GENERAL PHYSICS I (PHYS 151): A SUMMARY

I. Kinematics:

(a) Translational motion:

The (instantaneous) velocity of a point is defined as

$$\mathbf{v} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t},$$

where the displacement $\Delta \mathbf{r}$ is the difference between the final position vector $\mathbf{r}$ and the initial position vector $\mathbf{r}_0$ of the point:

$$\Delta \mathbf{r} = \mathbf{r} - \mathbf{r}_0,$$

and the elapsed time Δt is the difference between the final time t and the initial time t_0:

$$\Delta t = t - t_0.$$

Without taking the limit, one has the average velocity of the point during the elapsed time:

$$\bar{\mathbf{v}} = \frac{\Delta \mathbf{r}}{\Delta t}.$$

Similarly, we can define the (instantaneous) acceleration and average acceleration as

$$\mathbf{a} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t},$$

and

$$\bar{\mathbf{a}} = \frac{\Delta \mathbf{v}}{\Delta t}.$$

For constant $\mathbf{a}$, we have

$$v_x = v_{x0} + a_x \Delta t,$$

$$\Delta x = \bar{v}_x \Delta t = \frac{v_x + v_{x0}}{2} \Delta t = v_{x0} \Delta t + \frac{a_x}{2} (\Delta t)^2,$$

and

$$v_x^2 - v_{x0}^2 = 2 a_x \Delta x.$$

We can obtain the same set of equations for the y-direction motion if we replace x with y in the above equations. Also note that if we choose $t_0 = 0$ and $\mathbf{r}_0 = \mathbf{0}$, we will have x and t instead of Δx and Δt in the above expressions,

Furthermore, (instantaneous) speed is the magnitude of the velocity and average speed is ratio of the total distance covered and the elapsed time.

(b) Rotational motion:

The instantaneous rotational axis of an extended object is the intersect of two planes, with each drawn through a point on the object, perpendicular to the velocity of that point. The line can be either on the object or away from the object but always has a zero instantaneous velocity.

The angular displacement is defined as

$$\Delta\theta = \theta - \theta_0,$$

where θ_0 is the angular position at the time t_0 and θ at the time t. The magnitude of the angular displacement can be expressed in terms of the corresponding arc length Δs with

$$\Delta\theta = \frac{\Delta s}{r},$$

where r is the distance (radius) between the (instantaneous) rotational axis to the point under study and $\Delta\theta$ is measured in radians (rad) with $2\pi\,(\text{rad}) = 360°$. Note that $\Delta\theta \to 0$ is a vector whose direction can be determined either by the right-hand rule or from the direction of rotation [positive (+) if rotating counterclockwise and negative (−) if rotating clockwise].

The corresponding (instantaneous) angular velocity and acceleration are given by

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t},$$

whose direction is the same as $\Delta\theta \to 0$,

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t},$$

whose direction is the same as $\Delta\omega \to 0$.

For α being a constant vector, we have

$$\omega = \omega_0 + \alpha\Delta t,$$

$$\Delta\theta = \bar{\omega}\Delta t = \frac{\omega + \omega_0}{2}\Delta t = \omega_0\Delta t + \frac{\alpha}{2}(\Delta t)^2,$$

and

$$\omega^2 - \omega_0^2 = 2\alpha\Delta\theta.$$

Angular speed is the magnitude of the angular velocity.

(c) Relations between translational motion and rotational motion:

The period for rotational or circular motion is the time needed to complete one cycle with $\Delta\theta = 2\pi$ (rad). For constant angular velocity cases, we have the period

$$T = \frac{2\pi}{\omega},$$

where ω is the angular speed.

The speed of a point on a rigid body or in circular motion is

$$v = r\omega,$$

under any circumstance, with r being the distance between the instantaneous rotational axis and the point under study and ω being the angular speed of rotation or circular motion. A special case of this relation is the velocity of the center (of mass) in a pure (no sliding) rolling motion:

$$v_{\text{cm}} = r\omega,$$

where r is the distance between the instantaneous rotational axis (the contact point) and the point under study (the center).

For fixed-axis rotation or circular motion, the tangential and centripetal accelerations of the point are given by

$$a_{\text{t}} = r\alpha,$$

and

$$a_{\text{c}} = \frac{v^2}{r}.$$

(d) Relative motion:

The position, velocity, and acceleration relations between point 1 and point 2 are given by

$$\mathbf{r}_2 = \mathbf{r}_1 + \mathbf{r}_{21},$$

$$\mathbf{v}_2 = \mathbf{v}_1 + \mathbf{v}_{21},$$

and

$$\mathbf{a}_2 = \mathbf{a}_1 + \mathbf{a}_{21},$$

where the index 21 is for the relative measure from point 1 to point 2, or in other words, for the behavior of point 2 in reference to point 1. For example, $\mathbf{a}_{21}$ is the acceleration of point 2 in reference to point 1. A special case is a point on a pure rolling object with

$$\mathbf{r} = \mathbf{r}_{\text{cm}} + \mathbf{r}',$$

$$\mathbf{v} = \mathbf{v}_{\rm cm} + \mathbf{v}',$$

and

$$\mathbf{a} = \mathbf{a}_{\rm cm} + \mathbf{a}',$$

where the quantities with or without a prime are measures relative to the center (of mass) and the fixed ground and $\mathbf{r}_{\rm cm}$, $\mathbf{v}_{\rm cm}$, and $\mathbf{a}_{\rm cm}$ are the position, velocity, and acceleration of the center in reference to the fixed ground.

II. Dynamics:

(a) Forces and torques:

The response of point particle or the center of mass of an extended object to an external force $\mathbf{F}$ is given by Newton's second law

$$\mathbf{F} = m\mathbf{a}.$$

There are several common forces. Friction can be either kinetic (with relative motion) with

$$F_{\rm k} = \mu_{\rm k} F_{\rm N},$$

or static (with the tendency of relative motion) with its value and direction given by overcoming the tendency and its maximum value given by

$$F_{\rm s}^{\rm max} = \mu_{\rm s} F_{\rm N},$$

where $F_{\rm N}$ is the normal (component of the contact) force and $\mu_{\rm k}$ and $\mu_{\rm s}$ are the coefficients of kinetic and static frictions.

Tension can be built on an object (a rope or a surface, for example). So is a stress. Several common stresses have been discussed. For example, in the case of compressing or stretching a bar, we have

$$\frac{F}{A} = -Y\frac{X}{L},$$

where F is the restoring force, A is the cross section (the area on which the external force is applied perpendicularly), Y is Young's modulus of the material, X is the displacement of the end where the external force is applied, and L is the unstrained length of the bar. In the case of shear stress, we have

$$\frac{F}{A} = -S\frac{X}{L},$$

where F is the restoring force, A is the top section (the area to which the external force is applied parallel), S is shear modulus of the material, X is the displacement of the top section, and L is the unstrained height of the block. For a bulk material, we have

$$\Delta P = -B\frac{\Delta V}{V}$$

where P is the pressure, B is bulk modulus, and V is the volume of the system. Note that all the stresses have the same units $\mathrm{N/m^2}$, which are called pascals (Pa) for the case of pressure ($P = \Delta F/\Delta A$, is the force on a unit area of the surface of the bulk material under study).

Similarly, we have Hooke's law, the general expression for an elastic (restoring) force for any elastic medium (a spring or a metal bar, for example):

$$F = -kx$$

where k is called spring (elastic) constant.

Gravity is the force between any two objects that possess masses,

$$\mathbf{F}_{21} = -G\frac{m_1 m_2 \hat{\mathbf{r}}_{21}}{r_{21}^2}.$$

The consequence of gravity between a small object and Earth is the weight of the object:

$$W = mg,$$

which is always pointing to the ground.

The buoyant force (Archimedes' principle) on an object in a fluid is

$$F_{\mathrm{B}} = W_{\mathrm{D}},$$

which is always pointing upward with W_{D} being the weight of the displaced fluid.

If two objects are interacting with each other, we have

$$\mathbf{F}_{21} = -\mathbf{F}_{12},$$

a result of Newton's third law.

The effect of a force in case of rotation is through its corresponding torque:

$$\tau = lF$$

where l is the level arm of the force, the distance between the actual or imagined rotational axis to the extended line of the force on the object. The direction of the torque can be determined either by the right-hand rule or by its effect on rotation [positive (+) if rotating counterclockwise and negative (−) if rotating clockwise].

When the system is mechanical equilibrium (at rest or moving with a constant velocity (of the center of mass and a constant angular velocity about the center of mass), the total external force

$$\sum \mathbf{F} \equiv \mathbf{0},$$

a result of Newton's first law, and the total external torque

$$\sum \boldsymbol{\tau} \equiv \mathbf{0},$$

a result of no angular acceleration for an extended object. For fixed-axis rotation, similarity can be drawn between the rotational variables and translational variables, for example, τ and F, θ and x, ω and v, α and a, I and m, and p and L. Namely, we can write the rotational dynamical equation as

$$\tau = I\alpha,$$

where the moment of inertia is

$$I = \sum m_i r_i^2$$

with r_i being the distance between the mass element m_i and the rotational axis.

