcal{E} = \ell v_1 B$. d) $\mathcal{E} = \ell(v_1 + v_2)B$.
e) $\mathcal{E} = \ell(v_1 + v_2)$.

---

031 qmult 00410 1 1 3 easy memory: eddy currents 1

1075. Faraday law induced electric fields have electric field lines that form closed loops (or possibly extend to infinity). In fact, in cylindrically symmetry, the induced electric field's lines lines form circles (HRW-721). This is unlike electric field lines for charge-caused electric fields which only have non-zero field endings on charges or at infinity. There are no charges for the induced electric field's lines to start or end on. Charged particles in a region of induced electric fields will tend to move under the electric force as if in a whirlpool or a/an:

a) billy. b) mikey. c) eddy. d) fritzi. e) rudi.

---

031 qmult 00420 1 1 4 easy memory: eddy currents 2

1076. Pull a largish conductor through a region of non-uniform magnetic field and you will induce:

a) billy currents. b) mikey currents. c) fritzi currents. d) eddy currents. e) rudi currents.

---

031 qmult 00500 1 4 2 easy deducto-memory: electric generator

Extra keywords: mathematical physics

1077. "Let's play *Jeopardy*! For $100, the answer is: A device for turning macroscopic kinetic energy into electrical energy."

What is a/an _____, Alex?

a) electric motor b) electric generator c) inductor d) transformer e) magnetic core

---

031 qmult 00510 1 1 3 easy memory: electric generator emf curve

1078. The behavior of the emf of simple conventional electric generators with time is:

a) linear. b) quadratic. c) sinusoidal. d) inverse linear. e) cubic.

---

031 qmult 00600 1 4 2 easy deducto-memory: electric motor 1

Extra keywords: mathematical physics

1079. "Let's play *Jeopardy*! For $100, the answer is: It is a device that turns electric energy into macroscopic kinetic energy."

What is a/an _____, Alex?

a) electric generator b) electric motor c) commutator d) half-ring e) brush

---

031 qmult 00610 1 4 1 easy deducto-memory: electric motor 2

Extra keywords: mathematical physics

1080. "Let's play *Jeopardy*! For $100, the answer is: It is a device that consists of a rotating coil with an alternating current that is located in a magnetic field. It outputs kinetic energy."

What is a/an _____, Alex?

a) electric motor b) solenoid c) commutator d) half-ring e) brush

---

031 qmult 00620 1 4 4 easy deducto-memory: electric motor energy

Extra keywords: electric motor, induced emf, conservation of energy

1081. The physical reason why an electric motor does not give energy for free is:

a) the magnetic field induces a back emf in the rotating coil which forces the current source to expend energy ideally **JUST TWICE** what the motor puts out. Nature ends up **NOT CONSERVING** energy after all, but it does **NOT ALLOW** it to be transformed.

b) the magnetic field induces a back emf in the rotating coil which forces the current source to expend energy ideally **JUST EQUAL** to what the motor puts out. Nature ends up **CONSERVING** energy after all, but it does **NOT ALLOW** it to be transformed.

c) the magnetic field induces a back emf in the rotating coil which forces the current source to expend energy ideally **JUST EQUAL** to what the motor puts out. Nature ends up **NOT CONSERVING** energy after all, but it does **ALLOW** it to be transformed.

d) the magnetic field induces a back emf in the rotating coil which forces the current source to expend energy ideally **JUST EQUAL** to what the motor puts out. Nature ends up **CONSERVING** energy after all, but it does **ALLOW** it to be transformed.

e) the magnetic field induces a back emf in the rotating coil which forces the current source to expend energy ideally **JUST TWICE** what the motor puts out. Nature ends up **CONSERVING** energy after all, but it does **ALLOW** it to be transformed.

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031 qmult 10510 3 3 4 tough math: bike electric generator fullmult

1082.*** Consider a bike electric generator that powers a headlight. The emf of the generator is given by

$$\mathcal{E} = NAB\omega_{\text{gen}} \sin(\omega_{\text{gen}} t) \ ,$$

where $N = 100$ is the number of turns in the generator coil, $A = 1.2 \times 10^{-3}\,\text{m}^2$ is the coil area, $B = 0.10\,\text{T}$ is the magnitude of the magnetic field in the generator, ω_{gen} is the angular velocity of the the generator coil, and t is time. The maximum emf $\mathcal{E}_{\text{max}} = 4.0\,\text{V}$. The generator is cranked by a small wheel of radius $r_{\text{gen}} = 0.020\,\text{m}$ that is driven by the bike wheel of radius r_{bike}. There is a no-slip condition between the two wheels and the bike wheel and the ground. What is the translational speed of the bike?

a) 5.0 m/s. b) 4.5 m/s. c) 3.5 m/s. d) 6.7 m/s. e) 7.2 m/s.

Full-Answer Problems

031 qfull 00310 2 3 0 moderate math: motional emf for circular contour

1083. Since the motional emf Faraday's law is not actually a law but a derived result that does not apply in all cases, one actually has to prove it for new cases of application. One doesn't worry about this much in intro physics, but maybe one should. Here we prove the motional emf Faraday's law for a circular contour that has a varying radius r and is located in a uniform magnetic field perpendicular to the contour. The radial velocity of the contour is

$$v = \frac{dr}{dt} \ ,$$

This velocity is relative to an inertial frame. Let's say that the normal $\hat{n}$ to the contour points up and the magnetic field points down.

a) What is the motional emf around the contour in the counterclockwise direction looking down?

b) What is the magnetic flux linked by the contour. Remember the normal to the contour and the magnetic field direction are antiparallel.

c) For our special case prove the motional emf Faraday's law: i.e., prove

$$\mathcal{E} = -\frac{d\Phi}{dt} \ .$$

031 qfull 00320 3 3 0 tough math: proving motional emf faraday for rotating coil

1084. Since the motional emf Faraday's law is not actually a law but a derived result that does not apply in all cases, one actually has to prove it for new cases of application. One doesn't worry about this much in intro physics, but maybe one should. Here we prove the motional emf Faraday's law for a rotating planar contour in a uniform magnetic field. The contour is closed, of course, and the finite area linked by the contour is A. This is an important special case since it is a simple electric generator case.

a) Say that the magnetic field points in the positive x direction and the axis of rotation is the z axis. The contour is a planar rotating coil with angular velocity ω which does **NOT** have to be constant. The normal $\hat{n}$ to the contour is at polar coordinate θ from the x axis and for mental clarity can be thought of as having its tail on the z axis. Sketch a diagram of the system looking down from the positive z axis. Put the normal in the first quadrant.

b) Starting from the general differential motional emf

$$d\mathcal{E} = (\vec{v} \times \vec{B}) \cdot d\vec{s}$$

evaluate it in terms of B, ω, θ, the cylindrical radial coordinate r, and the differential dz. Note that there are two cases. The differential bit of contour on the side of the contour counterclockwise from $\hat{n}$ has velocity $-r\omega\hat{n}$ and that on the side of the contour clockwise from $\hat{n}$ has velocity in the direction $r\omega\hat{n}$. **HINT:** Note that $\hat{z}\cdot d\vec{s} = dz$ where dz could be implicitly positive for negative.

c) To absorb the two cases for the two sides of $\hat{n}$ in a single formula let's introduce coordinate y which is in the plane of the contour and perpendicular to the z axis. We let $y > 0$ for the counterclockwise side and $y < 0$ for the clockwise side. We can now right $\pm r$ as y. Give the formula for the differential motional emf now.

d) We want to integrate the differential emf over the contour by summing up the contributions to each dz interval first about each height z first and then summing these up for the total. We take dz to be positive for definiteness. Let's assume that we integrate around the contour in a counterclockwise direction when looking head-on at $\hat{n}$ or in other words in a right-hand rule sense with our right thumb along the $\hat{n}$ vector. The integration at first glance looks very tricky because our contour could be convoluted and can cross the height z numerous times. But it does collapse down to simplicity.

First, note that if you start following the contour from above z, then because the contour is closed you must cross z and even number of times to return to your start point.

Second, note that most lowest y crossing must be a downard crossing. The argument for this is tricky. Following the contour counterclockwise means that the inside (which is finite) is on your left as you follow the contour. In fact, this is really what going counterclockwise means for a general contour: after all at any point on the contour your direction in the plane could be any direction. Now say counterfactually that your most lowest y crossing was upward. This means that everything on your left is inside the contour, but there is, in fact, an infinite amount of area on your left below the z line that you can't return to close off since you are crossing the z line for the last time. The contradiction is resolved by admitting that the lowest y crossing was downward. The contour can now cross to the positiveward and continue to enclose finite area. Similarly the highest y crossing must be upward.

Third, what of cases where the contour just touches the z line and doesn't cross it. Well, these touches contribute zero volume to the integral and don't need to be worried about.

So the integration procedure is clear. For any height z, the contour crosses an even number of times. From the negative most y crossing, the crossings are always down followed by up. So add up the contributions in dz about z in down-up pairs started from the most negative y pair and going to the most positive y pair.

Enough preamble, now for the questions. Is the sum of each pair positive or negative? What is the sum of all the pairs? **HINT:** You can express this using dA the differential amount of area enclosed by the contour in the z interval dz.

e) Now integrate for the total motional emf. Express the result in terms of B, ω, θ, and contour area A.

f) Now find the magnetic flux linked by the contour and prove the motional emf Faraday's law for this case.

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031 qfull 00340 2 3 0 mod math: magnetic flux, growing loop 1: algebra course

Extra keywords: Algebra-based course. This is a motional emf problem. Needs fixing.

1085. There is a horizontal circular conducting loop (of one 1 turn) in a uniform magnetic field of magnitude B. (The field is applied: it is not due to the loop.) The magnetic field points straight up. The radius of the loop increases with time according to

$$r(t) = r_0\sqrt{\frac{t}{t_0}}\ ,$$

where r_0 is a fiducial (or reference) radius and t_0 is a fiducial (or reference) time. At $t = 0$, the loop has radius zero. At $t = t_0$, the loop has radius r_0.

a) What is the magnetic flux Φ due to the applied field linked by the loop in terms of the given variables B, r_0, t, and t_0?

b) What is magnitude of the induced emf $\mathcal{E}$ in the loop in terms of the given variables B, r_0, t, and t_0? What is the **SENSE** of the emf (i.e., what is the direction of current flow that it would cause) when viewing the loop in the direction of the applied B-field? **HINT:** Remember Lenz's law. Calculate the time-averaged induced emf first if you need to.

c) Given that the loop has constant resistance R, what is the power dissipated in the loop by the induced current in in terms of the given variables B, r_0, t, t_0, and R?

d) What is the total energy dissipated $E_{\rm dis}$ (i.e., lost to heat by the resistance) in the loop in the interval from $t = 0$ to t in terms of the given variables B, r_0, t, t_0 and R?

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031 qfull 00350 2 3 0 mod math: magnetic flux, growing loop 2

Extra keywords: The emf is motional emf not Faraday law emf

1086. There is a horizontal, infinitely thin, circular conducting loop in a uniform magnetic field $\vec{B}$. The field is constant in time. (The field is applied: it is not due to the loop.) The magnetic field points straight down. We take the normal to the planar area of the loop to point straight down too. The radius of the loop increases with time according to the formula $r = vt$. (At $t = 0$, the loop has zero radius.)

a) What is the magnetic flux linked by the loop as a **FUNCTION OF TIME** t?

b) What is induced emf in the loop as a **FUNCTION OF TIME** t? Give its direction when looking downward. Which version of Faraday's law is used to evaluate the emf? **HINT:** Remember Lenz's law and that an emf by definition is a quantity evaluated at one instant in time.

c) Given that the loop has constant resistance R (which is rather magical since the loop is growing), what is the current in the loop as a **FUNCTION OF TIME** t? Assume that Kirchhoff's voltage law and Ohm's law hold (which is true) even though there is not, in fact, any actual current-driving potential in this circuit. This means you can count effective voltage rises/drops across emfs going with/against the current and an effective potential drop given by Ohm's law occurs going with the current across a resistor.

d) What is the power dissipated in the loop by the induced current as a **FUNCTION OF TIME** t?

e) What is the total energy dissipated by the resistance in the loop in the interval from $t = 0$ to t?

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031 qfull 00360 2 3 0 moderate math: double resistor motional emf

1087. Consider two long parallel wires in the x-y plane. The wires are separated by distance $\ell = 0.500\,\text{m}$ and run in the x direction. There is uniform magnetic field that points in the $-z$ direction and has magnitude is $B = 2.00\,\text{T}$. A top view is in the $-z$ direction. We view the system from the top. A conducting bar is laid across the wires and is perpendicular to the wires (i.e., it is aligned with the y axis). Far to the left and right of the bar, the wires are connected by wire segments perpendicular to the wires (i.e., they are aligned with the y axis). The segment to the left has a resistor of resistance $R_1 = 3.00\,\Omega$ and the one to the right has resistance of $R_2 = 6.00\,\Omega$. The resistors give the only resistance in the system. The bar is moving to the right with constant velocity $v = 5.00\,\text{m/s}$. During the course of the problem, the bar never reaches rightward resistor.

a) Draw a diagram of the system.

b) Just for a moment let's speak generally. What is the formula for magnitude of the magnetic flux Φ for a uniform magnetic field of magnitude B over a surface area A perpendicular to the magnetic field direction? Now returning to our system of interest, what is the formula

$$\left|\frac{d\Phi}{dt}\right|$$

for our system in terms of B, v, and ℓ?

c) What is the numerical value of the current through resistor R_1 and what is its direction? What is the numerical value of the current through resistor R_2 and what is its direction? What is the numerical value of the current through the bar and what is its direction? Indicate the directions on the diagram.

d) What is the numerical value of the total power delivered to the circuit resistance?

e) What is the applied force that must be exerted on the bar by an outside source to keep the bar moving at a constant velocity? This force points in the direction of motion. There are no friction forces in the system.

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031 qfull 00510 2 3 0 moderate math: bike electric generator

1088. Consider a bike electric generator that powers a headlight. The emf of the generator is given by

$$\mathcal{E} = NAB\omega_{\text{gen}} \sin(\omega_{\text{gen}} t) \ ,$$

where N is the number of turns in the generator coil, A is the coil area, B is the magnitude of the magnetic field in the generator, ω_{gen} is the angular velocity of the the generator coil, and t is time. We assume $\omega_{\text{gen}} > 0$ to avoid otiose generality.

a) What is $\mathcal{E}_{\text{max}}$ (i.e., the maximum value of $\mathcal{E}$)?

b) What is ω_{gen} in terms of $\mathcal{E}_{\text{max}}$, N, A, and B?

c) The generator is cranked by a small wheel of radius r_{gen} that is driven by the bike wheel of radius r_{bike}. There is a no-slip condition between the two wheels. What is the formula for the speed of the bike wheel (i.e., v_{bike}) in terms of generator quantities?

d) What is the bike translational speed v_{trans} in terms of the generator quantities given the no-slip condition between ground and bike wheel?

e) What is the bike translational speed v_{trans} given $r_{\text{gen}} = 0.010\,\text{m}$, $\mathcal{E}_{\text{max}} = 5.0\,\text{V}$, $N = 100$, $B = 0.10\,\text{T}$, $A = 1.0 \times 10^{-3}\,\text{m}^2$?

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031 qfull 00520 2 3 0 mod math: electric generator, emf, power

1089. An electric generator has 200 turns on a circular coil of radius 0.5 m. The coil is planar: i.e., all the turns can be treated as overlapping in a single plane. The coil is in a uniform magnetic field of 4 T. The coil is rotated at a constant angular frequency ω on an axis that passes through a diameter of the coil and this axis is perpendicular to the magnetic field. (We assume $\omega > 0$ to avoid otiose generality.) The angular position of the coil θ is measured from the field direction to the normal to the coil (i.e., to the symmetry axis of the coil which is perpendicular to the axis of rotation). The frequency of rotation is 1000 rpm (revolutions per **MINUTE**).

a) Draw a diagram of the system.

b) What is the formula all in symbols for the magnetic flux Φ linked by the coil for a general angle θ?

c) What is amplitude of the generated emf? **HINT:** Use Faraday's law and remember that the rotation rate is constant.

d) Say one connected the generator to a resistive load of 100 kΩ. What is the maximum power generated in a cycle? Since the load is resistive, there is no phase difference between emf and the current.

e) What is the average power generated in a cycle given the 100 kΩ resistive load? **HINT:** You might find the trigonometric identity

$$\sin^2(x) = \frac{1}{2}\left[1 - \cos(2x)\right]$$

helpful.

f) Say the magnetic field was 1 gauss (10^{-4} T which is of order of the Earth's magnetic field), what would be the amplitude of the generated emf?

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031 qfull 00590 2 3 0 mod math: energy density in a big solenoid

1090. Say you had a big ideal solenoid with an inductance of $L = 10^6$ H and volume of $V_{\text{vol}} = 125\,\text{m}^3$. The current in the solenoid is $I = 100$ A.

a) Calculate the magnetic field energy density without calculating the B-field? **HINT:** This is easier than I thought.

b) Calculate the number of turns per unit length in symbols representing knowns (i.e., L, V_{vol}, and I) and numerically.

c) Calculate the magnetic field in terms of symbols representing knowns and numerically.

d) Now calculate the energy density from the magnetic field itself. Does the answer agree with the part (a) answer?

Chapt. 32 Inductance and Inductors

Multiple-Choice Problems

032 qmult 00100 1 1 3 easy memory: inductance

1091. The quantity _____ given the symbol L or M is a coefficient that appears in the formula relating Faraday law induced emf to the derivative of the current in a system. The formula itself is

$$\mathcal{E} = -L\frac{dI}{dt} \qquad \text{or} \qquad \mathcal{E} = -M\frac{dI}{dt} \ .$$

The formula is a special case of the Maxwell-Faraday equation version of Faraday's law. So the emf is only due induced electric fields. There is no motional emf contribution—at least not in what one ordinarily means by a system with _____.

a) resistance b) capacitance c) inductance d) emf e) reluctance

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032 qmult 00110 1 1 3 easy memory: self-inductance

1092. In an inductance system, when the contour for current current flow is the same as the contour for the emf, one has _____ and when not, one has _____.

a) mutual inductance; self-inductance b) self-inductance; joint inductance
c) self-inductance; mutual inductance d) real inductance; mutual inductance
e) self-righteous-inductance; mutual-fund inductance

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032 qmult 00200 1 1 1 easy memory: inductor defined 1

1093. A device designed to act only as self-inductance in system is called a/an:

a) inductor. b) distributor. c) contributor. d) caroller e) cruller

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032 qmult 00202 1 4 1 easy deducto-memory: inductor defined 2

1094. An inductor is a device designed:

a) to have a **SELF-INDUCED EMF** when a time-varying current (implying a time-varying magnetic field) **PASSES** through it.
b) to have a **SELF-INDUCED EMF** when a time-varying current (implying a time-varying magnetic field) **DOES NOT PASS** through it.
c) to have a **CAPACITANCE** when a time-varying current (implying a time-varying magnetic field) **PASSES** through it.
d) to have a **CAPACITANCE** when a time-varying current (implying a time-varying magnetic field) **DOES NOT PASS** through it.
e) for use by the draft board.

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032 qmult 00210 1 1 4 easy memory: mutual inductance transformer

1095. An example of mutual inductance device is a/an:

a) generator. b) distributor. c) contributor. d) transformer. e) informer

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032 qmult 00250 1 1 5 easy deducto-memory: inductor and current
Extra keywords: inductor and current

1096. An inductor resists changes in:

a) its immanence. b) its resistance. c) its capacitance. d) its inductance.
e) the current through it.

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032 qmult 00300 1 1 2 easy memory: ideal solenoid inductance

1097. The ideal solenoid has no magnetic field outside and magnetic field inside

$$\vec{B} = \mu_0 n I \hat{z} \ ,$$

where n is the number of turns per unit length, I is the current through the solenoid, and z is the symmetry axis with its sense determined by a right-hand rule from the current flow in the turns. From this formula, the solenoid length ℓ, area A, and volume $V_{\rm vol}$, and Faraday's law of induction

$$\mathcal{E} = -\frac{d\Phi}{dt} \ ,$$

one can easily calculate the ideal solenoid inductance to be:

a) $L = \mu_0 n^2 V_{\rm vol} \ell / A$. b) $L = \mu_0 n^2 V_{\rm vol}$. c) $L = \mu_0 n^2 V_{\rm vol} A / \ell$. d) $L = \mu_0 n^2$.
e) $L = \mu_0 n V_{\rm vol} A / \ell$.

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032 qmult 00410 1 1 1 easy memory: RL circuit behavior

1098. In an RC circuit the potential across a capacitor is proportional to the integral of current and in an RL circuit the potential across an inductor is proportional to:

a) the derivative of current. b) the 2nd derivative of current. c) the current.
d) the current squared. e) the square current.

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032 qmult 00520 1 1 5 easy deducto-memory: LC oscillator, radio emission
Extra keywords: LC oscillator, radio emission

1099. An important use of an LC circuit oscillator is in:

a) horticulture. b) literature. c) politics. d) neurasthenia.
e) radio emission.

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032 qmult 00530 1 3 2 math memory: LC oscillator, radio frequency

1100. Say you wanted to build an LC oscillator to produce a radio signal at 1.0×10^8 Hz (a hundred megacycles per second). You had a $1.0\,\mu$F capacitor. What size inductance do you need?

a) $L = 1.0 \times 10^8$ H. b) $L = 2.5 \times 10^{-12}$ H. c) $L = 1.0 \times 10^{-6}$ H.
d) $L = 2.5 \times 10^{-11}$ H. e) $L = 2.5 \times 10^8$ H.

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032 qmult 00600 1 1 1 easy memory: RLC circuit

1101. A current branch containing a resistor, inductor, and capacitor is a:

a) RLC circuit. b) resinca circuit. c) RIC circuit. d) RCI cirucit.
e) a no-go circuit.

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032 qmult 00610 1 1 2 easy memory: RLC damped circuits
Extra keywords: RLC damped circuits

1102. The simple RLC circuit (in which the RLC circuit is the whole circuit in fact with the devices in series) is analogous to the mechanical damped harmonic oscillator. It exhibits:

a) overdamping, critical damping, and linear damping.
b) overdamping, critical damping, and underdamping.
c) overdamping, critical damping, and quadratic damping.
d) superdamping, crucial damping, and underdamping.
e) superdamping, crucial damping, and Underdog damping.

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028 qmult 10310 3 3 3 tough thinking: infinite ladder inductor fullmult

1103.*** Consider an infinite ladder inductor. There are two terminals A and B at the left: A is the upper terminal; B is the lower terminal. From each terminal, a wire extends to infinity in the rightward direction. There are vertical wires at equal intervals connecting the A wire and the B wire: these are the rungs of the ladder. On each vertical wire (each rung) is an inductor of inductance L_2. On the A wire before each node (going rightward) is an inductor of inductance L_1. The branch inductors are ideal inductors and have no mutual inductances. Draw a diagram.

What is the inductance L_∞ of the infinite ladder (i.e., what is the inductance of the infinite ladder viewing A and B as the leads to an effective inductor)?

As Yoda would say, **HINT** you want? If you draw a vertical line just to the right of leftmost rung, the inductance of the ladder to the right of that line must be L_∞ also since the ladder is infinite.

a) $L_\infty = \dfrac{L_1\sqrt{5} - L_2}{2}$. b) $L_\infty = \dfrac{L_1\sqrt{5} + L_2}{2}$. c) $L_\infty = \dfrac{L_1 + \sqrt{L_1^2 + 4L_1L_2}}{2}$.

d) $L_\infty = \dfrac{L_1 - \sqrt{L_1^2 + 4L_1L_2}}{2}$. e) $L_\infty = L_1 + L_2$.

