Modern Physics: Physics 305, Section 1

NAME:

Homework 8: Spin, Magnetic Dipole Moments, and the Spin-Orbit Effect: Homeworks are due as posted on the course web site. They are **NOT** handed in. The student reports that it is completed and receives one point for this. Solutions are already posted, but students are only permitted to look at the solutions after completion. The solutions are intended to be (but not necessarily are) super-perfect and go beyond a complete answer expected on a test.

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014 qmult 00100 1 4 5 easy deducto-memory: spin defined

Extra keywords: mathematical physics

1. "Let's play *Jeopardy*! For \$100, the answer is: It is the intrinsic angular momentum of a fundamental (or fundamental-for-most-purposes) particle. It is invariant and its quantum number s is always an integer or half-integer.

   What is ____________, Alex?

   a) rotation b) quantum number c) magnetic moment d) orbital angular momentum e) spin

   SUGGESTED ANSWER: (e)

   Wrong answers:

   a) Well no, but not a bad guess.

   Redaction: Jeffery, 2008jan01

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014 qmult 00110 1 4 1 easy deducto-memory: Goudsmit and Uhlenbeck, spin

2. "Let's play *Jeopardy*! For \$100, the answer is: Goudsmit and Ulhenbeck."

   a) Who are the original proposers of electron spin in 1925, Alex?
   b) Who performed the Stern-Gerlach experiment, Alex?
   c) Who are Wolfgang Pauli's evil triplet brothers, Alex?
   d) What are two delightful Dutch cheeses, Alex?
   e) What were Rosencrantz and Gildenstern's first names, Alex?

   SUGGESTED ANSWER: (a) See Le-185 and ER-276. Actually Compton hinted at the idea in 1921, but didn't follow up on it.

   Wrong Answers:

   b) Now wouldn't you think Stern & Gerlach performed the Stern-Gerlach experiment? CDL-897 calls it the Stern-Gerlach experiment.
   e) Rosencrantz and Gildenstern were real people: members of the Danish embassy to England in 1592. Frederick (Fred) Rosenkrantz later met up with Johannes Kepler and thus provides the missing link between Kepler and Shakespeare. Rosenkrantz died tragically in a duel—trying to stop it, not fight it—but Shakespeare and Stoppard have made him immortal.

   Redaction: Jeffery, 2001jan01

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014 qmult 00120 1 1 1 easy memory: spin magnitude

3. A spin s particle's angular momentum vector magnitude (in the vector model picture) is

   a) $\sqrt{s(s+1)}\hbar$. b) $s\hbar$ c) $\sqrt{s(s-1)}\hbar$ d) $-s\hbar$ e) $s(s+1)\hbar^2$

   SUGGESTED ANSWER: (a)

   Wrong answers:

   e) This is the magnitude squared or the eigenvalue of the $S_{\rm op}^2$ spin operator.

   Redaction: Jeffery, 2008jan01

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014 qmult 00130 1 1 5 easy memory: eigenvalues of spin 1/2 particle

4. The eigenvalues of a component of the spin of a spin 1/2 particle are always:

   a) $\pm\hbar$. b) $\pm\dfrac{\hbar}{3}$. c) $\pm\dfrac{\hbar}{4}$. d) $\pm\dfrac{\hbar}{5}$. e) $\pm\dfrac{\hbar}{2}$.

   SUGGESTED ANSWER: (e)

   Wrong Answers:

   Redaction: Jeffery, 2001jan01

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014 qmult 00130 1 1 2 easy memory: eigenvalues of spin s particle

5. The quantum numbers for the component of the spin of a spin s particle are always:

   a) ± 1. b) $s, s-1, s-2, \ldots, -s+1, -s$. c) $\pm\dfrac{1}{2}$. d) ± 2. e) $\pm\dfrac{1}{4}$.

   SUGGESTED ANSWER: (b)

   Wrong Answers:
   c) This is only correct for electrons.

   Redaction: Jeffery, 2001jan01

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014 qmult 00140 1 4 2 easy deducto-memory: spin and environment

6. Is the spin (not spin component) of an electron dependent on the electron's environment?

   a) Always.
   b) No. Spin is an intrinsic, unchanging property of a particle.
   c) In atomic systems, no, but when free, yes.
   d) Both yes and no.
   e) It depends on a recount in Palm Beach.

