8.3 Question

In all four pictures the tension $T$ is causing the centripetal acceleration $\frac{V^2}{r}$.

$Ta = m \frac{V^2}{r} = m \frac{V^2}{r}$

$Tb = m \frac{V^2}{2r} = \frac{1}{2} m \frac{V^2}{r}$

$Tc = 2m \frac{V^2}{r} = 2 m \frac{V^2}{r}$

$Td = 2m \frac{V^2}{2r} = m \frac{V^2}{r}$

$Tc > Ta = Td > Tb$
8.8 A correctly banked road is designed so that \( f_s = 0 \) when cars drive the "correct" speed.

\[
\begin{align*}
\text{The normal force } \vec{N} &= N_x \hat{i} + N_y \hat{j} \\
\vec{N} &= -N \sin \theta \hat{i} + N \cos \theta \hat{j} \\
N \cos \theta &= mg \quad \Rightarrow \quad N = \frac{mg}{\cos \theta}
\end{align*}
\]

B. \( N \sin \theta \) is the force that causes the centripetal acceleration.

\[
N \sin \theta = \frac{m v^2}{r} = \frac{mg \sin \theta}{\cos \theta}
\]

\[
\frac{v^2}{r} = g \tan \theta
\]

\[
v = 90 \text{ km/hr} = \frac{75 \text{ m}}{s}
\]

\[
\gamma = 500 \text{ m}
\]

\[
\tan \theta = \frac{v^2}{r} = 0.125 \quad \Rightarrow \quad \theta = 7.1^\circ
\]
When the car is stationary, 

\[ T = mg \]

If the car goes over the hill with speed \( u \), the forces cause a centripetal acceleration \( \frac{v^2}{r} \), 

\[ mg - N = m \frac{v^2}{r} \quad \Rightarrow \quad N = mg - m \frac{v^2}{r} \]

So the faster the car goes, the smaller the normal force. The smallest \( N = 0 \) 

\[ 0 = mg - m \frac{v^2}{r} \quad \Rightarrow \quad v = \sqrt{gr} = \frac{22.4 m}{s} \]
\[d = 2\, m\]
\[m = 100\, g = 0.1\, kg\]
\[y = 1.0\, m\]
\[x = 30\, \text{cm} = 0.3\, m\]
\[\mu_k = 0.50\]

\[\vec{V} = \text{velocity as the object leaves the table}\quad V_x = V_0\quad \text{and}\quad V_y = 0\]

\[V_0\] causes the projectile motion during flight.

\[x = V_0\, t\]

\[y = -\frac{1}{2} g t^2\quad (\text{origin at edge of table})\]

\[T = \text{time to fall}\]

\[0.3 = V_0\, T\quad \rightarrow\quad T = \frac{0.3}{V_0}\]

\[-1.0 = -5\, T^2 = -5\left(\frac{0.3}{V_0}\right)^2\quad \rightarrow V_0^2 = 5(0.09)\]

\[V_0 = 0.67\, m/s\]

\[s_k = \mu_k\, N = \mu_k\, mg\]

\[s_k = 0.5\, N\]
The friction causes a deceleration

\[ F_k = ma \rightarrow a = \frac{0.5 \text{N}}{0.1 \text{kg}} = 5 \text{ m/s}^2 \]

When the acceleration is constant, the average velocity is
\[ \frac{1}{2} (v_i + v_f) = \bar{v} \]

In this problem, we know \( v_f = 0.67 \text{ m/s} \)
but \( v_i \) is unknown.

We also know how far the object slid, 20 m.

Distance = \( \bar{v} \cdot T \)

\[ 2.0 \text{m} = \frac{1}{2} (v_i + 0.67) T \]

2 unknowns

But the deceleration is known

\[ -a = -5 \text{ m/s}^2 = \frac{v_f - v_i}{T} = \frac{0.67 - v_i}{T} \]

\[ T = \frac{v_i - 0.67}{5} \]

4.0 = \((v_i + 0.67)(v_i - 0.67) = \Rightarrow \]

\[ 20 = v_i^2 - (0.67)^2 \cdot \frac{5}{5} \]

\[ v_i = 4.4 \text{ m/s} \]
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\[ u_0 = 11 \text{ m/s} \]

\[ \theta = 200 \]

\[ h = 2.0 \text{ m} \]

\[ l = 10 \text{ m} \]

\[ \mu_r = 0.02 \]

