

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V\psi = \cancel{E\psi} = i\hbar \frac{\partial \psi}{\partial t}$$

V accounts for the field energy of some system that exerts a force on the particle. And the "tension"

~~We generally just,~~

between kinetic energy and potential energy causes the time evolution (or stationarity of particle)

But in fact if the system exerts a potential (force) on the particle, the particle exerts ~~force~~ back a reaction force

4-482

- we've just idealized
the ~~back~~ reaction force
a negligible usually

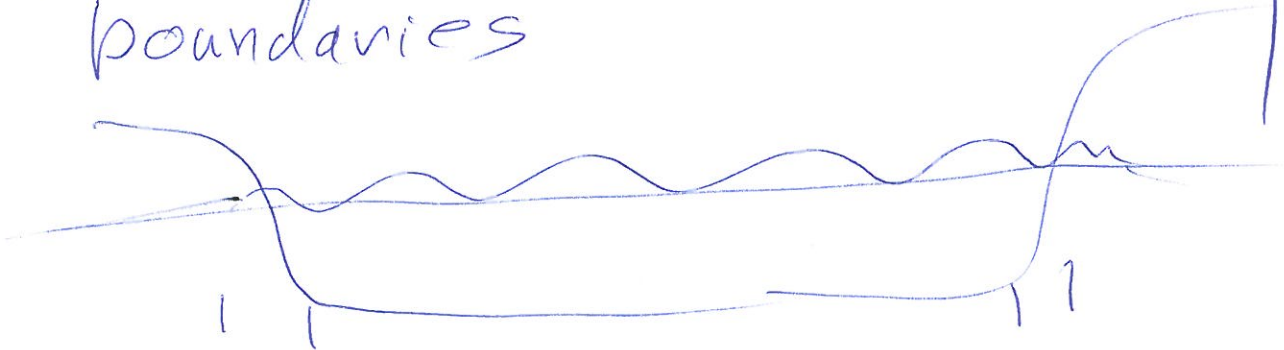
Not of course for
hydrogenic atom
where we considered
two interacting particles
explicitly

~~What~~ →

Where is the force / and
reaction force?

At those well disregarded
boundaries

Heaven
knows
where the
force
comes
from
in the
periodic
BC
case.

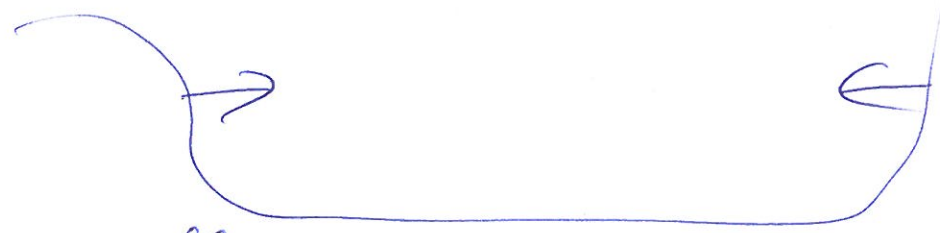


Recall we've approximated the electrons as non-interacting

At boundaries the collective force of ions ~~and the~~ combined with electron ~~at~~ Coulomb force we've sort of hidden act to confine the non-interacting electrons by a steep potential wall.

we've split this off from ~~the~~ "electrons" we've considered

The stationary states all change. The pressure comes from the electron in the occupied stationary states.



- wall presses in.
- NON-interacting electron
 - ↳ i.e., their kinetic energy pushes out.

5-584)

This is the emergent macroscopic ~~after~~ situation.

(the correspondance principle is being used)

So we can use macroscopic thermodynamics here.

~~(the by the correspondance)~~

Pressure

Recall

Changes of internal energy must come from Heat flow $dQ = Tds$ or macroscopic work $dW = -PdV$

$$dE_{\text{internal}} = Tds - PdV$$

total internal energy of sample temperature differential of entropy.

Say we keep $T=0$ and/or consider adiabatic (isentropic) changes $ds = 0$

Then

$$dE_{\text{internal}} = -PdV$$

5-585

For our free electron gas all internal energy is the sum of the microscopic kinetic energy

$$E_{\text{internal}} = \underbrace{\epsilon V}_{\substack{\text{our energy} \\ \text{per unit volume}}} \underbrace{V}_{\text{volume}}$$

$$\therefore d(\epsilon V) = -PdV$$

$$P = - \frac{d(\epsilon V)}{dV}$$

$$= -V \frac{d\epsilon}{dV} \quad \cancel{\epsilon}$$

$$= -V \frac{d\epsilon}{dn_e} \frac{dn_e}{dV} - \epsilon$$

We're hypothesizing the microscopic pressure must arise from the microscopic sum of KE which we've identified as our free electron KE. Seems OK

5-586

$n_e = \frac{N}{V}$ is electron density.

