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# Calibration of gamma-ray burst luminosity indicators

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## ABSTRACT

Several gamma-ray burst (GRB) luminosity indicators have been proposed which can be generally written in the form of $\hat{L} = c\prod x_i^{a_i}$, where $c$ is the coefficient, $x_i$ is the $i$th observable, and $a_i$ is its corresponding power-law index. Unlike in Type Ia supernovae, calibration of GRB luminosity indicators using a low-redshift sample is difficult. This is because the GRB rate drops rapidly at low redshifts, and some nearby GRBs may be different from their cosmological brethren. Calibrating the standard candles using GRBs in a narrow redshift range ($\Delta z$) near a fiducial redshift has been proposed recently. Here we elaborate such a possibility and propose to calibrate $\{a_i\}$ based on the Bayesian theory and to marginalize the $c$ value over a reasonable range of cosmological parameters. We take our newly discovered multivariable GRB luminosity indicator, $E_{\rm iso} = cE_{\rm p}^{a_1} t_{\rm b}^{a_2}$, as an example and test the validity of this approach through simulations, where $E_{\rm iso}$ is the isotropic energy of prompt gamma-rays, $E_{\rm p}$ is the spectral break energy, and $t_{\rm b}$ is the temporal break time of the optical afterglow light curve. We show that while $c$ strongly depends on the cosmological parameters, neither $a_1$ nor $a_2$ does as long as $\Delta z$ is small enough. The selection of $\Delta z$ for a particular GRB sample could be judged according to the size and the observational uncertainty of the sample. There is no preferable redshift to perform the calibration of the indices $\{a_i\}$, while a lower redshift is preferable for $c$-marginalization. The best strategy would be to collect GRBs within a narrow redshift bin around a fiducial intermediate redshift (e.g. $z_c \sim 1$ or $z_c \sim 2$), as the observed GRB redshift distribution is found to peak around this range. Our simulation suggests that with the current observational precisions of measuring $E_{\rm iso}$, $E_{\rm p}$ and $t_{\rm b}$, 25 GRBs within a redshift bin of $\Delta z \sim 0.30$ would give fine calibration to the Liang–Zhang luminosity indicator.

**Key words:** cosmological parameters – cosmology: observations – gamma-rays: bursts.

## 1 INTRODUCTION

The cosmological nature (Metzger et al. 1997) of long gamma-ray bursts (GRBs) and their association with star formation (e.g. Totani 1997; Paczynski 1998; Bromm & Loeb 2002) make GRBs a new probe of cosmology and galaxy evolution (e.g. Djorgovski et al. 2003). Gamma-ray photons, with energy from tens of keV to MeV, from GRBs are almost immune to dust extinction, and should be detectable out to a very high redshift (Lamb & Reichart 2000; Ciardi & Loeb 2000; Gou et al. 2004; Lin, Zhang & Li 2004). Several plausible GRB luminosity indicators have been proposed, including a luminosity–variability relation (Fenimore & Ramirez-Ruiz 2000; Reichart et al. 2001), a luminosity–spectral lag relation (Norris, Marani & Bonnell 2000), standard gamma-ray jet energy ($E_{\gamma,\rm jet}$: Frail et al. 2001; Bloom, Frail & Kulkarni 2003), an isotropic gamma-ray energy–peak spectral energy ($E_{\rm iso}$–$E_{\rm p}$) relation (Amati et al. 2002; Lloyd-Ronning & Ramirez-Ruiz 2002; Liang, Dai & Wu 2004), an $E_{\gamma,\rm jet} - E_{\rm p}$ relation (Ghirlanda, Ghisellini & Lazzati 2004b), and a multivariable relation among $E_{\rm iso}$, $E_{\rm p}$ and the break time of the optical afterglow light curves ($t_{\rm b}$ Liang & Zhang 2005). Attempts to use these luminosity indicators to constrain cosmological parameters have been made (e.g. Schaefer 2003; Bloom et al. 2003; Dai, Liang & Xu 2004; Ghirlanda et al. 2004a; Friedman & Bloom 2005; Firmani et al. 2005; Liang & Zhang 2005; Xu, Dai & Liang 2005; Xu 2005; Mortsell & Sollerman 2005; Bertolami & Silva 2006; Wang & Dai 2006). With the discovery of the tight Ghirlanda relation (Ghirlanda et al. 2004b) and the more empirical Liang–Zhang (LZ) relation (Liang & Zhang 2005), it is now highly expected that GRBs may become a promising standard candle to extend the traditional Type Ia SN standard candle to higher redshifts (e.g. Lamb et al. 2005b).