Full-Answer Problems

032 qfull 00310 2 5 0 moderate thinking: inductors in series and parallel

Extra keywords: inductors in series and parallel HRW-742-43,44

1104. Inductors can be put in series and parallel like other devices. Say you have a set of inductors $\{L_i\}$.

a) You have well-separated inductors in series. What is the net inductance of the set of inductors? Give a proof of your answer. How does the largest inductance compare to the net inductance?

b) Why must the inductors be well separated for the part (a) result to be valid in general? How would you expect the actual net inductance of not-well-separated inductors to compare to the combined inductance given the part (a) result for these inductors? Why would you expect this?

What can one say about the special case of ideal solenoid inductors in series? Do they need to be well separated for the simple net inductor formula to be valid? Explain your answer.

c) You have well-separated inductors in parallel. What is the net inductance of the set of inductors? Give a proof of your answer. How does the smallest inductance compare to the net inductance?

d) Why must the inductors be well separated for the part (d) result to be valid in general? How would you expect the actual net inductance of not-well-separated inductors to compare to the net inductance given the part (c) result for these inductors? Why would you expect this?

What can one say about the special case of ideal solenoid inductors in parallel? Do they need to be well separated for the simple net inductor formula to be valid? Explain your answer.

032 qfull 00410 2 3 0 mod math: RL circuit, RL time constant

1105. A simple RL circuit has been set up with an emf $\mathcal{E} = 10\,\text{V}$, resistor $1.0\,\text{k}\Omega$, and inductor $5\,\text{mH}$ in series. Initially there is zero current flowing, then a switch is thrown closing the circuit.

a) What is the current when the time $t = \tau_L$?

b) At what time (in numerals) does the current reach half its final value (i.e., half its value at time infinity)?

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032 qfull 00420 3 3 0 tough math: solving a DE for an RL circuit 1

1106. It's about time you-all solved a differential equation (DE). Consider the simple one-loop RL circuit DE with a constant driving emf $\mathcal{E}$ (i.e., an emf, resistor and inductor in series):

$$LI' + IR = \mathcal{E} \ .$$

a) Draw a diagram of the circuit.

b) It's easier to solve a homogeneous DE than an inhomogeneous one. Differentiate the DE to get a DE for I'.

c) Solve the DE of part (b) for the solution for I' and integrate it for the general solution for I. The solution should have two undetermined constants that set by boundary conditions of the problem. Define an e-folding time (or time contant) τ to be used in simplifying the solution.

d) What is the formula for the power outflow from the circuit to the inductor for the special case of our system. **HINT:** A negative power outflow is a power inflow.

e) Given $\mathcal{E} \neq 0$ and the initial current $I_0 = 0$, find the particular solution for I as a function of time. Assume I_0 is a known.

f) Given $\mathcal{E} = 0$ and the initial current $I_0 \neq 0$, find the particular solution for I as a function of time.

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032 qfull 00422 3 3 0 tough math: solving a DE for RL circuit 2

Extra keywords: This as sub-useful question. Maybe good for tests

1107. It's about time you-all solved a differential equation (DE). Consider the simple one-loop RL circuit DE with a fixed driving emf $\mathcal{E}$ (i.e., an emf, resistor and inductor in series):

$$LI' + IR = \mathcal{E} \ .$$

a) Draw a diagram of the circuit.

b) Step one divide the DE through by L. Step two find an integrating factor f. The f function satisfies equation

$$fI' + \frac{1}{\tau} fI = (fI)' \ ,$$

where $\tau = L/R$ is the RL circuit time constant or e-folding time. To find f expand $(fI)'$ using the chain rule. You will then get differential equation of a famous type

$$x' = kx \ .$$

This famous type can be solved by dividing through by x and then integrating both sides with respect to time. Solve for f. The scale of f is unimportant, and so $f(t = 0)$ should be set to 1.

c) Now multiply the DE (divided through by L) by f. Both sides of the DE can now be integrated from time zero to a general time. Do this and find the solution.

$$I = \frac{\mathcal{E}}{R}(1 - e^{-t/\tau}) \ .$$

The initial condition on the DE is that current is zero at time zero.

d) Sketch I as a function of time.

---

031 qfull 00424 3 5 0 tough thinking: RL circuit discharge
Extra keywords: a sub-useful question maybe good for tests

1108. You have a one-loop circuit with **ONLY** a resistor R and inductor L: there is no fixed emf source. At time zero, the current in the circuit is I_0.

a) Draw the circuit diagram. Indicate a fiducial (i.e., reference) current direction.

b) Write down Kirchhoff's voltage law for any time for this circuit. **HINTS:** The voltage drops across the resistor and inductor are both taken in the direction of current flow. If there is a actually a voltage rise then the drop turns out to be implicitly negative.

c) The equation in the part (b) answer is actually a differential equation for current. Solve this equation for I as function of time. **HINTS:** Remember the initial current is I_0: i.e., $I(t = 0) = 0$. Re-order the equation so that both sides of the equal sign can be integrated over time. Let $\tau = L/R$. You don't need an integrating factor this time. If your current goes to infinity, you may have made a sign error.

d) What is the expression for the initial energy stored in the inductor?

e) What is the expression for the power dissipated in the resistor at any time t in terms of variables including t explicitly?

f) Integrate the expression for power dissipated in the resistor from time zero to infinity. Has energy been conserved?

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032 qfull 00610 2 3 0 mod math: transformer step-up step-down
Extra keywords: transformer, step up, step down

1109. You want to step up an AC potential by a factor of 3.

a) What turn ratio $N_{\text{secondary}}/N_{\text{primary}}$ must you use?

b) What would be the ratio of the secondary to the primary current?

Chapt. 33 Alternating Current or AC

Multiple-Choice Problems

033 qmult 00100 1 1 3 easy memory: AC definition 1

1110. Alternating current or AC means that current and potential vary about a mean value (usually zero) and usually (but not always)

a) trapezoidally. b) parabolically. c) sinusoidally. d) geometrically. e) wretchedly.

033 qmult 00110 1 1 2 easy memory: sawtooth and square-wave

1111. Sawtooth and square-wave variation of current and potential can be referred to as _____, but they are not what one ususally means by the term.

a) DC b) AC c) HVDC d) HIFI e) WIFI

033 qmult 00120 1 1 5 easy memory: victory of AC

1112. AC won the "war of the currents" in the 1880s as the preferred method of large electrical energy transmission. But this wasn't because AC is more naturally produced by electrical generators and used by electrical motors than DC. Efficient conversions between AC and DC exist and saying that AC is more "natural" for generators and motors is arguable actually. No AC won because long-distance electrical energy transmission is most efficient at high potential and in the 1880s step-ups and step-downs from high potential were most efficiently done in AC using _____ which require AC. But AC's victory was never complete and, in fact, high efficiency DC step-up and step-down technology has been developed. Some experts think that high-voltage DC (HVDC) will replace AC in the power grid of tomorrow.

a) robots b) gigantic robots c) diodes d) capacitors e) transformers

033 qmult 00130 1 1 2 easy memory: frequency, period, and all that

1113. An AC potential can usually be described by a sinusoid: i.e., by

$$V = V_0 \cos(\omega t + \theta_0) \ ,$$

where V_0 is the amplitude, ω is the angular frequency, and θ_0 is a phase constant. If $\theta_0 = -90°$, then

$$V = V_0 \sin(\omega t) \ .$$

In actual fact, the value of θ_0 can usually be chosen for convenience (and the choice is often 0 or, so that one has a sine function, $-90°$) since the phase of the sinusoid at time zero is unimportant for the time average behavior or the behavior over a cycle which is usually all that is of interest. in describing an AC circuit in purely periodic operation. The angular frequency ω, _____ (cycles per unit time) f, and period p (not to be confused with power P) are related by

$$\omega = 2\pi f = \frac{2\pi}{p} \ , \qquad f = \frac{\omega}{2\pi} = \frac{1}{p} \ , \qquad p = \frac{1}{f} = \frac{2\pi}{\omega} \ .$$

a) RPM b) frequency c) fraction d) hertz e) function

033 qmult 00140 1 1 4 easy memory: unit of frequency

1114. The MKS unprefixed unit of frequency is the:

a) kilovolt (kV). b) volt (V) c) revolution per minute (RPM). d) hertz (Hz). e) kilohertz (kHz).

033 qmult 00150 1 1 3 easy memory: standard north american frequency

1115. The optimum choice of AC frequency for a particular application probably depends on many things. But for a power grid, one must have a standard frequency which is reasonably good for may applications. In the early days of power grids many frequencies were used by the pioneering power plants. Frequencies between 16.666 . . . Hz and 133.333 . . . Hz have used. The Niagara Falls hydropower plants generated 25 Hz for a long time which is a bit too low for lighting: there is a detectable flicker to the eye. Coventry, England once using the oddball value 87 Hz. But in the course of the 20th century, an overall optimum frequency of about 50 Hz was arrived at. In Europe, 50 Hz exactly became the standard probably because it seemed sort of metric. In North America, _____ exactly was settled probably since it seemed most Sumerian. Most of the world uses 50 Hz, in fact. Only North America, Central America, northern South America, and Liberia using _____. Japan uses both 50 Hz and _____. If there ever is a worldwide power grid, it may well be DC and there will be no frequency struggle.

a) 36 Hz b) 360 Hz c) 60 Hz d) 59.5 Hz e) 40 Hz

033 qmult 00160 1 1 1 easy memory: integral averages

1116. In AC work one often needs to know the time averages over one period p of $\cos^2(\omega t)$ $\sin^2(\omega t)$ and $\cos(\omega t)\sin(\omega t)$. The values, respectively, are:

a) 1/2, 1/2, 0. b) 1/2, 1/2, 1/2. c) 1/2, 0, 1/2. d) 0, 1/2, 0. e) 1/2, 0, 0.

033 qmult 00170 1 1 5 easy memory: sinusoidal RMS quantities

1117. If you have a sinusoidally vary quantity (like AC current or potential) with amplitude A_0, the RMS value for the quantity is:

a) $2A_0$. b) $1/(2A_0)$. c) $1/(A_0\sqrt{2})$. d) $A_0/2$. e) $A_0/\sqrt{2}$.

033 qmult 00210 1 1 3 easy memory: impedance magnitude

1118. For a given circuit branch (which could be a device or circuit system), the ratio of the amplitude of the potential drop across the branch to the amplitude of the current through it is the non-complex number form of the:

a) reflectance Z. b) frequency Z. c) impedance Z. d) reluctance Z. e) redolence Z.

033 qmult 00220 1 1 3 easy memory: complex number impedance

Extra keywords: Don't ask usually. Students may not have complex numbers.

1119. For a given circuit branch (which could be a device or circuit system), the ratio of the complex potential drop across the branch to the complex current through it is the:

a) reflectance Z. b) frequency Z. c) impedance Z. d) reluctance Z. e) redolance Z.

033 qmult 00230 1 1 2 easy memory: phase angle

1120. For a circuit branch (which could be a device or circuit system), you have potential drop across and current through given by, respectively,

$$V = V_0\cos(\omega t) \qquad \text{and} \qquad I = I_0\cos(\omega t - \phi)$$

The non-complex number impedance is _____ and the phase angle is _____. Formally, the current lags the potential by ϕ, but if $\phi = 0$ the two are in phase and if ϕ is negative then the current leads the potential.

a) $\frac{I_0}{V_0}$; ϕ b) $\frac{V_0}{I_0}$; ϕ c) $\frac{V_0}{I_0}$; ωt d) $\frac{V_0}{I_0}$; ω e) $\frac{I_0}{V_0}$; ω

---

033 qmult 00240 1 1 2 easy memory: AC power, phase angle

1121. For a circuit branch (which could be a device or circuit system), you have potential drop/rise across and current through given by, respectively,

$$V = V_0 \cos(\omega t) \qquad \text{and} \qquad I = I_0 \cos(\omega t - \phi) \ .$$

The time-average power output/input of the branch is given by:

a) $P = I_0 V_0$.
b) $P = I_{\rm rms} V_{\rm rms} \cos(\phi)$, where ϕ is the phase difference between potential and current.
c) $P = I_{\rm rms} V_{\rm rms}$.
d) $P = I_0 V_0 \cos(\phi)$, where ϕ is the phase difference between potential and current.
e) $P = I_{\rm rms}/V_{\rm rms}$.

---

033 qmult 00242 1 1 4 easy memory: phase angle of an AC circuit

1122. The time-average power supplied by a AC emf is 100 W. The RMS potential rise and currents for the device are, respectively 20 V and 10 A. What is the phase angle?

a) 0°. b) 30°. c) 45°. d) 60°. e) 90°.

---

033 qmult 00250 1 1 1 easy memory: resistor impedance

1123. Ohm's law is give the relationship between voltage drop across a resistor and current through it. The law is

$$V = IR$$

where V is voltage drop, I is current, and R is resistance. Given AC voltage drop

$$V = V_0 \cos(\omega t) \ ,$$

what is the non-complex number impedance and the phase angle for current?

a) R and 0°. b) $1/R$ and 0°. c) $1/R$ and 90°. d) $1/R$ and $-90°$.
e) R and 90°.

---

033 qmult 00260 1 1 3 easy memory: capacitor impedance

1124. The capacitor formula is

$$C = \frac{q}{V} \ ,$$

where V is voltage drop across the capacitor, q is the capacitor charge, and C is the capacitance. Given AC voltage drop

$$V = V_0 \cos(\omega t) \ ,$$

what is the non-complex number impedance (which is called the [non-complex] capacitive reactance) and the phase angle for current? Assume the current is flowing into positive plate of the capacitor.

a) $1/(\omega C)$ and 90°. b) ωC and 90°. c) $1/(\omega C)$ and $-90°$. d) ωC and $-90°$.
e) $1/\omega$ and $-90°$.

---

033 qmult 00270 1 1 5 easy memory: inductor impedance

1125. The inductor formula is

$$V = L\frac{dI}{dt} \ ,$$

where V is voltage drop across the capacitor, I is the current through the inductor, and L is the inductance. Given AC voltage drop

$$V = V_0 \cos(\omega t) \ ,$$

what is the non-complex number impedance (which is called the [non-complex] inductive reactance) and the phase angle for current?

a) L and $-90°$. b) L/ω and $-90°$. c) $1/(\omega L)$ and $90°$. d) ωL and $-90°$. e) ωL and $90°$.

---

033 qmult 00520 1 4 3 easy deducto-memory: driven RLC circuits, resonance

1126. The driven RLC circuit exhibits:

a) resonance at zero angular frequency.
b) correspondence at zero angular frequency.
c) resonance at angular frequency $\omega = 1/\sqrt{LC}$.
d) correspondence at angular frequency $\omega = 1/\sqrt{LC}$.
e) despondence at angular frequency $\omega = 1/\sqrt{LC}$.

---

033 qmult 00530 1 4 1 easy deducto-memory: RLC circuit, resonance

1127. A well known use of a RLC circuit is to:

a) tune in a desired frequency: for example a particular radio or television frequency.
b) change AC to DC.
c) change DC to AC.
d) multiply an undesired frequency.
e) divide a desired frequency.

---

033 qmult 00600 1 4 5 easy deducto-memory: transformer

Extra keywords: mathematical physics

1128. "Let's play *Jeopardy*! For \$100, the answer is: It is a device to transform (i.e., step-up or step-down) the voltage level of AC power."

What is a/an _____, Alex?

a) electric motor b) electric generator c) inductor d) magnetic core e) transformer

---

033 qmult 00610 1 1 3 easy memory: transformer

Extra keywords: transformer

1129. Transformers allow one to:

a) step out a potential. b) step through a potential.
c) step up and down a potential. d) step on a potential. e) overstep a potential.

---

033 qmult 10610 2 3 1 moderate math: transformer power efficiency mult

1130.** Using high transmission potential and low transmission current is energy efficient: there is less energy lost in heating the transmission wires.

i) A power station outputs power P at (RMS) potential $\mathcal{E}$. The transmission wire has resistance R. The power lost to heat in the wire is P_{loss} Find the formula for relative power loss P_{loss}/P in terms of P, R, and $\mathcal{E}$. **HINT:** Assume potential and current are in phase: i.e., assume that the formula $P = I\Delta V$ holds where for a device P is RMS power out/input, I is RMS current, and ΔV is RMS potential change.

ii) Say the power station produces 1.5 MW and transmits power to a town 15.0 km away. The resistance in the transmission line is $5.0 \times 10^{-2}\,\Omega$/km. What is the relative power loss using transmission potential 1200 V? Say a 100:1 step-up transformer is now inserted before transmission. What is the relative power loss in this case?

a) (i) $\dfrac{PR}{\mathcal{E}^2}$ (ii) 0.78; 7.8×10^{-5}. b) (i) $\dfrac{PR}{\mathcal{E}^2}$ (ii) 0.42; 4.2×10^{-5}.

c) (i) $\dfrac{PR}{\mathcal{E}}$ (ii) 0.50; 5.0×10^{-5}. d) (i) $\dfrac{PR}{\mathcal{E}}$ (ii) 0.42; 4.2×10^{-5}.

e) (i) $PR\mathcal{E}^2$ (ii) 0.50; 5.0×10^{-5}.

Full-Answer Problems

033 qfull 00540 2 3 0 moderate math: driven semi-parallel RLC circuit

1131. What can be called a driven semi-parallel RLC circuit consists of an AC emf

$$\mathcal{E} = \mathcal{E}_0 \cos(\omega t)$$

in series with a resistor R and a parallel combination inductor L and capacitor C. The parallel combination is connected back to the AC emf to complete the circuit.

a) Draw a diagram. In the main circuit branch, there is a current I. In the inductor branch, there is a current I_1. In the capacitor branch, there is a current I_2. The currents are considered unknowns in this problem. The other quantities are considered knowns.

b) Write down three equations that in principle can be used to solve for the currents I, I_1, and I_2.

c) The solution for current I is, in fact,

$$I = I_0 \cos(\omega t - \phi) = \frac{\mathcal{E}_0}{Z} \cos(\omega t - \phi) \ ,$$

where (non-complex) impedance

$$Z = \sqrt{R^2 + \frac{1}{[1/(\omega L) - \omega C]^2}}$$

and phase angle

$$\phi = \tan^{-1} \left\{ \frac{1}{[1/(\omega L) - \omega C]R} \right\} \ .$$

Since the denominator R for the phase angle is always positive, the phase angle is always in the 1st or 4th quadrant and never any need to add 180° due to the calculational ambiguity of inverse tangent. What happens to current I if ω is at the LC circuit angular frequency resonance ω_0?

d) At resonance, what is the current I_1 through the inductor? No differentiation or integration symbols should remain in the formula for I_1. What is the amplitude and the phase angle for the current I_1?

e) At resonance, what is the current I_2 through the capacitor? No differentiation or integration symbols should remain in the formula for I_2. What is the amplitude and the phase angle for the current I_2?

033 qfull 00550 2 3 0 moderate math: driven parallel RLC circuit

1132. Consider a driven parallel RLC circuit consists of an AC emf

$$\mathcal{E} = \mathcal{E}_0 \cos(\omega t)$$

that is put across a resistor R, an inductor L and capacitor C all in parallel.

a) Draw a diagram. In the main circuit branch, there is a current I. In the resistor branch, there is a current I_1. In the inductor branch, there is a current I_2. In the capacitor branch, there is a current I_3. The currents are considered unknowns in this problem. The other quantities are considered knowns.

b) Solve for the resistor current I_1 in terms of the given knowns $\mathcal{E}_0$, ω, R, L, and C. No differentiation or integration symbols should remain in the formula for I_1. What is the current amplitude and phase angle? **HINT:** Recall that if potential drop across a device is proportional to $\cos(\omega t)$, then the phase angle ϕ of the current is when current is expressed in terms of $\cos(\omega t - \phi)$.

c) Solve for the inductor current I_2 in terms of the given knowns $\mathcal{E}_0$, ω, R, L, and C. No differentiation or integration symbols should remain in the formula for I_2. What is the current amplitude and phase angle? **HINT:** Constants of integration vanish because one can assume that all currents time average to zero.

d) Solve for the capacitor current I_3 in terms of the given knowns $\mathcal{E}_0$, ω, R, L, and C. No differentiation or integration symbols should remain in the formula for I_3. What is the current amplitude and phase angle?

e) Solve for the main branch current I in terms of the given knowns $\mathcal{E}_0$, ω, R, L, and C. What is the current amplitude and phase angle? **HINT:** If you assume the form

$$A\cos(\omega t - \phi)$$

(where A is amplitude and ϕ is phase angle), you can expand to get

$$A[\cos(\omega t)\cos(\phi) + \sin(\omega t)\sin(\phi)\ .$$

Comparison with the result you first obtain allows a solution for A and ϕ.

f) What is current I at the LC resonance angular frequency

$$\omega_0 = \frac{1}{\sqrt{LC}}\ ?$$

---

033 qfull 00600 1 3 0 easy math: transformer equations

1133. A conventional transformer consists of a magnetic core (which schematically can be visualized as empty picture frame made of magnetic material [e.g., soft iron]) and two coils that wrap around opposite sides of the iron core. One coil is the primary which is attached to the ultimate power source and the other coil is the secondary to which is attached some circuit or network that is powered by the transformer.

a) In the ideal transformer, the magnetic flux through any turn of the primary is also the magnetic flux through any turn of the secondary. In reality, there must, of course, be some difference, but the magnetic core of a well designed transformer does a very good job of confining the magnetic field region to the magnetic core and thereby directing "all" the magnetic field lines through both primary and secondary. Given that the number of turns of the primary is N_p and that of the secondary is N_s, derive the relationship between the primary emf $\mathcal{E}_\mathrm{p}$ and the secondary emf $\mathcal{E}_\mathrm{s}$ for an ideal transformer. **HINT:** Use Faraday's law of induction.

b) In the ideal transformer, the power input to the primary is equal to the power output of the secondary. This is at each time point and in an RMS sense (TM-1005). The RMS version result can be derived just a consequence of energy conservation when there is no leakage and with the assumption of current an potential difference being in phase. Use the

power equality result to derive the relationship between the primary current $I_{\rm p}$ and the secondary current $I_{\rm s}$. Assume that the power in both coils obeys

$$P = I\Delta V \ ,$$

where I is the current through the device and ΔV is the potential change across the device (which for ideal coils is the emf).

c) Find the formula that relates $N_{\rm p}$, $I_{\rm p}$ $N_{\rm s}$, and $I_{\rm s}$. Then give a combined ratio formula for number of coil turns, coil emfs, and coil currents.

---

033 qfull 00610 1 3 0 easy math: transformer power efficiency

1134. Using high transmission potential and low transmission current is energy efficient: their is less energy lost in heating the transmission wires.

a) A power station outputs power P at (RMS) potential $\mathcal{E}$. The transmission wire has resistance R. The power lost to heat in the wire is $P_{\rm loss}$ Find the formula for relative power loss $P_{\rm loss}/P$ in terms of P, R, and $\mathcal{E}$. **HINT:** Assume potential and current are in phase: i.e., assume that the formula $P = I\Delta V$ holds where for a device P is RMS power out/input, I is RMS current, and ΔV is RMS potential change.

b) Given the result of part (a), how can one make relative power loss lower for fixed P. There must be some practical consideration that actually prevents reducing the relative loss to zero.

c) Say the power station produces 1.2 MW and transmits power to a town 10.0 km away. The resistance in the transmission line is $5.0 \times 10^{-2}\,\Omega$/km. What is the relative power loss using transmission potential 1200 V? Say a 100:1 step-up transformer is now inserted before transmission. What is the relative power loss in this case?