   SUGGESTED ANSWER: (b)

   Wrong Answers:
   e) Only for those who recall the American presidential election of 2000.

   Redaction: Jeffery, 2001jan01

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014 qmult 00900 1 1 3 easy memory: space and spin operators commute

7. A spatial operator and a spin operator commute:

   never. b) sometimes. c) always. d) always and never. e) to the office.

   SUGGESTED ANSWER: (c)

   Wrong Answers:
   e) I don't think this could reasonably be interpreted as a right answer.

   Redaction: Jeffery, 2001jan01

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014 qmult 01200 1 1 2 easy memory: Bohr magneton

8. What is

$$\mu_{\rm b} = \frac{e\hbar}{2m_e} = 9.27400915(26) \times 10^{-24}\,{\rm J/T} = 5.7883817555(79) \times 10^{-5}\,{\rm eV/T}\ ?$$

   a) The nuclear magneton, the characteristic magnetic moment of nuclear systems.
   b) The Bohr magneton, the characteristic magnetic moment of electronic systems.
   c) The intrinsic magnetic dipole moment of an electron.
   d) The coefficient of sliding friction.
   e) The zero-point energy of an electron.

   SUGGESTED ANSWER: (b)

   Wrong Answers:

a) The subscript "b" and the values should tell you this must be wrong.
c) No. For magnitude of the intrinsic magnetic dipole moment is $g\sqrt{3}/2$ times the Bohr magneton (ER-274).

Redaction: Jeffery, 2001jan01

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014 qmult 01210 1 1 3 easy memory: g factor

9. The g factor in quantum mechanics is the dimensionless factor for some system that multiplied by the appropriate magneton (e.g., Bohr magneton for electron systems) times the angular momentum of the system divided by $\hbar$ gives the magnetic moment of the system. For example for the electron, the intrinsic magnetic moment operator associated with intrinsic spin is given by

$$\vec{\mu}_{\rm op} = -g\mu_{\rm b}\frac{\vec{S}_{\rm op}}{\hbar} ,$$

where $\mu_{\rm b}$ is the Bohr magneton and $S_{\rm op}$ is the spin vector operator. What is g for the intrinsic magnetic moment operator of an electron to modern accuracy?

a) 1. b) 2. c) 2.0023193043622(15). d) 1/137. e) 137.

SUGGESTED ANSWER: (c)

Wrong answers:

b) This is what Dirac's relativistic quantum theory gives. Modern quantum electrodynamics gives to g to about 1 in 10^{12} (Wikipedia: 2008apr30: Precision tests of QED).

Redaction: Jeffery, 2008jan01

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014 qmult 01210 1 1 4 easy memory: magnetic moment precession

Extra keywords: The precession is also called Larmor precession (En-114)

10. An object in a uniform magnetic field with magnetic moment due to the object's angular momentum will both classically and quantum mechanically:

a) Lancy progress. b) Lorenzo regress. c) London recess. d) Larmor precess. e) Lamermoor transgress.

SUGGESTED ANSWER: (d)

Wrong Answers:

e) That too.

Redaction: Jeffery, 2001jan01

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018 qmult 00100 1 1 4 easy memory: spin-orbit interaction, hydrogenic atom

Extra keywords: spin-orbit interaction, hydrogenic atom

11. What is the main internal perturbation preventing the spinless hydrogenic eigenstates from being the actual ones?

a) The Stark effect. b) The Zeeman effect. c) The Stern-Gerlach effect. d) The spin-orbit interaction. e) The Goldhaber interaction.

SUGGESTED ANSWER: (d)

Wrong Answers:

b) The spin-orbit interaction isn't usually considered as a Zeeman effect although the physics is related: they are both magnetic interactions. The Zeeman effect is usually reserved for the effect of external magnetic fields and the expression for the interaction lacks the characteristic $\vec{L}\cdot\vec{S}$ factor of the spin-orbit interaction.
e) The Goldhabers are great old particle guys: Gerson and Maurice.