In order to achieve a cosmology-independent standard candle, one needs to calibrate any luminosity indicator. Otherwise, one inevitably encounters the so-called 'circularity problem' (e.g. Firmani et al. 2005; see Xu et al. 2005, for discussion). In the case of supernova cosmology, calibration is carried out with a sample of Type Ia

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SNe at very low redshift so that the brightnesses of the SNa are essentially independent of the cosmology parameters (e.g. Phillips 1993; Riess et al. 1995). In the case of GRBs, however, this is very difficult. The observed long-GRB rate falls off rapidly at low redshifts, as is expected if long GRBs follow global star formation. Furthermore, some nearby GRBs may be intrinsically different. Observations of GRB 980425, GRB 031203 and some other nearby GRBs indicate that they differ from typical GRBs by showing low isotropic energy, a simple light-curve, large spectral lag and dimmer afterglow flux (e.g. Norris 2002; Soderberg et al. 2004; Guetta et al. 2004; Liang & Zhang 2006). Although some GRBs in the optically dim sample of Liang & Zhang (2006) still follow the Ghirlanda and LZ relations, some very nearby events (e.g. GRB 980425 and GRB 031203) are clearly outliers of the Amati relation and also likely the outliers of the Ghirlanda and LZ relations. If at least some low-redshift GRBs are different from their cosmological brethren, it is very difficult to calibrate the GRB standard candle using a low-redshift sample.

Recently the possibility of calibrating the standard candles using GRBs in a narrow redshift range ($\Delta z$) near a fiducial redshift has been proposed (Lamb et al. 2005b; Ghirlanda et al. 2005).[1] In this paper we elaborate this method (Lamb et al. 2005b; Ghirlanda et al. 2005) further based on the Bayesian theory. We propose a detailed procedure to calibrate $\{a_i\}$ with a sample of GRBs in a narrow redshift range ($\Delta z$) without introducing a low-redshift GRB sample, and to marginalize the $c$ value over a reasonable range of cosmological parameters. The method is described in Section 2. We take our newly discovered GRB luminosity indicator as an example to test the approach through simulations (Section 3). The results are summarized in Section 4 with some discussion.

## 2 CALIBRATION METHOD

A GRB luminosity indicator can be generally written in the form of

$$\hat{L}(\Omega) = c(\Omega) Q(\Omega | X; A), \tag{1}$$

where $c(\Omega)$ is the coefficient, $\Omega$ is a set of cosmological parameters, and $Q(\Omega|X;A)$ is a model of the observables $X = \{x_i\}$ (measured in the cosmological proper rest frame) with the parameter set $A = \{a_i\}$, which is generally written in the form of $Q(\Omega\,|\,X;\,A) = \prod x_i^{a_i}|_\Omega$. Because in GRB luminosity indicators the parameters $\{x_i\}$ are usually direct observables (e.g. $E_{\rm p}$, $t_{\rm b}$, etc.) that only depend on $z$ and not on the cosmological parameters, the above expression naturally separates the $\Omega$-dependent part, $c(\Omega)$, from the $\Omega$-insensitive part, $Q(\Omega|X;\,A)$. This allows us to develop an approach to partially calibrate the luminosity indicators without requiring a low-redshift GRB sample. Our approach is based on the Bayesian theory, which is a method of predicting the future based on what one knows about the past. Our calibration process can be described as follows.

(i) Calibrate $A$ using a sample of GRBs that satisfy a luminosity indicator and are distributed in a narrow redshift range $z_0 \in z_c \pm \Delta z$. Luminosity distance as a function of redshift is non-linear, and the dependence of the luminosity distance on the cosmology model is different at different redshifts, so such a sample reduces this non-linear effect. The parameter set $A$ can then be derived by using a multiple regression method in a given cosmology $\bar{\Omega}$, $A(\bar{\Omega}, z_0)$. The goodness of the regression is measured by $\chi^2_{\rm min}(\bar{\Omega}, z_0)$,

$$\chi^2_{\rm min}(\bar{\Omega}, z_0) = \sum_i^N \frac{\left[\log \hat{L}^i(\bar{\Omega}, z_0) - \log L^i(\bar{\Omega}, z_0)\right]^2}{\sigma^2_{\log \hat{L}^i(\bar{\Omega}, z_0)}}, \tag{2}$$