~~$\frac{dE}{dn_e}$~~

$$\frac{dn_e}{dV} = -\frac{N}{V^2} = -\frac{n_e}{V}$$

$$-V \frac{dE}{dn_e} \frac{dn_e}{dV} = n_e \frac{dE}{dn_e}$$

From p. 5-579

$$P = n_e \frac{dE}{dn_e} - E$$

$$= \frac{h^2}{2m} \left(\frac{(2\pi)^d d}{4f} \right)^{2/d} \frac{d/2}{d/2+1} \left[\left(1 + \frac{2}{d}\right) - 1 \right]$$

$$\times \left(\frac{\rho}{\mu_e m_{amu}} \right)^{1+2/d}$$

$$= \frac{h^2}{2m} \left(\frac{(2\pi)^d d}{9f} \right)^{2/d} \frac{1}{d/2+1} \left(\frac{\rho}{\mu_e m_{amu}} \right)^{1+2/d}$$

For 3-d

$$P = \frac{h^2}{2m} (3\pi^2)^{2/3} \frac{2}{5} \left(\frac{\rho}{\mu_e m_{amu}} \right)^{5/3}$$

which is the equation of state (EOS) (5-587)
of $T=0$ free electron gas.

(Very important in modeling white dwarf stars)

$$P \propto \rho^{5/3} \text{ with no } T \text{ dependence}$$

Recall the ideal gas law

$$PV = NkT$$

$$P = nkT \text{ where } n = N/V$$

$$P \propto \rho T$$

is the number of particles

If $T=0$ no pressure

The free electron gas
(AKA Fermi gas
or degenerate gas)

5-588

has pressure at zero T.

This holds up White dwarfs and neutron stars (with neutrons instead of electrons) as they cool off forever (or until their nucleons decay)

Actually one needs to account for relativistic and temperature effects.

I used to know a lot about degenerate stars in the days when I was blowing up Type Ia supernova models.

The degenerate pressure is stiffer than an ideal gas since it goes as higher power of density ($^{5/3}$ rather than 1).

Recall from p. 5-586 for $d=3$

$$\begin{aligned}
 P &= \frac{h^2}{2m} (3\pi^2)^{2/3} \frac{2}{5} \left(\frac{\rho}{\mu_e m_{\text{atom}}} \right)^{5/3} \\
 &= 1.0035 \dots \times 10^{13} \frac{\text{dyne}}{\text{cm}^2} \left(\frac{\rho}{\mu_e} \right)^{5/3} \\
 &= 1.0035 \dots \times 10^{12} \text{ Pa} \left(\frac{\rho}{\mu_e} \right)^{5/3} \\
 &\cong 10^7 \text{ atm} \left(\frac{\rho}{\mu_e} \right)^{5/3}
 \end{aligned}$$

} P still in CGS i.e., g/cm^3

$$\rho_{\text{copper}} = 3.819 \times 10^{10} \frac{\text{dyne}}{\text{cm}^2}$$

$$= 3.819 \times 10^{10} \frac{\text{Pa}}{10}$$

$$\approx 4 \times 10^5 \text{ atm}$$

(Gr-198
Ans-14
agree)

These are huge pressures

— why aren't metals explosive?

Well the confining potential balances ~~and~~ the degeneracy pressure very stably.

— Recall Earth atmosphere pressure is close to zero for solids

→ Put them in vacuum and they expand only a little bit.

5-590

Bulk modulus

- Can we calculate this?

$$B \equiv -V \frac{dP}{dV}$$

a quantity with units of pressure

$$\frac{dV}{V} = -\frac{dP}{B}$$

Bulk modulus is minus one over

So Bulk modulus is a

characteristic pressure

for changing density or volume by a factor of $\frac{1}{2}$

Say B was constant

$$\ln \frac{V}{V_0} = -\frac{P-P_0}{B}$$

$$P = P_0 e^{-\frac{P-P_0}{B}}$$

So B is an e-folding pressure.

~~It measures the relative change in volume with a change in pressure per unit pressure~~

$$\text{But } \rho = \frac{M}{V}, \quad V = \frac{M}{\rho}$$

$$dV = \left(-\frac{M}{\rho^2}\right) d\rho$$

$$dV = -\frac{V}{\rho} d\rho$$

$$\frac{dV}{V} = -\frac{d\rho}{\rho}$$

$$\frac{d\rho}{\rho} = \frac{dP}{B}$$

So Bulk modulus is also one over the relative change in density per unit pressure.

$$B = \rho \frac{dP}{d\rho}$$

$$= \frac{h^2}{2m} (3T^2)^{2/3} \frac{2}{3} \left(\frac{\rho}{\rho_e m_{euler}} \right)^{5/3}$$

for 3-d

(from p. 5-586)

$$= \frac{5}{3} P$$

~~Copper~~

$$= 1.672... \times 10^{12} \frac{\text{dyne}}{\text{cm}^2} \left(\frac{\rho}{\rho_e} \right)^{5/3}$$

$$= 1.672... \times 10^{11} \text{ Pa} \left(\frac{\rho}{\rho_e} \right)^{5/3}$$

$$\approx 1.7 \times 10^6 \text{ atm} \left(\frac{\rho}{\rho_e} \right)^{5/3}$$

still in cgs

$$B_{\text{Copper}} = 6.36... \times 10^{11} \frac{\text{dyne}}{\text{cm}^2}$$

$$= 6.36... \times 10^{10} \text{ Pa}$$

$$= 6 \times 10^5 \text{ atm}$$

(Gr-223
Ans-141)
agrees.