Redaction: Jeffery, 2001jan01

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018 qfull 00500 1 3 0 easy math: fine-structure energy levels

12. The hydrogen atom energy level energies corrected for the fine structure perturbations (i.e., the relativistic and spin-orbit perturbations) is

$$E(n, \ell, \pm 1/2, j) = -\frac{E_{\rm Ryd}}{n^2}\frac{m}{m_e}\left[1 + \frac{\alpha^2}{n^2}\left(\frac{n}{j+1/2} - \frac{3}{4}\right)\right] ,$$

where n is the principal quantum number, ℓ is the orbital angular momentum quantum number, $\pm 1/2$ is allowed variations of j from ℓ, j (the total angular momentum quantum number) is a redundant parameter since $j = \max(\ell \pm 1/2, 1/2)$ (but it is a convenient one),

$$E_{\rm Ryd} = \frac{1}{2}m_e c^2\alpha^2$$

is the Rydberg energy, m_e is the electron mass, $\alpha \approx 1/137$ is the fine structure constant, and

$$m = \frac{m_e m_p}{m_e + m_p}$$

is the reduced mass with m_p being the proton mass. The bracketed perturbation correction term is

$$\frac{\alpha^2}{n^2}\left(\frac{n}{j+1/2} - \frac{3}{4}\right)$$

which is of order $\alpha^2 \approx 10^{-4}$ times smaller than the unperturbed energy. Show that the perturbation term is always negative and reduces the energy from the unperturbed energy: i.e., show that

$$\frac{n}{j+1/2} - \frac{3}{4} > 0$$

in all cases.

SUGGESTED ANSWER:

The correction term for a given n is smallest for largest the maximum possible j which is

$$j_{\rm max} = n - 1 + \frac{1}{2} = n - \frac{1}{2} ,$$

where we have used the fact that the largest ℓ for a given n is $n-1$ and that j is largest for the $+1/2$ case. Let us assume the inequality

$$\frac{n}{j+1/2} - \frac{3}{4} > 0$$

and use $j_{\rm max}$ in it and show that that leads to a correct result. Behold:

$$\begin{aligned}\frac{n}{j_{\rm max}+1/2} - \frac{3}{4} &> 0 ,\\ \frac{n}{n} - \frac{3}{4} &> 0 ,\\ 1 - \frac{3}{4} &> 0 ,\\ \frac{1}{4} &> 0 ,\end{aligned}$$

which is obviously correct. Reversing the steps verifies that the inequality is valid for $j_{\rm max}$, and therefore it is valid for all j.

We have proven the inequality, and thus that the fine-structure perturbation always causes a reduction from the energy of a given energy level.

Fortran-95 Code

Redaction: Jeffery, 2008jan01

Equation Sheet for Modern Physics

These equation sheets are intended for students writing tests or reviewing material. Therefore they are neither intended to be complete nor completely explicit. There are fewer symbols than variables, and so some symbols must be used for different things: context must distinguish.

The equations are mnemonic. Students are expected to understand how to interpret and use them.

1 Constants

$$\begin{aligned}
c &= 2.99792458 \times 10^8\,\mathrm{m/s} \approx 2.998 \times 10^8\,\mathrm{m/s} \approx 3 \times 10^8\,\mathrm{m/s} \approx 1\,\mathrm{lyr/yr} \approx 1\,\mathrm{ft/ns} \\
e &= 1.602176487(40) \times 10^{-19}\,\mathrm{C} \\
E_{\mathrm{Rydberg}} &= 13.60569193(34)\,\mathrm{eV} \\
g_e &= 2.0023193043622 \qquad \text{(electron g-factor)} \\
h &= 6.62606896(33) \times 10^{-34}\,\mathrm{J\,s} = 4.13566733(10) \times 10^{-15}\,\mathrm{eV\,s} \\
hc &= 12398.419\,\mathrm{eV\,Å} \approx 10^4\,\mathrm{eV\,Å} \\
\hbar &= 1.054571628(53) \times 10^{-34}\,\mathrm{J\,s} = 6.58211899(16) \times 10^{-16}\,\mathrm{eV\,s} \\
k &= 1.3806504(24) \times 10^{-23}\,\mathrm{J/K} = 0.8617343(15) \times 10^{-4}\,\mathrm{eV/K} \approx 10^{-4}\,eV/K \\
m_e &= 9.10938215(45) \times 10^{-31}\,\mathrm{kg} = 0.510998910(13)\,\mathrm{MeV} \\
m_p &= 1.672621637(83) \times 10^{-27}\,\mathrm{kg} = 938.272013(23), \mathrm{MeV} \\
\alpha &= e^2/(4\pi\epsilon_0\hbar c) = 7.2973525376(50) \times 10^{-3} = 1/137.035999679(94) \approx 1/137 \\
\lambda_{\mathrm{C}} &= h/(m_e c) = 2.4263102175(33) \times 10^{-12}\,\mathrm{m} = 0.0024263102175(33)\,\mathrm{Å} \\
\mu_{\mathrm{B}} &= 5.7883817555(79) \times 10^{-5}\,\mathrm{eV/T}
\end{aligned}$$