---

033 qfull 00620 2 3 0 moderate math: transformer circuit

1135. Consider two one-loop circuits connected by an ideal transformer which actually makes the loops a united circuit. The first loop (the primary loop) has an AC power source with emf $\mathcal{E}$ and a resister R_1 and the primary of the transformer with emf $\mathcal{E}_1$. The second loop (the secondary loop) has the secondary of the transformer with emf $\mathcal{E}_2$ and a resister R_2. The primary to secondary turn ratio is N_1/N_2. The emfs and currents in this problem are all RMS quantities. (That one can just use RMS quantities in a DC-like fashion is not something that one can do in general in AC circuits, but TM-1004–1005 show that one can in this special case.)

a) The ideal transformer obeys equations

$$\frac{\mathcal{E}_1}{N_1} = \frac{\mathcal{E}_2}{N_2} \qquad \text{and} \qquad I_1\mathcal{E}_1 = I_2\mathcal{E}_2 \ .$$

Find the relationship between the currents in terms of the turn numbers and prove that

$$\frac{N_2}{N_1} = \frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{I_1}{I_2} \ .$$

b) Determine $\mathcal{E}_1$ in terms of the turn ratio, I_2, and R_2. **HINT:** Use Kirchhoff's voltage law.

c) Determine $\mathcal{E}_1/I_1$ in terms of the turn ratio and R_2. This quantity is the equivalent resistance $R_{\rm eq}$ of secondary loop. Just viewed as a black box the secondary loop in an RMS sense looks like a resistor of resistance $R_{\rm eq}$.

d) Solve for I_1 and $\mathcal{E}_1$ in terms of $\mathcal{E}$, R_1, and $R_{\rm eq}$.

Chapt. 34 Maxwell's Equations, Electromagnetic Radiation and Special Relativity

Multiple-Choice Problems

034 qmult 00100 1 4 5 easy deducto-memory: Maxwell's equations

1136. "Let's play *Jeopardy*! For $100, the answer is: The 4 fundamental laws of classical electromagnetism."

What are _____, Alex?

a) magnets b) Faraday's and Lenz's laws c) Maxwell's demons
d) the cones of silence e) Maxwell's equations

034 qmult 00110 1 1 2 easy memory: maxwell equations explain all

1137. Maxwell's equations plus the electromagnetic force law account for all classical electromagnetic phenomena provided one takes the electromagnetic properties of materials as _____. Quantum mechanics is needed to understand these properties.

a) classically derivable results b) givens c) takens d) borrows e) nothings

034 qmult 00120 1 1 3 easy memory: maxwell's equation formulae

1138. In integral vacuum form, _____ are

$$\oint_A \vec{E} \cdot d\vec{A} = \frac{q_{\text{encl}}}{\varepsilon_0} \quad \text{Gauss's law ,}$$

$$\oint_A \vec{B} \cdot d\vec{A} = 0 \quad \text{Gauss's law for magnetism ,}$$

$$\oint_s \vec{E} \cdot d\vec{s} = -\frac{\partial \Phi_B}{\partial t} \quad \text{Faraday's law or the Maxwell-Faraday equation ,}$$

$$\oint_s \vec{B} \cdot d\vec{s} = \mu_0 I + \mu_0 \varepsilon \frac{\partial \Phi_E}{\partial t} \quad \text{Ampère's law .}$$

a) Einstein's field equations b) Faraday's equations c) Maxwell's equations
d) Maxwell's relations e) Schrödinger's equation

034 qmult 00130 1 1 5 easy memory: displacement current, electric flux

1139. The displacement current is:

a) a displaced current of electric charge.
b) the magnetic flux.
c) the electric flux.
d) $\mu_0 \varepsilon_0 d\Phi_E/dt$: a term Maxwell added to Ampère's law in order to account for the induction of **ELECTRIC** fields by time-varying magnetic flux.
e) $\mu_0 \varepsilon_0 d\Phi_E/dt$: a term Maxwell added to Ampère's law in order to account for the induction of **MAGNETIC** fields by time-varying electric flux.

034 qmult 00140 1 1 4 easy memory: maxwell's eqn and boundary conditions

1140. Maxwell's equations are the fundamental laws of classical electromagnetism. They are what is true at all times and places in the classical limit. But in order to understand how a particular

system behaves, one must solve the equations for electromagnetic fields. The particular solutions are determined by the laws together with particular _____ which for time-varying systems include _____ in time which are often initial conditions.

a) formulae b) middle conditions c) equations d) boundary conditions
e) laws

---

034 qmult 00200 1 1 1 easy memory: EMR wave equation

1141. In the absence of charge, Maxwell was able to manipulate the Maxwell's equations such that they yielded for one dimension in vacuum in scalar form the equations

$$\frac{\partial^2 E}{\partial x^2} = \mu_0\varepsilon_0\frac{\partial^2 E}{\partial t^2} \qquad \text{and} \qquad \frac{\partial^2 B}{\partial x^2} = \mu_0\varepsilon_0\frac{\partial^2 B}{\partial t^2} \ .$$

Despite appearances, the E-fields and B-fields satisfying these equations are coupled and one has in fact that

$$\frac{B}{E} = \sqrt{\mu_0\varepsilon_0} \ ,$$

Maxwell's recognized his equations as examples of the standard _____

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2} \ ,$$

where v was the phase speed of propagation. He thus identified

$$v = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

as the phase speed of self-propagating electromagnetic radiation. Since this speed agreed within error with the then known vacuum light speed, Maxwell concluded that light was _____.

a) wave equation; electromagnetic radiation
b) Schrödinger equation; electromagnetic radiation
c) wave equation; electromagnetism d) wave equation; electric radiation
e) Schrödinger equation; magnetic radiation

---

034 qmult 00202 1 1 3 easy memory: Maxwell's prediction of EM waves
Extra keywords: speed of light

1142. From his 4 equations, Maxwell was able to predict that light was:

a) made of particles that are now called photons.
b) electromagnetic waves with phase velocity $c = 1/\sqrt{LC}$.
c) light was electromagnetic waves with phase velocity $c = 1/\sqrt{\mu_0\varepsilon_0}$.
d) made of particles that are now called fluxons.
e) electromagnetic waves with phase velocity $c = \sqrt{T/\mu}$.

---

034 qmult 00220 1 1 4 easy memory: Hertz discovers radio

1143. In Maxwell's day it was already known that there were invisible forms of light (infrared and ultraviolet), but none of these form could be generated by macroscopic oscillations of charge. But one could—although I'm not sure if anyone did—predict that much lower frequency, longer wavelength electromagnetic radiation should exist and could be so generated In 1888, such electromagnetic radiation (which was soon called radio) was discovered by _____. This was a tremendous verification of Maxwell's equations.

a) Galileo (1564–1642) b) Isaac Newton (1643–1727)
c) James Clark Maxwell (1831–1879) d) Heinrich Hertz (1857–1894)
e) Albert Einstein (1879–1955)

---

034 qmult 00230 1 1 1 easy memory: electromagnetic radiation transverse 1

1144. Electromagnetic waves are _____ which means the oscillation is perpendicular to the propagation direction. The coupled electric and magnetic fields are also perpendicular to each other.

a) transverse b) longitudinal c) transverse-longitudinal d) matutinal
e) converse

---

034 qmult 00232 1 1 3 easy memory: electromagnetic radiation transverse 2 ; tests
Extra keywords: physci

1145. Electromagnetic waves are:

a) **LONGITUDINAL** waves with the electric and magnetic field vectors **PARALLEL** to each other.
b) **TRANSVERSE** waves with the electric and magnetic field vectors **PARALLEL** to each other.
c) **TRANSVERSE** waves with the electric and magnetic field vectors **PERPENDICULAR** to each other.
d) **LONGITUDINAL** waves with the electric and magnetic field vectors **PERPENDICULAR** to each other.
e) both **TRANSVERSE** and **TRANSVERSE** waves: the electric field vectors are longitudinal and the magnetic field vectors are transverse.

---

034 qmult 00240 1 1 2 easy memory: polarized light

1146. Natural light (which is just a name—there's no unnatural light) has an isotropic distribution of electric and magnetic field vector directions perpendicular to the propagation direction when looked at over space and time though not at one instant and place where, of course, the E-field and B-field vectors can only point in one direction. Light vectors that are not isotropically distributed are _____ to one degree or another in one way or another.

a) marginalized b) polarized c) metabolized d) pasteurized
e) metastasized

---

034 qmult 00310 1 1 3 easy memory: unique speed of light

1147. An oddity of the Maxwell's equations prediction that electromagnetic radiation in vacuum propagates at the vacuum speed light is that there is no reference frame specified for this speed. A single unique speed is derived from the equations. So a natural question to ask is with respect to what frame does light travel at the speed of light. Maxwell and his contemporaries postulated that there was form of matter that they called the _____ which was the medium for light just as air was the medium for sound. The _____ was what was oscillating and the speed of light was relative the _____. Of course, the _____ was a bit mysterious. It seemed to pervade all space (except maybe opaque materials) and had no other properties than to be the medium for light.

a) stuff b) luminiferous heather c) luminiferous ether d) strange matter
e) dark matter

---

034 qmult 00320 1 1 1 easy memory: Einstein and Maxwell's laws

1148. Einstein in the early 1900s became convinced that Maxwell's laws were indeed laws of nature and therefore should hold (i.e., have the same formulae) in all inertial frames of reference. But if that was true, then all inertial-frame observers should measure the same _____. But then all of classical mechanics had to be revised since the ordinary ideas of relative velocity were not correct.

a) vacuum light speed b) light mass c) dark mass d) light wavelength
e) light frequency

---

034 qmult 00330 1 4 5 easy deducto-memory: postulates of special relativity

1149. "Let's play *Jeopardy*! For $100, the answer is: (1) The laws of physics are the same (i.e., have the same formulae) in all inertial frames of reference. (2) The vacuum light speed is the same for all inertial-frame observers."

What are _____, Alex?

a) the laws of classical electrodynamics b) the principles of quantum mechanics
c) Euclid's axioms of geometry d) contradictory hypotheses
e) the two postulates of Einstein's special relativity

---

034 qmult 00340 1 1 5 easy memory: vacuum light speed
Extra keywords: physci KB-213-15

1150. The vacuum light speed relative to inertial frames depends on:

a) frequency. b) wavelength. c) distance traveled. d) amplitude.
e) nothing: it is a universal constant.

---

034 qmult 00350 1 2 4 moderate memory: Newton's laws, Maxwell's equations
Extra keywords: Lorentz invariance

1151. Which turned out to be more correct in relativistic physics: Newton's laws or Maxwell's equations and why?

a) They were both **EQUALLY INCORRECT** since they both changed form under **LORENTZ** transformations.
b) They were both **EQUALLY CORRECT** since they were both invariant under **LORENTZ** transformations.
c) Newton's laws were **MORE CORRECT** since they only were invariant under **LORENTZ** transformations.
d) Maxwell's equations were **MORE CORRECT** since they only were invariant under **LORENTZ** transformations.
e) Maxwell's equations were **MORE CORRECT** since they only were invariant under **GALILEAN** transformations.

---

034 qmult 00400 1 4 2 easy deducto-memory: speed of light, wavelength
Extra keywords: physci

1152. The relation between c, f, and λ is:

a) $c = f/\lambda$. b) $c = f\lambda$. c) $c = 1/\sqrt{f\lambda}$. d) $c = \sqrt{f\lambda}$. e) $c = f\lambda^2$.

---

034 qmult 00450 1 3 1 easy math: a radio wavelength calculation
Extra keywords: physci KB-214-37

1153. A radio station broadcasts at 600 kHz (kilohertz). Given that the speed of light in air is to good approximation 3.0×10^8 m/s, the wavelength of radio waves is

a) 500 m. b) 600 m. c) 0.002 m. d) 3.0×10^8 m. e) 0 m.

---

034 qmult 00510 1 4 3 easy deducto-memory: intensity definition

1154. "Let's play *Jeopardy*! For $100, the answer is: The energy per unit time per unit area in the direction of propagation of a beam of electromagnetic radiation."

What is the _____, Alex?

a) Doppler (or more moosely the effect) b) polarization (or more loosely the polar)
c) intensity (or more precisely the specific intensity integrated over wavelength)
d) Light e) electromagnetic spectrum

---

034 qmult 00520 1 1 2 easy memory: inverse-square law for point source

1155. The intensity (more properly the specific intensity integrated over wavelength) from an isotropic point source of light falls off with radius r like:

a) $1/r$. b) $1/r^2$. c) $1/r^3$. d) $1/r^4$. e) $1/r^5$.

---

034 qmult 00620 1 4 2 easy deducto-memory: colors of visible spectrum
Extra keywords: physci

1156. The colors of the visible spectrum in order of **INCREASING** wavelength are:

a) red, orange, yellow, green, blue, violet.
b) violet, blue, green, yellow, orange, red.
c) violet, green, blue yellow, red, orange.
d) violet, blue, forest-lawn, yellow, tangerine, red.
e) violent, stripe, forest-lawn, mouse, tangerine, exhaust.

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034 qmult 00630 2 5 3 moderate thinking: Canadian flag in red light
Extra keywords: physci KB-213-27

1157. The Canadian flag in ordinary lighting is red and white. When viewed in red light it is:

a) all white. b) red and black. c) all red. d) all black.
e) red and white still.

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034 qmult 00700 1 4 1 easy deducto-memory: blackbody radiation
Extra keywords: mathematical physics

1158. "Let's play *Jeopardy*! For $100, the answer is: The radiation emitted by a thermal process from dense bodies at uniform temperatures."

What is _____, Alex?

a) blackbody radiation b) visible light c) infrared radiation
d) ultraviolet radiation e) X-rays

---

034 qmult 00710 1 1 2 easy memory: human body temperature

1159. Human's radiate approximately blackbody radiation with a peak wavelength of about 9.34 μm (being a bit overly precise for the sake of a fiducial answer). Taking this wavelength as exact and the humand body as as an exact blackbody radiator, what is the human body temperature?

a) 37 K. b) 310 K. c) 98.6 K. d) 9.34 K. e) 2898 K.

---

034 qmult 00800 1 4 4 easy deducto-memory: Doppler effect definition
Extra keywords: mathematical physics

1160. "Let's play *Jeopardy*! For $100, the answer is: The change of frequency and wavelength of a wave phenomenon with change of frame of reference."

What is _____, Alex?

a) reflection b) spherical aberration c) the parallax effect
d) the Doppler effect e) refraction

---

034 qmult 00810 1 1 3 easy memory: Doppler effect calculation

1161. Having arrived at Pluto (formerly the ninth planet), you observe Earth with a spectrometer. You find that the wavelength of the hydrogen 656.28 nm emission (where the quoted wavelength is the laboratory frame value) varies by about 0.13 nm around 656.28 nm: i.e., it varies from about 0.065 nm below to 0.065 nm above. To understand this Doppler shift effect you draw a simple diagram that schematically illustrates that Pluto is far from the Sun and the Earth close to the Sun and thus that Earth-Pluto line is approximately parallel to the Sun-Pluto and that Pluto on its fairly circular orbit is moving approximately perpendicularly to these lines and thus that Pluto's velocity (which has a rather low magnitude of 4.666 km/s in any case) is not contributing significantly to the relative velocity causing the Doppler shift. So approximately only the Earth's velocity contributes the Doppler shift and this velocity will have a maximum magnitude that equals the Earth's orbital speed and this equality occurs when the Earth-Pluto

line is tangent to the Earth's circular orbit. So given all this preamble, about what is the orbital speed of Earth?

a) 90 km/s. b) 60 km/s. c) 30 km/s. d) 656 km/s. e) 1300 km/s.

---

034 qmult 00910 1 1 1 easy memory: photons, dangerous, organic molecules

Extra keywords: Needs to be corrected for fast electrons.

1162. Why is UV and higher frequency electromagnetic radiation dangerous to life?

a) The photons of such high frequency radiation **ARE** strong enough and often penetrating enough to break the bonds of organic molecules in cells. Such molecular damage (to DNA in particular) can destroy function and lead to mutations and cancers. Lower frequency radiation no matter how intense is usually much less dangerous because the photo-disassociation bond-breaking process is usually a single-photon absorption process.
b) The photons of such high frequency radiation **ARE NOT** strong enough and often **NOT** penetrating enough to break the bonds of organic molecules in cells. Such molecular damage (to DNA in particular) can destroy function and lead to mutations and cancers. Lower frequency radiation no matter how intense is usually much less dangerous because the bond breaking process is usually a single-photon absorption process.
c) Because its color is beyond violet.
d) Because its color is infrared.
e) Because it oscillates so slowly it shakes organic molecules to pieces.

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034 qmult 00920 2 5 2 moderate thinking: sunlight energy and momentum

Extra keywords: physci KB-214-19

1163. Electromagnetic radiation transports both energy and momentum in fact. Because it emits electromagnetic radiation without any outside energy resupply, the Sun must constantly decrease in total energy content. Why does the Sun not decrease in total momentum assuming that it is exactly spherically symmetric and is moving in a straight line?

a) Momentum is a **SCALAR** quantity. The emiting of momentum in all directions isotropically means the net momentum emitted is **NON-ZERO** and the Sun's total momentum does not change.
b) Momentum is a **VECTOR** quantity. The emiting of momentum in all directions isotropically means the net momentum emitted is **ZERO** and the Sun's total momentum does not change.
c) Momentum is a **VECTOR** quantity. The emiting of momentum in all directions isotropically means the net momentum emitted is **NON-ZERO** and the Sun's total momentum does not change.
d) Momentum is a **SCALAR** quantity. The emiting of momentum in all directions isotropically means the net momentum emitted is **ZERO** and the Sun's total momentum does not change.
e) The question is wrong. The Sun total angular momentum does change.

---

034 qmult 01010 1 1 2 easy memory: polarized light

1164. What is (partially) polarized light and completely polarized light?

a) Light with the electric field vectors partially and completely aligned **WITH** the propagation direction, respectively.
b) Light with the electric field vectors partially and completely aligned in a single direction **PERPENDICULAR** to the propagation direction, respectively.
c) Light with the electric field vectors partially and completely aligned in a direction **TANGENTIAL** to the propagation direction, respectively.
d) Light with the electric field vectors partially and completely aligned **WITHOUT** the propagation direction, respectively.
e) Light partially at the North Pole and completely at the North Pole.

034 qmult 10520 2 5 5 moderate thinking: solar constant and insolation fullmult

1165.** The solar constant $I = 1366\,\mathrm{W/m^2}$ on average (Wikipedia: Solar radiation, 2008feb18). In fact, it's not exactly constant, due to varying Earth-Sun distance, sunspots, and the 11-year solar cycle. But averaged over those variations, it is really very constant which is good for life on Earth. The solar constant is the power per unit area (or intensity) on a sphere surrounding the Sun at the Earth's distance from the Sun.

i) What is the **AVERAGE** solar power per unit area on the spherical top of the Earth's atmosphere? **HINT:** Remember the Earth's a rotating sphere. Think of its cross-sectional area in respect to the solar light flux and its spherical atmosphere surface area. To good approximation the top of the Earth's atmosphere has the same radius as the solid Earth. The significant atmosphere is a thin layer.

ii) The average solar insolation (not insulation) is about half the average power per unit area Earth. The insolation is the power per unit area that reaches the Earth's surface. Assume this average insolation applies to **DELAWARE** (area $6.452 \times 10^3\,\mathrm{km^2}$: note the units of kilometers squared)—yes I know what you are thinking, but it can be found on a map. Given photovoltaic solar panels of 10 % efficiency, what power could Delaware 50 % covered by panels generate?

a) (i) $170\,\mathrm{W/m^2}$ (ii) 0.11 TW.
b) (i) $342\,\mathrm{W/m^2}$ (ii) 0.11 TW.
c) (i) $170\,\mathrm{W/m^2}$ (ii) 0.055 TW.
d) (i) $342\,\mathrm{W/m^2}$ (ii) 2.44 TW.
e) (i) $342\,\mathrm{W/m^2}$ (ii) 0.055 TW.

034 qmult 10522 2 5 2 moderate thinking: solar constant and insolation saudi arabia fullmult

1166.** The solar constant $I = 1366\,\mathrm{W/m^2}$ on average (Wikipedia: Solar radiation, 2008feb18). In fact, it's not exactly constant, due to varying Earth-Sun distance, sunspots, and the 11-year solar cycle. But averaged over those variations, it is really very constant which is good for life on Earth. The solar constant is the power per unit area (or intensity) on a sphere surrounding the Sun at the Earth's distance from the Sun.

i) What is the **AVERAGE** solar power per unit area on the spherical top of the Earth's atmosphere? **HINT:** Remember the Earth's a rotating sphere. Think of its cross-sectional area in respect to the solar light flux and its spherical atmosphere surface area. To good approximation the top of the Earth's atmosphere has the same radius as the solid Earth. The significant atmosphere is a thin layer.

ii) The world average solar insolation (not insulation) is about half the average power per unit area Earth. But in **SAUDI ARABIA**, the average solar insolation is about 85 % of the average power per unit area Earth. The area of **SAUDI ARABIA** is $2.149690 \times 10^6\,\mathrm{km^2}$ (note the units are square kilometers). Given photovoltaic solar panels of 20 % efficiency, what power could **SAUDI ARABIA** 25 % covered by panels generate?

a) (i) $170\,\mathrm{W/m^2}$ (ii) 31 TW.
b) (i) $342\,\mathrm{W/m^2}$ (ii) 31 TW.
c) (i) $170\,\mathrm{W/m^2}$ (ii) 3.1 TW.
d) (i) $342\,\mathrm{W/m^2}$ (ii) 3.1 TW.
e) (i) $342\,\mathrm{W/m^2}$ (ii) 15 TW.

Full-Answer Problems

034 qfull 00410 2 3 0 mod math: frequency, period, Project Seafarer

1167. Project Seafarer (began circa 1980, ended 2004 as learnt from *The X-Files* website) was a program to construct an enormous underground antenna extending over $10,000\,\mathrm{km^2}$. The idea was to use very long wavelength radio waves to communicate with deeply submerged submarines. (More normal length radio waves don't penetrate deeply into conductors like sea water.)

a) If the wavelength were to be 10^4 Earth radii what would the frequency be? (The radius of the Earth is about 6370 km—that's kilometers not meters.)

b) What would the period be?

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034 qfull 00420 1 3 0 easy math: frequency width, 1st order approximation

1168. A particular helium-neon laser emits light at 632.8 nm with a wavelength width of 0.010 nm. What is the corresponding frequency width?

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034 qfull 00510 1 3 0 easy math: Solar constant, Pluto

1169. The total luminosity of the Sun is 3.86×10^{26} W and the mean Sun-Earth distance is 1.496×10^{11} m.

a) What is the intensity (i.e., energy per unit time per unit area) at the Earth?

b) Given that Pluto (formerly the ninth planet) is at about 40 AU (i.e., 40 astronomical units or 40 times the Sun-Earth distance), what approximately is the intensity at Pluto?

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034 qfull 00520 2 5 0 moderate thinking: solar constant and insolation

1170. The solar constant $I = 1366\,\mathrm{W/m^2}$ on average (Wikipedia: Solar radiation, 2008feb18). In fact, it's not exactly constant, due to varying Earth-Sun distance, sunspots, and the 11-year solar cycle. But averaged over those variations, it is really very constant which is good for life on Earth. The solar constant is the power per unit area (or intensity) on a sphere surrounding the Sun at the Earth's distance from the Sun.

a) What is the **AVERAGE** solar power per unit area on the spherical top of the Earth's atmosphere? **HINT:** Remember the Earth's a rotating sphere. Think of its cross-sectional area in respect to the solar light flux and its spherical atmosphere surface area. To good approximation the top of the Earth's atmosphere has the same radius as the solid Earth. The significant atmosphere is a thin layer.

b) The average solar insolation (not insulation) is about half the average power per unit area Earth. The insolation is the power per unit area that reaches the Earth's surface. Assume this average insolation applies to Nevada (area $2.86367 \times 10^5\,\mathrm{km^2}$). Given photovoltaic solar panels of 10 % efficiency, what power could Nevada 50 % covered by panels generate? How does this compare to the US total commercial power consumption (about 3.3 TW) and world total commercial power consumption (about 15 TW) (Wikipedia: 2008apr10)? **HINT:** A terawatt (TW) is 10^{12} W.