But the actual Copper bulk modulus is

~~1.31~~

$$= 13.4 \times 10^{10} \text{ Pa}$$

5-592

So our free electron
gas result is
off by a factor ~~of $\frac{1}{2}$~~
of $\sim \frac{1}{2}$.

Not so bad
considering we aren't
treating the actual
attractive and repulsive forces.

My guess is that one
can say the ions cancel
the ~~electron~~ free electron
charge still,

but they resist being
compressed together.

So squeezing requires both
increasing electron ∇E

Adds } and the positive PE of ion-ion
on extra resistance, } repulsion.
pressure and so increases
the Bulk Modulus.

Summary of Free electron gas | 5-593
 model Results for $T=0$

$$dN = \frac{g}{(2\pi)^d} f k^{d-1} dk \quad \left\{ \begin{array}{l} d=1, 2, 3 \\ f = \begin{cases} 1, & d=1 \\ 2\pi, & d=2 \\ 4\pi, & d=3 \end{cases} \end{array} \right.$$

(p. 5-570)

$$E = \frac{\hbar^2 k^2}{2m}$$

$$k = \left(\frac{2m}{\hbar^2}\right)^{1/2} E^{1/2}$$

$$dk = \left(\frac{2m}{\hbar^2}\right)^{1/2} \frac{1}{2} E^{-1/2} dE$$

} p. 5-570

$$dN = \frac{g}{(2\pi)^d} f \left(\frac{2m}{\hbar^2}\right)^{d/2} \frac{1}{2} E^{d/2-1} dE$$

} p. 5-571

Not it is volume independent.
 It is also shape independent.
 But that takes another proof.
 See p. 5-597

~~density~~ density of states per unit volume per unit energy
 — usually the density of states that one means (there are many density of states)

$$N_e = \frac{g f}{(2\pi)^d} \left(\frac{2m}{\hbar^2}\right)^{d/2} E_F^d \quad \text{p. 5-572}$$

5-574

$$n_e = \frac{N}{V} = \rho \sum_i \frac{x_i Z_i}{A_i M_{amu}} = \frac{\rho}{M_e M_{amu}}$$

$$\frac{1}{M_e} = \sum_i \frac{x_i Z_i}{A_i} \quad (\text{p. 5-573})$$

M_e is mean number
of nucleons per free electron

Fermi energy

$$E_F = \left(\frac{\hbar^2}{2m} \right) \left(\frac{(2\pi)^d}{g f} \right)^{2/d} \left(\frac{\rho}{M_e M_{amu}} \right)^{2/d} \quad \left\{ \begin{array}{l} \text{see} \\ \text{p. 5-574} \end{array} \right.$$

$$= \frac{\hbar^2}{2m} (3\pi^2)^{2/3} \left(\frac{\rho}{M_e M_{amu}} \right)^{2/3}$$

for $d=3$

$$= 4.166... \times 10^{-11} \text{ ergs} \left(\frac{\rho}{M_e} \right)^{2/3}$$

$$= 26.00 \text{ eV} \times \left(\frac{\rho}{M_e} \right)^{2/3}$$

Fermi Temperature

$$T_F = \frac{1}{k} \frac{\hbar^2}{2m} (3\pi^2)^{2/3} \left(\frac{\rho}{M_e M_{amu}} \right)^{2/3} \quad \text{for 3-d}$$

$$= 3.0176... \times 10^5 \text{ K} \left(\frac{\rho}{M_e} \right)^{2/3} \quad \left\{ \begin{array}{l} \text{p. 5-576} \end{array} \right.$$

$$\bar{E}_{\text{mean}} = \frac{d}{d+2} E_F$$

$$E_{\text{mean}} = \frac{3}{5} E_F \text{ for } 3\text{-d}$$

(5-594)

p. 5-578

free electron gas KE at $T=0$

~~$$E = \frac{\hbar^2}{2m} \left(\frac{(2\pi)^d d}{9f} \right)^{1+2/d} \left(\frac{\rho}{\mu_e m_{\text{amu}}} \right)^{1+2/d}$$~~

for $d=3$

~~$$E = \frac{\hbar^2}{2m}$$~~

$$E = \frac{\hbar^2}{2m} \left(\frac{(2\pi)^d d}{9f} \right)^{3/2} \frac{d/2}{d/2+1} \left(\frac{\rho}{\mu_e m_{\text{amu}}} \right)^{1+2/d}$$

free electron gas KE per unit volume

$$E = \frac{\hbar^2}{2m} \frac{3}{5} (3\pi^2)^{2/3} \left(\frac{\rho}{\mu_e m_{\text{amu}}} \right)^{5/3} \quad (5-579)$$

