Chapter 9 Problems 6, 8, 14, 16, 17, 21, 24, 25

(5 points)

(9.6) As stated in Ex 9-2, we create antisymmetric wavefunctions via the determinant: \( \Psi = \frac{1}{\sqrt{A}} \begin{vmatrix} \Psi_x(1) & \Psi_x(2) & \Psi_x(3) \\ \Psi_y(1) & \Psi_y(2) & \Psi_y(3) \\ \Psi_z(1) & \Psi_z(2) & \Psi_z(3) \end{vmatrix} \)

This determinant will yield 3 x 2 terms.

When computing \( \langle \Psi_A | \Psi_A \rangle = 1 = \frac{1}{A} \left[ \langle \Psi_x(1) | \Psi_x(1) \rangle \times \langle \Psi_y(2) | \Psi_y(2) \rangle \times \langle \Psi_z(3) | \Psi_z(3) \rangle + \text{other terms} \right] \)

All cross terms such as \( \langle \Psi_B(1) | \Psi_x(2) \rangle = 0 \) due to orthogonality.

\( \frac{1}{A} \left[ 6 \right] = 1 \Rightarrow A = \frac{1}{6} \Rightarrow \text{Normalized} \)

Constant = \( \frac{1}{\sqrt{6}} = \frac{1}{\sqrt{3!}} \)

\( \langle \Psi_x(1) | \Psi_y(1) \rangle = 1 \) etc.

\( \sum_{i,j,k=1,2,3} \langle \Psi_x(i) | \Psi_x(j) \rangle \langle \Psi_y(k) | \Psi_y(l) \rangle = 3 \times 2 = 6 \text{ possibilities} \)

\( = 1+1+1+1+1+1 = 6 \text{ over} \)

Here, we assumed that the wavefunctions are orthogonal normal.
(5 points)

\[ \alpha : (\mathbf{PP}) \Rightarrow P \text{ are antisymmetric w.r.t. exchange} \]

\[ \Rightarrow \psi = \frac{1}{\sqrt{2}} \left[ \psi_{p_1}(1) \psi_{p_2}(2) - \psi_{p_1}(2) \psi_{p_2}(1) \right] \times \frac{1}{\sqrt{2}} \times \]

\[ \times \left[ \psi_{\alpha_1}(3) \psi_{\alpha_2}(4) - \psi_{\alpha_1}(4) \psi_{\alpha_2}(3) \right] \]

Now, if the \( \alpha \)-particle is a collective Fermion, then exchanging one \( \alpha \) with another yields \(-1\). If it is a Boson, then:

\[ \psi_{\alpha_1}(1) \psi_{\alpha_2}(2) = + \psi_{\alpha_1}(2) \psi_{\alpha_2}(1) \]

Fermion:

\[ \psi_{\alpha_1}(1) \psi_{\alpha_2}(2) = - \psi_{\alpha_1}(2) \psi_{\alpha_2}(1) \]

Exchanging \( \alpha_1 \) with \( \alpha_2 \) implies that each \( P \) and each \( n \) MUST BE EXCHANGED! \(-1 \times (-1) \times (-1) \times (-1) \) for exchange of the 4 constituent particles (Fermions)

\[ \Rightarrow \psi_{\alpha_1}(1) \psi_{\alpha_2}(2) = (-1)^4 \psi_{\alpha_1}(2) \psi_{\alpha_2}(1) = (+1) \psi_{\alpha_1}(2) \psi_{\alpha_2}(1) \]

\( \Rightarrow \) The \( \alpha \)-Particle behaves as a BOSON!

QED
Phys 411  HW # 9  SOLNS (EX) [3]

9.14 (5 points)

\( E = - \frac{\mu Z_i^2 e^2}{2 \hbar^2 n^2} \) (eqn 9-27)

(a) 

\( Z_i \Rightarrow n=1 \Rightarrow E = -\frac{Z_i^2}{2} (13.6 \text{eV}) \)

Examining Fig 9-6, we see that \( E_1 = -79 \text{eV} \) (on the right) experimentally (most important).

\( \Rightarrow \) Solve for \( Z_n = \sqrt{\frac{79}{13.6}} = Z_n = 2.4 \)

(b) \( Z_i > Z (= 2) \).

(c) As \( Z_i \) is considerably greater than 2, we argue that \( Z_i \) is not very meaningful for an atom with only 2 electrons as one \( e^- \) poorly shields the second \( e^- \) from a statistical perspective.

