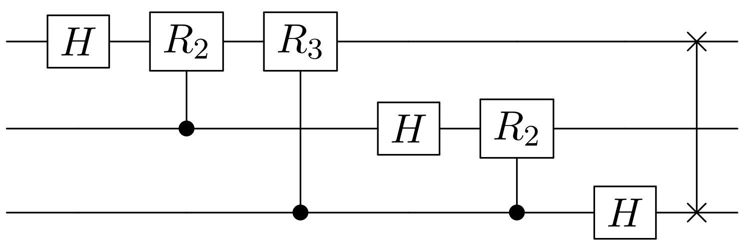
Class Project Due 5/3/2016 2:30 PM

(a) For a register containing n bits we pointed out, in the previous lecture that

$U_{FT}|\, b_{n-1}\, b_{n-2} \ldots b_1 b_0\rangle = \frac{1}{2^{n/2}}\, (\,|\,0\,\rangle + e^{i\,2\pi\,[.b_0]}\,|1\,\rangle)\,\,(\,|\,0\,\rangle + e^{i\,2\pi\,[.b_1\,b_0]}\,|1\,\rangle)\,\ldots\,(\,|\,0\,\rangle +$
$e^{i\,2\pi\,[.b_{n-1}\,b_{n-2}\ldots b_1\,b_0]}\,|1\,\rangle)$

and we showed <u>explicitly</u> for the case n=2 the validity of this identity. Do the same for the case n=3. Show all work.

(b) Consider the following circuit diagram



Build each of the seven components in that diagram. e.g. the first component would be the
3-qubit gate $H \otimes I \otimes I$ on the extreme left of the figure. Multiply each of these seven component gates, and
compare the result with the 3-Qubit QFT gate. Comment.

(c) Consider a 3 bit register $|x\,\rangle_3$ and the following function f: $\{0,1\}^{\otimes 3} \longrightarrow \{0,1\}$

x	f (x)
000	0
001	1
010	0
011	1
100	0
101	1
110	0
111	1

Construct an operator that has the property
$U_f|\,x\,\rangle_3|y\rangle = |\,x\,\rangle_3|\,f(x) + y\,\rangle$

(d) Now construct a gate W_f so that

$$W_f\,|\,0\,\rangle_3|0\rangle = \frac{1}{\sqrt{8}} \sum_{a=0}^{a=7} |\,a\,\rangle_3\,|f(a)\,\rangle$$

Write a Mathematica code, which incorporates the gate constructed above, that has as its input $|0\rangle_3|0\rangle$, and outputs the result for $\frac{1}{\sqrt{8}} \sum_{a=0}^{a=7} | a \rangle_3 |f(a) \rangle$.

(e) Now construct a simulation (*Mathematica* code) of a quantum computer that has as its input $| 000 \rangle_3 |0\rangle$. That input goes
through the gate constructed in part (d) above. Finally, that output goes through a 3-qubit QFT (acting on the first register) gate.
Perform measurements of the output in that register. Perform several runs and make a histogram of the output, comment on the significance of your results. (Remember: A simulation should behave just like a quantum computer !)