for 3-d

5-596

~~$$P = -V dE$$~~

$$P = n_e \frac{dE}{dn_e} - E$$

p. 5-586

$$= \frac{\hbar^2}{2m} \left(\frac{(2\pi)^d}{g_f} \right)^{2/d} \frac{1}{d/2+1} \left(\frac{\rho}{m_e m_{amu}} \right)^{1+2/d}$$

for 3-d

$$P = \frac{\hbar^2}{2m} (3\pi^2)^{2/3} \frac{2}{5} \left(\frac{\rho}{m_e m_{amu}} \right)^{5/3}$$

$$\propto \rho^{5/3}$$

a fermion

$T=0$ free electron gas result in the non-relativistic limit

$$= 1.0035 \times 10^{12} \text{ Pa} \left(\frac{\rho}{m_e} \right)^{5/3} \quad \left. \begin{array}{l} \text{p. 5-588} \\ P \text{ in CGS units.} \end{array} \right\}$$

$$B = -V \frac{dP}{dV} = \rho \frac{dP}{d\rho}$$

$$= \frac{5}{3} P$$

p. 5-591

$$B = \frac{\hbar^2}{2m} (3\pi^2)^{2/3} \frac{2}{3} \left(\frac{\rho}{M_e m_{\text{atom}}} \right)^{5/3}$$

$$= 1.672 \times 10^{11} \text{ Pa} \left(\frac{\rho}{M_e} \right)^{5/3}$$

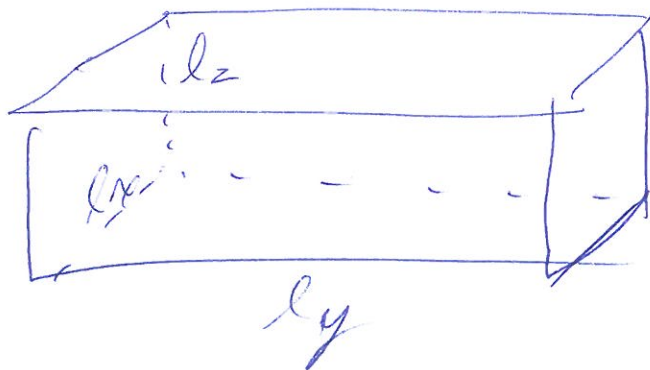
5-597

p. 5-591
with
P in
CGS
units

6) Any Shape

Solid for Free Electron Gas ~~Result~~ Formalism

We solved our free electron gas model assuming a rectangular box for both ISW & periodic BCs



~~Because this~~

The box could have any dimensions

$l_x, l_y, l_z,$

one could argue that

5-598

this could only be true
if the results in
the large asymptotic
limit ~~should be~~

and our large N results
(asymptotic results
for large)

were independent of
 l_x, l_y, l_z individually

and of their volume ~~$V = l_x l_y l_z$~~

Thus it is
plausible that our
results are shape independent.

In particular, that the
density of states is volume independent
is also shape independent.

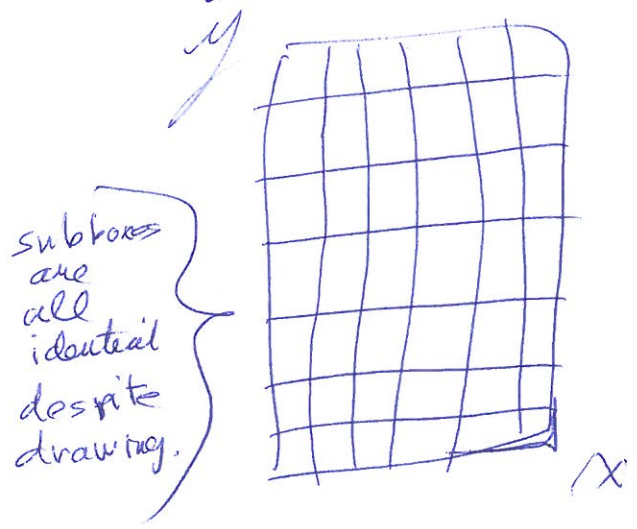
$$V = \prod_{i=1}^d l_i \quad \text{for all } d=1, 2, 3 \text{ cases.}$$

The 1-d case is simple.

It's always a rectangular box.
So we have to concern ourselves
with 2-d and 3-d.

The 2-d case actually suffices to explain shape independence

Consider a 2-rectangular box of ~~not~~ space space volume V



Each subbox is $\frac{1}{2}$ ~~(λ)~~ wavelength in the x and y direction for some k -state defined by (k_x, k_y)

- the $\frac{1}{2}$ is for I S W BCs
- the 1 is for periodic BC.

