Class: Physics 152 (section 001) *Homework Assignment # 7 SOLUTIONS*
Instructor: Dr. Michael Pravica
Date Due: 10/18/09

Homework Assignment #7 Solutions: Problems 21.2, 21.3, 21.8, 21.12, 21.16, 21.17, 21.21, 21.26, 21.27, 21.32, 21.34, 21.36, 21.37, 21.47, 21.48, 21.54, 21.60

2. ***REASONING*** The magnitude B of the magnetic field is given by $B = \dfrac{F}{|q_0| v \sin\theta}$ (Equation 21.1), and we will apply this expression directly to obtain B.

SOLUTION The charge $q_0 = -8.3 \times 10^{-6}$ C travels with a speed $v = 7.4 \times 10^6$ m/s at an angle of $\theta = 52°$ with respect to a magnetic field of magnitude B and experiences a force of magnitude $F = 5.4 \times 10^{-3}$ N. According to Equation 21.1, the field magnitude is

$$B = \frac{F}{|q_0| v \sin\theta} = \frac{5.4 \times 10^{-3}\ \text{N}}{\left|-8.3 \times 10^{-6}\ \text{C}\right| \left(7.4 \times 10^6\ \text{m/s}\right) \sin 52°} = \boxed{1.1 \times 10^{-4}\ \text{T}}$$

Note in particular that it is only the magnitude $|q_0|$ of the charge that appears in this calculation. The algebraic sign of the charge does not affect the result.

---

3. ***REASONING AND SOLUTION*** The speed of the electron can be determined using $eV = (1/2)mv^2$ so that

$$v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2\left(1.60 \times 10^{-19}\ \text{C}\right)\left(19\,000\ \text{V}\right)}{9.11 \times 10^{-31}\ \text{kg}}} = 8.17 \times 10^7\ \text{m/s}$$

The magnetic force is given by

$$F = |q| vB \sin\theta = (1.60 \times 10^{-19}\ \text{C})(8.17 \times 10^7\ \text{m/s})(0.28\ \text{T}) \sin 90.0° = \boxed{3.7 \times 10^{-12}\ \text{N}}$$

---

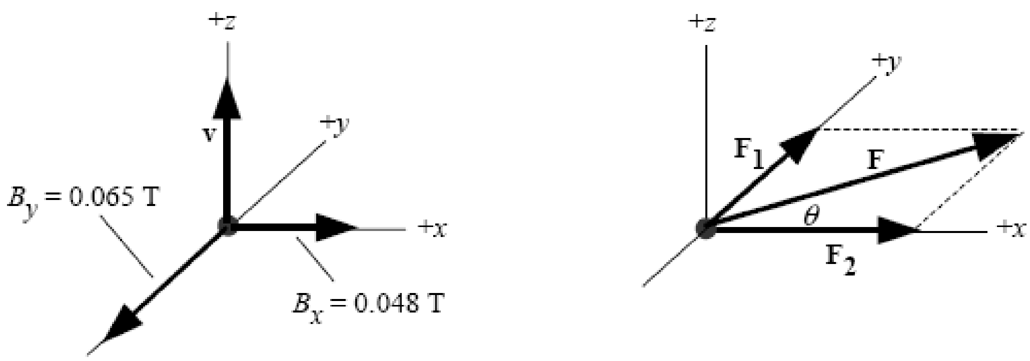
12. ***REASONING*** The radius r of the circular path is given by $r = \dfrac{mv}{|q| B}$ (Equation 21.2), where m and v are the mass and speed of the particle, respectively, $|q|$ is the magnitude of the charge, and B is the magnitude of the magnetic field. This expression can be solved directly for B, since r, m, and v are given and $q = +e$, where $e = 1.60 \times 10^{-19}$ C.

SOLUTION Solving Equation 21.2 for B gives

$$B = \frac{mv}{|q| r} = \frac{\left(3.06 \times 10^{-25}\ \text{kg}\right)\left(7.2 \times 10^3\ \text{m/s}\right)}{\left|+1.60 \times 10^{-19}\ \text{C}\right|\left(0.10\ \text{m}\right)} = \boxed{0.14\ \text{T}}$$

---

8. ***REASONING*** The drawing on the left shows the directions of the two magnetic fields, as well as the velocity **v** of the particle. Each component of the magnetic field is perpendicular to the velocity, so each exerts a magnetic force on the particle. The magnitude of the force is $F = |q_0| vB \sin\theta$ (Equation 21.1), and the direction can be determined by using Right-Hand Rule No. 1 (RHR-1). The magnitude and direction of the net force can be found by using trigonometry.



