4. (25 Points) Suppose you flip 50 fair coins.

(a) How many possible macrostates are there?
(b) How many possible microstates are there?
(c) What is the most likely macrostate(s) and what is its probability?
(d) What is the least likely macrostate(s) and what is its probability?

a) $51$ macrostates

b) $\mathcal{N}_{\text{tot}} = 2^{50} = 1.126 \times 10^{15}$

c) most likely is $25$ heads, $25$ tails

\[ P = \frac{\binom{50}{25}}{2^{50}} = 0.112 \Rightarrow 11.2\% \]

d) least likely is $0$ heads, or $0$ tails

\[ P = \frac{1}{\mathcal{N}_{\text{tot}}} = \frac{1}{2^{50}} = 8.88 \times 10^{-16} \]