for I S W $k_i l_i = n_i \pi$, $n_i = 1, 2, \dots$

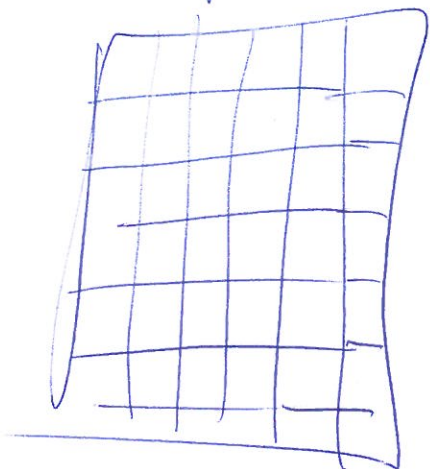
for periodic $k_i l_i = 2\pi n_i$, $n_i = 0, \pm 1, \pm 2, \dots$

and recall $\lambda_i = \frac{2\pi}{k_i}$

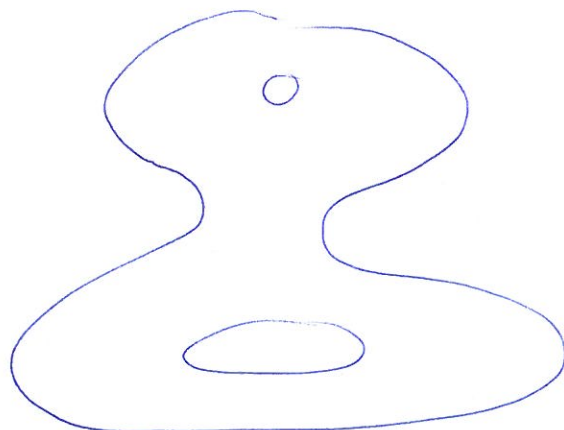
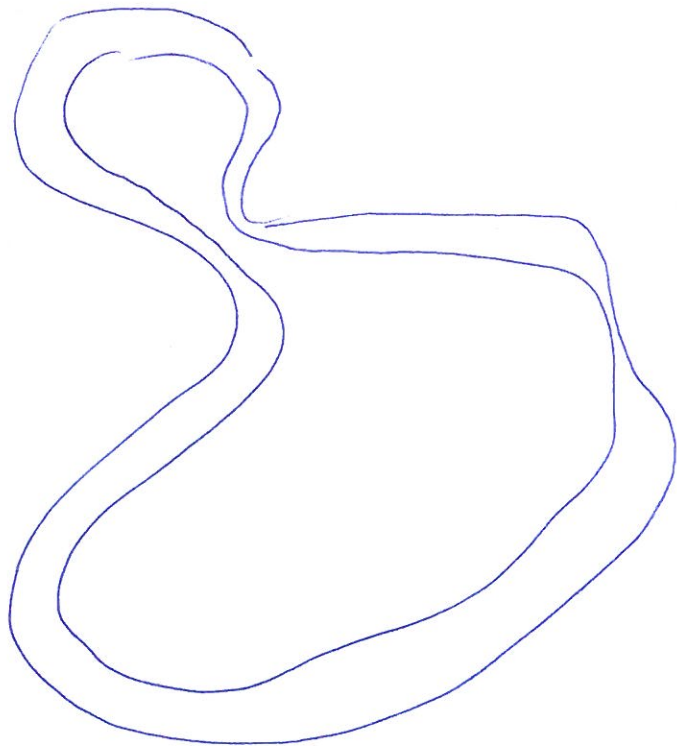
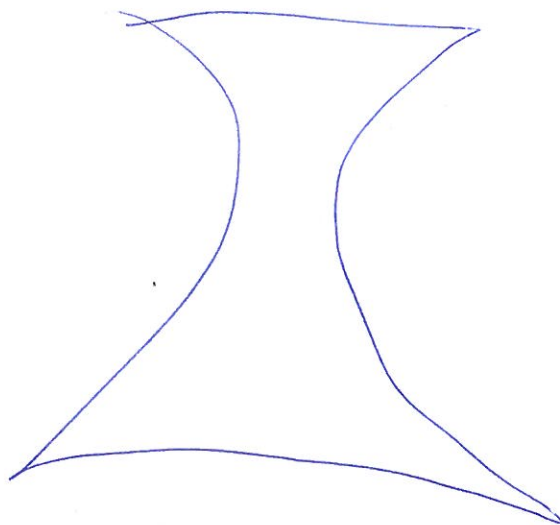
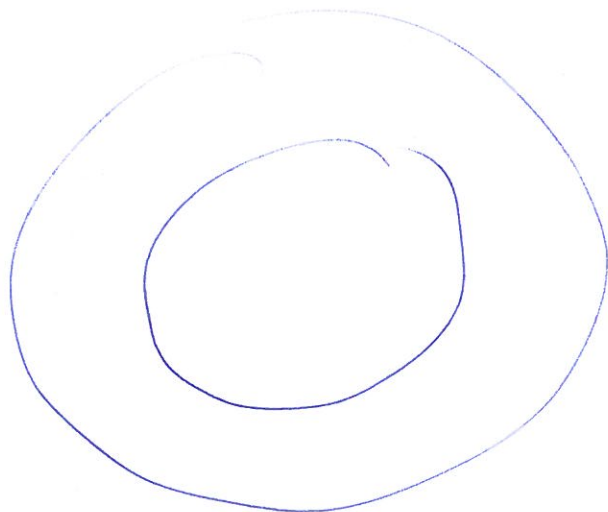
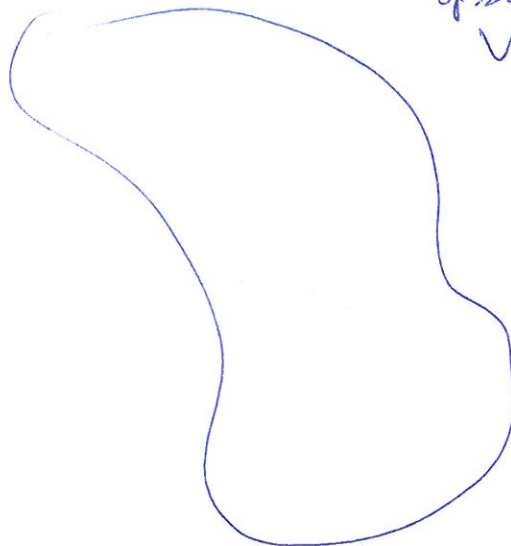
Now consider a general 2-d shape of the same volume