We need to find his speed at the end of the ramp.

\[ N = mg \cos \theta \]

\[ f_r = \mu_r N = \mu_r mg \cos \theta \]

Both \( f_r \) and \( mg \sin \theta \) are slowing him down.

\[ \mu_r mg \cos \theta + mg \sin \theta = ma \]

\[ a = \mu_r g \cos \theta + g \sin \theta = 3.6 \text{ m/s}^2 \]

\[ \sin \theta = \frac{h}{S} \]

\[ S = \frac{2.0 \text{ m}}{\sin 20^\circ} \]

\[ S = 5.8 \text{ m} = \text{length of ramp} \]
Like the previous problem,

\[ a = \frac{v_f - v_i}{t} \quad \text{and} \quad \bar{v} = \frac{v_f + v_i}{2} \]

but \[ S = \bar{v}t = \frac{v_f + v_i}{2} \times t \]

\[ t = \frac{v_f - v_i}{a} \]

\[ S = \frac{v_f^2 - v_i^2}{2a} \]

\[ a = -3.6 \text{ m/s}^2 \]

\[ S = 5.8 \text{ m} \]

\[ v_i = 11 \text{ m/s} \]

\[ v_f^2 = 2as + v_i^2 = 2(-3.6)(5.8) + 11^2 \]

\[ v_f = 8.9 \text{ m/s} \]

Now it is a projectile problem

\[ u_0 \]

\[ h = 2.0 \text{ m} \]

\[ y(+) = 0 + 3.0t - 5t^2, \ \text{when } y = -2 \]

\[-2 = 3.0T - 5T^2 \]

\[ 5T^2 - 3T - 2 = 0 \]

\[ (5T + 2)(T - 1) = 0 \]

\[ T = 1 \text{ s} \] or \[ T = -\frac{2}{5} \]

\[ x = 8.4 \times T = 8.4 \text{ m} \]

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\[ \theta = 44.7^\circ \]

\[ \text{Thrust} = T = 140,700 \text{N} \]
\[ M = 5000 \text{kg} \]
\[ T_x = T \cos \theta \]
\[ T_y = T \sin \theta \]

\[ \sum F_x = \text{Max} = T \cos \theta, \quad a_x = \frac{T \cos \theta}{M} \]
\[ \sum F_y = \text{My} = T \sin \theta - Mg = May \]
\[ a_y = \frac{T \sin \theta - g}{M} \]

\[ a_x = 20 \text{ m/s}^2, \quad a_y = 9.8 \text{ m/s}^2 \]

\[ x(t) = \frac{a_x}{2} t^2 \quad v_x(0) = 0 \]
\[ y(t) = \frac{a_y}{2} t^2 \quad v_y(0) = 0 \]

a) \[ \frac{y(t)}{x(t)} = \frac{a_y}{a_x} \approx \frac{1}{2} \quad y = \frac{1}{2} x \leftarrow \text{Straight Line} \]

b) \[ c) \quad v = \sqrt{v_x^2 + v_y^2} = 22.4 \text{ m/s}^2 \]
\[ v = at \rightarrow t = \frac{330 \text{ m/s}}{22.4} = 15 \text{ s} \]
\[ y = \frac{1}{2} a_y t^2 = 1125 \text{ m} \]
Car is moving into the paper. Maximum speed uses maximum static friction.

$$F_s = m g n$$

where $m = 1.00$ for rubber tires on concrete.

The centripetal acceleration is horizontal so want to break forces into horizontal forces.

$$\Sigma F_{\text{hor}} = \frac{m u^2}{n}$$

vertical

$$\Sigma F_{\text{ver}} = 0.$$  

$$\Sigma F_{\text{ver}} = n \cos \theta - F_s \sin \theta - Mg = 0$$

but $F_s = n \mu_s r$  

$$n = \frac{Mg}{\cos \theta - n \sin \theta}$$

$$\Sigma F_{\text{hor}} = F_s \cos \theta + n \sin \theta = n (\cos \theta + \sin \theta) = \frac{m u^2}{n}$$

$$\frac{Mg}{\cos \theta - n \sin \theta} (\cos \theta + n \sin \theta) = \frac{m u^2}{n}$$

