Magnetic-distortion-induced Ellipticity and Gravitational Wave Radiation of Neutron Stars: Millisecond Magnetars in Short GRBs, Galactic Pulsars, and Magnetars

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Abstract

Neutron stars may sustain a non-axisymmetric deformation due to magnetic distortion and are potential sources of continuous gravitational waves (GWs) for ground-based interferometric detectors. With decades of searches using available GW detectors, no evidence of a GW signal from any pulsar has been observed. Progressively stringent upper limits of ellipticity have been placed on Galactic pulsars. In this work, we use the ellipticity inferred from the putative millisecond magnetars in short gamma-ray bursts (SGRBs) to estimate their detectability by current and future GW detectors. For \(\sim 1\) ms magnetars inferred from the SGRB data, the detection horizon is \(\sim 30\) Mpc and \(\sim 600\) Mpc for the advanced LIGO (aLIGO) and Einstein Telescope (ET), respectively. Using the ellipticity of SGRB millisecond magnetars as calibration, we estimate the ellipticity and GW strain of Galactic pulsars and magnetars assuming that the ellipticity is magnetic-distortion-induced. We find that the results are consistent with the null detection results of Galactic pulsars and magnetars with the aLIGO O1. We further predict that the GW signals from these pulsars/magnetars may not be detectable by the currently designed aLIGO detector. The ET detector may be able to detect some relatively low-frequency signals (<50 Hz) from some of these pulsars. Limited by its design sensitivity, the eLISA detector seems to not be suitable for detecting the signals from Galactic pulsars and magnetars.

Key words: gamma-ray burst: general – gravitational waves – pulsars: general – stars: neutron

1. Introduction

The Laser Interferometer Gravitational-wave Observatory (LIGO) team have announced two direct detections of gravitational wave (GW) events (GW 150914 and GW 151226) from binary black hole (BH) mergers (Abbott et al. 2016a, 2016b). This marked the beginning of GW astronomy. In addition to the primary targets of inspiral and mergers of compact object binary (NS–NS, NS–BH, BH–BH) systems, another potential target of continuous GW emission for the ground-based GW detectors, such as the Advanced LIGO (Abbott et al. 2009), Advanced VIRGO (Acernese et al. 2008), and KAGRA (Kuroda & LCGT Collaboration 2010) interferometers, are rapidly rotating neutron stars (NSs), as long as the NSs sustain a significant non-axisymmetric deformation (Aasi et al. 2014, and reference therein).

Several mechanisms to induce NS asymmetries have been suggested in the literature. First, the crust of a NS is solid and elastic. The shape of the crust depends on many factors, such as the original formation history and accretion history of the NS, star quakes, and the equation of state (EoS) of the NS (Ushomirsky et al. 2000; Haskell et al. 2006). The deformation of the crust would not be easily smoothed under the effects of rotation, since it could be supported by anisotropic stresses in the solid. Second, gravitational radiation reactions (Owen & Lindblom 2002; Andersson 2003, for a review) or nuclear matter viscosity (Bonazzola et al. 1996, and references therein) may drive non-axisymmetric instabilities in rapidly rotating neutron stars, which could also produce asymmetries in neutron stars. Finally, neutron stars are known to have relatively large magnetic fields, and the anisotropy of the magnetic pressure would also distort the star. When the magnetic axis is not aligned with the rotation axis, the deformation would not be axisymmetric (Ostriker & Gunn 1969; Bonazzola & Gourgoulhon 1996; Konno et al. 2000; Ioka & Sasaki 2004; Stella et al. 2005; Tomimura & Eriguchi 2005; Haskell et al. 2008; Dall’Oso et al. 2009; Mastrano et al. 2011; García & Ranea-Sandoval 2015; de Araujo et al. 2016; Suvorov et al. 2016). This last mechanism likely plays an important role in defining \(\epsilon\) in various astrophysical contexts, especially in magnetars.