5-600

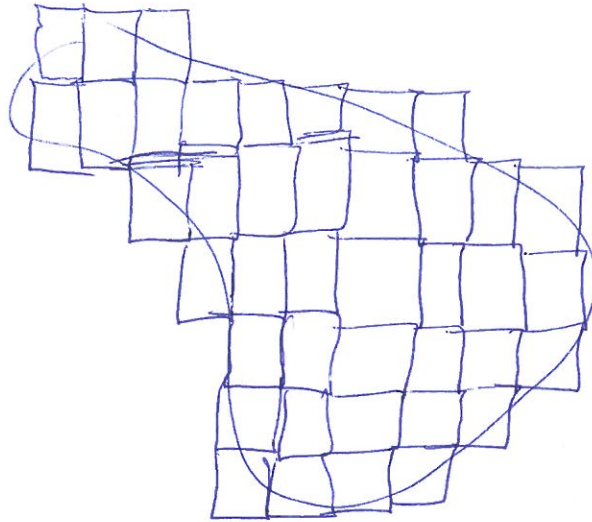
rectangular
Box of volume V



Other shapes all
of same
volume



We can take the subboxes 5-601
from the rectangular box ^{general}
and tile them over a ~~another~~ ^{arbitrary} shape.



We conserve
number
of
subboxes

The tiling is NOT perfect.

Some bits of ~~arbitrary~~ ^{the} general shape are uncovered,
and some bits of empty space
are covered.

But the tiling of general shape
approximates the general shape
and the (k_x, k_y) state is an
eigenstate of the tiled general shape.

5-602)

Now as k_x and k_y get bigger and bigger, λ_x and λ_y get smaller and smaller. The subboxes get smaller and smaller and we are able to make the tiled general shape approach the general shape more and more closely.

In the asymptotically large k_x & k_y ^{limit} ~~approx~~, the tiled general shape approaches the general shape arbitrarily closely.

So a large k_x & k_y state of the rectangular box approaches

being an k_x & k_y state of the general state. 5-603

The approach becomes arbitrarily close as k_x & k_y are made arbitrarily large.

Now small k_x or k_y states are NOT good approximations to states of the arbitrary shape. But for macroscopic (number of electrons), the large k_x and k_y states dominate both because they are more numerous & have more energy.

5-607)

Recall $d\mathcal{R} = \frac{g}{(2\pi)^d} \int k^{d-1} dk$ (see p. 5-573)

ρ_{kV} ~~density~~ number of k -states per ~~unit~~ unit k per volume (of a rectangular box)

$$k = \sqrt{k_x^2 + k_y^2}$$

So as k_x & $k_y \rightarrow$ large

$$\rho_{kV} \implies \frac{g}{(2\pi)^d} \int k^{d-1} \rightarrow \text{large}$$

for $d \geq 2$

So large k_x, k_y dominate number of k -states.

Also $\epsilon = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2}{2m} (k_x^2 + k_y^2)$
energy of a ~~state~~ k -state.

So we can expect large k_x & k_y states to dominate the behavior for ~~For~~ a rectangular box and general shape.

The same argument can be made in $d=3$

where it is even stronger

since $\rho_{kV} = \frac{g}{(2\pi)^d} k^{d-1}$

grows faster with k for larger d .

(i.e., faster for $d=3$ than for $d=2$)

Two points to ponder

- a) Can a general shape have more states than a rectangular box of the same volume.
 - i.e., higher density of states

5-600

Well I think NOT.

Well I know not
or all of
solid
state
physics
would
disappear.

But
what is
the
argument
?

~~If you had a good
tiling with~~

$$k_i l_{ij} = n_{ij} \pi$$

~~for some
straight path
across
direction,~~

~~then you'd also
get a good tiling for~~

$$k_{ij} l_{ij} = (n_{ij} \pm 1) \pi$$

I'm not sure I've got a
good argument.

~~First we~~ But the old college try,

First the general shape of volume V
must have quantized states.

Second say you started with
a rectangular box that

5-607

The tiling argument
is reasonably convincing
that the general shape
in the large k_x & k_y limit
has at least the density
of states of rectangular box
of the same volume.

