AST713: Final Review

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• This review includes all the lectures after the Mid-Term Exam. The students are requested to study the review before the Mid-Term. The final will be cumulative.
• There will be at least 10 multiple choice questions (1 pt each) in the final. Another 10 points will be either multiple-choice questions or problem solving. We'll discuss this again.
• Twenty points altogether for the final.
• Physical constants will be provided.
• Simple calculations are needed. Please bring a calculator with you.

You are supposed to know the following.

- The physical meaning of Lienard-Wichart potential, and the connection between particle acceleration and radiation.
- Poynting flux: $\mathbf{S} = (c/4\pi)(\mathbf{E} \times \mathbf{B})$.
- Magnetic energy density $B^2/8\pi$; electric energy density $E^2/8\pi$.
- Larmor formula - emission power of an non-relativistic charge: $P = 2q^2\dot{u}^2/3c^3 = 2\ddot{d}^2/3c^3$ (the second half is for dipole approximation). Angular distribution: $dP/d\Omega = (q^2\dot{u}^2/4\pi c^3)\sin^2\Theta = (\ddot{d}^2/4\pi c^3)\sin^2\Theta$.
- Two frames: lab frame and particle comoving frame; Three times: comoving time $\Delta t'$, lab-frame emission time Δt_e; lab-frame observing time Δt_{obs}.
- The relationship among the three times: $\Delta t_e = \gamma \Delta t'$, $\Delta t_{obs} = (1-\beta\cos\theta)\Delta t_e$, and $\Delta t_{obs} = \gamma(1-\beta\cos\theta)\Delta t' = \Delta t'/\mathcal{D}$.
- The Doppler factor $\mathcal{D} = [\gamma(1-\beta\cos\theta)]^{-1} = \gamma(1+\beta\cos\theta')$.
- Doppler boosts: $\omega_{obs} = \mathcal{D}\omega'$, $d\Omega = d\Omega'/\mathcal{D}^2$, $dP_{obs}/d\Omega = \mathcal{D}^4(dP'/d\Omega')$, $I_\nu = \mathcal{D}^3 I'_\nu$.
- Characteristic frequencies: cyclotron: $\omega_L = qB/m_0c$; synchrotron: $\omega_c = (3/2)\gamma^2\omega_L = (3/2)\gamma^2(qB/m_0c)$; curvature: $\omega_c = (3/2)\gamma^3(c/\rho)$, inverse Compton scattering $\omega_1 \simeq \min(\gamma^2\omega, \gamma mc^2)$.
- Emission power: cyclotron: $P = (4/3)\sigma_T c\beta^2 U_B$; synchrotron: $P = (4/3)\gamma^2\sigma_T c\beta^2 U_B$; inverse Compton scattering: $P = (4/3)\gamma^2\sigma_T c\beta^2 U_{ph}$; curvature: $P = (2/3)\gamma^4(q^2c/\rho^2)$.
- The spectral index of synchrotron emission below ω_c: 1/3 (as compared with the blackbody spectrum in the Rayleigh-Jeans regime, which is 2).
- If the electrons have a power law distribution with index p, i.e. $N(\gamma_e)d\gamma_e \propto \gamma_e^{-p}d\gamma_e$, the emission spectrum is also a power law (regardless of the radiation mechanism, say, synchrotron or inverse Compton scattering). The photon spectral index is $s = (p-1)/2$, i.e. $F_\nu \propto P_{tot}(\omega) \propto \nu^{-(p-1)/2}$.
- The cooling break in the electron spectrum is defined by $\tau = \gamma_c mc^2/(P(\gamma_c)$. The cooling emission frequency is then $\omega_c = (3/2)\gamma_c^2(qB/m_0c)$.

- Klein-Nishina regime: inverse Compton scattering cross section decreases when the scattering enters the K-N regime, i.e. within the rest frame of the electron the incident photon carries an energy comparable or higher than the electron rest mass. The scattered photon energy in this case is essentially the intial energy of the electron γmc^2.
- Bremstrahlung is free-free emission of electrons in the Coulomb field of ions.
- Difference between thermal emission and blackbody emission. The former requires electrons being in thermal equilibrium; while the latter requires both electrons and photons being in thermal equilibium. In other words, photons should be optically thick.