

These three problems are due on Oct. 28.

Problem 1. Imagine in a nearby solar system like our own (distance 10 kpc, i.e. 10 kilo-parsec) two planets collide. The first planet resembles Jupiter, with mass 1.0×10^{30} g and radius 7.0×10^9 cm. The second planet resembles Earth, with mass 6×10^{27} g and radius 6.4×10^8 cm. Estimate the total energy released during the collision. What is the maximum luminosity of the collision? How long does it take to release the total energy with the maximum luminosity? After the collision the Jupiter may temporarily possess a hot spot of the size of Earth. Assuming that the emission from the hot spot has a blackbody spectrum and the maximum luminosity, what is the temperature of the hot spot? At which frequency is the emission peaked? Assuming isotropic emission, what is the specific flux (in unit of $\text{ergs cm}^{-2} \text{ s}^{-1} \text{ Hz}^{-1}$) detected by the Earth astronomers? One parsec is 3.09×10^{18} cm.

Problem 2. At present time the universe is filled with an isotropic blackbody emission with temperature 2.7 Kelvin. What is the peak frequency and the peak wavelength of this emission? What is the photon energy density? Let's assume that the intergalactic medium number density is $\sim 10^{-6}$ protons per cm^3 . Assuming that matter is uniformly distributed in space (in reality matter is clustered in galaxies, and dark matter has more mass than normal matter), what is the typical baryon energy density? Comparing the baryon energy density with the photon energy density, is the current universe matter dominated or radiation dominated?

Problem 3. An equation of state can be generally expressed as $P = P(\rho, T, \text{composition})$. Under certain conditions, it can be expressed as $P \propto \rho^\Gamma$. Derive the equation of state for the following three cases. What are the Γ values for each case? (1) an isothermal ideal gas; (2) an adiabatic ideal gas both for the non-relativistic and relativistic regimes; (3) an degenerate gas both for the non-relativistic and relativistic regimes.