2 Geometrical Formulae

$$C_{\mathrm{cir}} = 2\pi r \qquad A_{\mathrm{cir}} = \pi r^2 \qquad A_{\mathrm{sph}} = 4\pi r^2 \qquad V_{\mathrm{sph}} = \frac{4}{3}\pi r^3$$

3 Trigonometry

$$\frac{x}{r} = \cos\theta \qquad \frac{y}{r} = \sin\theta \qquad \frac{y}{x} = \tan\theta \qquad \cos^2\theta + \sin^2\theta = 1$$

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b) \qquad \cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$\cos^2\theta = \frac{1}{2}[1+\cos(2\theta)] \qquad \sin^2\theta = \frac{1}{2}[1-\cos(2\theta)] \qquad \sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$\cos(a)\cos(b) = \frac{1}{2}\left[\cos(a-b) + \cos(a+b)\right] \qquad \sin(a)\sin(b) = \frac{1}{2}\left[\cos(a-b) - \cos(a+b)\right]$$

$$\sin(a)\cos(b) = \frac{1}{2}\left[\sin(a-b) + \sin(a+b)\right]$$

4 Blackbody Radiation

$$B_\nu = \frac{2h\nu^3}{c^2}\frac{1}{[e^{h\nu/(kT)}-1]} \qquad B_\lambda = \frac{2hc^2}{\lambda^5}\frac{1}{[e^{hc/(kT\lambda)}-1]}$$

$$B_\lambda\, d\lambda = B_\nu\, d\nu \qquad \nu\lambda = c \qquad \frac{d\nu}{d\lambda} = -\frac{c}{\lambda^2}$$

$$E = h\nu = \frac{hc}{\lambda} \qquad p = \frac{h}{\lambda}$$

$$F = \sigma T^4 \qquad \sigma = \frac{2\pi^5}{15}\frac{k^4}{c^2h^3} = 5.670400(40)\times 10^{-8}\,\mathrm{W/m^2/K^4}$$

$$\lambda_{\rm max}T = \text{constant} = \frac{hc}{kx_{\rm max}} \approx \frac{1.4387751\times 10^{-2}}{x_{\rm max}}$$

$$B_{\lambda,\rm Wien} = \frac{2hc^2}{\lambda^5}e^{-hc/(kT\lambda)} \qquad B_{\lambda,\rm Rayleigh-Jeans} = \frac{2ckT}{\lambda^4}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{c}\nu = \frac{\omega}{c} \qquad k_i = \frac{\pi}{L}n_i \quad \text{standing wave BCs} \qquad k_i = \frac{2\pi}{L}n_i \quad \text{periodic BCs}$$

$$n(k)\,dk = \frac{k^2}{\pi^2}\,dk = \pi\left(\frac{2}{c}\right)\nu^2\,d\nu = n(\nu)\,d\nu$$

$$\ln(z!) \approx \left(z+\frac{1}{2}\right)\ln(z) - z + \frac{1}{2}\ln(2\pi) + \frac{1}{12z} - \frac{1}{360z^3} + \frac{1}{1260z^5} - \ldots$$

$$\ln(N!) \approx N\ln(N) - N$$

$$\rho(E)\,dE = \frac{e^{-E/(kT)}}{kT}\,dE \qquad P(n) = (1-e^{-\alpha})e^{-n\alpha} \qquad \alpha = \frac{h\nu}{kT}$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \qquad f(x-vt) \qquad f(kx-\omega t)$$