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034 qfull 00710 2 3 0 moderate math: blackbody spectrum, Wien's law

Extra keywords: Wien's displacement law

1171. There is actually a simple formula for the blackbody specific intensity as a function of wavelength:

$$B_\lambda = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/(kT\lambda)} - 1} \ ,$$

where λ is wavelength, $h = 6.62606896(33) \times 10^{-34}\,\mathrm{J\,s}$ is Planck's constant, $c = 2.99792458 \times 10^8\,\mathrm{m/s}$ is the vacuum speed of light, $k = 1.3806504(24) \times 10^{-23}\,\mathrm{J/K}$ is Boltzmann's constant, and T is temperature on the absolute or Kelvin scale. Specific intensity to be precise is the energy flow per unit time per unit area perpendicular to the direction of flow per solid angle per wavelength. The MKS units of specific intensity are $\mathrm{W/(m^2\,m\,str)}$ where str stands for steradians the unit of solid angle. If one wants the energy delivered in some time dt, across some area dA, in some solid angle $d\Omega$, in some wavelength range $d\lambda$, it is

$$B_\lambda \, dt \, dA \, d\Omega \, d\lambda \ .$$

a) For analytic results, one frequently wants to use a dimensionless (i.e., unitless) variable in place of λ. The conventional choice is

$$x = \frac{hc}{kT\lambda} \ .$$

Write B_λ in terms of x eliminating all explicit λ's.

b) Find the formula for a non-zero, finite x that makes B_λ stationary with respect to λ. Use the chain rule. Note **NO** explicit analytic formula exists for x. The best you can do is find a simple implicit formula for x: i.e., implicit in this case means that x occurs on both sides of an equal sign. Is the stationary point going to be a maximum, minimum, or inflection point? Explain why.

c) By roughly plotting both sides of the formula found in the part (b) answer or by any other means determine the x that makes B_λ stationary.

d) From the results of foregoing parts, derive Wien's displacement law with an approximate value for Wien's displacement constant (i.e., the constant in Wien's displacement law).

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034 qfull 01010 1 3 0 easy math: polarized light transmission

1172. A completely polarized beam of light of intensity $10\,\mathrm{W/m^2}$ is sent through an ideal polarizer set at an angle of 45° from the polarization direction.

a) What is the intensity of the transmitted beam?

b) If the beam were completely unpolarized what would the intensity of the transmitted beam be?

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034 qfull 01210 3 5 0 tough thinking: tomb raider, radar

Extra keywords: (HRW-832:64)

1173. In your new job as a grave robber—ah, tomb raider—besides the necessary Kung Fu skill and ability to read Sumerian cuneiform, you must also be able to determine the dimensions of a tomb using radar (i.e., tomb radar). A radar pulse sent straight down will sent back a partial reflection (i.e., echo) at every interface. Say radar speed is 10.0 cm/ns in soil, 10.5 cm/ns in Pentelic marble, and 30 cm/ns in air (putrid, filthy air). While searching for the lost final resting place of Dracon the 7th century BCE archon of Athens—you've heard of Draconian legislation one assumes—you measure the following data—throwing out the uninteresting bits like the true scientist you are.

Echo Return Times Along a x-position Line

Δt (ns) \ x-position (m)	0	1	2	20	21	22
1	none	80.1	80.5	80.4	80.6	none
2		164.3	90.8	90.2	170.0	
3			111.0	112.0		
4			122.7	121.2		

a) What is the depth to the top of the tomb?

b) What is the depth to the bottom of the tomb?

c) How thick are the floor and ceiling of the tomb?

d) What are the height and length of hollow part of the death chamber?

e) How are you going to escape the ancient horror you have awoken? **HINT:** Kung Fu does not work on blood-drinking specters.

Chapt. 35 Geometrical Optics

Multiple-Choice Problems

035 qmult 00100 1 4 1 easy deducto-memory: geometrical optics definition
Extra keywords: mathematical physics

1174. "Let's play *Jeopardy*! For $100, the answer is: It is an approximate optical theory that treats light as traveling in straight line rays in homogeneous media and has special rules for dealing with reflection and refraction. The propagation of light is tracked by ray tracing. The theory usually applies whenever the wavelength of light is much smaller than the size scales of the objects light interacts with."

What is _____, Alex?

a) geometrical optics b) optometry c) Opticks
d) Maxwell's electromagnetism e) quantum electrodynamics

035 qmult 00110 1 1 4 easy memory: geometrical optics, classical particles

1175. Geometrical optics treats light in many respects as if light consisted of:

a) vector fields. b) scalar fields. c) clumps of electric charge.
d) classical particles. e) reindeer.

035 qmult 00120 1 1 5 easy memory: validity of geometrical optics

1176. Geometrical optics applies when:

a) $\lambda/D << 1$ where D is an irrelevant size scale.
b) $\lambda/D >> 1$ where D is an irrelevant size scale.
c) $\lambda/D >> 1$ where D is a size scale characteristic of the objects or apertures a light beam encounters.
d) $\lambda/D = 1$ where D is an irrelevant size scale.
e) $\lambda/D << 1$ where D is a size scale characteristic of the objects or apertures a light beam encounters.

035 qmult 00130 1 4 3 easy deducto-memory: rays

1177. In geometrical optics light propagation is treated using _____ which are idealized infinitely narrow beams of light that propagate in straight lines. Laser beams are actually quite a useful approximation to _____.

a) sams b) refractions c) rays d) waves e) reflections

035 qmult 00200 1 1 2 easy memory: law of reflection

1178. The law of reflection is:

a) $n_1 \sin\theta_1 = n_2 \sin\theta_2$. b) $\theta_{\text{incident}} = \theta_{\text{reflected}}$. c) $\dfrac{1}{f} = \dfrac{1}{d_o} + \dfrac{1}{d_i}$.
d) $m = \dfrac{h_i}{h_o} = -\dfrac{d_i}{d_o}$ e) $\theta_{\text{reflected}} = \sin\theta_{\text{incident}}$.

035 qmult 00210 1 1 4 easy memory: specular reflection

1179. Diffuse reflection is reflection from rough surfaces where the size scale of the surface features are of order the wavelength of light or larger. The opposite case is reflection from smooth surfaces where the size scale of surface features is significantly less than that of light. This kind of reflection is called _____ reflection.

a) special b) spectator c) spectacular d) specular e) spectrum

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035 qmult 00350 1 1 5 easy memory: corner reflector

1180. A _____ is a optical device consisting of three plane mirrors fixed together with each one perpendicular to the other two.

a) spherical mirror b) plane mirror c) pocket mirror d) half-silvered mirror e) corner reflector

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035 qmult 00420 1 1 3 easy memory: spherical mirror focal length

1181. The focal length f of a spherical mirror in the paraxial ray approximation is given by _____, where R is the radius of curvature and the upper case is for concave spherical mirrors and the lower case for convex spherical mirrors.

a) $f = \pm\frac{1}{2}R^2$ b) $f = \pm R$ c) $f = \pm\frac{1}{2}R$ d) $f = \pm R^2$ e) $f = \pm\frac{1}{R}$

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035 qmult 00430 1 1 3 easy memory: mirror equation

1182. The mirror equation is:

a) $n_1 \sin\theta_1 = n_2 \sin\theta_2$. b) $\theta_{\text{incident}} = \theta_{\text{reflected}}$. c) $\frac{1}{f} = \frac{1}{d_\text{o}} + \frac{1}{d_\text{i}}$.

d) $m = \frac{h_\text{i}}{h_\text{o}} = -\frac{d_\text{i}}{d_\text{o}}$ e) $\theta_{\text{reflected}} = \sin\theta_{\text{incident}}$.

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035 qmult 00440 2 3 2 moderate math: focal length inequality

1183. For a spherical mirror, the object distance d_o and image distance d_i are both positive. The mirror must be _____ and the focal length f must satisfy _____ and _____.

a) convex; $f < 0$; $f \geq \max(d_\text{o}, d_\text{i})$. b) concave; $f > 0$; $f \leq \min(d_\text{o}, d_\text{i})$.
c) concave; $f < 0$; $f \leq \min(d_\text{o}, d_\text{i})$. d) concave; $f > 0$; $f \geq \max(d_\text{o}, d_\text{i})$.
e) convex; $f > 0$; $f \leq \min(d_\text{o}, d_\text{i})$.

---

035 qmult 00450 1 3 4 easy math: mirror equation solved

1184. For a spherical mirror, we find image distance d_i to be given by:

a) $d_\text{i} = \frac{f^2}{d_\text{o} - f}$. b) $d_\text{i} = \frac{d_\text{o}^2}{d_\text{o} - f}$. c) $d_\text{i} = \frac{d_\text{o} f}{f - d_\text{o}}$. d) $d_\text{i} = \frac{d_\text{o} f}{d_\text{o} - f}$.
e) $d_\text{i} = \frac{d_\text{i} f}{d_\text{o} - f}$.

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035 qmult 00452 1 3 5 easy math: mirror equation solved for magnification

1185. For a spherical mirror, we find the magnification m in terms of d_0 and f to be given by:

a) $m = \frac{f^2}{d_\text{o} - f}$. b) $m = \frac{d_\text{o}^2}{d_\text{o} - f}$. c) $m = -\frac{d_\text{o}}{f - d_\text{o}}$. d) $m = \frac{d_\text{o} f}{d_\text{o} - f}$.
e) $m = -\frac{f}{d_\text{o} - f}$.

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035 qmult 00454 1 1 3 easy memory: mirror equation problem

1186. For a spherical mirror, the image distance $d_\text{i} = -10\,\text{cm}$ and the focal length is $f = -20\,\text{cm}$. What is the object distance?

a) 10 cm. b) -20 cm. c) 20 cm. d) There is no real object in this case.
e) 15 cm.

---

035 qmult 00456 1 1 4 easy memory: mirror equation problem
Extra keywords: a trick question

1187. For a spherical mirror, the image distance $d_\mathrm{i} = -30\,\mathrm{cm}$ and the focal length is $f = -20\,\mathrm{cm}$. What is the object distance?

a) 60 cm. b) -20 cm. c) 20 cm. d) There is no real object in this case. e) 30 cm.

---

035 qmult 00460 1 1 5 easy memory: van Eyck Arnolfini

1188. In the *Arnolfi Portrait* by Jan van Eyck right below the artist's signature (Jan van Eyck was here. 1434—"Just a moment ago it seems") is a small spherical mirror with distorted images including it seems the van Eyck himself in blue. The objects for the images are farther away the mirror than the absolute value of the focal length it seems—and they are upright. The mirror is:

a) contradicted. b) condescended. c) condensed. d) concave. e) convex.

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035 qmult 01000 1 1 3 easy memory: refraction

1189. The deflection of light rays at the interfaces of homogeneous media is:

a) diffraction. b) reflection. c) refraction. d) retraction. e) contraction.

---

035 qmult 01010 1 1 3 easy memory: index of refraction
Extra keywords: mathematical physics

1190. "Let's play *Jeopardy*! For $100, the answer is: For a given medium, it is $n = c/v$, where c is the vacuum light speed and v is the medium light speed."

What is the _____, Alex?

a) index reflection of or reflective of index
b) incidence of refraction or refractive incidence
c) index of refraction or refractive index
d) index of retraction or the retractive incidence
e) innocence of index or the innocuous incidence

---

035 qmult 01100 1 1 1 easy memory: law of refraction

1191. Snell's law or the law of refraction is:

a) $n_1 \sin\theta_1 = n_2 \sin\theta_2$. b) $\theta_\mathrm{incident} = \theta_\mathrm{reflected}$. c) $\dfrac{1}{f} = \dfrac{1}{d_\mathrm{o}} + \dfrac{1}{d_\mathrm{i}}$.

d) $m = \dfrac{h_\mathrm{i}}{h_\mathrm{o}} = -\dfrac{d_\mathrm{i}}{d_\mathrm{o}}$. e) $\theta_\mathrm{reflected} = \sin\theta_\mathrm{incident}$.

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035 qmult 01110 3 1 1 tough memory: Snell's law discovery

1192. Snell's law was first discovered (as far as history records) by _____ and first published (in printing at least) by _____.

a) Ibn Sahl (circa 984); René Descartes (1637)
b) Willebrord Snell (1621); Willebrord Snell (1621)
c) Ptolemy (circa 140 CE); Ibn Sahl (circa 984)
d) Thomas Harriot (1602); René Descartes (1637)
e) Ibn Sahl (circa 984); Ibn Sahl (984)

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035 qmult 01120 2 1 3 mod. memory: small-angle approximation for Snell's law

1193. The small-angle approximation for Snell's law which is _____. This approximation is not bad even for angles as large as $\sim 60°$ and as the angles go to zero it approaches being exact. It is often convenient for quick estimates.

a) $\dfrac{n_1}{\theta_1} \approx \dfrac{n_2}{\theta_2}$ b) $n_1 \sin\theta_1 \approx n_2 \sin\theta_2$ c) $n_1\theta_1 \approx n_2\theta_2$. d) $n_1\theta_1^2 \approx n_2\theta_2^2$
e) $n_1\theta_2 \approx n_2\theta_1$

035 qmult 01130 1 3 5 easy math: Snell's law calculation

1194. The index of refraction of air is 1.000293 (at 0° and 1 atm) and that of water is 1.333 (at 20°). The dependency on temperature and pressure are relatively weak. If a light ray in air is incident on water with incident angle 60°, what is the angle of the refracted ray according to Snell's law and the small-angle approximation to Snell's law?

a) 22.0°; 22.5°. b) 22.0°; 22.0°. c) 28.8°; 30.0°. d) 45.0°; 45.0°. e) 40.5°; 45.0°.

035 qmult 01140 1 4 5 easy deducto-memory: small-angle apparent depth formula

1195. "Let's play *Jeopardy*! For $100, the answer is:

$$\frac{d_{\text{apparent}}}{d} = \frac{n_1}{n_2} \ ,$$

where n_1 is the index of refraction in the medium in which the ray is observed and n_2 is the index of refraction of the medium in which the ray originates.

What is the _____, Alex?

a) small-angle apparent angle formula. b) all-angle apparent angle formula. c) apparent angle formula. d) all-angle apparent depth formula. e) small-angle apparent depth formula.

035 qmult 01150 1 4 1 easy deducto-memory: ray displacement

1196. "Let's play *Jeopardy*! For $100, the answer is: It is the angle change in ray direction when a light ray passes through a transparent homogeneous slab with parallel sides that is embedded in one homogeneous medium or more generally having media of the same index of refraction on each of its two sides."

What is _____, Alex?

a) zero b) 30° c) 60° d) 90° e) $\frac{\pi}{2}$

035 qmult 01200 1 4 2 easy deducto-memory: total internal reflection

1197. "Let's play *Jeopardy*! For $100, the answer is: It is the phenomenon in which rays fail to be transmitted into a transparent medium because its index of refraction n_2 is lower than that of the incident medium n_1 and the angle of incidence exceeds the critical angle given by

$$\theta_{\text{critical}} = \sin^{-1}\left(\frac{n_2}{n_1}\right) \ .$$"

What is _____, Alex?

a) refraction b) total internal reflection c) total internal refraction d) total external refraction e) none of the above

035 qmult 01350 1 1 3 easy memory: rainbow angles

1198. The mean angles of the primary and secondary rainbows from the antisolar point (the point opposite the Sun on the sky) are, respectively:

a) 42° and 82°. b) 51° and 42°. c) 42° and 51°. d) 45° and 90°. e) 51° and 138°.

035 qmult 01520 1 1 2 easy memory: converging and diverging lenses

1199. The simplest kind of converging less is _____ and simplest kind of diverging less is _____.

a) double concave; double convex b) double convex; double concave
c) spherical; antispherical d) planar; flat e) triple convex; triple concave

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035 qmult 01530 1 1 3 easy memory: thin-lens equation

1200. The thin-lens equation is:

a) $n_1 \sin\theta_1 = n_2 \sin\theta_2$. b) $\theta_{\text{incident}} = \theta_{\text{reflected}}$. c) $\dfrac{1}{f} = \dfrac{1}{d_\text{o}} + \dfrac{1}{d_\text{i}}$.

d) $m = \dfrac{h_\text{i}}{h_\text{o}} = -\dfrac{d_\text{i}}{d_\text{o}}$. e) $\theta_{\text{reflected}} = \sin\theta_{\text{incident}}$.

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035 qmult 01540 2 5 3 moderate thinking: converging lens image

1201. The object of a converging lens that is located farther from the focal point on one side of the lens causes a real image to be formed on the other side of the lens farther from the other side's focal point. But what do you see if you look at the object through the lens but are nearer to the lens than the image? **HINT:** What would you see if your eye were right close to the lens where it could be approximated as a planar slab of glass?

a) You would see an **UNINVERTED** image of the object which would be somewhat blurry depending on how well your eyes could focus it. Normal or corrected-to-normal vision is used to focusing parallel rays or **CONVERGING** rays from a source and might have difficulty with **DIVERGING** rays.
b) You would see an **INVERTED** image of the object which would be somewhat blurry depending on how well your eyes could focus it. Normal or corrected-to-normal vision is used to focusing parallel rays or **DIVERGING** rays from a source and might have difficulty with **CONVERGING** rays.
c) You would see an **UNINVERTED** image of the object which would be somewhat blurry depending on how well your eyes could focus it. Normal or corrected-to-normal vision is used to focusing parallel rays or **DIVERGING** rays from a source and might have difficulty with **CONVERGING** rays.
d) Everything would just be a blur.
e) You'd just see black. No light would get through.

---

035 qmult 01650 1 1 4 easy memory: near point and far point

1202. The _____ is the nearest to the eye that the eye can focus (about 25 cm for normal vision) and the _____ is the farthest to the eye that the eye can focus (about infinity for normal vision).

a) close point; far point b) solar point; antisolar point c) focus; counter-focus
d) near point; far point e) far point; near point

---

035 qmult 01700 1 1 4 easy memory: magnifying glass

1203. A magnifying glass just by itself is used primarily for:

a) colonoscopy. b) microscopy. c) astronomy.
d) close work (either to the eye or to the object of the work). e) frying ants.

---

035 qmult 01910 1 4 5 easy deducto-memory: Hans Lippershey

1204. "Let's play *Jeopardy*! For $100, the answer is: The probable inventor of the telescope or at least the earliest person known to have built one."

Who is _____, Alex?

a) William the Silent (1533–1584) b) Rembrandt van Rijn (1606–1669)
c) Galileo Galilei (1564–1642) d) Johannes Kepler (1571–1630)
e) Hans Lippeshey (1570–1619)

---

035 qmult 01920 1 4 4 easy deducto-memory: Keplerian telescope

1205. "Let's play *Jeopardy*! For $100, the answer is: The inventor of the _____ or astronomical telescope. He/she also showed the planetary orbits were nearly elliptical."

Who is _____, Alex?

a) Apollonios of Perga (circa 3rd century BCE)
b) Nicolaus Copernicus (1473–1543)
c) Galileo Galilei (1564–1642)
d) Johannes Kepler (1571–1630)
e) Caroline Herschel (1750–1848)

---

035 qmult 01930 1 1 1 easy memory: Keplerian telescope made

1206. You can make a Keplerian telescope from:

a) two converging lenses.
b) two diverging lenses.
c) a converging lens and a diverging lens.
d) three diverging lenses.
e) three converging lnses.

---

035 qmult 01940 1 1 2 easy memory: Keplerian telescope magnification

1207. The formula for the magnification of the Keplerian telescope is:

a) $M = -\dfrac{f_\mathrm{e}}{f_\mathrm{o}}$.
b) $M = -\dfrac{f_\mathrm{o}}{f_\mathrm{e}}$.
c) $M = -\dfrac{d_\mathrm{i}}{d_\mathrm{o}}$.
d) $M = -f_\mathrm{o} f_\mathrm{e}$.
e) $M = -d_\mathrm{i} d_\mathrm{o}$.

---

035 qmult 10350 3 5 2 tough thinking: Louis XIV mult

Extra keywords: Well really hard to solve, but not so hard to remember.

1208.*** You are Louis XIV (Louis Quatorze, Louis le Grand, le Roi Soliel, L'État, c'est moi, etc.) and it's midnight during the construction of Versailles and unable to sleep ("uneasy lies the head that wears the crown") you creep downstairs to the unfinished *Galerie des Glaces* (Hall of Mirrors) and there find yourself between two large plate mirrors a distance $2h$ apart in the y direction: the mirrors are parallel. Fascinated by the myriads of images you put your candle stick on a table a distance Δx away to your right. You are the origin of the x-y system *naturellement* which is *un système Cartésien naturellement*. The flame is equidistant from the mirrors as is yourself. The flame and your eyes are at the same height. Having studied Euclid's *Catoprics* under your personal geometry tutor, you know all about the geometrical optics of reflection.

i) What is the general formula for apparent x position of the **FLAME IMAGE** (i.e., the x position where the flame appears to be) in terms of the number of reflections n the image ray underwent?

ii) What is the general formula for apparent y position of **FLAME IMAGE** (i.e., the y position where the flame appears to be) in terms of the number of reflections n the image ray underwent?

a) (i) $x = 2n\Delta x$ (ii) $y = 2hn$.
b) (i) $x = \Delta x$ (ii) $y = 2hn$.
c) (i) $x = \Delta x$ (ii) $y = h$.
d) (i) $x = \Delta x/n$ (ii) $y = h/n$.
e) (i) $x = 2n\Delta x$ (ii) $y = h$.

Full-Answer Problems

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035 qfull 00350 3 5 0 tough thinking: Louis XIV and multiple mirror images

1209. You are Louis XIV (Louis Quatorze, Louis le Grand, le Roi Soliel, L'État, c'est moi, etc.) and it's midnight during the construction of Versailles and unable to sleep—"uneasy lies the head that wears the crown"—you creep downstairs to the unfinished *Galerie des Glaces* (Hall of Mirrors) and there find yourself between two large plate mirrors a distance $2h$ apart in the y-direction: the mirrors are parallel. Fascinated by the myriads of images, you put your candle

stick on a table at distance Δx to your right. You are the origin of the x-y system *naturellement* which is *un système Cartésien naturellement*. The flame is equidistant from the mirrors as is yourself. The flame and your eyes are at the same height. Having studied Euclid's *Catoprics* under your personal geometry tutor, you know all about the geometrical optics of reflection.

NOTE: There are parts a,b,c,d. The parts can be done somewhat independently and out of order. So do not stop if you cannot do any part first off.

a) You look straight at the mirror in the positive y-direction. Qualitatively, describe how the multiple image mirrors in the mirror appear to you and how you fit into the image in the mirror. **HINT:** Mirror within mirror. Also, you might have to correct your answer after completing the rest of the question.

b) Qualitatively describe how a ray from the candle gets to your eye for any image seen in the mirror in front of you. How many reflections are needed for the image m (the mth image)? Assume your eye is a single point at the origin facing the positive y-direction. Your head is not a point: i.e., you cannot see in the negative y-direction. **HINT:** Reflect on the law of reflection. Drawing a diagram helps a lot.

c) Derive formulae for δx (the x-position of the last reflection for image m), θ (the angle formed by the normal from your eye to the mirror and to the point of last reflection which has x-coordinate δx), and y_m (the apparent distance to image m. How far apart are the images in the apparent y distance? **HINT:** Drawing a diagram helps a lot.

d) Why should the candle image sizes decrease inverse proportionally to apparent distance y_m? **HINT:** Think of a high symmetry object instead of a candle.

