Solid State Relays:

Solid state relays serve the same function as mechanical relays but have no moving parts (hence the name solid state). They are semiconductor devices that either block or allow the load current to flow depending on the input to the relay. The inputs on most SSR's are isolated from the load. This means the circuit that triggers the relay doesn't have to be referenced to the same ground as the load (i.e. isolated). Since there is no electrical connection between the input and the output it is unlikely that the input circuit could be damaged by a spike in the output circuit.

A brief explanation of SSR's and a few pictures are at: http://en.wikipedia.org/wiki/Solid_state_relay

A simplified block diagram of an optically isolated SSR is shown to the left. Any voltage on the input above three or four volts sends enough current through the LED to generate light (photons). The photons travel a short distance in the relay and hit a photodiode, phototransistor, or other optical to electrical conversion device. The output switch is triggered by the current from the photodiode, phototransistor, or whatever.

The output switch is usually be made with a FET for a DC load and a TRIAC for an AC load. Note: Unlike a mechanical relay the output of a SSR will usually be specified for either an AC or DC load. Most SSR's can not switch both AC and DC loads.

A DC SSR will be similar to the FET's you've used except they handle more current and the input is isolated from the output. A DC SSR will have POS & NEG terminals indicating the direction for the load current. The SSR may be damaged if connected backwards. When switching inductive loads a flyback diode should be place across the SSR output terminals to clamp the inductive kickback.

Note: A TRIAC (Triode Alternating Current Switch) is two SCR's (Silicon Controlled Rectifier) connected back to back. You can think of a SCR as a three pin diode that doesn't start conducting until you trigger the gate (the third pin). Unlike a FET, once an SCR is triggered it will continue to conduct until the load current stops flowing (even if the gate is low). TRIAC's are used to switch AC loads since the current is constantly changing direction. Once the gate goes low the TRIAC will stop conducting within 1/120 of a second. The following link gives a reasonable explanation of this process: https://www.radio-electronics.com/info/data/semicond/triac/what-is-a-triac-basics-tutorial.php

Note: Don't connect a DC load to a TRIAC. Once turned on it will stay on. The only way to turn it off is to stop the current flow (i.e. disconnect the load).

Power dissipation:
Unlike mechanical relays that have extremely low metal to metal contact resistance FET's and TRIACS have a higher on resistance. The on resistance of a DC SSR (or a FET) will be listed as $R_{Dson}$. The power dissipated in the relay is: $P = I^2R$. An AC relay (or TRIAC) will usually list its ON voltage. Its power dissipation is $P = IV$. Always do a quick calculation to see if a heatsink is needed and if so how large it should be. Here are two links to datasheets for a AC SSR and an DC SSR: