Phys 467/667
Spring 2017
Exam 2
4/6/17
Time Limit: 120 Minutes

Name (Print): ______________________

Instructor ______________________

This exam contains 6 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may use your books, notes, and calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **If you use a "fundamental theorem" you must indicate this** and explain why the theorem may be applied.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

Do not write in the table to the right.

1. (10 points) Explain the following phenomons (use the graphs or equations if necessary)

   1. Consider the chemical reaction $2\,H_2(g) + O_2(g) = 2\,H_2O(l)$, it leads to an entropy decrease from left to right, but it is still favorable, why?
   2. Why does a city have a higher temperature at night than does the surrounding countryside?
   3. The use of catalyst can promote the chemical reaction and get more products. Is it correct or not? Why?
   4. If we mix two ideal gases made of different molecules, the entropy is increasing.
   5. A tray of ice cubes is placed in a food freezer having an ideal coefficient of performance (COP) of 9. If the room temperature is 40 °C, will the ice cubes remain frozen or will they melt? (hint: COP = Q_c/W)

2. (10 points) The equation of state for the Van der Waals gas (n = 1 mole) is

$$(P + \frac{\alpha}{V^2})(V - b) = RT \tag{1}$$

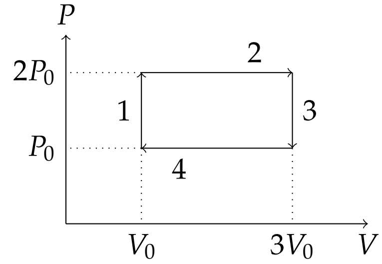
(a) (2 points) (Maxwell relation) Use the thermodynamic identity $dF = -SdT - PdV$ to prove that

$$(\frac{\partial S}{\partial V})_T = (\frac{\partial P}{\partial T})_V \tag{2}$$

(b) (5 points) Write dU as an exact differential with respect to T and V, and prove C_V is independent of V.

(c) (3 points) Evaluate $C_P - C_V = \frac{TV\beta^2}{\kappa}$ for this model (see problem 5.14 in the textbook for details) and figure out under which condition the result is close to R (which is valid for ideal gas)

3. (10 points) A heat engine is designed to pump the monoatomic ideal gas to undergo the cyclic process shown as follows. Write a table to summarize W, Q, ΔU for each step and calculate the effciency $e = \frac{W}{Q_h}$.



4. (10 points) In Boltzman statistics, the motion of molecules in the ideal gas follows a so called Maxwell distribution.

$$D(v) = (\frac{m}{2\pi kT})^{3/2} 4\pi v^2 e^{-mv^2/2kT} \tag{3}$$

where $D(v)$ is the probability function of the molecules with a speed of v. Caculate the average speed v, rms speed $\sqrt{\bar{v^2}}$ and the most likely speed $v_{\max}$, and sort them.

5. (10 points) Consider a solid contains many different sound waves, we pick one of these waves The sound wave could be described by a harmonic oscillator (called *phonon*) (with fixed frequency ω). Each ocillator carries energy $h\omega$. The probabilty of having n oscillators is

$$P(n) = \frac{1}{Z}\exp(-\frac{nh\omega}{k_B T}) \tag{4}$$

   (a) (2 points) Prove that

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots \tag{5}$$

   for what value of x does this series have a finite sum.

   (b) (3 points) Use the results of part (a) to solve Z.

   (c) (4 points) Find the average number of oscillators $\bar{n}$ at temperature T

   (d) (1 point) Find the average energy of these oscillators at temperature T.