Chapter 3 – Vectors and Coordinate Systems

As mentioned earlier in class, a vector is a quantity that has direction and magnitude, for example velocity. Vector quantities in the text are labeled with arrows above the letter. I will use bold letters to represent vectors. For example the vector \( \mathbf{A} \) is denoted as a bold capital A. The same symbol in regular type, \( A \) for example, represents the magnitude of the vector \( \mathbf{A} \). The magnitude is ALWAYS a positive number. The arrows below represent vectors \( \mathbf{A} \) and \( \mathbf{B} \).

The addition of two vectors, \( \mathbf{C} = \mathbf{A} + \mathbf{B} \), can be represented graphically by connecting the tail of \( \mathbf{B} \) to the head of \( \mathbf{A} \). The resulting vector \( \mathbf{C} \) is just the vector that connects the tail of \( \mathbf{A} \) to the head of \( \mathbf{B} \).

The subtraction of one vector from another, \( \mathbf{D} = \mathbf{A} - \mathbf{B} \), is the same as the addition of \( \mathbf{A} \) to the vector \( -\mathbf{B} \), \( \mathbf{D} = \mathbf{A} + (-\mathbf{B}) \), where \( -\mathbf{B} \) is the same as vector \( \mathbf{B} \) with its direction reversed! The diagram above shows both the addition and subtraction of vectors \( \mathbf{A} \) and \( \mathbf{B} \).

When actually doing algebra with vectors the easiest thing is to superimpose a Cartesian coordinate system over the vectors so that each vector can be represented by its decomposition into components in the \( x \) and \( y \) directions where \( \mathbf{i} \) and \( \mathbf{j} \) are unit vectors pointing in the positive coordinate direction. The Cartesian coordinates can be oriented in whatever direction makes sense for that particular problem.

\[
\mathbf{A} = A_x \mathbf{i} + A_y \mathbf{j} = A \cos 30^\circ \mathbf{i} - A \sin 30^\circ \mathbf{j} = 0.866 A \mathbf{i} - 0.500 A \mathbf{j}
\]
Note that I chose the standard orientation of the x and y axes in the above diagram. With that choice, the y-component of \( \mathbf{A} \) points in the negative j direction. The diagram ought to alert you to whether or not the components of a vector are in the positive or negative directions. Also notice that if you were given the vector \( \mathbf{A} = 0.866 \mathbf{A}_x - 0.500 \mathbf{A}_y \), the angle inside the triangle is given by the inverse tangent of the magnitude of \( \mathbf{A}_y \) divided by the magnitude of \( \mathbf{A}_x \).

Keep in mind that a different choice for the orientation of the axes would lead to different components, \( \mathbf{A}_x \) and \( \mathbf{A}_y \), for the same vector \( \mathbf{A} \)!

In terms of components, \( \mathbf{C} = \mathbf{A} + \mathbf{B} = (A_x \mathbf{i} + A_y \mathbf{j}) + (B_x \mathbf{i} + B_y \mathbf{j}) = (A_x + B_x) \mathbf{i} + (A_y + B_y) \mathbf{j} \) and \( \mathbf{C} = \mathbf{A} - \mathbf{B} = A_x \mathbf{i} + A_y \mathbf{j} - (B_x \mathbf{i} + B_y \mathbf{j}) = (A_x - B_x) \mathbf{i} + (A_y - B_y) \mathbf{j} \). For a vector to equal zero, ALL of its components have to be zero! The zero vector is given by \( \mathbf{0} = 0 \mathbf{i} + 0 \mathbf{j} \); the x and y-components have to each be equal to zero. Of course this follows from the definition of the sum of two vectors given above. If \( \mathbf{C} = \mathbf{A} + \mathbf{B} \), then \( C_x = A_x + B_x \) and \( C_y = A_y + B_y \). The vector equation is shorthand for the two separate equations: \( C_x = A_x + B_x \) and \( C_y = A_y + B_y \) each of which has to equal zero for \( \mathbf{C} \) to equal the zero vector.

Dealing with vectors is straightforward as long as you keep in mind that any vector in two dimensions can be written as the sum of a component in the x-direction and a component in the y-direction. And you can pick the x and y axes to be in directions that make the most sense for that particular problem. My suggestion is to make these choices explicitly by superimposing the axes on top of the diagram including the vectors to be decomposed into components.