SOLUTION

a. The magnitude F_1 of the magnetic force due to the 0.048-T magnetic field is

$$F_1 = |q_0| vB_x \sin 90.0° = \left(2.0\times10^{-5}\ \text{C}\right)\left(4.2\times10^{3}\ \text{m/s}\right)\left(0.048\ \text{T}\right) = 4.0\times10^{-3}\ \text{N}$$

The magnitude F_2 of the magnetic force due to the 0.065-T magnetic field is

$$F_2 = |q_0| vB_y \sin 90.0° = \left(2.0\times10^{-5}\ \text{C}\right)\left(4.2\times10^{3}\ \text{m/s}\right)\left(0.065\ \text{T}\right) = 5.5\times10^{-3}\ \text{N}$$

The directions of the forces are found using RHR-1, and they are indicated in the drawing on the right. Also shown is the net force **F**, as well as the angle θ that it makes with respect to the $+x$ axis. Since the forces are at right angles to each other, we can use the Pythagorean theorem to find the magnitude F of the net force:

$$F = \sqrt{F_1^2 + F_2^2} = \sqrt{\left(4.0\times10^{-3}\ \text{N}\right)^2 + \left(5.5\times10^{-3}\ \text{N}\right)^2} = \boxed{6.8\times10^{-3}\ \text{N}}$$

b. The angle θ can be determined by using the inverse tangent function:

$$\theta = \tan^{-1}\left(\frac{F_1}{F_2}\right) = \tan^{-1}\left(\frac{4.0\times10^{-3}\ \text{N}}{5.5\times10^{-3}\ \text{N}}\right) = \boxed{36°}$$

16. ***REASONING*** The speed of the α-particle can be obtained by applying the principle of conservation of energy, recognizing that the total energy is the sum of the particle's kinetic energy and electric potential energy, the gravitational potential energy being negligible in comparison. Once the speed is known, Equation 21.1 can be used to obtain the magnitude of the magnetic force that acts on the particle. Lastly, the radius of its circular path can be obtained directly from Equation 21.2.

SOLUTION

a. Using A and B to denote the initial positions, respectively, the principle of conservation of energy can be written as follows:

$$\underbrace{\tfrac{1}{2}mv_B^2}_{\substack{\text{Final kinetic}\\ \text{energy}}} + \underbrace{\text{EPE}_B}_{\substack{\text{Final electric}\\ \text{potential energy}}} = \underbrace{\tfrac{1}{2}mv_A^2}_{\substack{\text{Initial kinetic}\\ \text{energy}}} + \underbrace{\text{EPE}_A}_{\substack{\text{Initial electric}\\ \text{potential energy}}} \qquad (1)$$

Using Equation 19.3 to express the electric potential energy of the charge q_0 as $\text{EPE} = q_0 V$, where V is the electric potential, we find from Equation (1) that

$$\tfrac{1}{2}mv_B^2 + q_0 V_B = \tfrac{1}{2}mv_A^2 + q_0 V_A \qquad (2)$$

Since the particle starts from rest, we have that $v_A = 0$ m/s, and Equation (2) indicates that

$$v_B = \sqrt{\frac{2q_0\left(V_A - V_B\right)}{m}} = \sqrt{\frac{2\left[2\left(+1.60\times10^{-19}\ \text{C}\right)\right]\left(1.20\times10^{6}\ \text{V}\right)}{6.64\times10^{-27}\ \text{kg}}} = \boxed{1.08\times10^{7}\ \text{m/s}}$$

b. According to Equation 21.1, the magnitude of the magnetic force that acts on the particle is

$$F = \left|q_0\right| v_B B \sin\theta = \left|2\left(1.60\times10^{-19}\ \text{C}\right)\right|\left(1.08\times10^{7}\ \text{m/s}\right)\left(2.20\ \text{T}\right)\sin 90.0^\circ = \boxed{7.60\times10^{-12}\ \text{N}}$$

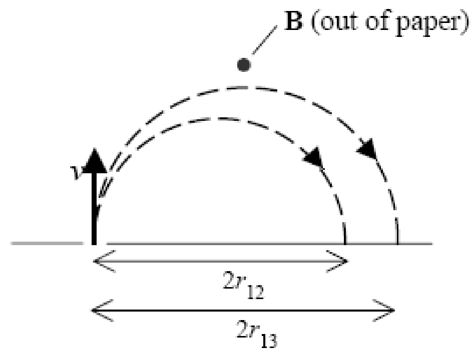
where $\theta = 90.0°$, since the particle travels perpendicular to the field at all times.