Could it have a higher density
of states?

Well all solid state physics
would be wrong if it did.

But one would like a convincing
theoretical argument.

And I don't know it.

5-608

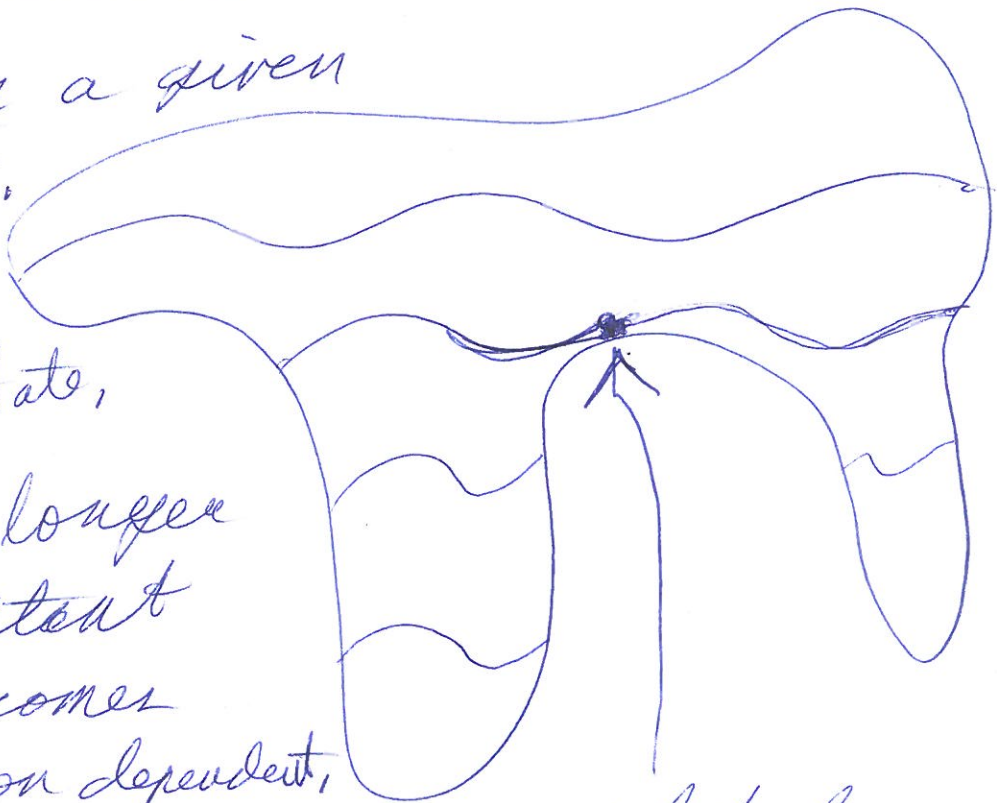
But there are ^{three} ~~a couple~~ of points

i) The general shape must have quantized stationary states

ii) For a given k -state.

Say an X -state, k_x is no longer a constant but becomes position dependent,

$$k_x(y)$$



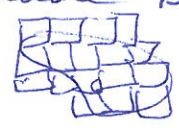
- what happens at this point?

- discontinuously the sharp infinite wall BC changes.

- argument breaks off inconclusively

iii) Maybe the tiling argument can be made better or run backwards.

- argument breaks off inconclusively



cover with squares of $n \times n$ number?

d_s k_F large enough 5-609
and $\lambda_F = \frac{2\pi}{k_F}$

large enough in
real cases for our
large k -shape independence
to be valid?

$\lambda_F \ll l$ characteristic
of sample.

~~Q~~ Recall p. 5-570

$$d\mathcal{N} = \frac{g}{(2\pi)^d} k^{d-1} dk$$

$$\therefore n_e = \frac{N}{V} = \frac{g}{(2\pi)^d} \int_0^{k_F} k^{d-1} dk$$

$$= \frac{g}{(2\pi)^d} \frac{k_F^d}{d}$$

$$\therefore k_F = \left(\frac{(2\pi)^d d}{g} n_e \right)^{1/d} n_e^{1/d}$$

5-600)

$$k_F = \left(\frac{(2\pi)^d d}{g f} \right)^{1/d} \left(\frac{\rho}{\mu_e M_{amu}} \right)^{1/d}$$

$$\lambda_F = \frac{2\pi}{k_F} = 2\pi \left(\frac{g f}{(2\pi)^d d} \right)^{1/d} \left(\frac{\mu_e M_{amu}}{\rho} \right)^{1/d}$$

$$\text{For } d=3, \lambda_F = 2\pi \left(\frac{1}{3\pi^2} \right)^{1/3} \left(\frac{\mu_e M_{amu}}{\rho} \right)^{1/3}$$

$$= 3.663 \dots \left(\frac{\mu_e}{\rho} \right)^{1/3}$$

$$= \dots$$

$$= 3.663 \dots \left(\frac{\mu_e}{\rho} \right)^{1/3}$$

$$= 3.663 \dots \left(\frac{\mu_e}{\rho} \right)^{1/3}$$

$$\lambda_{F, \text{copper}} = 7.04 \dots \text{ \AA}$$

These seem small enough for the typing argument to hold on the face of it.