---

035 qfull 01110 3 5 0 mod thinking: refraction proofs

1210. There are **THREE** relatively simple refraction situations which can be proven.

a) Prove Snell's law. Assume (1) that the wave fronts are bent but not broken as they cross the interface, (2) the wave frequency is the same in both media, and (3) the index of refraction for a medium is $n = c/v$ where v is the speed of light in the medium. **HINT:** Draw a diagram and focus on the segment of the interfaces between two wave fronts.

b) Prove the displacement result for a light ray that travels through a transparent slab with parallel sides embedded in a medium of uniform index of refraction: i.e., prove that the ray emerges from the slab on line path parallel to its original line path but displaced from that original line path. **HINT:** Draw a diagram.

c) Using Snell's law (and **NOT** using the the general apparent depth formula), prove the **SMALL-ANGLE APPARENT DEPTH FORMULA**

$$\frac{d_{\text{apparent}}}{d} = \frac{n_1}{n_2} \, ,$$

where d is the actual depth of an object in medium 2, d_{apparent} is the apparent depth of the object as seen from medium 1, n_1 is the index of refraction in medium 1, and n_2 is the index of refraction in medium 2. Make sure you consider both the case where $n_1 < n_2$ and the case where $n_1 > n_2$. **HINT:** A common example of apparent depth differing from actual depth is when you look down into a glass of water and see a straw seemingly bent at the water interface. Draw a diagram.

---

035 qfull 10200 2 5 0 moderate thinking: 2-d corner reflector

1211. You have corner reflector. This consists of two mirrors attached to each other at a right angle.

a) Say a beam of light from infinity enters the corner reflector in a plane perpendicular to the two mirrors: i.e., the problem is only a two-dimensional one. The beam reflects **TWICE** and then exits going off to infinity What is the total angular deflection of the beam from its initial path? (Note angular deflection is the angle between the path the beam would have

followed if not deflected and the path it actually does follow. Ask if this is not perfectly clear.) A general result is required: you are **NOT** allowed to assume that beam is deflected at each mirror by 90°. **HINT:** What is the angular deflection on each reflection? How can they be related by geometry? Sum these angles making sure that you count them in the same sense: i.e., counterclockwise or clockwise, but not both ways.

b) Now say that you tip your corner reflector so that the corner is at the bottom and the reflector is symmetrical about a vertical axis. You fill the reflector with water assuming there is some end panes to hold the water in. In this case what is the two-dimensional deflection angle for a beam coming in from infinity, reflected twice, and deflected out to infinity? **HINT:** Reduce the problem to that of part (a) and make use of the law of refraction.

---

035 qfull 10300 3 5 0 tough thinking: the rainbow, focusing
Extra keywords: stationary points

1212. You have a spherical drop of water and draw a horizontal axis through its center. A beam of light travels to the right and is parallel to the axis. It strikes the drop at an angle θ relative to the normal to the drop. Inside the drop the beam is deflected at an angle θ_{refr} relative to the normal using the usual incident/refracted beam conventions. Draw a big, clear diagram.

a) Using geometry (e.g., the isosceles triangle theorem), and the laws of reflection and refraction find an expression in terms of θ only for the total angular deflection θ_{tot} of the beam from its initial path for the case that the beam reflects once inside the drop and then emerges. **HINT:** Find the deflection from the current path for each event (i.e., refraction or reflection) and sum these.

b) Find the expression for θ that makes θ_{tot} stationary. Is this stationary point a maximum or minimum or neither? Now find an expression for the stationary θ_{tot}. **HINT:** You will need the formula

$$\frac{d}{dx}\sin^{-1}(u) = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx} \ .$$

c) Given $n_{\text{air}} = 1.00029$ (at STP) and $n_{\text{water}} = 1.333$ (at 20°), what is the numerical stationary θ_{tot} in degrees. Also what is $180° - \theta_{\text{tot}}$.

d) At any stationary point, does a function vary comparatively quickly or slowly with respect to its argument?

e) If a range of beams of varying angle are redirected into a narrower beam, what would that be called?

f) Describe how the drop's effect on beams of light results in a rainbow.

Chapt. 36 Interference

Multiple-Choice Problems

036 qmult 00100 1 4 5 easy deducto-memory: interference

1213. "Let's play *Jeopardy*! For $100, the answer is: It happens when you mix waves of just about any wave phenomena."

a) What is friction, Alex?
b) What is a contact force, Alex?
c) What is polarization, Alex?
d) What is the prime directive, Alex?
e) What is interference, Alex?

036 qmult 00200 1 1 1 easy memory: superposition principle

1214. The superposition principle is name for the fact that waves that are sufficiently small can be added:

a) linearly.
b) quadratically.
c) inversely.
d) negatively.
e) sincerely.

036 qmult 00300 1 1 4 easy memory: interference criterion

1215. Interference and diffraction effects are small for a wave front breaking obstacle or aperture of characteristic size d when:

a) $\lambda/d >> 1$.
b) $\lambda/d = 1$.
c) $\lambda/d \approx 1$.
d) $\lambda/d << 1$.
e) $\lambda d >> 1$.

036 qmult 00400 1 1 1 easy memory: visible range

1216. The wavelength range of human vision is:

a) ~ 0.4–$0.7\,\mu$m.
b) ~ 0.1–$1.0\,\mu$m.
c) ~ 0.4–0.7 m.
d) ~ 0.4–0.7 km.
e) ~ 0.7–$2.5\,\mu$m.

036 qmult 00500 1 1 2 easy memory: quantum electrodynamics

1217. The best theory of electromagnetism known is

a) Classical or Maxwellian electrodynamics.
b) quantum electrodynamics (QED).

c) Huygens' principle.
d) the Huygens-Fresnel principle.
e) Kirchhoff's scalar theory of light.

---

036 qmult 00600 1 1 5 easy memory: photons and wave trains

1218. Quantum mechanical photons or coherent groups of photons (produced by stimulated emission such as occurs massively in lasers) can be identified with the classical:

a) wave horses.
b) wave iron horses.
c) wave donkeys.
d) wave chariots.
e) wave trains.

---

036 qmult 00700 1 1 3 easy memory: coherence

1219. Coherence (in regard to wave phenomena) is the property of having:

a) logical consistency.
b) a random phase relation in space and/or time.
c) a definite phase relation in space and/or time.
d) stickiness.
e) logical inconsistency.

---

036 qmult 00800 1 1 3 easy memory: in phase

1220. If waves are in phase, they

a) are incoherent.
b) have a non-zero phase difference.
c) have a zero phase difference.
d) have a periodic time-varying phase difference.
e) have a good working relationship.

---

036 qmult 00900 1 4 5 easy deducto-memory: complete destructive interference

1221. "Let's play *Jeopardy*! For $100, the answer is: zero."

a) What is less than, Alex?
b) What is a cipher, Alex?
c) What is oreth spelled backwards, Alex?
d) What is the intensity of a constructive interference, Alex?
e) What is the intensity of a complete destructive interference, Alex?

---

036 qmult 01000 1 1 1 easy memory: two-slit pattern

1222. The two-slit interference pattern for visible light and for low orders typically appears as

a) roughly evenly spaced bright and dark fringes.
b) roughly evenly spaced bright and red fringes.
c) fringes with the bright ones much narrower than the dark ones.
d) Airy rings.
e) a continuous spectrum.

---

036 qmult 01100 1 1 4 easy memory: two-slit formula

1223. The formula for the maxima of a two-slit interference pattern (for slit separation of d) is probably most easily remembered (maybe because it can be instantly derived) in the form:

a) $d \sin\theta = n$.

b) $d\cos\theta = n\lambda$.
c) $d^2\sin\theta = n\lambda$.
d) $d\sin\theta = n\lambda$.
e) $\sin\theta = n$.

Full-Answer Problems

036 qfull 00100 1 3 0 easy math: sodium line, index of refraction
Extra keywords: (HRW-884:1E)

1224. The wavelength of the yellow sodium D lines in air is approximately 0.589 μm. Note, the index of refraction of air at 0°C and 1 atm is 1.000293 (HZ-38).

a) What is the frequency of this light?

b) What is wavelength in glass with index of refraction $n = 1.52$?

c) What is the speed of light in this glass?

036 qfull 00200 1 5 0 easy thinking: waves on beach
Extra keywords: (HRW-884:5P)

1225. Consider ocean waves moving at 4.0 m/s incident on a beach at angle of 40° to the normal to the beach. (Here the angle refers to the direction that waves are moving in, not to the angle set by the line of their crests.) Suddenly they cross into a shallower region where the wave speed is only 3.0 m/s. **NOTE:** I derived this problem from HRW. I assume they know what they are talking about.

a) Assuming the law of refraction applies to water waves just as it does to light, what is the angle to the normal of the waves in the shallower water. **HINT:** Draw a diagram.

b) Give an explanation as to why waves often come the shore at an angle of incidence of nearly zero.

036 qfull 00400 3 5 0 tough thinking: variational principle
Extra keywords: law of reflection and law of refraction (Snell's law)

1226. In the Huygens-Fresnel principle each point of a wave front acts as a sourcelet for a new wavelet. This principle has a lot of truth in it. But alone it isn't the whole story because for one thing we know those sourcelets don't radiate isotropically. Incorporating anisotropy, however, is well beyond our scope. Another problem is how does one deal with a continuum of sourcelets? At this point what is called a variational principle will come to our aid and give us some insight if still not the whole story.

The optical variational principle (often called Fermat's principle) has a long history: it was first formulated by Hero of Alexandria (circa 150–250 AD) and reformulated by Pierre de Fermat (1608–1665) in 1657 (HZ-67). We won't consider those antique versions here. The modern version is that significant light energy strong tends to follow the path in space that is stationary with respect to arbitrary variations in optical path or phase change (which is just 2π times the optical path). An optical path length u between points A and B is

$$u = \int_A^B \frac{ds}{\lambda(s)} \ ,$$

where s is some path coordinate and $\lambda(s)$ is the wavelength of the radiation at s. The wavelength can vary with s due to varying index of refraction:

$$\lambda(s) = \frac{\lambda_{\text{vacuum}}}{n} \ ,$$

where $n = c/v(s)$ is the index of refraction which is always greater than 1, except in a vacuum.

What does stationary mean here? Just what it means in calculus: the path is constant with respect to first order perturbations. All the light starts in phase from A and flows along the stationary path from A to B and along those continuum of paths that are close to the stationary path will arrive with nearly the same phase. This light will interfere mostly constructively. Light that comes along other paths will arrive with a wide range of phases and will strongly trend to cancel out—I think the trend to cancellation is tricky to show in general, but nature has no problem doing it. Note we also have principle of reversibility: a stationary path from A to B is also one from B to A.

It is beyond our scope to study methods for finding the stationary path in general. That involves variational calculus. Suffice it to say here that if index of fraction is constant the stationary path between two points is a straight line. Perhaps it is better to say that one stationary path is a straight line.

a) Consider A and B in a medium with constant index of refraction. There is a horizontal mirror below the points and the mirror surface defines an x axis: the perpendicular axis to the x axis is the y axis. The point A is at $(0, y_1)$ and B is at (ℓ, y_2). One stationary path between A and B is the straight line that joins them. This is why when you stand at B you see a source at A when looking straight at A. But don't you see the A source somewhere else in this case? The mirror acts as an excellent scatterer of light: it doesn't absorb or transmit a lot. Consider a straight-line path from A to the mirror at arbitrary position x and another straight-line path from x to B. Show that the stationary "reflection" optical path is the one that gives the law of reflection: $\theta_{\rm inc} = \theta_{\rm refl}$, where $\theta_{\rm inc}$ and $\theta_{\rm refl}$ are the angles of incidence and reflection measured from the normal to the mirror surface. **HINT:** Draw a diagram.

b) Now what about the law of refraction (Snell's law)? Change the mirror into and interface between medium 1 with index of refraction n_1 and medium 2 with index of refraction n_2. Let y_2 for B be negative. Show that the stationary optical path is the one that yields

$$n_1 \sin(\theta_{\rm inc}) = n_2 \sin(\theta_{\rm refr}) \ .$$

where $\theta_{\rm refr}$ is the angle of refraction measured from the normal to the interface. **HINT:** Draw a diagram.

NOTE: The optical variational principle, I hope, has given some insight into the paths through space on which light energy flows: these paths can be considered a continuum of sourcelets. But note it does not say how much light goes backward or forward at each sourcelet. The wave variational principle does not tell us that. And the sourcelets are not all the same. The sourcelets in vacuum, in the middle of media, and at interfaces will all give somewhat different behavior. To understand that behavior one needs a more detailed theory of electromagnetic radiation and its interaction with matter. On the other hand, the optical variational principle can be (and has been) generalized to other kinds of waves: e.g., water waves I think. One important example of a generalization is Richard Feynman's formulation of quantum mechanics. This formulation is equivalent to other quantum mechanical formulations, but is based on variational principle for matter waves. As an equivalent formulation, it in a sense says nothing new, but in another sense it gives one a new perspective on the physics.

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036 qfull 00500 2 3 0 moderate math: reflected two-slit interference

1227. An instructor (but probably not yours) sets up a two-slit interference demonstration with a 632.8 nm laser at the front of the class. The laser beam points straight out into the class and passes through the two slits creating a pattern. The pattern reflects off a mirror 20.0 m away from the slits and travels back to a screen at the front of the class that is 22.0 m from the mirror.

The two-slit device, mirror, and screen are all perpendicular to the optical axis established by the beam.

a) Draw a diagram of the setup. Include the location of the virtual image of the screen pattern in the region behind the mirror.

b) What simple mirror symmetry trick allows you to analyze the interference pattern easily: i.e., what trick reduces the problem to an already solved problem?

c) If the bright fringe maxima on the screen are all separated by about 10.0 cm, what is the slit separation determined using the (1st order) small angle approximation? Justify using the small angle approximation both a priori and a posteriori.

---

036 qfull 00600 2 5 0 moderate thinking: slightly disaligned slits

1228. For simple developments we usually think of the two-slit device as exactly perpendicular to the optical axis (defined by the incoming beam direction). But in point of fact in any undergraduate experiment such exact aligned probably can't be obtained. There will always be some disalignment. But it turns out this disalignment does not matter to 1st order in small angles in the Huygens-Fresnel principle picture.

a) Draw a good diagram with a horizontal beam aimed at a two-slit device (with slit separation d). The normal to the slit device is at θ_{dis} above the horizontal. Two paths come from infinity horizontally and pass through the slits then head off at an angle θ' above the normal to the device to a point P at infinity. Mark these two paths on the diagram and show the two triangles on either side of slit device mark out the lengthening of the bottom path.

b) Remember the phase change for traveling a distance r is $\phi = kr$, where $k = 2\pi/\lambda$ is the wavenumber. What is the phase difference between the paths described in part (a)? Note

$$\theta' = \theta - \theta_{\text{dis}} \ ,$$

where θ is the angle from the horizontal.

c) Make the small angle approximation for the phase difference of part (b) and show that the result is the same as the phase difference for an exactly vertical slit device in the small angle approximation. What does this demonstration mean for undergraduate experiments?

d) Why can't the agreement of results found in part (c) be exactly right even in the small angle limit?

---

036 qfull 00610 2 5 0 moderate thinking: stupid disaligned slits

Extra keywords: Stupid question: misses the obvious, see 00600

1229. For simple developments we usually think of the two-slit device as exactly perpendicular to the optical axis (defined by the incoming beam direction). But in point of fact in any undergraduate experiment such exact aligned probably can't be obtained. There will always be some disalignment. This disalignment (in the Huygens-Fresnel principle picture) amounts to there being a fixed phase difference ϕ_{dif} between the two slit sources for the interference pattern. Thus the effective phase difference for outgoing beams at infinity is

$$\phi = kd\sin(\theta') - \phi_{\text{dif}} \ ,$$

where $k = 2\pi/\lambda$ is the wavenumber and θ' is measured two-slit axis (i.e., from the axis that is perpendicular to the two-slit device). The relation of θ' to θ (the angle from the optical axis) is

$$\theta' = \theta - \Delta\theta_{\text{dis}} \ ,$$

where $\Delta\theta_{\text{dis}}$ is the angle of two-slit axis from the optical axis.

a) Draw a diagram of the system.

b) Solve for θ'.

c) For the small angle approximation, define an new angle θ'' (in terms of both θ' and θ'') such that we you have a formula for θ'' that exactly resembles the formula for θ in the exact alignment case.

d) Why don't we ever worry about disalignment in undergraduate experiments?

---

036 qfull 00700 2 5 0 moderate thinking: FWHM for two slit

1230. The intensity function for the far-field two-slit interference pattern is

$$I = 4\frac{I_0}{r^2}\cos^2(\alpha) \ ,$$

where r is the distance from the slits to the screen, I_0/r^2 is the intensity pattern for a single infinitely narrow slit, and

$$\alpha = \frac{\phi}{2} = \pi\frac{d}{\lambda}\sin\theta \ .$$

The angle θ is the angle measured from the optical axis, of course. The full width half maximum (FWHM) is the spacing between the two points on either side of an intensity peak where the height is half of the peak height. Show that the FWHM for the central bright fringe of the two-slit setup is

$$\Delta\theta = \frac{1}{2}\frac{\lambda}{d}$$

assuming the small angle approximation and that r can be treated as constant.

Chapt. 37 Diffraction

Multiple-Choice Problems

037 qmult 00100 1 1 2 easy memory: diffraction defined

1231. Diffraction and interference are essentially the same physical phenomenon. It is customary though to use the term diffraction when there are:

a) few sources.
b) many sources or even a continuum of them.
c) no sources.
d) 2 sources.
e) different sources.

037 qmult 00200 1 4 2 easy deducto-memory: shadows

1232. "Let's play *Jeopardy*! For $100, the answer is: A large region of strong (but not necessarily complete) destructive interference."

a) What is a substance, Alex?
b) What is a shadow, Alex?
c) What is a dark fringe of a two-slit interference pattern, Alex?
d) What is the path of the Huns, Alex?
e) What is the Shadow, Alex?

037 qmult 00300 1 1 1 easy memory: one-slit pattern

1233. The one-slit diffraction pattern has:

a) a broad, bright central fringe.
b) a broad, dark central fringe.
c) three broad bright central fringes.
d) a narrow, dim central fringe.
e) a broad, shaggy central fringe.

037 qmult 00400 1 1 4 easy memory: one-slit zeros

1234. The formula for the zeros of a one-slit diffraction pattern (for a slit of width a) is probably most easily remembered (maybe because it so similar to the two-slit maxima formula) in the form:

a) $a \sin\theta = n$.
b) $a \cos\theta = n\lambda$.
c) $a^2 \sin\theta = n\lambda$.
d) $a \sin\theta = n\lambda$.
e) $\sin\theta = n$.

037 qmult 00500 1 1 1 easy memory: enveloping patterns

1235. The two-slit interference pattern (slit center separation d) will be enveloped by the one-slit diffraction patterns of its two slits (widths a) if

a) $a << d$.

b) $a = d$.
c) $a >> d$.
d) $a = 2d$.
e) $a = 3d$.

---

037 qmult 00600 1 4 4 easy deducto-memory: diffraction grating

1236. "Let's play *Jeopardy*! For $100, the answer is: An optical device consisting of many narrow slits (called rulings) or reflective grooves (also called rulings)."

a) What is a Michelson interferometer, Alex?
b) What is a converging lens, Alex?
c) What is a compact disk, Alex?
d) What is a diffraction grating, Alex?
e) What is a laser, Alex?

---

037 qmult 00700 1 1 5 easy memory: Rittenhouse

1237. The diffraction grating was apparently invented (not that anyone noticed much) in 1785 by:

a) Ben Franklin.
b) Benjamin Thompson (ennobled as Graf Rumford).
c) Thomas Jefferson.
d) Sam Adams.
e) David Rittenhouse.

---

037 qmult 00800 1 1 5 easy memory: use of diffraction grating

1238. The diffraction grating is an important device in:

a) microscopy.
b) sewer systems.
c) cameras.
d) compact disks.
e) spectroscopy.

---

037 qmult 00900 1 1 1 easy memory: diffraction grating resolution

1239. The bright fringes of a diffraction grating diffraction pattern are:

a) very high and narrow and this allows high wavelength resolution.
b) very low and broad and this allows low wavelength resolution.
c) are exactly like two-slit bright fringes.
d) are more or less like two-slit bright fringes.
e) very high and narrow and this allows low wavelength resolution.

Full-Answer Problems

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037 qfull 00010 1 3 0 easy math:

1240. Wavelength 0.633 μm light is incident on a single slit.

a) Qualitatively sketch the diffraction pattern intensity as a function of θ.

b) What is the analytic expression for the angular difference between the nth order minima on the opposite sides of the central maximum?

c) The two first order minima are separated by 3.0°. What is the slit width a?

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037 qfull 00100 1 3 0 easy math: sound diffraction, concert A

Extra keywords: (HRW-913:6p)

1241. As far as patterns go (but not speed, intensity, reflection, transmission) a lot of wave phenomena behave quite alike—they'd scarcely all be called wave phenomena if they didn't.

a) The speed of sound in air at 20° and 1 atm is 343 m/s (HRW-400). The frequency range of human hearing is approximately 30-30, 000 Hz (a large dynamic range: CAC-66). What is the range of human hearing in wavelength?

b) What is the wavelength corresponding to concert A ($f = 440$ Hz: CAC-67)?

c) Say someone (who shall be nameless here) is clarinetting—any word can be verbed— at concert A with amplification in one chamber and the sound waves produced are plane waves that intercept head-on a tall (very tall) door that is 4 m wide beyond which is another chamber (with echo-proof walls) that stretches $\ell = 100$ m straight away from the door to a wall at which a solitary (and lonely) listener is seated right at the 2nd order diffraction minimum and having trouble hearing—which is maybe all for the best—is adjusting her hearing aid, but of course that does not help unless she moves from her unfortunate location at x meters from the axis that runs normal from the door to the wall that she is seated at—and, wouldn't you know it, the poser of this conundrum wonders what x would be, full stop, period, hang-up, roger over and out.

d) What would x from the part (c) be if the door were only 1 m wide? Explain your answer.

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037 qfull 00110 1 3 0 easy math: digital television

1242. Conventional television **BROADCAST** signals have a wavelength of $\sim$ 50 cm. Digital television according plans (circa 2001) will have a wavelength of $\sim$ 10 mm.

a) Will the change to digital increase or decrease the diffraction of television signals into the shadow regions of geometrical optics? Briefly justify your answer.

b) Say the gap of 5 m between two buildings approximates a single slit—which the probably don't do very well because of their finite thickness. For the conventional television wavelength, what is the angular width (in degrees) of the central diffraction bright fringe **BETWEEN** the first zeros (i.e., points of complete destructive interference)?

c) Repeat part (b), except now use the digital television wavelength. Could lack of diffraction be a problem for digital television? Why?

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037 qfull 00200 3 5 0 tough thinking: Babinet's principle

1243. Say you had some kind of aperture beyond which is a region only illuminated by the light through that aperture.

a) You have two masks which both cover the whole aperture. The first is completely transparent where the second is opaque and opaque where the second is completely transparent: i.e., the masks are complementary. When there is no mask on the aperture, the electric field (considered a scalar field as are all the others in this problem) in the beyond-region is $E(\vec{r})$. When the first mask is put on the aperture, the electric field in the beyond-region is $E_1(\vec{r})$. When the second mask is put on the aperture, the electric field in the beyond-region is $E_2(\vec{r})$. Using the Huygens-Fresnel principle theory find the relation satisfied by $E(\vec{r})$, $E_1(\vec{r})$, and $E_2(\vec{r})$. Explain why the relation occurs. The relation is gloriously called Babinet's principle.

b) Why can't Babinet's principle be exactly correct?

c) Taking Babinet's principle to be exact what is the relation between the intensities caused by the masks (i.e., $I_1(\vec{r})$ and $I_2(\vec{r})$) in a region where the unmasked aperture gives zero

intensity $I(\vec{r}) = 0$: e.g., a region of shadow. What can you say about the diffraction patterns cast by the two masks in regions of shadow.

