AST713: Mid-Term Exam

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Oct. 28, 2008

2:30-4:30 pm

- One point each for the multiple-choice questions (only one correct answer);
- Five points for each problem;
- Thirty points altogether

Multiple Choice Questions

1. The specific intensity of a blackbody spectrum ($I_\nu = B_\nu$) in the Rayleigh-Jeans regime is a power law with an index of
(a) 0
(b) 1/3
(c) 1
(d) 2

2. Which of the following statements is incorrect
(a) an O2 II star is hotter than a B1 I star
(b) a G2 V star is intrinsically brighter than a K2 II star
(c) an A5 II star is cooler than an O3 III star
(d) a B2 II star is intrinsically brighter than a G2 V star

3. Which of the following regarding the reason of the existence of a Chandrasekhar mass limit in degenerate stars is correct
(a) It has something to do with the electron degeneracy pressure but not neutron degeneracy pressure
(b) Because degenerate stars are smaller when their masses are larger, and because a degenerate star cannot be smaller than the size of a black hole
(c) Because the relativistic equation of state of a degenerate gas is different from that of a non-relativistic equation of state
(d) Because Chandrasekhar derived a limit.

4. Which of the following type of supernova is not due to core collapse of a massive star?
(a) Type Ia
(b) Type Ib
(c) Type Ic
(d) Type II

5. Which of the following statements is incorrect
(a) A Strömgren radius of a massive star is defined by the balance between the photoionization rate and recombination rate of hydrogen atoms in the interstellar medium around the star.
(b) Stars will guarantee to form as long as the Jeans instability condition is satisfied.
(c) Elliptical galaxies have less star formation activities than spiral and irregular galaxies
(d) The current universe is largely ionized.

6. If the source and the observer switch locations in a gravitational lens, the size of the Einstein radius is very likely
(a) the same
(b) different
(c) one cannot tell
(d) none of the above

7. For a cosmological object at a redshift z, which of the following distances is the largest
(a) angular separation distance
(b) proper distance
(c) luminosity distance
(d) they are all the same

8. The first Friedmann equation can be effectively written as $\Omega_m + \Omega_k + \Omega_\Lambda = 1$. In a universe where all three terms exist, the time sequence for the domination of the three terms are
(a) $(\Omega_m, \Omega_k, \Omega_\Lambda)$
(b) $(\Omega_k, \Omega_m, \Omega_\Lambda)$
(c) $(\Omega_k, \Omega_\Lambda, \Omega_m)$
(d) $(\Omega_\Lambda, \Omega_k, \Omega_m)$

9. The final product of the evolution of a 30 $M_\odot$ main sequence star is very likely a
(a) white dwarf
(b) neutron star
(c) quark star
(d) black hole

10. A compact star with 1.8 $M_\odot$ mass is very likely a
(a) white dwarf
(b) neutron star
(c) black hole
(d) one cannot tell

Problems

1. The X-ray spectrum of a distant pulsar can be well fit by a blackbody emission component. The best fit temperature is 3.1 million Kelvin. The measured X-ray blackbody luminosity is 4.9×10^{28} erg s^{-1}. How large is the emission area? A neutron star has a radius of about 10 km. What fraction of the neutron star surface is emitting X-rays?

The emergent flux from an blackbody emitting surface is $F = L/A = \sigma T^4$*, So the emission area is*

$$A = \frac{L}{\sigma T^4} = \frac{4.9 \times 10^{28}}{5.67 \times 10^{-5} \times (3.1 \times 10^6)^4} = 9.4 \times 10^6 \text{ cm}^2 \ . \tag{1}$$

The full surface area of the neutron star is $A_{\rm ns} = 4\pi(10^6)^2 = 1.26 \times 10^{13}$ cm^2*. The area fraction of the hot spot is* $f = A/A_{\rm ns} = 7.4 \times 10^{-7}$*. This is a very small hot spot on the neutron star.*

2. The stellar core temperature is $\sim 1.7 \times 10^7$ K in order to trigger p-p chain nuclear fusion. The bounding condition requires that the radiation pressure is lower than the gas pressure. Compare the two pressures at the stellar core and set a lower limit of the mass density at the core.

