

These three problems are due on Dec. 2, 2008.

Problem 4. In an astronomical source, electrons are radiating in a random magnetic field via synchrotron radiation. The electron distribution is non-thermal power law: $N(\gamma)d\gamma \propto \gamma^{-p}d\gamma$, with the spectral index $p \sim 2.2$ and the minimum electron Lorentz factor $\gamma_m \sim 100$. The average strength of the magnetic field is about $B \sim 1$ mG (10^{-3} Gauss, Gauss is the unit of magnetic fields in cgs units). What is the synchrotron frequency of the electrons with minimum energy? Neglecting the self-absorption effect, what is the observed emission spectrum below and above this frequency? The observed power law spectrum cuts off above the frequency $\sim 10^{12}$ Hz. Assuming that this break is caused by synchrotron cooling, estimate the age of the source.

Problem 5. This is the famous “compactness” problem of gamma-ray bursts (GRBs). Astronomers usually use the time variability scale δt to estimate the size of the emission region $l = c\delta t$. GRBs are known to happen at cosmological distances (say, $d \sim 10^{28}$ cm). They have a typical time variability scale of $\delta t = 10$ ms. The gamma-ray fluence detected on earth is $\sim 10^{-6}$ erg cm^{-2} . What is the total energy of the burst assuming isotropic emission? The observed typical photon energy is roughly the energy of the rest electron energy $m_e c^2$ (by coincidence). What is the photon number density? At this energy photons interact with each other to produce electron-positron pairs. The cross section is roughly the Thomson cross section σ_T . What is the optical depth of this interaction? (The answer is a huge number.) Is it optically thin or thick? (The answer is definitely optically-thick.) The observed spectrum, however, is an optically-thin power law spectrum. This is the compactness problem. In order to solve the problem, GRBs must be relativistic and beam towards us (you don’t need to prove this, but you can have a try).

Problem 6. Pulsars are neutron stars that emit broad-band spectrum of radiation, including radio waves, X-rays and gamma-rays. Near the magnetic polar cap, the magnetic field strength is $B \sim 10^{12}$ G. The field line is curved with a curvature radius of about $\rho \sim 10^7$ cm. Two populations of relativistic electrons are generated near the polar cap: The “primary” particles have a typical Lorentz factor of $\gamma_1 \sim 10^7$ (beaming along the magnetic field line); while the “secondary” particles have a typical Lorentz factor of $\gamma_2 \sim 100$ (with both the parallel and the perpendicular components with respect to the magnetic field line). The polar cap is hot with a temperature of about $T \sim 10^6$ K, and it is likely that there is an oscillating inner accelerator that excites electro-magnetic low frequency waves with frequency $\sim 10^6$ Hz near the polar cap region. Try to find out as many as possible mechanisms to power the observed radio emission in the $\sim 10^8$ - 10^{11} Hz band, and the observed gamma-ray emission in the 1 MeV - 10 GeV band.