c. According to Equation 21.2, the radius of the circular path on which the particle travels is

$$r = \frac{mv_B}{\left|q_0\right| B} = \frac{\left(6.64\times10^{-27}\ \text{kg}\right)\left(1.08\times10^{7}\ \text{m/s}\right)}{\left|2\left(1.60\times10^{-19}\ \text{C}\right)\right|\left(2.20\ \text{T}\right)} = \boxed{0.102\ \text{m}}$$

---

17. ***REASONING*** The drawing shows the velocity **v** of the carbon atoms as they enter the magnetic field **B**. The diameter of the circular path followed by the carbon-12 atoms is labeled as $2r_{12}$, and that of the carbon-13 atoms as $2r_{13}$, where r denotes the radius of the path. The radius is given by Equation 21.2 as $r = mv/(|q|B)$, where q is the charge on the ion (q = $+e$). The difference Δd in the diameters is $\Delta d = 2r_{13} - 2r_{12}$ (see the drawing).



SOLUTION The spatial separation between the two isotopes after they have traveled though a half-circle is

$$\Delta d = 2r_{13} - 2r_{12} = 2\left(\frac{m_{13}v}{eB}\right) - 2\left(\frac{m_{12}v}{eB}\right) = \frac{2v}{eB}\left(m_{13} - m_{12}\right)$$

$$= \frac{2\left(6.667\times10^{5}\text{ m/s}\right)}{\left(1.60\times10^{-19}\text{ C}\right)\left(0.8500\text{ T}\right)}\left(21.59\times10^{-27}\text{ kg} - 19.93\times10^{-27}\text{ kg}\right) = \boxed{1.63\times10^{-2}\text{ m}}$$

---

21. ***REASONING*** When the proton moves in the magnetic field, its trajectory is a circular path. The proton will just miss the opposite plate if the distance between the plates is equal to the radius of the path. The radius is given by Equation 21.2 as $r = mv/(|q|B)$. This relation can be used to find the magnitude B of the magnetic field, since values for all the other variables are known.

SOLUTION Solving the relation $r = mv/(|q|B)$ for the magnitude of the magnetic field, and realizing that the radius is equal to the plate separation, we find that

$$B = \frac{mv}{|q|r} = \frac{\left(1.67\times10^{-27}\text{ kg}\right)\left(3.5\times10^{6}\text{ m/s}\right)}{\left(1.60\times10^{-19}\text{ C}\right)\left(0.23\text{ m}\right)} = \boxed{0.16\text{ T}}$$

The values for the mass and the magnitude of the charge (which is the same as that of the electron) have been taken from the inside of the front cover.

---

26. ***REASONING*** The magnitude F of the magnetic force experienced by the wire is given by $F = ILB\sin\theta$ (Equation 21.3), where I is the current, L is the length of the wire, B is the magnitude of the earth's magnetic field, and θ is the angle between the direction of the current and the magnetic field. Since all the variables are known except B, we can use this relation to find its value.

SOLUTION Solving $F = ILB\sin\theta$ for the magnitude of the magnetic field, we have

$$B = \frac{F}{IL\sin\theta} = \frac{0.15\text{ N}}{\left(75\text{ A}\right)\left(45\text{ m}\right)\sin 60.0^\circ} = \boxed{5.1\times10^{-5}\text{ T}}$$

---

27. SSM ***REASONING AND SOLUTION*** The force on a current-carrying wire is given by Equation 21.3: $F = ILB\sin\theta$. Solving for the angle θ, we find that the angle between the wire and the magnetic field is

$$\theta = \sin^{-1}\left(\frac{F}{ILB}\right) = \sin^{-1}\left[\frac{5.46 \text{ N}}{(21.0 \text{ A})(0.655 \text{ m})(0.470 \text{ T})}\right] = \boxed{57.6°}$$

---

32. ***REASONING*** A maximum magnetic force is exerted on the wire by the field components that are perpendicular to the wire, and no magnetic force is exerted by field components that are parallel to the wire. Thus, the wire experiences a force only from the x- and y-components of the field. The z-component of the field may be ignored, since it is parallel to the wire. We can use the Pythagorean theorem to find the net field in the x, y plane. This net field, then, is perpendicular to the wire and makes an angle of $\theta = 90°$ with respect to the wire. Equation 21.3 can be used to calculate the magnitude of the magnetic force that this net field applies to the wire.