where $N$ is the size of the sample, $\sigma_{\log \hat{L}^i(\bar{\Omega}, z_0)}$ is the error of the empirical luminosity from the observational errors of observables, and $\log L^i(\bar{\Omega}, z_0)$ is the theoretical luminosity. The smaller the reduced $\chi^2_{\rm min}(\bar{\Omega}, z_0)$, the better the regression, and hence, the higher the probability that $A(\bar{\Omega}, z_0)$ is intrinsic. Assuming that the $\chi^2_{\rm min}(\bar{\Omega}, z_0)$ follows a normal distribution, the probability can be calculated by

$$P(\bar{\Omega}, z_0) \propto {\rm e}^{-\chi^2_{\rm min}(\bar{\Omega}, z_0)/2}. \tag{3}$$

The calibrated $A$ with a sample distributed around $z_0$ is thus given by

$$A_0 = \frac{\int_\Omega A(\bar{\Omega}, z_0) P(\bar{\Omega}, z_0)\, {\rm d}\bar{\Omega}}{\int_\Omega P(\bar{\Omega}, z_0)\, {\rm d}\bar{\Omega}}, \tag{4}$$

and its rms could be estimated by

$$\delta A_0^2 = \frac{\int_\Omega [A(\bar{\Omega}, z_0) - \bar{A}(z_0)]^2 P(\bar{\Omega}, z_0)\, {\rm d}\bar{\Omega}}{\int_\Omega P(\bar{\Omega}, z_0)\, {\rm d}\bar{\Omega}}, \tag{5}$$

where $\bar{A}_0(z_0)$ is the unweighted mean of $A(\bar{\Omega}, z_0)$ in different $\bar{\Omega}$.

(ii) Marginalize the $c$ value over a reasonable range for a given GRB sample. The $c$ value depends strongly on the cosmological parameters, so it can only be calibrated with a low-redshift sample. Because of the reasons discussed above, such a low-$z$ sample is hard to collect. We therefore do not calibrate the $c$ value but rather marginalize it over a reasonable range of cosmological parameters for a given GRB sample. For a given value of $c$, one can derive an empirical luminosity $\hat{L}(\bar{\Omega}, c, A_0, z_0)$ from the luminosity indicator and its error. The $\chi^2(\bar{\Omega}, c, A_0, z_0)$ and the corresponding probability $P(\bar{\Omega}, c, A_0, z_0)$ can be then calculated with the formulae similar to equations (2) and (3), respectively. Therefore, the calibrated luminosity is derived by

$$\hat{L_0} = \frac{\int_c \int_\Omega \hat{L}(\bar{\Omega}, c, A_0, z_0) P(\bar{\Omega}, c, A_0, z_0)\, {\rm d}\bar{\Omega}\, {\rm d}c}{\int_c \int_\Omega P(\bar{\Omega}, c, A_0, z_0)\, {\rm d}\bar{\Omega}\, dc}, \tag{6}$$

and its rms is estimated by

$$\delta\hat{L_0}^2 = \frac{\int_c \int_\Omega [\hat{L}(\bar{\Omega}, c, A_0, z_0) - \bar{L}(z_0)]^2 P(\bar{\Omega}, c, A_0, z_0)\, {\rm d}\bar{\Omega}\, {\rm d}c}{\int_c \int_\Omega P(\bar{\Omega}, c, A_0, z_0)\, {\rm d}\bar{\Omega}\, {\rm d}c}, \tag{7}$$

where $\bar{L}(z_0)$ is the unweighted mean of $\hat{L_0}(\bar{\Omega}, c, A_0, z_0)$ in different $c$ and $\bar{\Omega}$ values.

## 3 SIMULATION TESTS

The current GRB samples that favour various luminosity indicators are very small, so that one cannot directly utilize our approach to perform the calibration. The calibrations would nonetheless become possible in the future when enough data are accumulated. We therefore simulate a large sample of GRBs to examine our approach. The simulations aim to address questions such as how many bursts are needed, and how narrow the redshift bin used should be, etc., given a particular observed sample. We take the LZ relation (Liang & Zhang 2005) as an example, which reads

$$E_{\gamma,{\rm iso}} = c E'^{a_1}_{\rm p} t'^{a_2}_{\rm b}, \tag{8}$$

where $t'_{\rm b}$ and $E'_{\rm p}$ are measured in the cosmic rest frame of the burst proper. We simulate $10^6$ GRBs. Each simulated GRB is characterized by a set of parameters denoted by ($z$, $E_{\rm p}$, $E_{\rm iso}$, $t_{\rm b}$). It is

[1]The similar idea was also discussed in an earlier version of Liang & Zhang (2005).