5 Photons

$$KE = h\nu - w \qquad \Delta\lambda = \lambda_{\rm scat} - \lambda_{\rm inc} = \lambda_{\rm C}(1-\cos\theta)$$

$$\ell = \frac{1}{n\sigma} \qquad \rho = \frac{e^{-s/\ell}}{\ell} \qquad \langle s^m\rangle = \ell^m m!$$

6 Matter Waves

$$\lambda = \frac{h}{p} \qquad p = \hbar k \qquad \Delta x \Delta p \geq \frac{\hbar}{2} \qquad \Delta E \Delta t \geq \frac{\hbar}{2}$$

$$\Psi(x,t) = \int_{-\infty}^{\infty} \phi(k)\Psi_k(x,t)\,dk \qquad \phi(k) = \int_{-\infty}^{\infty} \Psi(x,0)\frac{e^{-ikx}}{\sqrt{2\pi}}\,dk$$

$$v_{\rm g} = \left.\frac{d\omega}{dk}\right|_{k_0} = \frac{\hbar k_0}{m} = \frac{p_0}{m} = v_{\rm clas,0}$$

7 Non-Relativistic Quantum Mechanics

$$H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V \qquad T = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} \qquad H\Psi = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V\Psi = i\hbar\frac{\partial\Psi}{\partial t}$$

$$\rho = \Psi^*\Psi \qquad \rho\,dx = \Psi^*\Psi\,dx$$

$$A\phi_i = a_i\phi_i \qquad f(x) = \sum_i c_i\phi_i \qquad \int_a^b \phi_i^*\phi_j\,dx = \delta_{ij} \qquad c_j = \int_a^b \phi_j^* f(x)\,dx$$

$$[A,B] = AB - BA$$

$$P_i = |c_i|^2 \qquad \langle A\rangle = \int_{-\infty}^{\infty} \Psi^* A\Psi\,dx = \sum_i |c_i|^2 a_i \qquad H\psi = E\psi \qquad \Psi(x,t) = \psi(x)e^{-i\omega t}$$

$$p_{\rm op}\phi = \frac{\hbar}{i}\frac{\partial\phi}{\partial x} = p\phi \qquad \phi = \frac{e^{ikx}}{\sqrt{2\pi}} \qquad \frac{\partial^2\psi}{\partial x^2} = \frac{2m}{\hbar^2}(V-E)\psi$$

$$|\Psi\rangle \qquad \langle\Psi| \qquad \langle x|\Psi\rangle = \Psi(x) \qquad \langle\vec r|\Psi\rangle = \Psi(\vec r) \qquad \langle k|\Psi\rangle = \Psi(k) \qquad \langle\Psi_i|\Psi_j\rangle = \langle\Psi_j|\Psi_i\rangle^*$$

$$\langle\phi_i|\Psi\rangle = c_i \qquad 1_{\rm op} = \sum_i |\phi_i\rangle\langle\phi_i| \qquad |\Psi\rangle = \sum_i |\phi_i\rangle\langle\phi_i|\Psi\rangle = \sum_i c_i|\phi_i\rangle$$

$$1_{\rm op} = \int_{-\infty}^{\infty} dx\,|x\rangle\langle x| \qquad \langle\Psi_i|\Psi_j\rangle = \int_{-\infty}^{\infty} dx\,\langle\Psi_i|x\rangle\langle x|\Psi_j\rangle \qquad A_{ij} = \langle\phi_i|A|\phi_j\rangle$$

$$Pf(x) = f(-x) \qquad P\frac{df(x)}{dx} = \frac{df(-x)}{d(-x)} = -\frac{df(-x)}{dx} \qquad Pf_{\rm e/o}(x) = \pm f_{\rm e/o}(x)$$

$$P\frac{df_{\rm e/o}(x)}{dx} = \mp\frac{df_{\rm e/o}(x)}{dx}$$

8 Spherical Harmonics

$$Y_{0,0} = \frac{1}{\sqrt{4\pi}} \qquad Y_{1,0} = \left(\frac{3}{4\pi}\right)^{1/2} \cos(\theta) \qquad Y_{1,\pm 1} = \mp \left(\frac{3}{8\pi}\right)^{1/2} \sin(\theta) e^{\pm i\phi}$$

$$L^2 Y_{\ell m} = \ell(\ell+1)\hbar^2 Y_{\ell m} \qquad L_z Y_{\ell m} = m\hbar Y_{\ell m} \qquad |m| \leq \ell \qquad m = -\ell, -\ell+1, \ldots, \ell-1, \ell$$

0	1	2	3	4	5	6	...
s	p	d	f	g	h	i	...