$$u^2 = \frac{(\cos \theta + \sin \theta)g r}{(\cos \theta - \sin \theta)}$$

Put in numbers: $\theta = 15^\circ$, $g = 9.8$, $r = 70m$

$$u = 34.5 \text{ m/s}$$
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\[ l = 1.0 \text{ m} \]
\[ r = 20 \text{ cm} = 0.2 \text{ m} \]
\[ m = 500 \text{ g} = 0.5 \text{ kg} \]

\[ \theta = \sin^{-1} \frac{r}{l} = 11.5^\circ \]

\[ \sin \theta = \frac{r}{l} \]
\[ \cos \theta = \frac{\sqrt{l^2 - r^2}}{l} \]

\[ \Sigma F_y = 0 \rightarrow T \cos \theta = mg \]
\[ \Sigma F_x = T \sin \theta = \frac{mu^2}{r} \]

\[ 5.1N = \begin{vmatrix} T = \frac{mg \sin \theta}{\cos \theta} = \frac{mg}{\sqrt{l^2 - r^2}} \end{vmatrix} \]

Makes sense when \( r = 0 \)
\[ \frac{T}{\cos \theta} = \frac{mg}{\sqrt{l^2 - r^2}} \]

\[ \frac{T \sin \theta}{T \cos \theta} = \frac{mu^2}{r} \]
\[ \tan \theta = \frac{u^2}{rg} \rightarrow u = \sqrt{rg \tan \theta} = 0.64 \frac{m}{s} \]

\[ \tan \theta = \frac{r}{\sqrt{l^2 - r^2}} \]

\[ (u \text{ is } \frac{m}{s}) \left( \frac{1 \text{ rev}}{2 \pi \text{ rad}} \right) \left( \frac{60 \text{ s}}{1 \text{ min}} \right) \]
\[ = 30 \text{ rpm} \]
\[ r = 2.5 \text{ m} \]

\[ 0.6 \leq \mu_s \leq 1.0 \]

\[ \mu_k \rightarrow \text{irrelevant, we hope!} \]

Minimum \( M = 30 \text{ Kg} \)

Free-body diagram

- \( N \) causes centripetal acceleration

\[ N = m \frac{v^2}{r} = mw^2 r \]

\[ F_s = mg = F_s, \text{max} = \mu_s N = \mu_s mw^2 r \]

\[ F = \mu_s w^2 r \rightarrow w^2 = \frac{F}{\mu_s r} \]

\( \omega \) has to be big enough to keep "slippery" customers from "sliding" to their doom!

\[ \omega_{\text{min}} = \sqrt{\frac{10}{0.6 (2.5)}} = 2.6 \text{ rad/s} \]

\[ = 25 \text{ rpm} \]
Projectile problem — mass moves horizontally to begin with.

\[ U_x(0) = ? \]
\[ U_y(0) = 0 \]

\[ T = 5 \text{ N} \]
\[ m = 0.1 \text{ Kg} \]
\[ r = 0.6 \text{ m} \]
\[ T - mg = \frac{mu^2}{r} \]
\[ u^2 = \frac{rT - rg}{m} = 24 \text{ m}^2/\text{s}^2 \]
\[ u = 4.9 \text{ m/s} \]

\[ y(t) = 0 + 0 - \frac{1}{2} gt^2 \]

What is \( t \) when \( y(t) = -1.4 \text{ m} \)

\[ -1.4 = -5t^2, \quad t = \sqrt{\frac{1.4}{5}} = 0.53 \text{ s} \]

\[ x(t) = U_x(0) t \]
\[ x(0.53) = (4.9)(0.53) = 2.6 \text{ m} \]
\[ T = \frac{T \cdot \cos 20^\circ - f_k}{m}, \quad f_k = \mu_k N \]

\[ G_T = \frac{\text{Thrust} \cdot \cos 20^\circ - \mu_k g}{m} = 0.58 \text{ m/s} \]

\[ \alpha = \frac{G_T}{r} = 0.29 \text{ rad/s} \]

\[ \theta = \frac{1}{2} \times t^2 \Rightarrow t = \sqrt{\frac{2 \times 2\pi}{0.29}} = 21 \text{ sec} \]

Starting from rest:
\[ \omega = \alpha t = (0.29) \times 21 = 6.1 \text{ rad/s} \]

\[ b) \ T = \text{Tension} = mW^2r - \text{Thrust} \cdot \sin 20^\circ = 36 \text{ N} \]