Quadrupole deformation of the neutron star is characterized by the ellipticity \(\epsilon\), which is defined by

\[
\epsilon = \frac{\text{equatorial radius} - \text{polar radius}}{\text{mean radius}}. \quad (1)
\]

A neutron star with a rotation period \(P\) and ellipticity \(\epsilon\) radiates GWs at a frequency \(f = 2/P\) with an energy loss rate \(^{5}\)

\[
\dot{E}_{GW} = - \frac{32G\ell^2c^7\nu^6}{5c^5}, \quad (2)
\]

where \(\Omega = 2\pi/P\) is the angular frequency, and \(I\) is the moment of inertia of the NS.

In principle, the value of \(\epsilon\) may be measured from observations once the GW radiation from one particular NS is detected. Decades of searches with various GW detectors

In general, gravitational radiation is emitted at the spin frequency and its octave, and the total energy loss rate is \(\dot{E}_{GW} = - \frac{32G\ell^2c^7\nu^6}{5c^5}\sin^2\chi(1 + 15\sin^2\chi)\), where \(\chi\) is the tilt angle between the spin axis and the non-symmetric axis (Cutler & Jones 2001, and reference therein). For simplicity, in this work we adopt an orthogonal rotator \(\chi = \pi/2\), and assume that GWs only emit at twice the spin frequency.
(e.g., initial LIGO, Virgo, GEO 600, and the first observing run of the Advanced LIGO (aLIGO) detectors) targeted on a selection of known Galactic pulsars (Aasi et al. 2014; Abbott et al. 2017, and references therein), however, did not detect any GW signals due to the sensitivity limitation of current detectors. More and more stringent upper limits on the value of \( \epsilon \) have been set for these pulsars. For eight pulsars, the resulting upper limits already surpass their spin-down limits (which attribute all the spin-down luminosity to lost GW radiation; Abbott et al. 2017).

Recently, an indirect method has been proposed to estimate the \( \epsilon \) value for a particular class of NSs, i.e., the rapidly spinning, strongly magnetized, supramassive NSs (henceforth, millisecond magnetar). A millisecond magnetar has long been proposed to be a possible central engine for gamma-ray bursts (GRBs; Usov 1992; Dai & Lu 1998; Zhang & Mészáros 2001; Dai et al. 2006; Gao & Fan 2006; Metzger et al. 2011) and electromagnetic counterparts of NS–NS mergers (Gao et al. 2013; Yu et al. 2013; Zhang 2013; Metzger & Piro 2014). The model is especially relevant to some short GRBs with soft \( \gamma \)-ray extended emission (Norris & Bonnell 2006; Sakamoto et al. 2011) or GRBs with an internal X-ray plateau followed by very rapid decay (Troja et al. 2007; Rowlinson et al. 2010; Lü et al. 2015). These features mark the abrupt cessation of the central engine, likely due to the collapse of a supramassive NS into a BH (Rowlinson et al. 2010; Lü & Zhang 2014; Lü et al. 2015). When modeling the X-ray plateau of short GRBs, Fan et al. (2013) noticed that the observed duration of the internal X-ray plateau is shorter than that expected in the magnetic dipole radiation scenario. They then suggested that GW radiation likely dominates the loss of rotational energy for these millisecond magnetars. By investigating some particular cases of short GRBs, they suggested that the ellipticity and dipole magnetic field strength \( B_{\text{dip}} \) for the supramassive magnetars are around 0.01 and \( 10^{15} \) G, respectively. They also claimed that the GWs from such sources may be detectable with the proposed Einstein Telescope (ET; Punturo et al. 2010). Later, Gao et al. (2016) used the statistical observational properties of \textit{Swift} short gamma-ray bursts (SGRBs) and the mass distribution of Galactic double neutron star systems to systematically place constraints on the neutron star EoS and the properties of the post-merger product. They found that when the SGRB central engine is a supramassive NS, in order to reproduce the distributions of internal X-ray plateau luminosity and break time, the ellipticity of the millisecond magnetar needs to be in the range of 0.004–0.007, and the dipole magnetic field strength of the NS is typically \( 10^{15} \) G. Significant GW radiation is expected to be released after the merger. This conclusion applies to a range of EoSs (Li et al. 2016). Lasky & Glampedakis (2016) explored the physically motivated \( \epsilon \) via the spin-flip mechanism. Even though the relatively large value \( \epsilon \sim 0.01 \) inferred by Fan et al. (2013) may not be physically attainable, the value \( \epsilon \sim 0.004–0.007 \) inferred by Gao et al. (2016) is marginally consistent with the range of \( \epsilon \) suggested by Lasky & Glampedakis (2016).