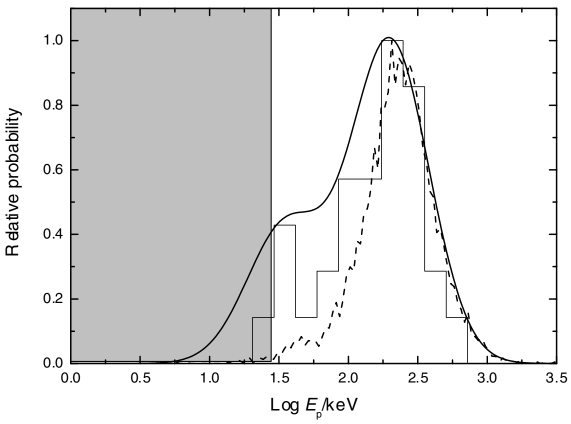


**Figure 1.** The observed $E_{\rm p}$ distribution: dashed line – derived from bright GRB sample (Preece et al. 2000); step-line – *Swift* data (Zhang et al. 2006); smoothed-curve – our model with bimodal Gaussian distribution (equation 9). The dark region marks the cut-off at $E_{\rm p} < 30$ keV due to the instrument threshold limit.

well known that the $E_{\rm p}$ distribution of a bright Burst And Transient Source Experiment (BATSE) GRBs presented by Preece et al. (2000) is well modelled by a Gaussian function. The *High Energy Transient Explorer* (*HETE-2*) and *Swift* observations of X-ray rich GRBs and X-ray flashes (XRFs: Heise et al. 2000; Lamb et al. 2005a) have considerably extended the $E_{\rm p}$ distribution to a softer band. Liang & Dai (2004) studied the observed $E_{\rm p}$ distribution of GRBs and XRFs, combined with both *HETE-2* and BATSE observations, and found that the observed $E_{\rm p}$ distribution for GRBs/XRFs is bimodal with peaks at ∼30 keV and ∼200 keV. The ∼30 keV peak has a sharp cut-off at the low energy end, likely being due to the instrument threshold limit. A recent study of a *Swift* burst sample marginally reveals such a bimodal distribution (Zhang et al. 2006). We therefore model the $E_{\rm p}$ distribution by combining the observations of BATSE and *Swift*, i.e.

$$\frac{{\rm d}p}{{\rm d}\log E_{\rm p}} = \frac{0.70}{0.56\sqrt{\pi/2}}\exp\left[-2\left(\frac{\log E_{\rm p}-2.30}{0.56}\right)^2\right] \quad (9)$$

$$+\frac{0.30}{0.56\sqrt{\pi/2}}\exp\left[-2\left(\frac{\log E_{\rm p}-1.55}{0.56}\right)^2\right] \quad (10)$$

with a cut-off at $E_{\rm p} = 30$ keV (see Fig. 1). The $E_{\rm iso}$ distribution is obtained from the current sample of GRBs with known redshifts. Since the $E_{\rm iso}$ distribution suffers observational bias at the low $E_{\rm iso}$ end, we consider only those bursts with $E_{\rm iso} > 10^{51.5}$ erg, and get[2] the differential probability of the $E_{\rm iso}$ as ${\rm d}p/{\rm d}\ \log E_{\rm iso} \propto -0.3 \log E_{\rm iso}$. The redshift distribution is assumed following the global star-forming history of the Universe. The model SF2 of Porciani & Madau (2001) is used. We truncate the redshift distribution at 10. A fluence threshold of $S_\gamma = 10^{-7}$ erg cm$^{-2}$ is adopted.

We assume that these GRBs satisfy the LZ relation and derive $t_{\rm b}$ from the simulated $E'_{\rm p}$ and $E_{\rm iso}$. Since the observed $t_{\rm b}$ is in the range of ∼0.4–6 d, we also require that $t_{\rm b}$ is in the same range to account for the selection effect to measure an optical light-curve break. Because the observed $\sigma_x/x$ is about 10–20 per cent, the simulated errors of these observables are assigned as $\sigma_x/x = 0.25k$ with a lower limit

[2]Our simulations do not sensitively depend on the $E_{\rm iso}$ distribution. We have used a random distribution between $10^{51.5} \sim 10^{54.5}$ erg, and found that the characteristics of our simulated GRBs are not significantly changed.