SOLUTION According to Equation 21.3, the magnetic force has a magnitude of $F = ILB \sin\theta$, where I is the current, B is the magnitude of the magnetic field, L is the length of the wire, and θ is the angle of the wire with respect to the field. Using the Pythagorean theorem, we find that the net field in the x, y plane is

$$B = \sqrt{B_x^2 + B_y^2}$$

Using this field in Equation 21.3, we calculate the magnitude of the magnetic force to be

$$F = ILB \sin\theta = IL\sqrt{B_x^2 + B_y^2}\ \sin\theta$$

$$= (4.3 \text{ A})(0.25 \text{ m})\sqrt{(0.10 \text{ T})^2 + (0.15 \text{ T})^2}\ \sin 90° = \boxed{0.19 \text{ N}}$$

---

34. ***REASONING*** Since the rod does not rotate about the axis at P, the net torque relative to that axis must be zero; $\Sigma\tau = 0$ (Equation 9.2). There are two torques that must be considered, one due to the magnetic force and another due to the weight of the rod. We consider both of these to act at the rod's center of gravity, which is at the geometrical center of the rod (length = L), because the rod is uniform. According to Right-Hand Rule No. 1, the magnetic force acts perpendicular to the rod and is directed up and to the left in the drawing. Therefore, the magnetic torque is a counterclockwise (positive) torque. Equation 21.3 gives the magnitude F of the magnetic force as $F = ILB \sin 90.0°$, since the current is perpendicular to the magnetic field. The weight is mg and acts downward, producing a clockwise (negative) torque. The magnitude of each torque is the magnitude of the force times the lever arm (Equation 9.1). Thus, we have for the torques:

$$\tau_{\text{magnetic}} = +\underbrace{(ILB)}_{\text{force}}\underbrace{(L/2)}_{\text{lever arm}} \qquad \text{and} \qquad \tau_{\text{weight}} = -\underbrace{(mg)}_{\text{force}}\underbrace{\left[(L/2)\cos\theta\right]}_{\text{lever arm}}$$

Setting the sum of these torques equal to zero will enable us to find the angle θ that the rod makes with the ground.

SOLUTION Setting the sum of the torques equal to zero gives $\Sigma\tau = \tau_{\text{magnetic}} + \tau_{\text{weight}} = 0$, and we have

$$+(ILB)(L/2) - (mg)\left[(L/2)\cos\theta\right] = 0 \qquad \text{or} \qquad \cos\theta = \frac{ILB}{mg}$$

$$\theta = \cos^{-1}\left[\frac{(4.1\text{ A})(0.45\text{ m})(0.36\text{ T})}{(0.094\text{ kg})(9.80\text{ m/s}^2)}\right] = \boxed{44°}$$

---

47. ***REASONING AND SOLUTION*** The magnitude B of the magnetic field at a distance r from a long straight wire carrying a current I is $B = \mu_0 I/(2\pi r)$. Thus, the distance is

$$r = \frac{\mu_0 I}{2\pi B} = \frac{(4\pi \times 10^{-7}\text{ T}\cdot\text{m/A})(48\text{ A})}{2\pi(8.0 \times 10^{-5}\text{ T})} = \boxed{0.12\text{ m}} \qquad (21.5)$$

---

36. ***REASONING*** According to Equation 21.4, the torque τ that the circular coil experiences is $\tau = NIAB\sin\phi$, where N is the number of turns, I is the current, A is the area of the circle, B is the magnetic field strength, and ϕ is the angle between the normal to the coil and the magnetic field. To use this expression, we need the area of the circle, which is πr^2, where r is the radius. We do not know the radius, but we know the length L of the wire, which must equal the circumference of the single turn. Thus, $L = 2\pi r$, which can be solved for the radius.

SOLUTION Using Equation 21.4 and the fact that the area A of a circle is $A = \pi r^2$, we have that

$$\tau = NIAB\sin\phi = NI\left(\pi r^2\right)B\sin\phi \qquad (1)$$

Since the length of the wire is the circumference of the circle, or $L = 2\pi r$, it follows that the radius of the circle is $r = \frac{L}{2\pi}$. Substituting this result into Equation (1) gives

$$\tau = NI\left(\pi r^2\right)B\sin\phi = NI\left[\pi\left(\frac{L}{2\pi}\right)^2\right]B\sin\phi = \frac{NIL^2B}{4\pi}\sin\phi$$

The maximum torque τ_{max} occurs when $\phi = 90.0°$, so that

$$\tau_{max} = \frac{NIL^2B}{4\pi}\sin 90.0° = \frac{(1)(4.30\text{ A})\left(7.00\times10^{-2}\text{ m}\right)^2(2.50\text{ T})}{4\pi}\sin 90.0° = \boxed{4.19\times10^{-3}\text{ N}\cdot\text{m}}$$

---

37. SSM ***REASONING*** The magnitude τ of the torque that acts on a current-carrying coil placed in a magnetic field is specified by $\tau = NIAB\sin\phi$ (Equation 21.4), where N is the number of loops in the coil, I is the current, A is the area of one loop, B is the magnitude of the magnetic field, and ϕ is the angle between the normal to the coil and the magnetic field. All the variables in this relation are known except for the current, which can, therefore, be obtained.