This method of inferring \( \epsilon \) is based on the electromagnetic observations of SGRB X-ray afterglows. It is of great interest to investigate the consistency between this result with the GW observations of Galactic pulsars and magnetars. This is the purpose of this paper.

\section{General Formalism}

Among various mechanisms, magnetic distortion likely plays a dominant role in maintaining a relatively large \( \epsilon \) for a millisecond magnetar. We focus on this possibility. According to previous analytical and numerical studies, the magnetic distortion of an NS depends on the strength and the configuration of the magnetic fields (including the inclination angle and the toroidal-to-poloidal ratio) (Bonazzola & Gourgoulhon 1996; Haskell et al. 2008). In general, one may parameterize that

\begin{equation}
\epsilon = \beta B^2, \tag{3}
\end{equation}

where

\begin{equation}
B^2 = \frac{1}{V} \int B^2 dV \tag{4}
\end{equation}

scales with the volume average of magnetic pressure, \( V \) is the volume of the star, and the coefficient \( \beta \) contains the information of the magnetic field configurations. For simplicity, we connect \( B \) and \( B_{\text{dip}} \) by defining

\begin{equation}
B_{\text{dip}} = \eta B, \tag{5}
\end{equation}

with \( 0 \leq \eta \leq 1 \), where \( \eta = 0 \) and 1 represent a star with a purely toroidal and poloidal field component, respectively.

Given the dipole magnetic field strength \( B_{\text{dip,m}} \) and the ellipticity \( \epsilon_m \) for the millisecond magnetars, the ellipticity for a Galactic pulsar \( \epsilon_p \) with dipole magnetic field strength \( B_{\text{dip,p}} \) may be estimated as (assuming that magnetic distortion is the dominant mechanism deforming their respective \( \epsilon \))

\begin{equation}
\epsilon_p = 5 \times 10^{-9} \frac{\epsilon_m}{0.005} \left( \frac{\eta_m}{\eta_p} \right) \left( \frac{B_{\text{dip,p,12}}}{B_{\text{dip,m,15}}} \right)^2, \tag{6}
\end{equation}

where \( \eta_p \) and \( \eta_m \) represent the configurations of the magnetic fields for the pulsar and the millisecond magnetar, respectively.

It is worth pointing out that the value of \( \epsilon_p \) must not be larger than the spin-down limit \( \epsilon_{\text{sd}} \), which is determined by equating the power radiated through GWs to the observed spin-down luminosity of the pulsar, i.e., \( \dot{E}_{\text{sd}} = I \Omega \dot{\Omega} = \dot{E}_{\text{GW}} \). This gives

\begin{equation}
\epsilon_{\text{sd}} = \frac{5c^3 \Omega}{32G \Omega_p^2}, \tag{7}
\end{equation}

\begin{equation}
= 1.9 \times 10^{-7} \left( \frac{P}{1 \text{ ms}} \right)^{3/2} \left( \frac{\dot{P}}{10^{-15}} \right)^{1/2},
\end{equation}

where \( P \) and \( \dot{P} \) are the period and period derivative of the pulsar, respectively.

The characteristic GW amplitude of a rotating magnetized NS with an ellipticity \( \epsilon \) and rotation frequency \( \Omega \) can be estimated as (Corsi & Mészáros 2009)

\begin{equation}
h_\epsilon = \frac{\hbar(t)}{\sqrt{df}}, \tag{8}
\end{equation}

where \( f = \Omega/\pi \),

\begin{equation}
h(t) = \frac{4G \Omega_p^2}{c^3 d} \epsilon, \tag{9}
\end{equation}

and \( d \) is the distance to the source.