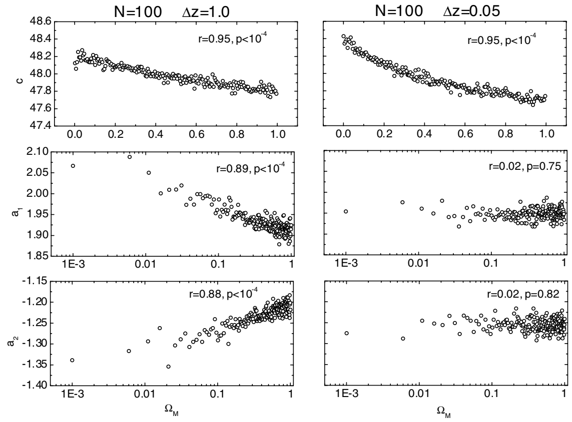


**Figure 2.** Comparison of the dependencies of $c$, $a_1$, and $a_2$ on $\Omega_M$ for a sample of 100 GRBs distributed in $z = 2.0 \pm 1.0$ (left-hand panels) and in $z = 2.00 \pm 0.05$ (right-hand panels), respectively. Current observational errors are introduced for the simulated bursts. The dependencies are measured by the Spearman correlation, and the correlation coefficient ($r$) and its chance probability ($P$) are marked in each panel.

of $\sigma_x/x > 5$ per cent, where $x$ is one of the observables $E_{\rm p}$, $S_\gamma$ or $t_{\rm b}$, and $k$ is a random number between $0 \sim 1$. Our simulation procedure is the same as that presented in Liang & Zhang (2005).

With the simulated GRB sample we examine the plausibility of our calibration approach. We consider only a flat universe with a varying $\Omega_M$. We picked up two samples with 100 GRBs in each group.[3] The first group has a narrow redshift bin (i.e. $z = 2.0 \pm 0.05$) and the second group has a wide redshift bin (i.e. $z = 2.0 \pm 1.0$). We then derive the parameters $c$, $a_1$ and $a_2$ using the multivariable regression analysis (Liang & Zhang 2005) for different cosmological parameters ($\Omega_M$) and evaluate the dependencies of the derived parameters on $\Omega_M$. The dependencies of these quantities on $\Omega_M$ are quantified by the Spearman correlation, and the results are presented in Fig. 2. It is found that $c$ strongly depends on $\Omega_M$ regardless of the value of $\Delta z$, as is expected. On the other hand, while $a_1$ and $a_2$ are strongly correlated with $\Omega$ for the case of $\Delta z = 1.0$, they are essentially independent of $\Omega_M$ for $\Delta z = 0.05$. These results suggest that once $\Delta z$ is small enough, the influence of cosmological parameters on both $a_1$ and $a_2$ becomes significant lower than the observational uncertainty and the statistical fluctuation. This makes the calibration of both $a_1$ and $a_2$ possible with a GRB sample within a narrow redshift bin.

The selection of $\Delta z$ is essential to establish the calibration sample most optimally. Two effects are needed to be taken into consideration when selecting $\Delta z$, i.e. the observational errors of the sample and the statistical fluctuation effect. The most optimal calibration sample requires that the variations of the standard candle parameters caused by varying cosmology should be comparable to the variations caused by these two effects. In such a case we could establish a sample with a large enough $N$ to reduce the fluctuation effect but with a small enough $\Delta z$ so that the dependencies of both $a_1$ and $a_2$ on $\Omega_M$ are not dominant. As the relation between $a_1$ (or $a_2$) and $\log \Omega_M$ is roughly fitted by a linear function (see Fig. 2), we measure the dependence by the chance probability ($P$) of the Spearman correlation. If $P < 10^{-4}$, the dependence is statistically significant, and the sample is inappropriate for the calibration purpose. Fig. 3 shows the distributions of $\log P$ for $a_1$ (left) and $a_2$ (right) in the

[3] To avoid the statistical fluctuation effect we use a large sample.