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037 qfull 00300 1 3 0 easy math: gyrinid beetle
Extra keywords: (HRW-916:36E)

1244. Tropical gyrinid beetles (whirligig beetles) have scales whose alignment makes up a scattering (or reflection) diffraction grating. The diffraction pattern from the grating may confuse predators. Say 0.555 μm light (green light: HZ-56) is incident normal to the beetle grating and a 1st order maximum is seen at an angle of 26° from the normal.

a) What is the grating spacing?

b) What angle of the 2nd order maximum?

c) Ordinarily the grating will scatter white light. What would you see if you projected this light on a screen? What would you see if you were looking at the beetle? Assume the beetle grating works like an ideal grating. Real beetles may be less than ideal gratings.

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037 qfull 00400 1 3 0 easy math: X-ray diffraction, Bragg law
Extra keywords: (HRW-917:53E)

1245. You shine 1.2 Å X-rays on a crystal of lithium floride.

a) The 2nd order reflection occurs at a Bragg angle of 28°. What is the interplanar spacing in Angstroms for this reflection? **HINT:** Recall that the convention in X-ray diffraction is to measure angle from the surface of scattering, not from the normal as in visual optics.

b) What is the angle of the 1st order reflection for the interplanar spacing?

Chapt. 38 Special Relativity

Multiple-Choice Problems

038 qmult 00100 1 1 2 easy memory: Galilean transformations

1246. Classical Newtonian laws of physics are inertial frame covariant under:

a) the Lorentz transformations. b) the Galilean transformations. c) no transformations at all. d) the Keplerian transformations. e) the Maxwellian transformations.

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038 qmult 00110 1 4 1 easy deducto-memory: Heinrich Hertz

1247. Heinrich Hertz (1857–1894) experimentally verified the existence of:

a) electromagnetic waves. b) light. c) an upper limit to all physical velocities. d) gravity. e) gravitational radiation.

---

038 qmult 00200 1 1 1 easy memory: discovery special relativity

Extra keywords: physci

1248. Einstein first published his theory of special relativity in:

a) 1905. b) 1879. c) 1955. d) 1776. e) 1066.

---

038 qmult 00300 1 1 2 easy memory: 2 special relativity postulates

Extra keywords: physci KB-94-19

1249. Einstein developed special relativity starting from _____ basic postulates—at least in the accounts of most textbooks—which may be thinking of the ideal Einstein rather than of the historical Einstein.

a) zero b) two c) three d) four e) infinite

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038 qmult 00310 1 1 3 easy memory: relativity postulate 1

Extra keywords: physci KB-92-11, KB-94-19

1250. According to the principle (or postulate) of relativity of special relativity, the laws of physics or, more exactly, the expressions of physical law are the same in:

a) all frames of reference.
b) most frames of reference.
c) all inertial frames (i.e., fundamentally unaccelerated frames) of reference.
d) all non-inertial frames (i.e., fundamentally accelerated frames) of reference.
e) no frames of reference

---

038 qmult 00320 1 4 5 easy deducto-memory: relativity postulate 2

Extra keywords: physci KB-92-11, KB-94-19

1251. "Let's play *Jeopardy*! For $100, the answer is: That the laws of physics are the same for observers in all inertial frames."

What is _____, Alex?

a) Einstein's vacuum speed of light postulate b) postulate of the invariance of coincidence c) $\vec{F}_{\text{net}} = m\vec{a}$ d) horse sense e) Einstein's relativity postulate in special relativity

---

038 qmult 00330 1 1 4 easy memory: light speed postulate
Extra keywords: physci KB-94-19

1252. The vacuum speed of light has the same value, labeled by c, in all directions in:

a) some inertial frames. b) frames moving with the Earth only. c) no frames. d) all inertial frames. e) no inertial frames.

---

038 qmult 00400 1 4 2 easy deducto-memory: revising Newtonian mechanics
Extra keywords: physci

1253. Einstein's two special relativity postulates implied that classical Newtonian physics:

a) needed no revision. b) needed revision. c) was wildly wrong in all cases of interest. d) was not correct now, but would be in the future. e) was completely useless.

---

038 qmult 00500 1 1 5 easy memory: Lorentz transformations

1254. In special relativity, the spacetime coordinates in different inertial frames are related by the _____ transformations.

a) Galilean b) Keplerian c) Newtonian d) Maxwellian e) Lorentz

---

038 qmult 00510 1 4 4 easy deducto-memory: deriving the Lorentz trans.

1255. In deriving the Lorentz transformations the condition is imposed that the:

a) vacuum light speed varies with inertial frame.
b) non-vacuum light speed varies with inertial frame.
c) non-vacuum light speed is inertial-frame invariant.
d) vacuum light speed is inertial-frame invariant.
e) vacuum light speed is 1 m/s.

---

038 qmult 00610 1 4 1 easy deducto-memory: Fitzgerald contraction

1256. The proper or rest-frame length L_0 parallel to the direction of motion of an object moving uniformly relative to an inertial frame S is related to the length L measured by an observer at rest in S by the Fitzgerald contraction formula:

a) $L = L_0\sqrt{1-\beta^2}$. b) $L = L_0/\sqrt{1-\beta^2}$. c) $L = L_0$. d) $L = 1/L_0$. e) $L = L_0^2$.

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038 qmult 00700 1 4 4 easy deducto-memory: relativistic mass increase
Extra keywords: physci KB-92-12

1257. As an object's velocity increases relative to some observer, the mass of the object as measured by that observer:

a) goes to zero. b) decreases. c) stays constant. d) increases and approaches infinity as the speed approaches the vacuum light speed. e) is infinite.

---

038 qmult 00800 1 1 2 easy memory: time dilation mnemonic
Extra keywords: physci

1258. The mnemonic for the time dilation effect is "moving clocks:

a) run fast." b) run slow." c) are stopped." d) are right twice a day." e) run backward."

---

038 qmult 00900 1 4 3 easy deducto-memory: twins paradox
Extra keywords: physci

1259. In the twins paradox, the twin who ages least is the one who is:

a) not accelerated. b) not Fitzgerald contracted. c) accelerated. d) Fitzgerald contracted. e) hermetically sealed in a time capsule.

---

038 qmult 01000 1 4 3 easy deducto-memory: Minkowski diagrams

1260. "Let's play *Jeopardy*! For $100, the answer is: It can be divided into Past, Future, Now, and Elsewhen, and on it world-lines evolve."

a) What is _____, Alex?

a) a conformal mapping b) a Feynman diagram c) a Minkowski diagram d) history e) life

---

038 qmult 01010 1 4 1 easy deducto-memory: world line

1261. A world line in a Minkowski diagram is:

a) the trajectory of an object in spacetime.
b) the trajectory of an object in space alone.
c) trajectory of an object in time alone.
d) the arrow of time.
e) a telephone that is a party line for the entire world—recommended for exhibitionists.

---

038 qmult 01020 1 5 5 easy thinking: light cone Alpha Centauri

1262. We know already that a tricky political situation will arise on Alpha Centauri 6 (4 light-years distant) in a election 3 years hence. In our Terracentric way, we the people of Earth are going to make our views known to the Alpha Centaurs. What is our chance of influencing their decision?

a) It's possible: Alpha Centauri 3 years hence is **INSIDE** our light cone. A radio signal can be sent and received and the Alpha Centaurs will conclude that there is life out there—arguably intelligent.
b) So-so.
c) Immense: nothing is more cherished than unasked for advice from those incompetent to give it.
d) Nil: Alpha Centaurs would rather take their own bad advice, than someone else's good advice: they're pretty normal that way—explaining the horse's hind end is another matter.
e) Nil: Alpha Centauri 3 years hence is **OUTSIDE** of our light cone. Nothing we do now can have the slightest effect on them then. Jump up and down, scream, shout, giggle inopportunely—nothing can help.

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038 qmult 01100 1 1 5 easy memory: time travel

1263. If you could travel instantly over a finite distance in any inertial reference frame, you could:

a) do anything. b) write the great American novel. c) play a didjeridu. d) ride a unicycle and spit backwards at the same time. e) time travel.

---

038 qmult 01200 1 1 5 easy memory: Einstein's equation presented
Extra keywords: physci

1264. Einstein's equation (or mass-energy equivalence equation) is

a) $E = mc^3$. b) $E = m/c^3$. c) $E = m/c^2$. d) $E = mc^4$. e) $E = mc^2$.

---

038 qmult 01210 2 1 5 moderate memory: Einstein's equation in general

Extra keywords: physci

1265. Einstein's equation $E = mc^2$:

a) applies only to rest mass.
b) applies only to electromagnetic radiation.
c) applies only to nuclear reactions.
d) is only valid in non-inertial frames.
e) is general. All forms of energy have inertial mass and a gravitational effect. Rest mass is itself a special form of energy like electromagnetic field, thermal, or kinetic energies.

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038 qmult 01220 1 3 1 easy math: E=mc**2 applied to electron

Extra keywords: physci KB-93-33

1266. The mass of an electron is 9.1×10^{-31} kg. The energy equivalent of this mass is approximately:

a) 10^{-13} J. b) 10^{-47} J. c) 3×10^{-22} J. d) 3×10^{-38} J. e) 1 J.

---

038 qmult 01230 1 3 3 easy math: E=mc**2 applied to dynamite

Extra keywords: physci KB-96-31

1267. About 5.4×10^6 J of chemical energy is released as explosion energy when 1 kg of dynamite explodes. What fraction of the total rest mass energy of the dynamite is this?

a) 9.0×10^{16}. b) 4.8×10^{23}. c) 6.0×10^{-11}. d) 0. e) infinite.

Full-Answer Problems

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038 qfull 00100 2 3 0 moderate math: relativistic identities

Extra keywords: special relativity formulae

1268. This is problem on special relativity formulae. There are parts a,b,c,d,e,f.

a) Given

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \ ,$$

find β as a function of the Lorentz factor γ.

b) Given $p = \gamma m_0 v = \gamma m_0 c\beta$, derive β as a function of $p/(m_0 c)$ alone: i.e., $p/(m_0 c)$ should be the only relativistic variable in the function.

c) Given $E = \gamma m_0 c^2$, show that $E = \sqrt{p^2c^2 + (m_0c^2)^2}$.

d) Given $E = \gamma m_0 c^2$, derive β as a function of $E/(m_0 c^2)$ alone: i.e., $E/(m_0 c^2)$ should be the only relativistic variable in the function.

e) Given the result of part (d), derive β as a function of kinetic energy K.

f) From Taylor's series, verify that $\beta = \sqrt{2K/(m_0c^2)}$ to 1st order in $K/(m_0c^2)$. The 1st order expression is, of course, the classical (i.e., non-relativistic result) result.

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038 qfull 00200 2 5 0 moderate thinking: Stacy Fitzgerald contraction

1269. Stacy Dragila, Pocatello's pride and 2000 Olympic gold-medal winner, is barreling along at a relativistic speed with her pole. The pole is 15 ft long in rest length. She runs through a barn 10 ft long in rest length and open at both ends. Stacy's going so fast that to an observer at rest in the ground inertial frame, her pole is Fitzgerald contracted to less than 10 ft. Thus the ground-frame observer sees Stacy and pole entirely vanish into the barn for a brief time. But to Stacy it's the barn that is Fitzgerald contracted and she and pole are never entirely inside the barn. Resolve the paradox qualitatively. **HINT:** The answer isn't very long.

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038 qfull 00300 2 3 0 moderate math: alternate Fitzgerald contraction

1270. The Fitzgerald contraction formula is

$$\ell = \ell_0 \sqrt{1-\beta^2} \ ,$$

where ℓ_0 is the proper length of object (i.e., length measured in the rest frame of the object) moving at speed β relative to some other inertial frame (let's call it the lab frame) and ℓ is the corresponding length in the lab frame: note the lengths are both parallel to the direction of motion. The formula is derived from the Lorentz transformations assuming that the measurement of the location of the ends of the length was simultaneous in the lab frame. But there is another way one can measure the length of the moving object in the lab frame: measure the time $\Delta\tau$ (i.e., the time in the "length form" equal to $c\Delta t$) between the front end of the object passing a fixed point in the lab frame and the rear end passing the same point, and multiply by the speed of the object. Call this length ℓ_*, where

$$\ell_* = \beta \Delta\tau \ .$$

Using the Lorentz transformations determine ℓ_* as a function of ℓ_0. **HINT:** The ends of the object passing the fixed point constitute two events with different, but related, spacetime coordinates. Also, observers in both frames see the measurements happening on the endpoints of the object. In the frame of the object, the measurement events are on an object at rest.

---

038 qfull 00400 3 5 0 hard thinking: colliding star ships

1271. Federation star ships Egregious and Execrable are on a head-on collision course. Egregious is moving at velocity $\beta_1 = 0.6$ and Execrable at velocity $\beta_2 = -0.8$: both velocities are relative to the more-or-less inertial frame of the Galaxy.

a) What is the velocity β_2' of Execrable in the inertial frame of the Egregious?

b) Good ship Execrable has proper length 500 m. What are it's lengths in the Galaxy frame and in the Egregious frame?

c) In the Galaxy frame Egregious and Execrable start off a light-year apart ($1\,\text{lyr} = 9.460 \times 10^{15}\,\text{m}$). How long in **YEARS** till they collide in the Galaxy frame? **HINT:** Calculate their relative velocity as seen in the Galaxy frame.

d) In part (c), you obtained a superluminal velocity, but in special relativity we say c (speed of light in vacuum) is the ultimate physical speed. Both the part (c) answer and saying c is the ultimate physical speed are correct. Resolve the paradox? **HINT:** Note, the word "physical" before speed.

e) By the simplest means calculate the time to collision in the rest frame of the Egregious.

f) Now for the challenge. Calculate the time to collision in the rest frame of the Egregious, using velocities, lengths, and times as measured in the Egregious frame. Is the answer the same as that in part (e)? **HINT:** First, find Execrable's spacetime coordinates in the Egregious frame.

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038 qfull 00500 2 3 0 moderate math: Smaug, Frodo, and time dilation

1272. You and your loyal hound Frodo are traveling through space in your star ship Smaug. You are limited to going at less than the ultimate physical speed $c = 1\,\text{lyr/yr}$ or $\beta = 1$, of course. But other than that Smaug can go at any speed and you and Frodo are real tough and can stand any acceleration.

NOTE: There are parts a,b,c. Some of the can be done independently. So don't stop if you can't do a part.

a) Say Frodo wants to travel from Earth to Sirius (distance $d = 8.7\,\text{lyr}$) and you want to make the trip in 1 year for **YOU**. At what β value would Smaug have to go? **HINTS:** You will have to do some algebra. Recall the Galaxy-frame travel time is $d/(c\beta)$ and the time dilation formula is $t' = t\sqrt{1-\beta^2}$.

b) Stirred by your first conquest of space, you and Frodo—well Frodo is less eager—now want to cross the bulge to the far end of the disk of Galaxy: distance $100,000\,\text{lyr}$. But you want to live to see the journey's end. What physical effect makes it possible to survive the journey?

c) How much **LESS** than the speed of light (in units of c: i.e., in terms of β) do you need to go at to make the pan-Galactic journey of $100,000\,\text{lyr}$ in 1 year for **YOU**? **HINT:** Recall the 1st order Taylor expansion

$$\frac{1}{\sqrt{1+x}} \approx 1 - \frac{1}{2}x$$

which is valid for $x << 1$.

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038 qfull 00502 2 3 0 moderate math: Smaug, Frodo, and dog years: joke question

1273. As continuation of the last question. How long are each of your the trips in dog years for Frodo? And why does Frodo want to stay at Sirius?

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038 qfull 00510 2 5 0 moderate thinking: Frodo and the black hole

1274. Once again you and your trusty hound Frodo are voyaging through Galactic space on your star ship Smaug.

a) Unbeknownst to you, Frodo has an inner life in which he in comtemplates the universe. He asks himself—since it's no use asking you–"if the gravitational force is always attractive and dominates large-scale structure, why doesn't the whole Galaxy of stars collapse to form a super-massive blackhole?" Assuming Frodo answers himself correctly, what is his answer?

b) While Frodo is musing, you are doing a few calculations. Your speed is a lazy $3.000 \times 10^4\,\text{km/s}$—kilometers per second mark you—relative to the more-or-less inertial frame of the Galaxy and Smaug's rest mass is $1.000 \times 10^5\,\text{kg}$. What is Smaug's **NON-RELATIVISTIC** kinetic energy?

c) Calculate the kinetic energy of Smaug in special relativity. Does the special relativistic calculation of kinetic energy yield a significantly different value (say 0.1 % different) from the non-relativistic calculation?

d) Just as Frodo is begging to go for a space walk, you realize from your space chart that you are on a straight line path for a black hole—your controls are jammed—could be Frodo was gnawing on some vital connection—the mass of the black hole is 1 solar mass (1 solar mass is $1.9891 \times 10^{30}\,\text{kg}$) and it is $1.5 \times 10^{11}\,\text{m}$ away. From a Newtonian calculation at what distance will you reach the speed of light! **NOTE**: Because of special and general relativistic effects this calculation is not really valid, but it does lead to some insight into what must happen.

e) There's only one thing left to do—get the last word. So when your Lorentz factor reaches 10^{10}—just before your gravitational shielding (a pure plot device) fails and your atoms become permanently uncooperative—you send a proper time 3 minute video message to all your friends. Just from special relativistic physics, how long will your video message run in **YEARS** in the more-or-less inertial frame of the Galaxy? **NOTE**: General relativity will also affect time and in reality you won't be at a constant velocity.

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038 qfull 00600 1 3 0 easy math: fast protons

1275. The proton mass in energy units is 938.272029 MeV or approximately 1 GeV. The proton synchrotron at Fermilab near Chicago can (or could) accelerate a proton to total energy (i.e., kinetic energy plus rest mass energy) of about 500 Gev.

a) What is the kinetic energy of the proton to the same order of accuracy that the total energy was quoted to?

b) What is the Lorentz factor of the proton?

c) What is the velocity β of the proton?

Chapt. 39 Photons and Matter Waves

Multiple-Choice Problems

039 qmult 00100 1 1 3 easy memory: quantum mechanics
Extra keywords: physci
1276. The physical theory that deals mainly with microscopic phenomena is:

a) quartz mechanics. b) quarks mechanics. c) quantum mechanics. d) quantum jump mechanics. e) quasi-mechanics.

039 qmult 00110 1 4 3 easy deducto memory: QM probabilistic
Extra keywords: physci
1277. In standard quantum mechanics, nature is fundamentally:

a) unlikely. b) ballistic. c) probabilistic. d) deterministic. e) neither fish nor fowl.

039 qmult 00200 1 1 1 easy memory: photon energy
1278. The photon, the quantum of electromagnetic radiation, has **ENERGY**:

a) $hf = \hbar\omega$. b) h/λ. c) $\hbar k$. d) $h^2 f$. e) hf^2.

039 qmult 00210 1 1 1 easy memory: photon energy
Extra keywords: physci This differs from 00210 only in answer (a).
1279. The photon, the quantum of electromagnetic radiation, has **ENERGY**:

a) hf. b) h/λ. c) $\hbar k$. d) $h^2 f$. e) hf^2.

039 qmult 00220 1 1 4 easy memory: photon rest mass
Extra keywords: physci KB-280-5
1280. The rest mass of a photon is:

a) the same as that of an electron. b) 1 atomic mass unit. c) negative. d) 0. e) depends on frequency.

039 qmult 00100 1 3 5 easy math: eye detectable photons
Extra keywords: physci BK-283-3
1281. The human eye may be able to detect as little as 10^{-18} J of light energy. How many photons of frequency 5×10^{14} Hz does it take to make up just about this amount of energy?

a) 10^{-18}. b) 5×10^{14}. c) 10. d) 1. e) 3.

039 qmult 00300 1 1 4 easy memory: photoelectric effect
Extra keywords: physci
1282. A key piece of evidence for the wave-particle duality of light is the:

a) photograph effect. b) electrodynamics of Maxwell summarized in the four Maxwell's equations. c) frequency of red light. d) photoelectric effect. e) photomagnetic effect.

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039 qmult 00310 2 1 2 moderate memory: photoelectron energy
Extra keywords: physci KB-280-1

1283. The kinetic energy of the electrons emitted in a photoelectric effect device depends on the _____ of the light incident on the metal.

a) intensity b) frequency c) speed d) resonance e) resolve

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039 qmult 00400 1 1 1 easy memory: Compton effect

1284. Einstein predicted and Compton proved that photons:

a) have linear momentum.
b) do not have linear momentum.
c) sometimes have linear momentum.
d) both have and do not have linear momentum at the same time.
e) neither have nor have not linear momentum.

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039 qmult 00500 1 1 2 easy memory: wave-particle duality

1285. The principle that all microscopic physical entities have both wave and particle properties is called the wave-particle:

a) singularity. b) duality. c) triality. d) infinitality. e) nullility.

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039 qmult 00510 1 4 5 easy deducto-memory: Louis de Broglie
Extra keywords: physci

1286. "Let's play *Jeopardy*! For $100, the answer is: He was the first to propose that particles with rest mass also had a wave nature."

Who was _____, Alex?

a) Isaac Newton (1643–1727) b) James Clerk Maxwell (1831–1879) c) Albert Einstein (1879–1955) d) Erwin Schrödinger (1887–1961) e) Louis de Broglie (1892–1987)

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039 qmult 00520 1 3 1 easy math: de Broglie wavelength calculation
Extra keywords: physci KB-283-11

1287. What is the de Broglie wavelength of an electron of speed 10^6 m/s?

a) 7.27×10^{-10} m. b) 6.626×10^{-34} m. c) 9.109×10^{-31} m. d) 3.6×10^{-11} m. e) 0.

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039 qmult 00600 1 4 5 easy deducto-memory: Schrödinger eqn
Extra keywords: physci

1288. "Let's play *Jeopardy*! For $100, the answer is: The equation/equations that governs the time evolution of quantum mechanical systems in the non-relativistic approximation."

What is/are _____, Alex?

a) Newton's 2nd law, $\vec{F}_{\text{net}} = m\vec{a}$ b) Maxwell's equations c) Einstein's field equations of general relativity d) Dirac equation e) Schrödinger's equation

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039 qmult 00700 1 1 1 easy memory: QM probability density
Extra keywords: physci

1289. In the statistical interpretation of wave function Ψ, $|\Psi|^2$ is:

a) a probability density. b) a probability amplitude. c) 1. d) 0. e) a negative probability.

039 qmult 00800 1 1 1 easy memory: stationary state

1290. A system in a stationary state will:

a) not evolve in time.
b) evolve in time.
c) both evolve and not evolve in time.
d) occasionally evolve in time.
e) violate the Heisenberg uncertainty principle.

039 qmult 00900 1 1 3 easy memory: free particle

1291. The free particle stationary states

a) can be occupied by a particle.
b) can be occupied by two particles.
c) cannot actually be occupied by a particle.
d) are unknown.
e) are normalizable.

039 qmult 01000 1 4 5 easy deducto-memory: Heisenberg uncertainty

1292. "Let's play *Jeopardy*! For $100, the answer is: It describes a fundamental limitation on the accuracy with which we can know position and momentum simultaneously."

a) What is Tarkovsky's doubtful thesis, Alex?
b) What is Rublev's ambiguous postulate, Alex?
c) What is Kelvin's vague zeroth law, Alex?
d) What is Schrödinger's wild expostulate, Alex?
e) What is Heisenberg's uncertainty principle, Alex?

Full-Answer Problems

039 qfull 00400 2 3 0 moderate math: helium gas balloon

1293. You have a helium gas balloon at typical room temperature ($T = 300\,\mathrm{K}$) and pressure ($1\,\mathrm{atm} = 1.01 \times 10^5\,\mathrm{Pa}$.

a) Estimate the average de Broglie wavelength of a helium atom in meters and Angstroms given that the mean kinetic energy of Maxwellian gas is $K = (3/2)kT$, where $k = 1.381 \times 10^{-23}\,\mathrm{J\,K^{-1}}$ is Boltzmann's constant and T is in Kelvin. You will also need the atomic mass of helium, the amu, and Planck's constant: these are, respectively, $A = 4.0026$, $m_{\mathrm{amu}} = 1.661 \times 10^{-27}\,\mathrm{kg}$, and $h = 6.626 \times 10^{-34}\,\mathrm{J\,s}$.

b) Calculate the particle density of the gas and inverse of particle density (i.e., the specific volume) in MKS units and in terms of Angstroms.

c) What is the typical distance between the atoms in Angstroms if you consider them as classical particles.

d) Can the gas be treated as classical gas of particles? **HINT:** You will probably be able to guess the criterion, but likely not the true justification for the criterion.

