Quantum Mechanics

Homework 9: Time-Independent Perturbation Theory

1. At time $t = 0$, an electron of charge $\tilde{e}$ is in the n eigenstate of an infinite square well with potential

$$V(x) = \begin{cases} 0, & x \in [0, a]; \\ \infty & x > a. \end{cases}$$

At that time, a constant electric field $\tilde{E}$ pointed in the positive x direction is suddenly applied. (Note the tildes on charge and electric field are to distinguish these quantities from the natural log base and energy.) **NOTE:** The 1-d infinite square-well eigenfunctions and eigen-energies are, respectively

$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a} x\right) \qquad \text{and} \qquad E_n = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2}{2m}\left(\frac{\pi}{a}\right)^2 n^2 ,$$

where $n = 1, 2, 3, \ldots$ The sinusoidal eigenfunctions can be expressed as exponentials: let $z = \pi x/a$, and then

$$\sin(nz) = \frac{e^{inz} - e^{-inz}}{2i} .$$

a) Use 1st order time-dependent perturbation theory to calculate the transition probabilities to all **OTHER** states m as a function of time. You should evaluate the matrix elements as explicitly: this is where all the work is naturally.

b) How do the transition probabilities vary with the energy separation between states n and m?

c) Now what is the 1st order probability of staying in the same state n?

2. "Let's play *Jeopardy*! For \$100, the answer is: An effect discovered by Einstein by means of a thermodynamic equilibrium detailed balance argument."

What is ____________, Alex?

a) spontaneous emission b) special relativity c) the photoelectric effect
d) stimulated emission e) spontaneous omission

3. Typically, strong atomic and molecular transitions are ____________ transitions.

a) electric quadrupole b) electric dipole c) magnetic dipole d) electric monopole
e) magnetic metropole

4. Say we had a classical simple harmonic oscillator (SHO) consisting of a particle with mass m and charge e and a restoring force $m\omega_0^2$ where ω_0 is the simple harmonic oscillator frequency. This SHO is subject to driving force caused by traveling electromagnetic field (i.e., light):

$$\vec{F}_{\text{drive}} = e\vec{E}_0 e^{i\omega t} ,$$

where $\vec{E}_0$ is the amplitude, ω is the driving frequency, and we have used the complex exponential form for mathematical convenience: the real part of this force is the real force. The magnetic force can be neglected for non-relativistic velocities. The Lorentz force is

$$\vec{F} = e\left(\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\right)$$

(Ja-238) and $\vec{E}$ and $\vec{B}$ are comparable in size for electromagnetic radiation, and so the magnetic force is of order v/c smaller than the electric force. (See also MEL-130.) An oscillating charge is an accelerating charge and will radiate electromagnetic radiation. The power radiated classically is

$$P = \frac{2e^2 a^2}{3c^3} ,$$

where $\vec{a}$ is the charge acceleration. This radiation causes an effective damping force given approximately by

$$\vec{F}_{\text{damp}} = -m\gamma\vec{v} ,$$

where

$$\gamma = \frac{2e^2\omega_0^2}{3mc^3} \ .$$

The full classical equation of motion of the particle is

$$m\vec{a} = -m\omega_0^2\vec{r} + e\vec{E}_0 e^{i\omega t} - m\gamma\vec{v} \ .$$

a) Solve the equation of motion for $\vec{r}$ and $\vec{a}$. **HINTS:** The old trial solution approach works. Don't forget to take the real parts although no need to work out the real part explicitly: i.e., Re[solution] is good enough for the moment.

b) Now solve for the time average of the power radiated by the particle. **HINT:** You will need the explicit real acceleration now.

c) The average power radiated must equal the average power absorbed. Let's say that the particle is in radiation flux from a single direction with specific intensity

$$I_0 = \frac{cE_0^2}{8\pi}$$

(Mi-9), where time averaging is assumed *como usual*. The power absorbed from this flux is $\sigma(\omega)I_0$, where $\sigma(\omega)$ is the cross section for energy removed. Solve for $\sigma(\omega)$ and then find show that it can be approximated by a Lorentzian function of ω with a coefficient $\pi e^2/(mc)$. **HINT:** It is convenient to absorb some of the annoying constants into another factor of γ.

d) Now rewrite the cross section as a function of $\nu = \omega/(2\pi)$ (i.e., the ordinary frequency) and then integrate over ν to get the frequency integrated cross section $\sigma_{\nu\ \mathrm{int}}$ of the system. What is the remarkable thing about $\sigma_{\nu\ \mathrm{int}}$? Think about how it relates to the system from which we derived it. Evaluate this frequency integrated cross section for an electron. **HINT:** The following constants might be useful

$$\alpha = \frac{e^2}{\hbar c} = \frac{1}{137.036} \ , \qquad \hbar = 1.05457 \times 10^{-27}\,\mathrm{erg\,s} \ , \qquad \text{and} \qquad m_e = 9.10939 \times 10^{-28}\,\mathrm{g} \ .$$