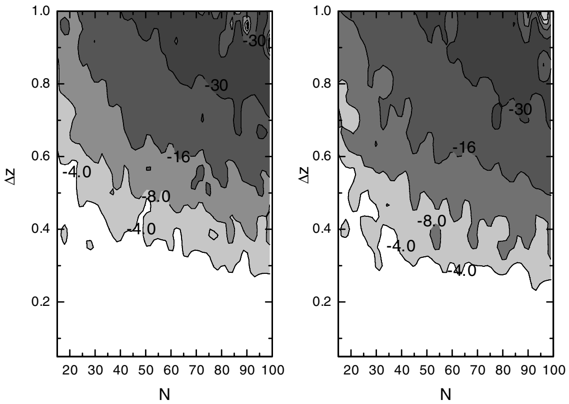


**Figure 3.** Distribution of $\log P$ in the ($N$, $\Delta z$) plane. The grey contours mark the areas where the dependencies of $a_1$ and $a_2$ on $\Omega_M$ are statistical significant ($P < 10^{-4}$). The white region is suitable for the calibration purpose.

$\Delta z$–$N$ plane, assuming the current observational errors for the observables. The grey contours mark the areas that the dependences of $a_1$ and $a_2$ on $\Omega_M$ are statistically significant. We find that $P$ dramatically decreases as $\Delta z$ increases for a given $N$. Given a $P$ value, $\Delta z$ initially decreases rapidly as $N$ increases but flattens at $N > 50$. This indicates that the statistical fluctuation effect is much lower than the observational errors for a sample with $N > 50$. We can see that with the current observational precision, $\Delta z \sim 0.3$ is robust enough to calibrate both $a_1$ and $a_2$. Increasing the GRB sample size alone does not improve the calibration when $N > 50$, as the $a_1$ and $a_2$ errors are dominated by the observational uncertainties in the data. In order to improve calibration further, higher observational precision of $E_{\rm iso}$, $E_{\rm p}$ and $t_{\rm b}$ is needed, which requires a broad-band $\gamma$-ray detector and good temporal coverage of the afterglow observations.

The observed GRB redshift distribution ranges from 0.0085 to 6.3. We examine if there exists a preferable redshift range for the calibration purpose. We randomly select a sample of 25 GRBs at $z_c \pm 0.3$, and perform the multivariable regression analysis to derive $a_1$ and $a_2$ from this sample by assuming a flat universe with $\Omega_M = 0.28$. The derived $a_1$ and $a_2$ are plotted as a function of $z_c$ in Fig. 4. We find that they are not correlated with $z_c$, and their variations are essentially unchanged, i.e. $\sim$0.15. This indicates that there is no evidence for a vantage redshift range when calibrating $a_1$ and $a_2$. It is therefore sufficient to select a sample at any redshift bin to

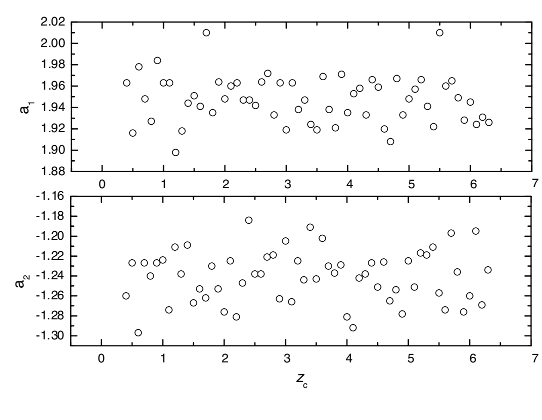


**Figure 4.** The variations of $a_1$ and $a_2$ as a function of $z_c$. The calibration sample consists of 25 simulated GRBs at $z_c \pm 0.3$. $\Omega_M = 0.28$ is adopted.