039 qfull 00500 3 5 0 tough thinking: Rutherford's nucleus
Extra keywords: (HRW-977:62P) Also in qmp

1294. Rutherford discovered the nucleus in 1911 by bombarding metal foils with alpha particles now known to be helium nuclei (atomic mass 4.0026). An alpha particle has positive charge $2e$. He

expected the alpha particles to pass right through the foils with only small deviations. Most did, but some scattered off a very large angles. Using a classical particle picture of the alpha particles and the entities they were scattering off of he came to the conclusion that atoms contained most of their mass and positive charge inside a region with a size scale of $\sim 10^{-15}\,\mathrm{m} = 1\,\mathrm{fm}$: this 10^{-5} times smaller than the atomic size. (Note fm stands officially for femtometer, but physicists call this unit a fermi.) Rutherford concluded that there must be a dense little core to an atom: the nucleus.

a) Why did the alpha particles scatter off the nucleus, but not off the electrons? **HINTS:** Think dense core and diffuse cloud. What is the force causing the scattering?

b) If the alpha particles have kinetic energy 7.5 Mev, what is their de Broglie wavelength?

c) The closest approach of the alpha particles to the nucleus was of order 30 fm. Would the wave nature of the alpha particles have had any effect? Note, the wave-particle duality was not even suspected for the massive particles in 1911.

---

039 qfull 00600 2 3 0 moderate math: complex numbers
Extra keywords: (HRW-977:66E)

1295. A complex number z is defined by

$$z = x + iy \ ,$$

where x and y are real numbers and $i = \sqrt{-1}$ is imaginary. The real part of z is $x = \mathrm{Re}[z]$ and the imaginary part is $y = \mathrm{Im}[z]$ The complex conjugate of z is defined by

$$z^* = x - iy \ .$$

a) What is zz^*?

b) Prove that

$$(z_1 z_2)^* = z_1^* z_2^* \ .$$

c) Prove that

$$\left(\frac{1}{z}\right)^* = \frac{1}{z^*} \ .$$

d) Prove that

$$\left(\frac{z_1}{z_2}\right)^* = \frac{z_1^*}{z_2^*} \ .$$

---

039 qfull 00700 2 5 0 moderate thinking: Schrödinger eqn.

1296. Do the following.

a) Verify that the full time-dependent wave function

$$\Psi(x,t) = \psi(x) e^{-iEt/\hbar}$$

reduces the full time-dependent Schrödinger equation

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V(x)\Psi = i\hbar\frac{\partial \Psi}{\partial t}$$

to the time-independent Schrödinger equation

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V(x)\psi = E\psi$$

for the stationary states ψ.

b) For $V = 0$ (i.e., the free particle system), verify that

$$\psi = e^{ikx}$$

are the unnormalized stationary states, where

$$k = \pm\sqrt{\frac{2mE}{\hbar^2}} \ .$$

Why can't these states be normalized?

c) The infinite square well has

$$V = \begin{cases} 0 \ , & \text{for } 0 \le x \le a; \\ \infty \ , & \text{for } x < 0 \text{ and } x > a. \end{cases}$$

Verify that

$$\psi = \sqrt{\frac{2}{a}}\sin(kx)$$

are the normalized solutions with boundary conditions $\psi(0) = \psi(a) = 0$. Also show that

$$k = \frac{n\pi}{a} \qquad \text{and} \qquad E = \frac{\hbar^2}{2m}\left(\frac{\pi}{a}\right)^2 n^2 \ .$$

---

039 qfull 02000 3 5 0 tough thinking: complete set of states

Extra keywords: A good challenge question.

1297. It has been mathematically shown that the set of stationary state wave functions for a system constitute a **COMPLETE SET**: i.e., any function that obeys the same boundary conditions as the stationary state wave functions can be expanded as a linear combination of those stationary state wave functions. For example say ψ is a general wave function and $\{\phi_i\}$ are set of stationary state wave functions or eigenfunctions for short: the curly brackets imply set of. Then we can expand ψ thus

$$\psi = \sum_i a_i\phi_i \ ,$$

where the sum in general could be over infinitely many eigenfunctions and a_i is an expansion coefficient which may in general be complex. It is also true that the eigenfunctions can always be **ORTHONORMALIZED**. This means that

$$\int \phi_j^*\phi_i \, dv = \delta_{ji} \ ,$$

where where the integral is over all space and

$$\delta_{ji} = \begin{cases} 1 \ , & \text{for } j = i; \\ 0 \ , & \text{for } j \ne i \end{cases}$$

is the Kronecker delta function. Let us assume that $\{\phi_i\}$ is orthonormalized.

a) Show that

$$a_j = \int \phi_j^*\psi \, dv \ .$$

b) Given that

$$\int \psi^* \psi \, dv = 1$$

(i.e., ψ is normalized), show that

$$\sum_i |a_i|^2 = 1 \ ,$$

where $|a_i|^2 = a_i^* a_i$.

c) The time-independent Schrödinger equation for our system can be written

$$H\phi_i = E_i \phi_i \ ,$$

where H is called the Hamiltonian operator or just **HAMILTONIAN**. For one dimensional systems

$$H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x) \ .$$

The Hamiltonian acting on the eigenfunction ϕ_i changes it into itself times E_i. Note E_i is just a **CONSTANT NUMBER** and can be taken out of any integral. Now it turns out that the mean energy E_{mean} of a system with H and wave function ψ has a value given by

$$E_{\text{mean}} = \int \psi^* H \psi \, dv \ .$$

Show that

$$E_{\text{mean}} = \sum_i |a_i|^2 E_i$$

also. For no good reason mean values in quantum mechanics are called expection values. So E_{mean} is the expectation value of the Hamiltonian.

---

039 qfull 03000 3 5 0 tough thinking: Einstein, Runyon

1298. "God does not play dice"—Einstein. Discuss.

Chapt. 40 More About Matter Waves

Multiple-Choice Problems

040 qmult 00100 1 1 1 easy memory: infinite square well

1299. In quantum mechanics, the infinite square well can be regarded as the prototype of:

a) all bound systems.
b) all unbound systems.
c) both bound and unbound systems.
d) neither bound nor unbound systems.
e) Prometheus unbound.

040 qmult 00200 1 1 3 easy memory: boundary conditions

1300. Meeting the boundary conditions of quantum mechanical bound systems imposes:

a) Heisenberg's uncertainty principle.
b) Schrödinger's equation.
c) quantization.
d) a vector potential.
e) a time-dependent potential.

040 qmult 00300 1 1 4 easy memory: macro system in a stationary state

1301. Except for certain special cases (superconductors, superfluids, and Bose-Einstein condensates), no macroscopic system can be in a:

a) mixed state.
b) vastly mixed state.
c) classical state.
d) stationary state.
e) state of the union.

040 qmult 00400 1 1 2 easy memory: transitions

1302. Transitions between stationary states (sometimes, but actually rarely, called quantum jumps) are:

a) only collisional.
b) both collisional and radiative.
c) only radiative.
d) neither collisional nor radiative.
e) only collisional to higher energy stationary states and only radiative to lower energy stationary states.

040 qmult 00500 1 4 3 easy deducto-memory: lasers, stimulated emission

1303. "Let's play *Jeopardy*! For $100, the answer is: It is the basis for lasers and masers."

a) What spontaneous radiative emission, Alex?
b) What desultory radiative emission, Alex?

c) What stimulated radiative emission, Alex?
d) What the laser force, Alex?
e) What the laser potential, Alex?

---

040 qmult 00600 1 1 5 easy memory: continuum of unbound states

1304. At energies higher than the bound stationary states there:

a) are between one and several tens of unbound states.
b) are only two unbound states.
c) is a single unbound state.
d) are no states.
e) is a continuum of unbound states.

---

040 qmult 00700 1 4 2 easy deducto-memory: tunneling

1305. "Let's play *Jeopardy*! For $100, the answer is: This effect occurs because wave functions can extend (in an exponentially decreasing way albeit) into the classically forbidden region: i.e., the region where a classical particle would have negative kinetic energy."

a) What is stimulated radiative emission, Alex?
b) What is quantum mechanical tunneling, Alex?
c) What is quantization, Alex?
d) What is symmetrization, Alex?
e) What is normalization, Alex?

---

040 qmult 00800 1 1 1 easy memory: hydrogen atom, 2-body

1306. The hydrogen atom is the simplest of all neutral atoms because:

a) it is a 2-body system.
b) it is a 3-body system.
c) it has no electrons.
d) it has many electrons.
e) it is the most abundant element in the universe.

---

040 qmult 00900 1 4 3 easy deducto-memory: Bohr atom

1307. "Let's play *Jeopardy*! For $100, the answer is: This model of an atom is of historical and pedagogical interest, but it is of no use in modern practical calculations and from the modern standpoint is probably misleading rather than insight-giving."

a) What is Shrödinger's model of the hydrogen atom, Alex?
b) What the Thomas-Fermi model of a many electron atom, Alex?
c) What is Bohr's model of the hydrogen atom, Alex?
d) What is the liquid drop model of the atom, Alex?
e) What is model hydrogen atom of Leucippos and Democritos's , Alex?

---

040 qmult 01100 1 1 5 easy memory: Greek atomists

1308. The atomic theory was first proposed by the ancient Greeks Leucippos (5th century BC) and Democritos (5th to 4th century BC: he reputedly lived to be 100). The term atomos means uncut: e.g., the grass is atomos. The atomists started from a philosophical position that there had to be something to give stability to nature: obviously the macroscopic world was full of change: therefore what was imperishable or uncutable—atoms—must be below perception. The modern quantum theory does indeed bear out some of their thinking. Microscopic particles can be created and destroyed, of course, but the members of a class are much more identical than macroscopic objects can ever be: fundamental particles like electrons and quarks are thought to be absolutely identical. Thus the forms particles can take are apparently eternal: a hydrogen atom today is the same in theory as one at any time in universal history.

The atomists tried to work out an atomic understanding of existence in general. For instance they constructed a cosmology using atoms that bears some resemblance to modern eternal inflationary cosmology in which there are infinitely many universes that are born out of primordial space-time foam and perhaps return to that—foam to foam. Unfortunately, the atomists got off on the wrong foot on the shape of the Earth: they were still flat Earthers when the round Earth theory was being established. Quite obviously to us, the atomists were badly non-experimental. Much of their thinking can be called rational myth. To a degree they were lucky in happening to be attracted to an essentially right idea.

The atomists were eventually stigmatized as atheists: they did not deny that gods exist, but didn't leave anything for the gods to do. This may have been their downfall. The more orthodox and popular philosophies of Plato, Aristotle, and the Stoics rejected atomism probably, among other things, for its seeming atheism. Christianity followed suit in this regard. The writings of the atomists only exist in fragments—and Democritos seems to have been as famous as Plato in his day. The Epicurean philosophers adopted atomism, but also suffered the stigmatization as atheists—and also hedonists who are, of course, the worst. But the atom idea lingered on through the centuries: Leucippos and Democritus, Epicurus, Lucretius (his surviving poem *De Rerum Natura* [*On Nature*] expounds atomism), Gassendi (17th century), Newton, Dalton: the chain is unbroken: it is not true that modern atomism has no historical or essential connection to ancient atomism.

A good account of ancient atomism can be found in David Furley's *The Greek Cosmologists*.

Now, without recurring to the top of this preamble, atomism was invented in:

a) the early 19th century.
b) the 17th century by Gassendi.
c) the 10th century AD.
d) the 5th century AD.
e) the 5th century BC.

---

040 qmult 01200 1 1 4 easy memory: hydrogenic energy formula

1309. The formula

$$E_n = -\frac{1}{2} m_e c^2 \alpha^2 \frac{Z^2}{n^2}$$

gives the main energy levels of:

a) positronium.
b) magnesium deboride.
c) the hydrogen molecule.
d) the hydrogenic atom.
e) the infinite square well.

---

040 qmult 01300 1 1 2 easy memory: hydrogen wave functions

1310. The hydrogen wave functions contain a factor that causes them to:

a) increase exponentially with radius.
b) decrease exponentially with radius.
c) increase logarithmically with radius.
d) increase quadratically with radius.
e) increase linearly with wavelength.

---

040 qmult 01400 1 4 5 easy deducto-memory: nucleus

1311. "Let's play *Jeopardy*! For $100, the answer is: This bound quantum mechanical system consists of protons and neutrons held together by the strong nuclear force."

a) What is an infinite square well, Alex?
b) What is a finite square well, Alex?
c) What is a finite squarish well, Alex?
d) What is an atom, Alex?
e) What is a nucleus, Alex?

---

040 qmult 01400 1 1 1 easy memory: causality, relativity

1312. Einstein ruled out faster than light signaling because:

a) it would cause irresolvable causality paradoxes.
b) it would not cause irresolvable causality paradoxes.
c) it led to irresolvable paradoxes in quantum mechanics.
d) it would destroy the universe.
e) it had been experimentally verified.

---

040 qmult 01500 1 1 3 easy memory: EPR paradox

1313. The Einstein-Podolsky-Rosen (EPR) paradox was proposed to show that ordinary quantum mechanics implied superluminal signaling and therefore was:

a) more or less correct.
b) absolutely correct.
c) defective.
d) wrong in all its predictions.
e) never wrong in its predictions.

---

040 qmult 01600 1 4 3 easy deducto-memory: Bell's theorem

1314. "Let's play *Jeopardy*! For $100, the answer is: This theorem (if it is indeed inescapably correct) and the subsequent experiments on the effect the theorem dealt with show that quantum mechanical signaling exceeds the speed of light."

a) What is Dark's theorem, Alex?
b) What is Midnight's theorem, Alex?
c) What is Bell's theorem, Alex?
d) What is Book's theorem, Alex?
e) What is Candle's theorem, Alex?

Full-Answer Problems

---

040 qfull 00100 2 3 0 moderate math: real exponent exponentials

1315. Consider the time-independent Schrödinger equation

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi = E\psi$$

with V constant and $E < V$.

a) Verify that the exponential solutions

$$\psi = e^{\pm\kappa x}$$

satisfy Schrödinger's equation when

$$\kappa = \sqrt{\frac{2m(V-E)}{\hbar^2}}\ .$$

b) Can the real exponent exponential solutions be the solutions for a complete wave function? Why or why not?

c) Of what use are the real exponent exponential solutions.

---

040 qfull 00200 3 5 0 tough thinking: box quantization
Extra keywords: Needs to be finished: I need to understand

1316. The statistical mechanics treatment of gases (and quasi-gases: i.e., things that arn't really gases, but are amenable to analogous formalism) often begins with finding the stationary states of a rectangular box with zero potential (or constant potential) inside it and periodic boundary conditions. This box quantization gives good results even when nothing like a box is around to contain the gas. The simple reason is that for a large box, interior behavior must be mostly unaffected by what is outside the box even if its just more gas: so you can always imagine a big box (of constant interior potential) drawn in some much bigger region of gas (that may not have constant potential behavior). The use of periodic boundary conditions is similarly motivated. The boundary conditions of real container may be very complex: e.g., boundary conditions of a jar, the sides of metal slab (for an electron gas inside a metal), or nothing at all for an imagined box embedded in a bigger region of gas. But if the box is large enough the boundary conditions should not affect the interior much. Thus periodic boundary conditions are as good a choice as any other and they are simple to use. Of course, if you really have a small container in some sense, the actual size and boundary conditions of the container might become important.

a)

---

040 qfull 00300 2 3 0 moderate math: recoiling hydrogen atom
Extra keywords: (HRW-1004:42P)

1317. A hydrogen atom at **REST** emits a Lyα photon: i.e., a photon from a transition from the $n = 2$ to the $n = 1$ level. Recall the hydrogenic atom energy level formula

$$E = -\frac{1}{2}mc^2\alpha^2\frac{Z^2}{n^2} = -E_{\rm Ryd}\frac{Z^2}{n^2} \ ,$$

where $E_{\rm Ryd} = 13.6\,{\rm eV}$, n is the principal quantum number, and Z is the nuclear charge.

a) What is the energy and wavelength of the emitted photon? HINT: A useful number in such calculations is $hc = 12398\,{\rm eV\text{-}Å}$.

b) Assuming that the hydrogen atom can be treated as a classical, non-relativistic particle, find an expression for its recoil kinetic energy in terms of the energy the photon and the rest mass of the hydrogen atom. **HINT:** What conservation law can be applied to the system.

c) What is the recoil kinetic energy of the hydrogen atom. Where must this kinetic energy have come from? How does this kinetic energy compare to the photon's energy and to the thermal kinetic energy of an atom at 300 K? What is the speed of the recoiling hydrogen? Were we justified in treating this recoil non-relativistically? **HINT:** You might find the following constants helpful: $e = 1.602 \times 10^{-19}$, $c = 2.998 \times 10^8\,{\rm m/s}$, $A_{\rm hyd} = 1.00797$, $m_{\rm amu} = 1.66 \times 10^{-27}$, $m_{\rm amu}c^2 = 931.5 \times 10^6\,{\rm eV}$, and $k = 0.8617 \times 10^{-4}\,{\rm eV\,K^{-11}}$.

---

040 qfull 00500 2 3 0 moderate math: H atom radial probability
Extra keywords: (HRW-1005:55P)

1318. The ground state of the hydrogen atom has a isotropic wave function: i.e., it has no angular dependence. This wave function (time-independent version) is

$$\psi(r) = \frac{1}{\sqrt{4\pi}}\frac{2}{a^{3/2}}e^{-r/a} \ ,$$

where $a = 0.529\,\text{Å}$ is the Bohr radius. The radial probability density (i.e., the probability density integrated over all solid angle times r^2) is

$$\rho(r) = \oint |\psi|^2\, r^2 d\Omega = \frac{4r^2}{a^3} e^{-2r/a} \ .$$

The probability of finding the electron in a spherical shell of thickness dr is then

$$dP = \frac{4r^2}{a^3} e^{-2r/a}\, dr \ .$$

a) What is the most probable radius r for finding the electron.

b) Verify that the ground state wave function is normalized. **HINT:** You might find the factorial integral useful:

$$n! = \int_0^\infty e^{-x} x^n\, dx \ .$$

c) Assume that our wave function is accurate right down to $r = 0$. (This can't be exactly true since the potential must change from a point charge form when one is very close to the proton or effectively inside of it.) What is the probability of finding the electron inside the nuclear region? Hydrogen, of course, has a boring one proton nucleus. We can take the outer radius of the nucleus to be a proton's rms radius of $r = 8 \times 10^{-6}\,\text{Å}$. **HINT:** It seems perfectly reasonable to make a zeroth order expansion of the exponential factor in the probability density.

Chapt. 41 Nuclear Physics

Multiple-Choice Problems

043 qmult 00100 1 1 2 easy memory: Rutherford atom
Extra keywords: physci KB-249-1

1319. In the Rutherford model of the atom, the positive charge is:

a) spread throughout the atom. b) concentrated in a small central core called the nucleus. c) outside the atom in a nucleus. d) at the bottom of the atom in a nucleus. e) not present.

043 qmult 00200 1 4 5 easy deducto-memory: atomic number
Extra keywords: physci KB-249-3

1320. "Let's play *Jeopardy*! For $100, the answer is: It is the atomic number of an element."

What are the number of _____ in the nucleus, Alex?

a) triangles b) isotopes c) neutrons d) electrons e) protons

043 qmult 00210 1 1 2 moderate memory: reading nuclide symbols 1
Extra keywords: physci KB-251-5

1321. How many neutrons are there in ${}^{6}_{3}\mathrm{Li}$, ${}^{13}_{6}\mathrm{C}$, ${}^{31}_{15}\mathrm{P}$, and ${}^{94}_{40}\mathrm{Zr}$.

a) 3, 6, 15, and 40. b) 3, 7, 16, and 54. c) 6, 13, 31, and 94. d) 9, 19, 46, and 141. e) 0, 1, 2, and 3.

043 qmult 00220 1 1 2 moderate memory: reading nuclide symbols 2
Extra keywords: physci KB-251-6

1322. How many neutrons are there in ${}^{18}_{8}\mathrm{O}$, ${}^{26}_{12}\mathrm{Mg}$, ${}^{57}_{26}\mathrm{Fe}$, and ${}^{109}_{47}\mathrm{Ag}$.

a) 18, 26, 57, and 109. b) 8, 12, 26, and 47. c) 10, 14, 31, and 62. d) 26, 38, 83, and 156. e) 0, 1, 2, and 3.

043 qmult 00400 1 1 1 easy memory: three radiation particles
Extra keywords: physci

1323. What are three kinds of nuclear radiation "particles"?

a) alpha, beta, gamma. b) alpha, beta, omega. c) alpha, psi, omega. d) beta, psi, omega. e) Alpher, Behte, Gamow.

043 qmult 00410 1 1 3 easy memory: alpha particle
Extra keywords: physci KB-249-6

1324. An alpha particle is a:

a) positron or electron. b) helium-3 (${}^{3}_{2}\mathrm{He}$) nucleus. c) helium-4 (${}^{4}_{2}\mathrm{He}$) nucleus. d) gamma-ray. e) neutrino.

043 qmult 00420 1 1 4 easy memory: alpha particle changes

Extra keywords: physci KB-251-11

1325. When an alpha particle is emitted, the atomic number and mass number of a nucleus:

a) are unchanged. b) become equal. c) increase by 2 and 4, respectively. d) decrease by 2 and 4, respectively. e) both decrease by 2.

---

043 qmult 00500 1 1 5 easy memory: beta decay particle

Extra keywords: physci KB-249-7

1326. In a beta decay process, the decaying nucleus emits a/an:

a) helium-4 ($^4_2$He) nucleus. b) alpha particle. c) gamma-ray. d) beta particle which is **ALWAYS** a fast positron. e) beta particle: i.e., a fast electron or positron.

---

043 qmult 00700 1 1 5 easy memory: spontaneous fission defined 1

Extra keywords: KB-250-20

1327. The spontaneous splitting of a radioactive nucleus (e.g., $^{235}_{92}$U) into two or more large fragment nuclei plus possibly possibly smaller emissions called:

a) spontaneous fusion. b) a chain reaction. c) diffusion. d) a rope reaction. e) spontaneous fission.

---

043 qmult 00720 3 5 2 tough thinking: uranium decay to lead

Extra keywords: physci KB-252-5

1328. Uranium-235 ($^{235}_{92}$U) decays eventually to a stable lead isotope by emitting 7 alpha particles and 4 electrons (i.e., 4 negative beta particles). The process goes through intermediate stages. What is the nuclide symbol of the resulting nucleus?

a) $^{207}_{78}$Pb. b) $^{207}_{82}$Pb. c) $^{208}_{82}$Pb. d) $^{235}_{82}$Pb. e) $^{207}_{82}$U.

---

043 qmult 00810 2 1 4 moderate memory: nucleons after decay

Extra keywords: physci KB-250-12

1329. After a radioactive decay the number of nucleons in the nucleus is _____ the original number.

a) always equal to b) always more than c) never less than d) never more than e) always less than

---

043 qmult 00900 1 4 2 easy deducto-memory: half-life defined

Extra keywords: physci KB-250-14

1330. "Let's play *Jeopardy*! For $100, the answer is: The time for a half on average of a sample of a radioactive species to decay."