For a given pulsar, once its characteristic GW amplitude \( h_\epsilon \) is detected, the \( \epsilon \) value can be directly measured. On the other
hand, if no GW signal is detected, an upper limit on \( \epsilon \) can be set by comparing \( \hat{h} \) with the noise level of the GW detector,

\[
\hat{h}_{\text{rms}} = \sqrt{\langle S_n(f) \rangle},
\]

where \( S_n(f) \) is the power spectral density (PSD) of the detector noise. We consider the aLIGO, ET, and eLISA (Amaro-Seoane et al. 2012) detectors for a single detector analysis. The PSD for aLIGO O1 and the designed PSD for eLISA are adopted from the respective websites of these collaborations.\(^6\) For the designed PSD of aLIGO, we adopt the analytical model (Arun et al. 2005; Sun et al. 2015a)

\[
S_n(f) = S_0 \left[ x^{-4.14} - 5x^{-2} + \frac{111(1 - x^2 + x^2/2)}{1 + x^2/2} \right],
\]

for \( f \geq 20 \) Hz, where \( x = f/f_0 \), \( f_0 = 215 \) Hz, and \( S_0 = 10^{-49} \) Hz\(^{-1}\). When \( f < 20 \) Hz, \( S_n(f) = \infty \) is adopted.

For the designed PSD of ET, we adopt the analytical model (Mishra et al. 2010; Sun et al. 2015a):

\[
S_n(f) = S_0 \left[ 2.39 \times 10^{-27}x^{-15.67} + 0.349x^{-2.145} + 1.76x^{-0.12} + 0.409x^{1.12} \right]
\]

for \( f \geq 10 \) Hz, where \( x = f/f_0 \), \( f_0 = 100 \) Hz, and \( S_0 = 10^{-50} \) Hz\(^{-1}\). When \( f < 10 \) Hz, \( S_n(f) = \infty \) is adopted.

3. Results

3.1. Millisecond Magnets in SGRBs

Based on the results from Gao et al. (2016), we adopt \( B_{\text{dip,m}} = 10^{15} \) G and \( \epsilon_m = 0.005 \) as the dipole magnetic field strength and the ellipticity for the millisecond magnetars. With Equations (8) and (9), one can estimate the characteristic GW amplitude \( h_c \) for millisecond magnetars in SGRBs. Comparing the value of \( h_c \) with the noise level of the GW detectors, \( \hat{h}_{\text{rms}} \), one can estimate the detection horizon of GW signals from these millisecond magnetars, i.e.,

\[
d \leq \left( \frac{5IG}{Pc^3} \right)^{1/2} h_{\text{rms}}^{-1}
\]

\[\lesssim 360 \text{ Mpc} \left( \frac{h_{\text{rms}}}{10^{-25}} \right)^{-1} \left( \frac{I}{10^{45} \text{ g cm}^2} \right)^{1/2} \left( \frac{P}{1 \text{ ms}} \right)^{-1/2}.
\]

Substituting Equations (11) and (12), we plot the detection horizon of GW signals from millisecond magnetars for aLIGO and ET (Figure 1). We can see that the aLIGO horizon for such a signal could be up to 400 Mpc, while the ET horizon could be up to 3 Gpc, both for relatively slowly spinning magnetars \( (p \geq 8 \) ms) for \( \sim 1 \) ms magnetars, as inferred from the SGRB data (Gao et al. 2016) the detection horizons for aLIGO and ET are \( \sim 30 \) Mpc and \( \sim 600 \) Mpc, respectively. The corresponding SGRB detection rate (Wanderman & Piran 2015; Sun et al. 2015b) is low for aLIGO, but is reasonably high for ET (Figure 1).

3.2. Galactic Pulsars and Magnetars

With the calibration from millisecond magnetars inferred from SGRB data (Gao et al. 2016), the ellipticity for a pulsar \( \epsilon_p \) with dipole magnetic field strength \( B_{\text{dip,p}} \) could be extrapolated from Equation (6).

In Figure 2, we plot the extrapolated results for different \( \eta_m/\eta_p \) values. Comparing with the aLIGO O1 results, we find that the millisecond magnetar ellipticity value inferred from the SGRB data would be consistent with the aLIGO O1 results, as long as \( \eta_m \) is not larger than \( \eta_p \) by more than one order of magnitude. Since the toroidal field is more important (smaller \( \eta \)) in rapid rotators, it is essentially impossible to have \( \eta_m/\eta_p > 1 \). Our results therefore suggest that the non-detection of Galactic pulsars by aLIGO O1 is naturally expected given the \( \epsilon_m \) inferred from the SGRB data.