9 Hydrogenic Atom

$$\psi_{n\ell m} = R_{n\ell}(r) Y_{\ell m}(\theta, \phi) \qquad \ell \leq n-1 \qquad \ell = 0, 1, 2, \ldots, n-1$$

$$a_z = \frac{a_0}{Z} \left(\frac{m_e}{m_{\text{reduced}}}\right) \qquad a_0 = \frac{\hbar}{m_e c\alpha} = \frac{\lambda_{\text{C}}}{2\pi\alpha} \qquad m_{\text{reduced}} = \frac{m_1 m_2}{m_1 + m_2}$$

$$R_{10} = 2 a_Z^{-3/2} e^{-r/a_Z} \qquad R_{20} = \frac{1}{\sqrt{2}} a_Z^{-3/2} \left(1 - \frac{1}{2}\frac{r}{a_Z}\right) e^{-r/(2a_Z)}$$

$$R_{21} = \frac{1}{\sqrt{24}} a_Z^{-3/2} \frac{r}{a_Z} e^{-r/(2a_Z)}$$

$$R_{n\ell} = -\left\{\left(\frac{2}{n a_Z}\right)^3 \frac{(n-\ell-1)!}{2n[(n+\ell)!]^3}\right\}^{1/2} e^{-\rho/2} \rho^\ell L_{n+\ell}^{2\ell+1}(\rho) \qquad \rho = \frac{2r}{n r_Z}$$

$$L_q(x) = e^x \left(\frac{d}{dx}\right)^q \left(e^{-x} x^q\right) \quad \text{Rodrigues's formula for the Laguerre polynomials}$$

$$L_q^j(x) = \left(\frac{d}{dx}\right)^j L_q(x) \qquad \text{Associated Laguerre polynomials}$$

$$\langle r \rangle_{n\ell m} = \frac{a_Z}{2}\left[3n^2 - \ell(\ell+1)\right]$$

$$\text{Nodes} = (n-1) - \ell \qquad \text{not counting zero or infinity}$$

$$E_n = -\frac{1}{2} m_e c^2 \alpha^2 \frac{Z^2}{n^2} \frac{m_{\text{reduced}}}{m_e} = -E_{\text{Ryd}} \frac{Z^2}{n^2} \frac{m_{\text{reduced}}}{m_e} \approx -13.606 \times \frac{Z^2}{n^2} \frac{m_{\text{reduced}}}{m_e} \text{ eV}$$

10 Spin, Magnetic Dipole Moment, Spin-Orbit Interaction

$$S_{\text{op}}^2 = \frac{3}{4}\hbar \begin{pmatrix} 1 & 0 \\ 01 & \end{pmatrix} \qquad s = \frac{1}{2} \qquad s(s+1) = \frac{3}{4} \qquad S = \sqrt{s(s+1)}\hbar = \frac{\sqrt{3}}{2}\hbar$$

$$S_{z,\text{op}} = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad m_s = \pm\frac{1}{2} \qquad \chi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \chi_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\mu_\text{b} = \frac{e\hbar}{2m_e} = 9.27400915(26) \times 10^{-24}\,\text{J/T} = 5.7883817555(79) \times 10^{-5}\,\text{eV/T}$$

$$\mu_\text{nuclear} = \frac{e\hbar}{2m_p} = 5.05078324(13) \times 10^{-27}\,\text{J/T} = 3.1524512326(45) \times 10^{-8}\,\text{eV/T}$$