What is _____, Alex?

a) quarter-life b) half-life c) half-time d) halfling e) half done

---

043 qmult 00910 1 5 5 easy thinking: half-life calculation 1

Extra keywords: physci KB-252-7

1331. A radioactive isotope sample of 100 g is 75 % decayed after 10 years. It's half-life must be:

a) 10 years. b) 20 years. c) 75 years. d) 2 years. e) 5 years.

---

043 qmult 01000 1 1 1 easy memory: MeV categorized

Extra keywords: physci KB-250-17

1332. The mega-electronvolt (MeV) is a unit of:

a) energy. b) momentum. c) charge. d) retreat. e) debacle.

---

043 qmult 01110 2 3 3 moderate math: binding energy of He-4
Extra keywords: physci KB-252-15

1333. The mass of $^4_2$He is 4.0026 u, where u is the atomic mass unit. Recall that 1 u = 931.5 MeV, that a hydrogen atom has mass 1.0078 u, and neutron has mass 1.0087 u. What is the **BINDING ENERGY** and **BINDING ENERGY PER NUCLEON** of a $^4_2$He in that order assuming one starts with 2 hydrogen nuclei and 2 neutrons?

a) 931.5 MeV and 233 MeV/nucleon. b) 4.0026 u and 1.0078 u/nucleon. c) 28.3 MeV and 7.08 MeV/nucleon. d) 7.08 MeV and 28.3 MeV/nucleon. e) 1.0 MeV and 0 MeV/nucleon.

---

043 qmult 01210 2 4 5 moderate deducto-memory: nuclear binding energy maximum
Extra keywords: physci KB-250-19

1334. The nuclear binding energy per nucleon is greatest for nuclei of:

a) reduced mass. b) no mass. c) least mass. d) greatest mass. e) mass near iron (i.e., iron peak nuclei).

---

043 qmult 01220 2 4 3 moderate deducto-memory: most stable nucleus
Extra keywords: physci KB-251-17 but I've had to correct the question.

1335. "Let's play *Jeopardy*! For $100, the answer is: Because it is has the 3rd great binding energy per nucleon among stable nuclei and is much more abundant than those nuclides with the greatest and 2nd greatest binding energy per nucleon, it is the most stable abundant nuclide in terrestrial and many astrophysical environments."

What is _____, Alex?

a) $^{235}_{92}$U b) $^2_1$H c) $^{56}_{26}$Fe d) $^{13}_6$C e) the bare neutron

---

044 qmult 01300 1 1 2 easy memory: fission defined 2
Extra keywords: physci KB-250-20

1336. The splitting of a nucleus into two or more large fragment nuclei plus possibly other smaller emissions is called:

a) fusion. b) fission. c) beta decay. d) a chain reaction. e) nucleation.

---

044 qmult 01320 1 1 1 easy memory: exothermic fusion and fission
Extra keywords: physci KB-252-19

1337. Exothermic fusion and fission processes are possible because nuclei have:

a) different binding energies per nucleon. b) the same binding energy per nucleon. c) mass. d) no mass. e) neutrons.

---

044 qmult 01400 1 4 4 easy deducto-memory: chain reaction defined
Extra keywords: physci KB-250-21

1338. "Let's play *Jeopardy*! For $100, the answer is: In this nuclear process, neutrons emitted during fission induce further fissions."

What is a _____, Alex?

a) gamma-ray decay b) beta decay c) fusion reaction d) chain reaction e) chain gang reaction

---

044 qmult 01520 2 4 3 moderate deducto-memory: plutonium reactors
Extra keywords: physci KB-252-23

1339. In addition to uranium-235 ($^{235}_{92}$U), fission reactors and fission bombs can use as a fuel:

a) deuterium (${}^{2}_{1}\mathrm{H}$). b) tritium (${}^{3}_{1}\mathrm{H}$). c) plutonium-239 (${}^{239}_{94}\mathrm{Pu}$). d) iron-56 (${}^{56}_{26}\mathrm{Fe}$). e) helium-4 (${}^{4}_{2}\mathrm{He}$)

---

044 qmult 01530 1 1 5 easy memory: reactor mass loss

Extra keywords: physci KB-253-17

1340. A nuclear reactor generates 1.0 GW of power. About how much mass does it lose per day? **HINT:** Recall $E = mc^2$.

a) 10^3 kg. b) 10^{14} kg. c) 10^9 kg. d) 10^{-8} kg e) 10^{-3} kg.

---

044 qmult 01610 1 1 1 easy memory: fusion reactor fuel

Extra keywords: physci KB-250-23

1341. For fuel, fusion reactors will probably use

a) ${}^{2}_{1}\mathrm{H}$ (deuterium) and ${}^{3}_{1}\mathrm{H}$ (tritium). b) ${}^{1}_{1}\mathrm{H}$ (ordinary hydrogen). c) ${}^{235}_{92}\mathrm{U}$ (uranium-235). d) ${}^{239}_{94}\mathrm{Pu}$ (plutonium-239). e) gasoline.

Appendix 1 Introductory Physics Equation Sheet

Note: In this chapter equation sheets are presented suitable for the physics and astronomy part of a physical sciences course, introductory physics algebra-based, and introductory physics calculus-based.

Equation Sheet for Physical Sciences Courses

This equation sheet is intended for students writing tests or reviewing material. Therefore it is neither intended to be complete nor completely explicit. There are fewer symbols than variables, and so some symbols must be used for different things: context must distinguish.

The equations are mnemonic. Students are expected to understand how to interpret and use them. Usually, non-vector forms have been presented: i.e., forms suitable for one-dimensional calculations.

1342 **Geometry**

$$C_{\text{cir}} = 2\pi r \qquad A_{\text{cir}} = \pi r^2 \qquad A_{\text{sph}} = 4\pi r^2 \qquad V_{\text{sph}} = \frac{4}{3}\pi r^3$$

$$c^2 = a^2 + b^2 \quad \text{Pyth.Thm.}$$

1343 **Kinematics**

$$d = vt \qquad v_{\text{ave}} = \frac{d_{\text{final}} - d_{\text{initial}}}{t} \qquad v = at \qquad a_{\text{ave}} = \frac{v_{\text{final}} - v_{\text{initial}}}{t}$$

$$\text{Amount} = \text{Constant Rate} \times \text{time} \qquad \text{time} = \frac{\text{Amount}}{\text{Constant Rate}} \qquad a_{\text{centripetal}} = \frac{v^2}{r}$$

1344 **Dynamics**

$$F_{\text{net}} = ma \qquad 1\,\text{N} \approx 0.225\,\text{lb} \qquad F_{\text{centripetal}} = \frac{mv^2}{r} \qquad p = mv$$

1345 **Gravity**

$$F = \frac{Gm_1m_2}{r^2} \qquad F_g = mg \qquad v_{\text{circular}} = \sqrt{\frac{GM}{r}} \qquad v_{\text{escape}} = \sqrt{\frac{2GM}{r}}$$

$$G = 6.67430(15) \times 10^{-11}\,\text{N}\,\text{m}^2/\text{kg}^2 (\text{circa} 2025)$$

$$g = 9.80\,\text{m/s}^2 \quad (\text{latitude range } \sim 9.78030\text{–}9.8322\,\text{m/s}^2\ [\text{CAC-72}])$$

1346 **Energy and Work**

$$W = Fd \qquad 1\,\text{J} = 1\,\text{N} \cdot \text{m} \qquad P = \frac{W}{t} \qquad K = \frac{1}{2}mv^2 \qquad U_{\text{gravity}} = mgy$$

$$c = 2.99792458 \times 10^8\,\text{m/s} \approx 2.998 \times 10^8\,\text{m/s} \approx 3 \times 10^8\,\text{m/s}$$

$$E = mc^2 \qquad E_{\text{rest}} = m_{\text{rest}}c^2 \qquad \Delta t_{\text{proper}} = \Delta t\sqrt{1 - (v/c)^2}$$

1347 **Thermodynamics and Buoyancy**

$$T_{\text{absolute}} = T_{\text{Celsius}} + 273.15 \qquad T_{\text{Fahrenheit}} = \frac{9}{5}T_{\text{Celsius}} + 32 \qquad \Delta Q = C_{\text{specific}}m\Delta T$$

$$\rho = \frac{m}{V} \qquad n = \frac{N}{V} \qquad p = \frac{F}{A} \qquad p = p_{\text{surface}} + \rho gy \qquad F_{\text{buoyant}} = m_{\text{dis}}g = \rho_{\text{fluid}}V_{\text{dis}}g$$

$$\rho_{\text{fluid}}V_{\text{dis}} = m_{\text{floating}} \qquad PV = NkT \qquad k = 1.3806505 \times 10^{-23}\,\text{J/K}$$

$$\varepsilon = \frac{W_{\text{done}}}{Q_{\text{absorbed}}} \qquad \varepsilon_{\text{upperlimit}} = 1 - \frac{T_{\text{cold}}}{T_{\text{hot}}}$$

1348 **Electricity and Magnetism**

$$F = \frac{kQ_1Q_2}{r_{12}} \qquad k = 8.99 \times 10^9\,\text{N}\,\text{m}^2/\text{C}^2 \qquad e = 1.60217733 \times 10^{-19}\,\text{C}$$

$$1 \text{ ampere (A)} = 1\ \frac{\text{coulomb (C)}}{\text{second (s)}}$$

$$1 \text{ volt (V)} = 1\ \frac{\text{joule (J)}}{\text{coulomb (C)}} \qquad 1 \text{ ohm } (\Omega) = 1\ \frac{\text{volt (V)}}{\text{ampere (A)}}$$

$$\sum \Delta V_{\text{rise}} = \sum \Delta V_{\text{drop}} \qquad V = IR \qquad P = VI$$

1349 **Waves**

$$v = f\lambda \qquad p = 1/f \qquad n\frac{\lambda_n}{2} = L \qquad \lambda_n = \frac{2L}{n} \qquad f_n = \frac{v}{2L}n$$

$$v_{\text{sound } 20^\circ\text{C } 1\,\text{atm}} = 343\,\text{m/s} \qquad v_{\text{sound } 0^\circ\text{C } 1\,\text{atm}} = 331\,\text{m/s}$$

1350 **Nuclear Physics**

$${}^{A}_{Z}X \qquad n(t) = \frac{N_0}{2^{t/t_{1/2}}} \qquad 1\,\text{amu} = 931.494043\,\text{MeV}$$

1351 **Quantum Mechanics**

$$h = 6.6260693 \times 10^{-34}\,\text{J}\,\text{s} \qquad m_e = 9.1093826 \times 10^{-31}\,\text{kg} \qquad E = hf \qquad \lambda = \frac{h}{p} = \frac{h}{mv}$$

$$K_{\text{photoelectron}} = hf - w \qquad H\Psi = \frac{ih}{2\pi}\frac{\partial\Psi}{\partial t}$$

1352 **Astronomy**

$$v = Hd \qquad H = 71^{+4}_{-3}\,\frac{\text{km/s}}{\text{Mpc}} \quad \text{(circa 2004)}$$

Equation Sheet for Introductory Physics Algebra-Based

This equation sheet is intended for students writing tests or reviewing material. Therefore it is neither intended to be complete nor completely explicit. There are fewer symbols than variables, and so some symbols must be used for different things: context must distinguish.

The equations are mnemonic. Students are expected to understand how to interpret and use them.

1353 Constants

$$c = 2.99792458 \times 10^8\,\mathrm{m/s} \approx 2.998 \times 10^8\,\mathrm{m/s} \approx 3 \times 10^8\,\mathrm{m/s} \approx 1\,\mathrm{lyr/yr} \approx 1\,\mathrm{ft/ns} \qquad \text{exact by definition}$$

$$e = 1.602176487(40) \times 10^{-19}\,\mathrm{C}$$

$$G = 6.67430(15) \times 10^{-11}\,\mathrm{N\,m^2/kg^2} \qquad \text{(circa 2025)}$$

$$g = 9.8\,\mathrm{m/s^2} \qquad \text{fiducial value}$$

$$k = \frac{1}{4\pi\varepsilon_0} = 8.987551787\ldots \times 10^9 \approx 8.99 \times 10^9 \approx 10^{10}\,\mathrm{N\,m^2/C^2}$$

$$k_{\mathrm{Boltzmann}} = 1.3806504(24) \times 10^{-23}\,\mathrm{J/K} = 0.8617343(15) \times 10^{-4}\,\mathrm{eV/K} \approx 10^{-4}\,\mathrm{eV/K}$$

$$m_e = 9.10938215(45) \times 10^{-31}\,\mathrm{kg} = 0.510998910(13)\,\mathrm{MeV}$$

$$m_p = 1.672621637(83) \times 10^{-27}\,\mathrm{kg} = 938.272013(23),\mathrm{MeV}$$

$$\varepsilon_0 = \frac{1}{\mu_0 c^2} = 8.8541878176\ldots \times 10^{-12}\,\mathrm{C^2/(N\,m^2)} \approx 10^{-11} \qquad \text{vacuum permittivity or electric constant}$$

$$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N/A^2} \qquad \text{exact by definition}$$

1354 Geometrical Formulae

$$C_{\mathrm{cir}} = 2\pi r \qquad A_{\mathrm{cir}} = \pi r^2 \qquad A_{\mathrm{sph}} = 4\pi r^2 \qquad V_{\mathrm{sph}} = \frac{4}{3}\pi r^3$$

$$c = \sqrt{a^2 + b^2 - 2ab\cos\theta_c} \qquad \frac{\sin\theta_a}{a} = \frac{\sin\theta_b}{b} = \frac{\sin\theta_c}{c}$$

1355 Trigonometry Formulae

$$\frac{x}{r} = \cos\theta \qquad \frac{y}{r} = \sin\theta \qquad \frac{y}{x} = \tan\theta \qquad \cos^2\theta + \sin^2\theta = 1$$

$$\cos^2\theta = \frac{1}{2}[1 + \cos(2\theta)] \qquad \sin^2\theta = \frac{1}{2}[1 - \cos(2\theta)] \qquad \sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b) \qquad \cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$360^\circ = 2\pi \qquad \text{degrees to radians} = \frac{\pi}{180^\circ} \qquad \text{radians to degrees} = \frac{180^\circ}{\pi}$$

1356 Quadratic Formula

If $0 = ax^2 + bx + c$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = -\dfrac{b}{2a} \pm \sqrt{\left(\dfrac{b}{2a}\right)^2 - \dfrac{c}{a}}$

1357 **Vector Formulae**

$$\vec{a} = (a_x, a_y, a_z) = a_x\hat{x} + a_y\hat{y} + a_z\hat{z} \qquad \vec{a} + \vec{b} = (a_x + b_x, a_y + b_y, a_z + b_z)$$

$$\vec{a} \cdot \vec{b} = ab\cos\theta = a_x b_x + a_y b_y + a_z b_z \qquad \vec{a} \times \vec{b} = (ab\sin\theta)\hat{n}$$

1358 **One-Dimensional Kinematics**

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} \qquad v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \qquad a_{\text{avg}} = \frac{\Delta v}{\Delta t} \qquad a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

$$v = v_0 + at \qquad x = x_0 + v_0 t + \frac{1}{2}at^2 \qquad v^2 = v_0^2 + 2a(x - x_0)$$

$$x = x_0 + \frac{1}{2}(v_0 + v)t \qquad x = x_0 + vt - \frac{1}{2}at^2 \qquad g = 9.8\,\text{m/s}^2$$

1359 **Two- and Three-Dimensional Kinematics: General**

$$\vec{v}_{\text{avg}} = \frac{\Delta \vec{r}}{\Delta t} \qquad \vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} \qquad \vec{a}_{\text{avg}} = \frac{\Delta \vec{v}}{\Delta t} \qquad \vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$

$$\vec{a}_{\text{centripetal}} = \frac{v^2}{r}(-\hat{r}) \qquad a_{\text{centripetal}} = \frac{v^2}{r}$$

1360 **Projectile Motion**

$$x = v_{x,0}t \qquad y = y_0 + v_{y,0}t - \frac{1}{2}gt^2 \qquad v_{x,0} = v_0\cos\theta \qquad v_{y,0} = v_0\sin\theta$$

$$x_{\text{max}} = \frac{v_0^2}{g}\sin(2\theta) \qquad y_{\text{max}} = \frac{v_0^2}{2g}\sin^2\theta$$

1361 **Very Basic Newtonian Physics**

$$\vec{F}_{\text{net}} = m\vec{a} \qquad \vec{F}_{\text{opp}} = -\vec{F} \qquad F_g = mg \qquad g = 9.8\,\text{m/s}^2 \qquad F = -kx$$

$$F_{\text{f static}} = \min[F_{\text{applied}}, F_{\text{f static max}}] \qquad F_{\text{f static max}} = \mu_{\text{static}} F_{\text{N}} \qquad F_{\text{f kinetic}} = \mu_{\text{kinetic}} F_{\text{N}}$$

$$\vec{F}_{\text{centripetal}} = m\frac{v^2}{r}(-\hat{r}) \qquad F_{\text{centripetal}} = m\frac{v^2}{r}$$

Gravity

$$G = 6.67430(15) \times 10^{-11}\,\mathrm{N\,m^2/kg^{-2}} \quad \text{(circa 2025)} \qquad \vec{F}_{1\text{ on }2} = \frac{Gm_1m_2}{r_{12}^2}(-\hat{r}_{12})$$

$$F_{12} = \frac{Gm_1m_2}{r_{12}^2} \qquad \vec{f}_g = \frac{GM}{r^2}(-\hat{r}) \qquad f_g = \frac{GM}{r^2}$$

$$v_{\text{escape}} = \sqrt{\frac{2GM}{r}} \qquad v_{\text{orbit}} = \sqrt{\frac{GM}{r}}$$

$$P^2 = \left(\frac{4\pi^2}{GM}\right) r^3 \qquad P = \left(\frac{2\pi}{\sqrt{GM}}\right) r^{3/2}$$

Work and Energy

$$\Delta W = \vec{F} \cdot \Delta\vec{s} = F\Delta s\cos\theta \qquad K = \frac{1}{2}mv^2 \qquad U = mgy \qquad E_{\text{mech}} = K + U$$

$$\Delta E_{\text{mech}} = \Delta K + \Delta U$$

$$\Delta K = W_{\text{net}} \qquad \text{Work-Kinetic-Energy Theorem}$$

$$\Delta U_{\text{conservative}} = -W_{\text{conservative}}$$

$$\Delta E_{\text{mech}} = W_{\text{nonconservative}} \qquad \text{Work-Energy Theorem}$$

$$P_{\text{avg}} = \frac{\Delta W}{\Delta t} \qquad P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} \qquad P = \vec{F} \cdot \vec{v}_s$$

Systems of Particles

$$\vec{r}_{\text{cm}} = \frac{\sum_i m_i\vec{r}_i}{m_{\text{total}}} \qquad \vec{v}_{\text{cm}} = \frac{\sum_i m_i\vec{v}_i}{m_{\text{total}}} \qquad \vec{a}_{\text{cm}} = \frac{\sum_i m_i\vec{a}_i}{m_{\text{total}}} \qquad \vec{F}_{\text{net ext}} = m\vec{a}_{\text{cm}}$$

$$\vec{p} = m\vec{v} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} \qquad \vec{F}_{\text{net ext}} = \lim_{\Delta t \to 0} \frac{\Delta\vec{p}_{\text{total}}}{\Delta t}$$

Collisions

$$\vec{J} = \vec{F}_{\text{avg}}\Delta t \qquad \vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f} \qquad \vec{v}_{\text{cm}} = \frac{\vec{p}_1 + \vec{p}_2}{m_{\text{total}}}$$

$$K_{\text{total }f} = K_{\text{total }i} \qquad \text{1-d Elastic Collision Expression}$$

$$v_{1f} = \frac{(m_1 - m_2)v_{1i} + 2m_2v_{2i}}{m_1 + m_2} \qquad \text{1-d Elastic Collision Expression}$$

$$v_{2f} - v_{1f} = -(v_{2i} - v_{1i}) \qquad v_{\text{rel } f} = -v_{\text{rel } i} \qquad \text{1-d Elastic Collision Expressions}$$

1366 **Rotational Kinematics**

$$\theta = \frac{s}{r} \qquad \omega = \frac{v}{r} \qquad \alpha = \frac{a}{r}$$

$$\omega_{\text{avg}} = \frac{\Delta\theta}{\Delta t} \qquad \omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} \qquad \alpha_{\text{avg}} = \frac{\Delta\omega}{\Delta t} \qquad \alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t}$$

$$\omega = \alpha t + \omega_0 \qquad \theta = \frac{1}{2}\alpha t^2 + \omega_0 t \qquad \theta = \frac{1}{2}(\omega_0 + \omega)t \qquad \omega^2 = \omega_0^2 + 2\alpha\theta \qquad \theta = -\frac{1}{2}\alpha t^2 + \omega t$$

1367 **Simple Harmonic Motion**

$$P = \frac{1}{f} \qquad \omega = 2\pi f \qquad F = -kx \qquad U = \frac{1}{2}kx^2$$

$$\omega = \sqrt{\frac{k}{m}} \qquad P = 2\pi\sqrt{\frac{m}{k}} \qquad E_{\text{mech total}} = \frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}kx_{\text{max}}^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$x(t) = x_{\text{max}}\cos(\omega t) \qquad v(t) = -\omega x_{\text{max}}\sin(\omega t) \qquad a(t) = -\omega^2 x_{\text{max}}\cos(\omega t)$$

$$v_{\text{max}} = \omega x_{\text{max}} \qquad a_{\text{max}} = \omega^2 x_{\text{max}}$$

$$P = 2\pi\sqrt{\frac{\ell}{g}}$$

1368 **Fluids**

$$\rho = \frac{m}{V} \qquad P = \frac{F}{A} \qquad P = P_0 + \rho g y_{\text{depth}} \qquad P = P_0 - \rho g y_{\text{height}}$$

$$\rho_{\text{air}} = 1.21\,\text{kg/m}^3 \qquad \rho_{\text{water}} = 998\,\text{kg/m}^3 \qquad \text{at } 20^\circ\text{C and 1 atm}$$

$$1\,\text{atm} = 1.01325 \times 10^5\,\text{Pa} = 101.325\,\text{kPa} = 14.696\,\text{psi} = 760\,\text{torr}$$

Pascal's principle	$P = P_{\text{ext}} - \rho g(y - y_{\text{ext}}) \qquad \Delta P = \Delta P_{\text{ext}}$
Archimedes's principle	$F_{\text{buoy}} = m_{\text{fluid dis}}g = \rho_{\text{fluid}}V_{\text{fluid dis}}g$
Floating condition	$f = \dfrac{V_1}{V} = \dfrac{\rho_2 - \rho_{\text{obj}}}{\rho_2 - \rho_1}$
equation of continuity for ideal fluid	$R_V = Av = \text{Constant}$
Bernoulli's equation	$P + \dfrac{1}{2}\rho v^2 + \rho g y = \text{Constant}$

1369 **Waves**

$$f = \frac{1}{P} \qquad f\lambda = v \qquad f\lambda = c \qquad v = \sqrt{\frac{F_\mathrm{T}}{\mu}} \qquad \mu = \frac{m}{L}$$

$$c = 2.99792458 \times 10^8\,\mathrm{m/s} \approx 2.998 \times 10^8\,\mathrm{m/s} \approx 3.00 \times 10^8\,\mathrm{m/s}$$

$$P = \frac{F}{A} \qquad I = \frac{P}{4\pi r^2} \qquad \beta = (10\,\mathrm{dB}) \times \log\left(\frac{I}{I_\mathrm{threshold}}\right) \qquad I_\mathrm{threshold} = 10^{-12}\,\mathrm{W/m^2}$$

$$f_\mathrm{medium} = \frac{f_\mathrm{frame}}{1 - v_\mathrm{frame}/v_\mathrm{sound}} \qquad f_\mathrm{frame} = f_\mathrm{medium}(1 - v_\mathrm{frame}/v_\mathrm{sound})$$