(b) Linear momentum:

The impulse–momentum theorem,

$$\Delta\mathbf{P} = \mathbf{I},$$

is the result of Newton's second law $\mathbf{F} = m\mathbf{a}$ or

$$\mathbf{F} = \lim_{\Delta t \to 0} \frac{\Delta\mathbf{P}}{\Delta t},$$

where $\mathbf{P} = \sum m_i \mathbf{v}_i$ is the total momentum of the system and $\mathbf{I} = \Delta t \sum \mathbf{F}_i^{\rm ext}$ is the total external impulse. Note that $\mathbf{F} = \sum \mathbf{F}_i^{\rm ext}$ is the total external force. One can find (the position or coordinate of) the center of mass of the system from

$$\mathbf{r}_{\rm cm} = \frac{1}{m} \sum m_i \mathbf{r}_i,$$

with the total mass of the system $m = \sum m_i$. From the position of the center of mass, one can find its velocity and acceleration

$$\mathbf{v}_{\rm cm} = \lim_{\Delta t \to 0} \frac{\Delta\mathbf{r}_{\rm cm}}{\Delta t},$$

and

$$\mathbf{a}_{\rm cm} = \lim_{\Delta t \to 0} \frac{\Delta\mathbf{v}_{\rm cm}}{\Delta t}.$$

Several facts result from the definition of the center of mass, for example,

$$\mathbf{P} = m\mathbf{v}_{\text{cm}},$$

and

$$\mathbf{F} = m\mathbf{a}_{\text{cm}}.$$

Then if the external impulse (or force) is zero, the momentum of the system is conserved, which is case for any collision process:

$$\mathbf{P}_{\text{f}} = \mathbf{P}_{\text{i}}.$$

If the collision is elastic (an ideal case), the kinetic energy of the system is also conserved.

(c) Angular momentum:

Similarly we have

$$\boldsymbol{\tau} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{L}}{\Delta t}.$$

where τ is the total external torque and $\mathbf{L}$ is the angular momentum of the system. For fixed-axis rotation, we have

$$L = I\omega,$$

with the moment of inertia given by

$$I = \sum m_i r_i^2,$$

where r_i is the distance between the mass element m_i and the rotational axis. If $\boldsymbol{\tau} = \mathbf{0}$, the angular momentum of the system is conserved with

$$\mathbf{L}_{\text{f}} = \mathbf{L}_{\text{i}}.$$

(d) Work–energy theorem:

The work done by a force is given by

$$W = FS\cos\theta$$

where F is the force, S is the displacement, and θ is the angle between them. For a fixed-axis rotational motion, we have

$$W = \tau\Delta\theta.$$

The work done by all the forces and/or torques is equal to the change of the kinetic energy of the system:

$$W = \Delta E_{\mathrm{k}},$$

where the kinetic energy is given by

$$E_{\mathrm{k}} = \frac{m}{2}v^2 + \frac{I}{2}\omega^2 = \frac{I_x}{2}\omega^2$$

where v is the velocity of the particle (or center of mass), I is the momentum of inertia with rotation around an axis through the center of mass, and $I_x = I + md^2$ is the moment of inertia around the instantaneous rotational axis that is d away from the center of mass.

If we separate the work done by conservative forces (gravity and elastic force for example) from that done the nonconservative forces (friction and air resistance for example), we have

$$W = W_{\mathrm{c}} + W_{\mathrm{nc}}$$

with

$$W_{\mathrm{nc}} = \Delta E.$$

Here the total energy E is given by

$$E = E_{\mathrm{k}} + E_{\mathrm{p}}.$$

The potential energy E_{p} can contain one or several types of potential energies. The gravitational potential energy is given by

$$E_{\mathrm{p}} = mgy$$

and the elastic potential energy is given by

$$E_{\mathrm{p}} = \frac{k}{2}x^2.$$

The average total (thermal) energy at different temperature is called the internal energy (U) of the system. For an ideal gas

$$U = \frac{3}{2}Nk_{\mathrm{B}}T,$$

where N is the number of particles in the system and k_{B} is the Boltzmann constant.

III. Harmonic motion:

All harmonic motions can be viewed as a mass–spring system with

$$ma = -kx$$

where x is the displacement of the mass from its equilibrium position with positive direction along the stretching of the spring. Then we have

$$x = A\cos(\omega t + \phi),$$

and

$$v = -\omega A \sin(\omega t + \phi),$$

where the amplitude A and the initial phase ϕ are given by the initial conditions (initial displacement and initial velocity), and the angular frequency is given by

$$\omega = \sqrt{\frac{k}{m}}.$$

The frequency of the oscillation of vibration is

$$f = \frac{\omega}{2\pi} = \frac{1}{T},$$

with T being the period of the oscillation. Harmonic motions can also be viewed as the projection (the x-coordinate) of a uniform circular motion with A being the radius of the circle, ω being the angular velocity, and ϕ being the initial angle. Other oscillatory motions that have the same acceleration–displacement relation,