The radiation pressure is

$$P_{rad} = \frac{1}{3} a T^4 = \frac{1}{3} \times 7.56 \times 10^{-15} \times (1.7 \times 10^7)^4 \simeq 2.1 \times 10^{14} \text{ cgs} \; . \tag{2}$$

The gas pressure is

$$P_{gas} = nkT = n \times 1.38 \times 10^{-16} \times 1.7 \times 10^7 \simeq 2.3 \times 10^{-9} n \tag{3}$$

The condition $P_{gas} > P_{rad}$ leads to

$$n > 0.9 \times 10^{23} \text{ cm}^{-3} \tag{4}$$

The average mass in the stars is

$$\bar{m} = \frac{2}{1 + 3X + Y/2} m_H \sim 0.6 m_H \; , \tag{5}$$

The lower limit of the mass density is

$$\rho = n\bar{m} > 0.9 \times 10^{23} \times 0.6 \times 1.67 \times 10^{-24} = 0.09 \text{ g cm}^{-3} \; . \tag{6}$$

Without the 0.6 factor ($\rho > 0.15$ g cm^{-3}) is also fine.

3. Derive the Eddington luminosity. The inner accretion disk radius r_{in} is 3 times of the Schwarzschild radius of a non-spinning black hole. What is the mass accretion rate $\dot{m}$ at r_{in} that sustains the Eddington luminosity?

Consider balance between the gravity force and the radiation force of a proton in an accreting system. The gravity force is

$$F_g = \frac{GMm_p}{r^2}, \tag{7}$$

where M is the mass of the gravity source, r is the distance of the proton from the center of mass. The radiation force is

$$F_r = (F/c)\sigma_T \ , \tag{8}$$

where $F = L/(4\pi r^2)$ is the flux (F/c is the radiation pressure). One can then derive

$$L_{Edd} = \frac{4\pi cGMm_p}{\sigma_T} = 1.26 \times 10^{38}(M/M_\odot) \text{ erg s}^{-1}. \tag{9}$$

The Schwarzschild radius of a black hole is $r_s = 2GM/c^2$. So the inner accretion disk radius is

$$r_{in} = 3r_s = \frac{6GM}{c^2} \simeq (9 \text{ km}) \ \frac{M}{M_\odot} \ . \tag{10}$$

The accretion luminosity is

$$L_{acc} = \frac{1}{2}\frac{GM\dot{m}}{r_{in}} = \frac{1}{12}\dot{m}c^2 \ . \tag{11}$$

For $L_{acc} = L_{Edd}$, one requires

$$\dot{m} = \frac{12L_{Edd}}{c^2} = (1.7 \times 10^{18} \text{ g s}^{-1}) \ \frac{M}{M_\odot} \ . \tag{12}$$

4. What is the Hubble's law? Prove that the Hubble's law suggest that the expansion has no center.

The Hubble's law states that the receding velocity of the distant galaxies is proportional to their distances, i.e.

$$\mathbf{v} = H_0 \mathbf{r} \ , \tag{13}$$

where $H_0 = (70 \pm 10)$ km s^{-1} Mpc^{-1} *is the Hubble's constant.*

Suppose we observe two galaxies at distances $\mathbf{r_1}$ *and* $\mathbf{r_2}$*. According to the Hubble's law, one can write*

$$\mathbf{v_1} = H_0 \mathbf{r_1} \ , \tag{14}$$

and

$$\mathbf{v_2} = H_0 \mathbf{r_2} \ , \tag{15}$$

so that

$$\mathbf{v_2} - \mathbf{v_1} = H_0 (\mathbf{r_2} - \mathbf{r_1}) \ . \tag{16}$$

This suggests that the Hubble's law is also valid at Galaxy 1. The same applies to Galaxy 2 and any other galaxies. Therefore the Hubble's law suggests that the expansion has no center.

Physical constants (all in cgs units, not every one is used):

- Speed of light: $c = 3 \times 10^{10}$ cm s^{-1}
- Planck's constant: $h = 6.63 \times 10^{-27}$ erg s
- Gravitational constant: $G = 6.67 \times 10^{-8}$ dyn cm^2 g^{-2}
- Electron charge: $e = 4.8 \times 10^{-10}$ e.s.u.
- Electron mass: $m_e = 9.1 \times 10^{-28}$ g
- Proton mass: $m_p = 1.67 \times 10^{-24}$ g
- Boltzmann's constant: $k = 1.38 \times 10^{-16}$ erg K^{-1}
- Thomson cross section: $\sigma_{\rm T} = 6.65 \times 10^{-25}$ cm^2
- Stefan-Boltzmann's constant: $\sigma = 5.67 \times 10^{-5}$ erg cm^{-2} K^{-4} s^{-1}
- Boltzmann's energy density constant: $a = 7.56 \times 10^{-15}$ erg cm^{-3} K^{-4}
- Solar mass: $M_\odot = 2.0 \times 10^{33}$ g
- Solar radius: $R_\odot = 7 \times 10^{10}$ cm
- Astronomical unit: 1 AU $= 1.5 \times 10^{13}$ cm