\(^6\) For aLIGO, see https://dcc.ligo.org/LIGO-G1600150/public; and for eLISA, see https://www.elisascience.org/articles/elisa-mission/lisa-white-paper.
GW detectors, i.e., aLIGO, Galactic pulsars and magnetars, compared against the noise curves of various sensitivity, Figure 3. The Astrophysical Journal, 57(9), 2005. We estimate the characteristic GW amplitude \( h_c \) for known pulsars and investigate their detectability with current and future GW detectors. We estimate the expected \( h_c \) values for all known pulsars listed in the ATNF pulsar catalog (Manchester et al. 2005) and all known magnetars listed in the McGill magnetar catalog (Olausen & Kaspi 2014). For each pulsar we apply Equation (6) to estimate its ellipticity (and make sure that it does not exceed Equation (7)), and then apply Equations (8) and (9) to calculate its characteristic GW amplitude \( h_c \). In the estimations, \( B_{\text{dip}}, P, \dot{P} \), and the distance \( d \) of each pulsar are used.

We plot the estimated \( h_c \) values (with \( \eta_m/\eta_p = 1 \)) for all the pulsars and magnetars in Figure 3. This is compared against the sensitivities of the GW detectors, i.e., aLIGO, ET, and eLISA, for a single detector analysis. We find that for \( \eta_m/\eta_p = 1 \), the GW signals from these pulsars are not detectable for the aLIGO detector at the full design (e.g., reaching the designed capability by the LIGO Scientific Collaboration (LIGO Scientific Collaboration 2015)). The eLISA detector, limited by its designed sensitivity, is also not suitable for detecting the signals from Galactic magnetars or known pulsars. The ET detector may be able to detect some relatively low-frequency signals (<50 Hz) from some of these pulsars. It is worth noting that although the magnetic field strengths of the Galactic magnetars are similar to those of millisecond magnetars, their characteristic GW amplitudes are quite low due to their much slower spin period.

In the above analysis we only compare the \( h_c \), defined in Equation (8) and the detector sensitivity defined in Equation (10) to estimate the detectability of the GW signal. It is possible to implement a more comprehensive coherent data analysis procedure (Cutler & Schutz 2005; Dupuis & Woon 2005; Astone et al. 2010) to improve the GW signal detection (Aasi et al. 2015). With such a procedure, a few of the pulsars shown in Figure 3 could become detectable by aLIGO. Considering that the LIGO detectors are escalating and the ET detectors are still in the stage of conceptual development, we would like to leave a detailed investigation of such an effect to future work.

4. Conclusion and Discussion

Rapidly rotating neutron stars are potential sources of continuous GWs for ground-based interferometric GW detectors, if the neutron stars sustain a non-axisymmetric deformation. Recently, \( \epsilon \sim 0.005 \) for rapidly spinning, strongly magnetized, supramassive neutron stars (millisecond magnetars) has been inferred from the statistical observational properties of Swift SGRBs. We estimate the detection horizon of such millisecond magnetars with current (aLIGO) and future (ET) GW detectors. For fast rotators (\( P \sim 1 \) ms), the horizons are \( \sim 30 \) Mpc and \( \sim 600 \) Mpc, respectively, for aLIGO and ET. For slow rotators (e.g., \( P \sim 8 \) ms), the horizon can be extended to \( \sim 400 \) Mpc and \( \sim 3 \) Gpc, respectively. The non-detections of such millisecond magnetars from SGRBs by aLIGO are consistent with the inferred short period (\( \sim 1 \) ms) of these magnetars (Gao et al. 2016).

Assuming that magnetic distortion is the main origin of ellipticity, in this work we show that these values are consistent with the non-detection results of Galactic pulsars by aLIGO O1, as long as \( \eta_m/\eta_p \) is not greater than 1 by more than one order of magnitude. We further estimate the characteristic GW amplitude \( h_c \) for known pulsars and normal magnetars and find that the GW signals from these pulsars are not detectable by the aLIGO detector full design and by eLISA (assuming \( \eta_m/\eta_p = 1 \)). The ET detector may be able to detect the relatively low-frequency signals (<50 Hz) from some of these pulsars.