\( e^- e^- \) later:

\( e^- e^- \) partial shielding.
9.16 (5 points) \[ n=1 \quad \frac{3}{2} \frac{3}{3} \] \[ z_n = \frac{1}{16} \]

From p. 326, \( z \approx \frac{n^2 a_0}{z_n} \)

\[ \Rightarrow \quad \bar{r}_1 = \langle \bar{r}_1 \rangle \approx \frac{1^2 a_0}{16} = \frac{a_0}{16} \]

\[ \bar{r}_2 = \langle \bar{r}_2 \rangle \approx \frac{2^2 a_0}{8} = \frac{a_0}{2} = 0.5a_0 \]

\[ \bar{r}_3 = \langle \bar{r}_3 \rangle \approx \frac{3^2 a_0}{3} = 3a_0 = \langle \bar{r}_3 \rangle \]

b. From Fig 9.10, \( \langle r_1 \rangle \approx 0.06a_0 \Rightarrow \) Good agreement!

\( \langle r_2 \rangle \approx 0.45a_0 \ (l=1) \Rightarrow \) Good agreement!

\( \langle r_3 \rangle \approx 1.2a_0 \ (l=0) \Rightarrow \) Not so good

Note that Ar has 3 shells \( \Rightarrow \) Hartree Fock works well for inner shells but not outer shell.
(5 points)

Z = 110  My guess:

1s  2s  2p  3s  3p  3d  4s  4p  4d  4f
2  2  6  2  6  10  2  6  10  14  \[ = 60 \text{ e}\] 
5s  5p  5d  6s  5f  6p  7s  6d
2  6  10  2  14  6  2  8 \[ = 50 \text{ e}\] 

110 \text{ e}^+ \text{ TOTAL}

5. p

\text{transition level METAL —}

\text{Similar perhaps to Ni, Pd, Pt (low P/ resistivity)}

\text{high thermal conductivity}

\text{Radioactive — maybe! Not necessarily but probably}

\text{If it has a long half-life (tr2), the perhaps in}

\text{meteorites or alloyed with Pd, Pt in Traces —}

\text{Amounts! Synthesized in supernovae or in an accelerator}

(5 points)

\[ \text{74 W} \]

\[ ^{74} \text{W} \text{ } K_\alpha \Rightarrow \lambda = 0.21 \AA \Rightarrow 9.5 \times 10^{-15} = 59.2 \text{ keV} \]

\[ K_\beta \Rightarrow \lambda = 0.184 \AA \Rightarrow 1.1 \times 10^{-14} = 67.5 \text{ keV} \]

\[ K_\gamma \Rightarrow \lambda = 0.179 \AA \Rightarrow 1.11 \times 10^{-14} = 69.4 \text{ keV} \]

\text{K absorption edge = 0.178} \AA \Rightarrow 69.8 \text{ keV} \rightarrow
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9.24 cont'd. So we have the following diagram:

- n=4 \Rightarrow -400\text{eV}
- n=3 \Rightarrow -2.3\text{keV}
- n=2 \Rightarrow -10.6\text{keV} \Rightarrow E_2
- n=1 \Rightarrow -69.8\text{keV} \Rightarrow E_1

\[ E_\alpha = E_2 - E_1 = 59.2\text{keV} \Rightarrow E_2 = -10.6\text{keV} \]
\[ E_\beta = E_3 - E_1 = 67.5\text{keV} \Rightarrow E_3 = -2.3\text{keV} \]
\[ E_\gamma = E_4 - E_1 = 69.4\text{keV} \Rightarrow E_4 = -0.4\text{keV} \]
(5 Points) \( 26F{e} \rightarrow Z = 26 \)

(9) \( L_x \rightarrow E_3 - E_2 = \Delta E = k\alpha \)

Following the hint (ex 9-5): \( Z_2 = Z - 10 = 26 - 10 = 16 \)

\( Z_3 = 18 - 15 = 3 \) for \(^{18}Ar \rightarrow Z = 15\) concept

\( \Rightarrow Z_3 = 26 - 15 = Z = 15 \) for \(^{26}Fe \) (\( Z = 26 \))

As in ex 9-8 \( V_{\text{min}} \text{ must eject } n = 2 \) electron

\( E_2 - E_\infty = E_2 = \frac{-13.6 Z_2^2}{2^2} \) (eV) =

\( = -13.6 \left( \frac{16}{2^2} \right)^2 = -13.6 \left( \frac{26 - 10}{4} \right)^2 = -870 \text{ eV} \)

\( \Rightarrow 870 \text{ V required} \)

Finally \( E_3 = \frac{-13.6(Z_3)^2}{2^n} \) eV; \( Z_3 = 26 - 15 = 11 \)

\( \Rightarrow E_3 = -13.6(11)^2 = -183 \text{ eV} \)

\( \Rightarrow E_2 - E_3 = (870 - 183) \text{ eV} = 687 \text{ eV} \)

\( \Rightarrow 1.81 \text{ nm} \)