SOLUTION Solving the equation $\tau = NIAB\sin\phi$ for the current I and noting that $\phi = 90.0°$ since τ is specified to be the maximum torque, we have

$$I = \frac{\tau}{NAB\sin\phi} = \frac{5.8\text{ N}\cdot\text{m}}{(1200)\left(1.1\times10^{-2}\text{ m}^2\right)(0.20\text{ T})\sin 90.0°} = \boxed{2.2\text{ A}}$$

---

48. ***REASONING*** The magnitude B of the magnetic field in the interior of a solenoid that has a length much greater than its diameter is given by $B = \mu_0 n I$ (Equation 21.7), where $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$ is the permeability of free space, n is the number of turns per meter of the solenoid's length, and I is the current in the wire of the solenoid. Since B and I are given, we can solve Equation 21.7 for n.

SOLUTION Solving Equation 21.7 for n, we find that the number of turns per meter of length is

$$n = \frac{B}{\mu_0 I} = \frac{7.0\ \mathrm{T}}{\left(4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}\right)\left(2.0 \times 10^{2}\ \mathrm{A}\right)} = \boxed{2.8 \times 10^{4}\ \text{turns/m}}$$

---

54. ***REASONING AND SOLUTION*** The net force on the wire loop is a sum of the forces on each segment of the loop. The forces on the two segments perpendicular to the long straight wire cancel each other out. The net force on the loop is therefore the sum of the forces on the parallel segments (near and far). These are

$$F_{\text{near}} = \mu_o I_1 I_2 L/(2\pi d_{\text{near}}) = \mu_0(12\ \mathrm{A})(25\ \mathrm{A})(0.50\ \mathrm{m})/[2\pi\ (0.11\ \mathrm{m})] = 2.7 \times 10^{-4}\ \mathrm{N}$$

$$F_{\text{far}} = \mu_o I_1 I_2 L/(2\pi d_{\text{far}}) = \mu_0(12\ \mathrm{A})(25\ \mathrm{A})(0.50\ \mathrm{m})/[2\pi\ (0.26\ \mathrm{m})] = 1.2 \times 10^{-4}\ \mathrm{N}$$

Note: F_{near} is a force of attraction, while F_{far} is a repulsive one. The magnitude of the net force is, therefore,

$$F = F_{\text{near}} - F_{\text{far}} = 2.7 \times 10^{-4}\ \mathrm{N} - 1.2 \times 10^{-4}\ \mathrm{N} = \boxed{1.5 \times 10^{-4}\ \mathrm{N}}$$

---

60. ***REASONING*** Since the two wires are next to each other, the net magnetic field is everywhere parallel to $\Delta\ell$ in Figure 21.40. Moreover, the net magnetic field **B** has the same magnitude B at each point along the circular path, because each point is at the same distance from the wires. Thus, in Ampère's law (Equation 21.8), $B_{\|} = B$, $I = I_1 + I_2$, and we have

$$\Sigma B_{\|}\Delta\ell = B\left(\Sigma\Delta\ell\right) = \mu_0\left(I_1 + I_2\right)$$

But $\Sigma\Delta\ell$ is just the circumference ($2\pi r$) of the circle, so Ampère's law becomes

$$B\left(2\pi\,r\right) = \mu_0\left(I_1 + I_2\right)$$

This expression can be solved for B.

SOLUTION

a. When the currents are in the same direction, we find that

$$B = \frac{\mu_0\left(I_1 + I_2\right)}{2\pi\,r} = \frac{\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)\left(28\ \mathrm{A} + 12\ \mathrm{A}\right)}{2\pi\left(0.72\ \mathrm{m}\right)} = \boxed{1.1\times10^{-5}\ \mathrm{T}}$$

b. When the currents have opposite directions, a similar calculation shows that

$$B = \frac{\mu_0\left(I_1 - I_2\right)}{2\pi\,r} = \frac{\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)\left(28\ \mathrm{A} - 12\ \mathrm{A}\right)}{2\pi\left(0.72\ \mathrm{m}\right)} = \boxed{4.4\times10^{-6}\ \mathrm{T}}$$