For 3-d,

5-611

$$\lambda_F = 2.409... \text{ \AA} \left(\frac{\mu_e}{\rho}\right)^{1/3}$$

$$\lambda_{F_{\text{copper}}} = 4.624... \text{ \AA}$$

To me on the face of it, these seem small enough for the tiling argument to be valid.

And, of course, experiment confirms ~~the rule~~ that volume and shape independence hold.

5-612

7) Statistical Mechanics

— a hodge podge treatment mixing classical and QM ideas.

— Results justify the mixed treatment.

↳ experiment shows the mixture is the right one to some degree of approximation.

a) Consider an arbitrary General Potential Well.

— the well has quantized states.

In fact, ^{many} ~~many~~ results 5-613
~~do~~ turn out to be independent
of quantization;

So there could be a
continuum of states just
as for free particles
— which is one of the
cases of interest.

You can imagine in those
cases that we've just
gone to the continuum limit.

Each state has its own
single-particle energy

But many states are energy degenerate.

Let all states of energy E_i
constitute energy level i

5-619

Note the term energy level is used in various ways, context must decide.

Let all states of the same energy constitute an energy level i

The energy of the level i is E_i

The degeneracy of level i is g_i

Number of single-particle states in level

The number of ~~identical~~ particles in level i is N_i

The total energy of the system is

$$E = \sum_i N_i E_i$$

and the total number of particles is

$$N = \sum_i N_i$$

For our developments, 5-615
we assume E and N
are conserved. One can
generalize to cases where
they aren't, but that's
beyond our scope.

A set of $\{N_i\}$ consistent
with conservation of particles
is a configuration.

The number of distinct states
that correspond to
a configuration is
the weight W of the
configuration

$$W = W(\{N_i\})$$

5-616

The fundamental axiom
of equilibrium ^{statistical mechanics} ~~thermodynamics~~

is that the probability
of the system

being in a configuration $\{N_i\}$
is proportional to $W(\{N_i\})$.

this axiom or postulate oddly
doesn't seem to have fixed name
(but maybe I just don't know it).

Note "equilibrium statistical mechanics".

In non-equilibrium statistical mechanics
the state of the system must be
determined by solving rate equations
e.g., for level i

$$n_i \sum_j R_{ij} = \sum_j n_j R_{ji} \quad \left\{ \begin{array}{l} \text{assembly} \\ \text{time independence.} \end{array} \right.$$

Valid
by
all
of
history

This is
a time
average,
Not an
ensemble
average
But for
an ensemble,

it
will be
an
ensemble
average too
Ergodic
hypothesis
(Wik)

Not
a wave
function
collapse
probability

n_i is the occupation number
of state i

5-617

R_{ij} is the rate of transition
from state i to state j .

— One has a large matrix
equation to solve.

↳ which is frequently ~~not~~
part of a self-consistent
convergence problem since

R_{ij} often depend on n_i
and other varying parameters

In equilibrium stat mech,
we assume there are processes
that causes transitions of
particles between states
and the system between
configurations — but
we never need to specify
them other than that they exist.

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The fundamental axiom
obviates the need.

If there were no transitions
a system could
be put in an arbitrary
configuration and it
would just stay there

b) Determining $W(\{N_i, \Xi\})$
for Distinct or classical
Identical Particles

— one has to count
up the ~~configurations~~
distinct states for
configuration $\{N_i, \Xi\}$.

classical
identical
particles
are
distinct
in the
sense
they can
always
be told apart
somehow.

— There are several ways
to ~~do~~ do it or think about it.

My Way

5-619

Put the energy levels in
a mental line

$$\frac{N_1}{1} \quad \frac{N_2}{2} \quad \dots \quad \frac{N_i}{i}$$

There are N_1 particles in level 1
 N_2 in level 2
 N_i in level i

The particles are lined up to,

$$N! = C \prod N_i!$$

number of
ways of
ordering particles
on a line

distinct
ways of
slotting
particles in to levels

ordering of
particles in
a level ~~is not~~
is NOT
counted here

5-620

$$C = \frac{N!}{\prod_i N_i!}$$

distinct ways of slotting particles in to levels to get configuration $\{N_i\}$

But now consider the N_i particles in level i .

The degeneracy of the level i is g_i

You have g_i choices for putting each particle into a state.

Example states 1, 2, 3 and particles a and b
 $3^2 = 9$

1	a	1	1
1	a	2	1
1	a	3	1
1	b	1	1
1	b	2	1
1	b	3	1
2	a	1	1
2	a	2	1
2	a	3	1
2	b	1	1
2	b	2	1
2	b	3	1

For a given number of slot-ways there are $\prod_i g_i^{N_i}$ distinct states
 $W = C \prod_i g_i^{N_i}$

$g_i^{N_i}$ is the number of distinct states for a level for a particular set of N_i distinct particles

$$W = \frac{N!}{\prod_i N_i!} \prod_i g_i^{N_i}$$