$$a = -\omega^2 x,$$

have the same solution as the mass–spring system. For example, in a simple pendulum with small angular displacement, we have

$$\alpha \simeq -\frac{g}{l}\theta.$$

Then the solution must be

$$\theta = A\cos(\omega_0 t + \phi),$$

and

$$\omega = -\omega_0 A \sin(\omega_0 t + \phi),$$

with

$$\omega_0 = \sqrt{\frac{g}{l}}.$$

Note that g/l in the pendulum plays the role of k/m of the mass–spring system and ω_0 is used for the angular frequency to avoid confusion with the angular velocity of the mass in the pendulum.

IV. Fluids:

The density of a bulk (fluid of solid) material is defined as

$$\rho = \frac{m}{V}$$

with m being the mass and V being the volume of the system.

For an ideal fluid (incompressible, irrotational, and nonviscous) in steady motion or stationary, we have Bernoulli's equation:

$$P + \frac{\rho}{2}v^2 + \rho g y = \text{constant},$$

with $v = 0$ being the static case.

The mass conservation in a steady flow is given by the equation of continuity,

$$Av\rho = \text{constant}.$$

An important principle, Pascal's principle, states that the pressure inside a fluid is transferable from one point to another point and is used extensively in design of hydraulic systems such as a hydraulic car lift.

The buoyant force (Archimedes' principle) on an object in a fluid is

$$F_{\text{B}} = W_{\text{D}},$$

which is always pointing upward with W_D being the weight of the displaced fluid.

The viscosity of a fluid is determined from experiment with

$$\frac{F}{A} = \eta \frac{v}{y}$$

where η is the coefficient of viscosity of the fluid given in Pa s or poises (p). The governing law of a steady flow in a circular pine is Poiseuille's law with

$$v_0 = \frac{(P_2 - P_1)R^2}{4\eta L},$$

or

$$\frac{\Delta m}{\Delta t} = \frac{\pi\rho(P_2 - P_1)R^4}{8\eta L}.$$

V. Thermal physics:

The relations among different temperature scales (Fahrenheit, Celsius, and Kelvin) are given by

$$T(^\circ\mathrm{F}) = \frac{9}{5}T(^\circ\mathrm{C}) + 32,$$

and

$$T(\mathrm{K}) = T(^\circ\mathrm{C}) + 273.15.$$

The freezing and boiling temperatures of water are 0°C and 100°C, respectively.

The heat transferred into the system due to the temperature change is

$$\Delta Q = cm\Delta T$$

where c is the specific heat capacity of the material.

The heat transferred into the system due to fusion is

$$\Delta Q_\mathrm{f} = L_\mathrm{f} m$$

where L_f is the latent heat of fusion of the material.

The heat transferred into the system due to vaporization is

$$\Delta Q_\mathrm{v} = L_\mathrm{v} m$$

where L_v is the latent heat of vaporization of the material.

Materials usually expand with the temperature increase. The linear and bulk thermal expansions are given by

$$\Delta L/L = \alpha\Delta T,$$

and

$$\Delta V/V = \beta\Delta T,$$

where the linear expansion coefficient α and bulk thermal expansion coefficient β are related through $\beta = 3\alpha$ for the same material.

The relative humidity of the air is given by

$$H = \frac{P_x}{P}$$

where P_x is the partial pressure of the water vapor in the air and P is the saturated vapor pressure of water at the same temperature.

Thermal conduction is governed by

$$\Delta Q = \kappa\frac{A}{L}\Delta T\,\Delta t,$$

where ΔQ is the heat transferred, κ is the thermal conductivity of the material, A and L are cross section and length of the bar, respectively, ΔT is the temperature difference, and Δt is the elapsed time.

An object will radiate or absorb light depending on the temperatures of the object and its environment. The net emission power is given by

$$P_{\text{net}} = e\sigma A(T^4 - T_0^4) = \Delta Q/\Delta t,$$

where T is the temperature of the object, T_0 is the temperature of the environment, σ is the Stefan–Boltzmann constant, and e is the emissivity of the specific surface of the object.

The equation of state for an ideal gas is given by

$$PV = nRT = Nk_{\text{B}}T$$

where R is the gas constant and k_{B} is the Boltzmann constant. The internal energy of such an ideal gas is

$$U = \frac{3}{2}Nk_{\text{B}}T.$$

Materials can be transported through diffusion similar to the heat. The diffusion equation is given by

$$\Delta m = D\frac{A}{L}\Delta c\,\Delta t.$$

where Δc is the concentration difference and D is the diffusion coefficient.