Chapter 2 – Kinematics in One Dimension

This chapter introduces the notions of “instantaneous” velocity and acceleration. These new concepts allow us to analyze situations where the velocity of the particle changes from instant–to-instant as it moves from one spot to the next. For one-dimensional motion let’s represent the location of the particle by some function $s(t)$, where $s(t)$ locates the position of the particle as measured from some origin at the time $t$. Usually we take the earliest time in a given problem as $t = 0$. So $s(0)$ locates the initial position of the particle. For example, suppose $s(0) = 10$ meters. The origin is the spot where $s = 0$, so $s(0) = 10$ meters says the particle is initially 10 meters to the right of the origin where “to the right” is the usual definition of the positive direction.

Newton, the co-inventor of calculus, most likely invented it because he was studying motion and needed a way to think quantitatively about instantaneous values of velocity and acceleration. Consequently this is an opportune time to see how calculus arises quite naturally from the study of motion.

As we saw last chapter, the average velocity of a particle moving in one dimension as it travels from time $t_1$ to time $t_2$ is just $(s(t_2) - s(t_1))/(t_2 - t_1)$ or more succintly $\Delta s/\Delta t$. If we let $t_2$ get closer and closer to time $t_1$, the interval over which the averaging is done becomes smaller and smaller. At some point the interval will become so small that the average velocity remains essentially constant as $t_2$ continues to get closer to $t_1$. At that point, we call the average over the tiny interval the instantaneous velocity at time $t_1$. In calculus parlance, the instantaneous velocity at time $t_2$ is just the limiting value of $(s(t_2) - s(t_1))/(t_2 - t_1)$ as $t_1$ approaches $t_2$ and that limiting value is called the derivative of the function $s(t)$ with respect to $t$ evaluated at the time $t_2$. In calculus notation the instantaneous velocity at time $t$ is given by, $v(t) = ds/dt$.

It should be clear that the “normal” definition of the derivative of a function $f(x)$ with respect to $x$ is embedded in the above paragraph, $df/dx$ equals the limit as $\Delta x$ approaches zero of the ratio $[f(x + \Delta x) - f(x)]/\Delta x$. This definition can be used to find the derivative of any function that can be written as $x$ to some power $n$, $x^n$. The text notes that $d/dx(x^n) = n x^{n-1}$. You should try to use the definition of the derivative to “prove” that $d/dx(x^n) = n x^{n-1}$.

All the important equations in kinematics can be easily derived using calculus. If $s(t)$ is known, then the instantaneous velocity is just, $v(t) = ds/dt$, and the instantaneous
acceleration is just, \( a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2} \). On the other hand, if the instantaneous acceleration is known, \( a(t) \), the instantaneous velocity and distance covered during some interval of time can easily be found by integrating the acceleration once to get the velocity and then again to get the displacement.

\[ a(t) = \frac{dv}{dt} \text{ so that the small change in velocity during the tiny time interval } dt \text{ is just } dv = a \, dt. \]

Integrating this equation from some initial time, usually zero, to time \( t \), gives the velocity at time \( t \),

\[ \int_0^t dv = v(t) - v(0) = \int_0^t a \, dt. \]

Solve for \( v(t) \) to get \( v(t) = v(0) + \int_0^t a \, dt \).

Analogously, once \( v(t) \) is known we can use \( v(t) = \frac{ds}{dt} \) to find the small displacement caused by \( v(t) \) during the little time increment \( dt \), \( ds = v \, dt \), that happens during the tiny time \( dt \). Now add all those small displacements from time \( t = 0 \) to time \( t \) to get the position of the particle at time \( t \),

\[ \int_0^t ds = s(t) - s(0) = \int_0^t v \, dt. \]

Solve for \( s(t) \) to get \( s(t) = s(0) + \int_0^t v \, dt \).

As practice, show that when the acceleration is constant, \( a = \text{constant acceleration} \), the instantaneous velocity is just,

\[ v(t) = v(0) + a \, t. \]

Now integrate \( v(t) \) for the special case that the acceleration is constant to get the displacement during the time interval from \( t = 0 \) to time \( t \).

\[ s(t) = s(0) + v(0) \, t + \frac{1}{2} a \, t^2. \]

Remember these last two equations are only true when the acceleration is constant. As it turns out, objects falling near the surface of earth fall with constant acceleration. Consequently, for most examples of motion in the textbook which take place near the surface of earth, the acceleration can be assumed to be constant and the above equations apply.
The concepts in this chapter, position, velocity, and acceleration are easy to visualize and related by straightforward mathematical expressions. Consequently, this is a particularly good chapter to practice solving word problems and to develop more skill in thinking graphically about problems. Later in this course, and more so in PHYS 181 and PHYS 182, the concepts become less easy to visualize and the analytic skills learned now will become more vital in solving problems.