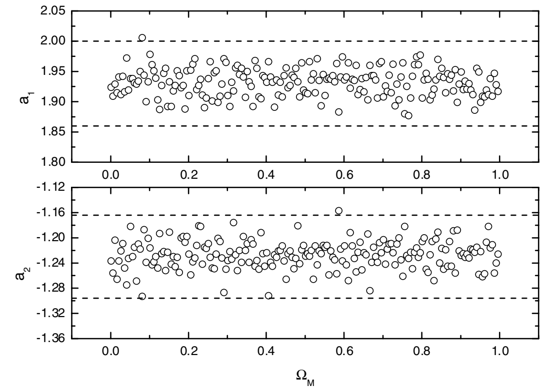


**Figure 5.** The variations of $a_1$ and $a_2$ as a function of $\Omega_M$. The calibration sample consists of 25 simulated GRBs distributed in the redshift bin $z = 1 \pm 0.3$. The dashed lines enclose the $3\sigma$ significance regions.

calibrate $a_1$ and $a_2$. Such a sample is likely to be established with GRBs at $z_c = (1 - 2.5)$, as the observed redshift distribution peaks in this range. The cosmological dependence is less significant at lower redshifts. Thus a lower redshift (e.g. $z_c = 1$) sample is preferred for $c$ marginalization.

According to Fig. 3, the best strategy to perform GRB standard candle calibration is to establish a moderate GRB sample (e.g. 25 bursts) within a redshift bin of $\Delta z \sim 0.3$ at a fiducial intermediate redshift (e.g. $z_c \sim 1$ or $z_c \sim 2$). We simulate a sample of GRB with $N = 25$, $z = 1 \pm 0.3$, and derive $a_1$ and $a_2$ as a function of $\Omega_M$ in Fig. 5. The calibrated $a_1$ and $a_2$ are $1.93 \pm 0.07$ and $-1.23 \pm 0.07$, respectively, where the quoted errors are at the $3\sigma$ significance level.

## 4 CONCLUSIONS AND DISCUSSION

We have explored in detail an approach to calibrate the GRB luminosity indicators, $\hat{L}(\Omega) = c(\Omega)Q(\Omega|X; A)$, based on the Bayesian theory without a low-redshift GRB sample. The essential points of our approach include, (i) calibrate $A$ with a sample of GRBs in a narrow redshift bin $\Delta z$; and (ii) marginalize the $c$ value over a reasonable range of cosmological parameters for a given GRB sample. We take our newly discovered multivariable GRB luminosity indicator $E_{\rm iso} = cE_{\rm p}^{a_1} t_{\rm b}^{a_2}$ (LZ relation) as an example to test the above approach through simulations. We show that while $c$ strongly depends on cosmological parameters, both $a_1$ and $a_2$ do not, if $\Delta z$ is small enough. The selection of $\Delta z$ depends on the size and the observational uncertainty of the sample. For the current observational precision, we find $\Delta z \sim 0.3$ is adequate to perform the calibration.

It is also found that the calibrations for both $a_1$ and $a_2$ are equivalent for samples at any redshift bin. The best strategy would be to collect GRBs within a narrow redshift bin around a fiducial intermediate redshift (e.g. $z_c \sim 1$ or $z_c \sim 2$), as the observed GRB redshift distribution is found to peak in this range. Our simulation suggests that with the current observational precision of measuring GRB isotropic energy ($E_{\rm iso}$), spectral break energy ($E_{\rm p}$) and the optical temporal break time ($t_{\rm b}$), 25 GRBs within a redshift bin of $\Delta z \sim 0.30$ would give fine calibrations to the LZ relation. Inspecting the current GRB sample that satisfies the LZ relation, we find that nine GRBs, i.e. 970828 ($z = 0.9578$), 980703 ($z = 0.966$), 990705 ($z = 0.8424$), 991216 ($z = 1.02$), 020405 ($z = 0.69$), 020813 ($z = 1.25$), 021211 ($z = 1.006$), 041006 ($z = 0.716$) and 050408 ($z = 1.24$) are roughly distributed in the redshift range $z = 1.0 \pm 0.3$.

We expect roughly 15 more bursts to form an adequate sample to calibrate the LZ relation.

The observed redshift distribution for the current long GRB sample covers from 0.0085 to 6.29. There have been suggestions that GRB properties may evolve with redshift (e.g. Lloyd-Ronning, Fryer & Ramirez-Ruiz 2002; Amati et al. 2002; Wei & Gao 2003; Graziani et al. 2004; Yonetoku et al. 2004). Among the proposed GRB luminosity indicators, the cosmological evolution effect has not been considered. With the current GRB sample with known redshifts, it is difficult to access whether and how GRBs evolve with redshift. Nonetheless, since our calibration approach makes use of a GRB sample in a narrow redshift bin, the evolution effect essentially does not affect on the calibration of the parameter set $A$, the set of the power index (indices) in the luminosity indicators. However, it could significantly impact on the $c$-marginalization.

## ACKNOWLEDGMENTS

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