According to the Sackur-Tetrode equation, the entropy of a monoatomic ideal gas can become negative when its temperature (and hence its energy) is sufficiently low. Of course this doesn’t happen in reality (it will liquify first). Calculate the temperature \( T \) where the Sackur-Tetrode equation predicts that the entropy of He gas will be negative for temperatures lower than \( T \). Assume that the helium starts at room temperature and atmospheric pressure and that the density does NOT change.

To make life easier use the following form for the Sackur-Tetrode equation

\[
S = Nk \ln \left[ \frac{V}{N} e^{5/2} \left( \frac{4\pi mU}{3Nh^2} \right)^{3/2} \right]
\]

\( S \) will become negative below \( T \) where numbers inside \( [ ] \) = 1 so \( \ln 1 = 0 \) won’t

\[
\frac{kT}{\rho}, e^{\left( \frac{4\pi m 2NkT}{3Nh^2} \right)^{3/2}} = 1
\]

but

\[
PV = NkT
\]

\[
\frac{N}{V} = \frac{P_0}{kT_0} = \text{constant}
\]

\[
U = \frac{3}{2} NkT
\]

\[
\frac{kT}{\rho}, e^{\left( \frac{4\pi m 2NkT}{3Nh^2} \right)^{3/2}} = 1
\]

\[
\left( \frac{2\pi m kT}{h^2} \right)^{3/2} = e^{\frac{-5/2}{kT_0}}
\]

\[
T = \left( e^{\frac{-5/2}{kT_0}} \right)^{2/3} \cdot \frac{h^2}{2\pi mk}
\]

\[
= e^{\frac{-5/2}{\left( \frac{10^5 P_0}{(1.38 \times 10^{-23} T_0/8)(298 K)} \right)^{3/2}}}
\]

\[
\approx \frac{(6.63 \times 10^{-34} s)}{2\pi (4\pi)(1.66 \times 10^{-37} k)}
\]

\[
= 0.012 K
\]