It is possible that the ellipticity of the millisecond magnetar is not mainly contributed by magnetic deformation. For non-magnetic distortions, the distortion is usually more significant for rapid rotators, so, given the same \( \epsilon \) inferred from the millisecond magnetars in SGRBs, the \( \epsilon \) for Galactic pulsars/magnetars could be even lower than the \( \eta_m/\eta_p = 1 \) extrapolation shown in Figure 2. This would be even more consistent with the aLIGO O1 non-detection result, and the detectability of Galactic NSs by future GW detectors would be more unlikely.

It is worth pointing out that the SGRB-data-inferred ellipticity value for millisecond magnetars could be inconsistent with the aLIGO O1 results if \( \eta_p \) is smaller than \( \eta_m \) by more than one order of magnitude. However, according to previous studies (Bonazzola & Gourgoulhon 1996; Konno et al. 2000; Stella et al. 2005; Haskell et al. 2008; Mastrano et al. 2011), in order to achieve \( \epsilon \sim 0.005 \), a very high strength (10\(^{16-17}\) G) is needed, implying that the internal (toroidal) field of the millisecond magnetar may be more than 1–2 orders of magnitude stronger than the dipole field value 10\(^{15}\) G, namely \( \eta_m \sim 0.01–0.1 \). In this case, the toroidal field of the Galactic pulsars needs to be more than three orders of magnitude stronger than the dipole field value (\( \eta_p \) being smaller than 0.001) in order to invalidate \( \epsilon \sim 0.005 \) for millisecond magnetars. This is essentially impossible. There is no evidence of a significant toroidal magnetic field component for radio pulsars. Even though it is conjectured that a toroidal component exists for Galactic magnetars, the degree of twisting must be much weaker than that of millisecond magnetars, since the magnetar activities (quiescent emission and soft \( \gamma \)-ray bursts) are believed to be powered by magnetar untwisting (Thompson

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7 http://www.atnf.csiro.au/research/pulsar/psrcat/


& Duncan 2001). Instead, it is very likely that $\eta_m/\eta_n$ is less than unity, so the characteristic GW amplitudes $h$, for Galactic pulsars and magnetars shown in Figure 3 are overestimated. Even ET might not be capable of detecting these sources.

When estimating the detection probability of Galactic pulsars and magnetars for aLIGO’s full design, ET, and eLISA, we simply compare the characteristic GW amplitude $h_\text{c}$ of the sources with the analytical noise curve of the detectors. In reality, the noise curves may become more complicated due to some additional noise (see the aLIGO O1 curve above the analytical aLIGO full design curve in Figure 3). This would drop the signals that are only slightly above the noise curve, rendering them not detectable (see Abbott et al. 2017 for examples). On the other hand, a more comprehensive coherent data analysis procedure would improve the GW signal detection probability. It is possible that a few GWs from the pulsars shown in Figure 3 may become detectable even by aLIGO with the help of such a procedure.

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References

Andersson, N. 2003, CQGra, 20, R105
Cutler, C., & Jones, D. I. 2001, PhRvD, 63, 024002
Cutler, C., & Schutz, B. F. 2005, PhRvD, 72, 063006
Dupuis, R. J., & Woan, G. 2005, PhRvD, 72, 102002
Fan, Y.-Z., Wu, X.-F., & Wei, D.-M. 2013, PhRvD, 88, 067304
Gao, H., Zhang, B., & Li, H.-J. 2016, PhRvD, 93, 044065
Gao, W.-H., & Fan, Y.-Z. 2006, ChJAA, 6, 513
Kuroda, K. & LCGT Collaboration 2010, CQGra, 27, 084004
Owen, B. J., & Lindblom, L. 2002, CQGra, 19, 1247
Punturo, M., Abernathy, M., Acernese, F., et al. 2010, CQGra, 27, 194002
Shapiro, S. L., & Teukolsky, S. A. 1985, in Research Supported by the National Science Foundation (New York: Wiley-Interscience), 663
Sun, B., Cao, Z., Wang, Y., & Yeh, H.-C. 2015a, PhRvD, 92, 044034
Usov, V. V. 1992, Natur, 357, 472
Yu, Y.-W., Zhang, B., & Gao, H. 2013, ApJL, 776, L40