$$\vec{\mu}_\ell = -g_\ell \mu_\text{b} \frac{\vec{L}}{\hbar} \qquad \mu_\ell = g_\ell \mu_\text{b} \ell(\ell+1) \qquad \mu_{\ell,z} = -g_\ell \mu_\text{b} \frac{L_z}{\hbar} \qquad \mu_{\ell,z} = -g_\ell \mu_\text{b} m_\ell$$

$$\vec{\tau} = \vec{\mu} \times \vec{B} \qquad PE = -\vec{\mu} \cdot \vec{B} \qquad \vec{F} = \Delta(\vec{\mu} \cdot \vec{B}) \qquad F_z = \sum_j \mu_j \frac{\partial B_j}{\partial z} \qquad \vec{\omega} = \frac{g_\ell \mu_\text{b}}{\hbar}\vec{B}$$

$$\vec{J} = \vec{L} + \vec{S} \qquad J = \sqrt{j(j+1)}\hbar \qquad j = |\ell - s|, |\ell - s + 1|, \ldots, \ell + s \quad \text{triangle rule}$$

$$J_z = m_j \hbar \qquad m_j = -j, -j+1, \ldots, j-1, j$$

$$E(n, \ell, \pm 1/2, j) = -\frac{E_\text{Ryd}}{n^2}\frac{m}{m_e}\left[1 + \frac{\alpha^2}{n^2}\left(\frac{n}{j+1/2} - \frac{3}{4}\right)\right]$$

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11 Special Relativity

$$c = 2.99792458 \times 10^8\,\text{m/s} \approx 2.998 \times 10^8\,\text{m/s} \approx 3 \times 10^8\,\text{m/s} \approx 1\,\text{lyr/yr} \approx 1\,\text{ft/ns}$$

$$\beta = \frac{v}{c} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}} \qquad \gamma(\beta << 1) = 1 + \frac{1}{2}\beta^2 \qquad \tau = ct$$

Galilean Transformations	Lorentz Transformations
$x' = x - \beta\tau$	$x' = \gamma(x - \beta\tau)$
$y' = y$	$y' = y$
$z' = z$	$z' = z$
$\tau' = \tau$	$\tau' = \gamma(\tau - \beta x)$
$\beta'_\text{obj} = \beta_\text{obj} - \beta$	$\beta'_\text{obj} = \dfrac{\beta_\text{obj} - \beta}{1 - \beta\beta_\text{obj}}$

$$\ell = \ell_\text{proper}\sqrt{1-\beta^2} \qquad \Delta\tau_\text{proper} = \Delta\tau\sqrt{1-\beta^2}$$

$$m = \gamma m_0 \qquad p = mv = \gamma m_0 c\beta \qquad E_0 = m_0 c^2 \qquad E = \gamma E_0 = \gamma m_0 c^2 = mc^2$$

$$E = mc^2 \qquad E = \sqrt{(pc)^2 + (m_0c^2)^2}$$

$$KE = E - E_0 = \sqrt{(pc)^2 + (m_0c^2)^2} - m_0c^2 = (\gamma - 1)m_0c^2$$

$$f = f_{\text{proper}}\sqrt{\frac{1-\beta}{1+\beta}} \quad \text{for source and detector separating}$$

$$f(\beta << 1) = f_{\text{proper}}\left(1 - \beta + \frac{1}{2}\beta^2\right)$$

$$f_{\text{trans}} = f_{\text{proper}}\sqrt{1-\beta^2} \qquad f_{\text{trans}}(\beta << 1) = f_{\text{proper}}\left(1 - \frac{1}{2}\beta^2\right)$$

$$\tau = \beta x + \gamma^{-1}\tau' \quad \text{for lines of constant } \tau'$$

$$\tau = \frac{x - \gamma^{-1}x'}{\beta} \quad \text{for lines of constant } x'$$

$$x' = \frac{x_{\text{intersection}}}{\gamma} = x'_{x\text{ scale}}\sqrt{\frac{1-\beta^2}{1+\beta^2}} \qquad \tau' = \frac{\tau_{\text{intersection}}}{\gamma} = \tau'_{\tau\text{ scale}}\sqrt{\frac{1-\beta^2}{1+\beta^2}}$$

$$\theta_{\text{Mink}} = \tan^{-1}(\beta)$$