Modern Physics: Physics 305, Section 1

NAME:

Homework 3: Photons Homeworks are due as posted on the course web site. They are **NOT** handed in. The student reports that it is completed and receives one point for this. Solutions are already posted, but students are only permitted to look at the solutions after completion. The solutions are intended to be (but not necessarily are) super-perfect and go beyond a complete answer expected on a test.

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003 qmult 10030 1 1 2 easy memory: GR accelerated motion
Extra keywords: Gre-67

1. In general relativity, free-fall with no other forces acting is at least in Gre-67's interpretation (and this interpretation is different from one used in most physics contexts):

   a) accelerated motion. b) unaccelerated motion. c) simple harmonic oscillation. d) anharmonic oscillation. e) static equilibrium.

   SUGGESTED ANSWER: (b) Probably, all relativists agree with Greene, but you never know: some may interpret things differently.

   Wrong answers:
   c) A nonsense answer.

   Redaction: Jeffery, 2008jan01

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003 qmult 11010 1 4 3 easy deducto-memory: general relativity 1
Extra keywords: Gre-498

2. "Let's play *Jeopardy*! For $100, the answer is: It is a theory in which mass-energy determines the geometry of spacetime and in which the geometry of spacetime (which is the cause of gravity) plus other forces determine the motion of mass-energy."

   What is ____________, Alex?

   a) quantum mechanics b) special relativity c) general relativity d) Newtonian physics e) Maxwellian electromagnetism

   SUGGESTED ANSWER: (c)

   Wrong answers:
   b) As Lurch would say AAAARGH.

   Redaction: Jeffery, 2008jan01

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003 qmult 13030 1 1 4 easy memory: dynamic spacetime
Extra keywords: Gre-75

3. Because spacetime responds to mass-energy in general relativity, one can say that in general relativity spacetime is:

   a) static. b) ellipsoidal. c) hyperbolical. d) dynamic. e) flat.

   SUGGESTED ANSWER: (d) By saying a thing is dynamic, one means in this context that the thing can be acted upon by something or responds to something.

   Wrong answers:
   e) At present, space seems flat, but this isn't a valid completion of the sentence.

   Redaction: Jeffery, 2008jan01

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002 qmult 00210 1 4 5 easy deducto-memory: photoelectric effect

4. "Let's play *Jeopardy*! For $100, the answer is: It is the emission of electrons from matter caused by the absorption of photons. The effect in some sense includes photoionization as a subcategory since photoionization agrees with the definition, but other cases such as emission of non-localized electrons in materials are also included in the effect and are what one usually thinks of when one says the name of the effect."

What is the ____________, Alex?

a) Mössbauer effect b) Hall effect c) quantum Hall effect d) Zeeman effect
e) photoelectric effect

SUGGESTED ANSWER: (e)

Wrong answers:
b) Well no.

Redaction: Jeffery, 2008jan01

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002 qmult 00320 1 3 1 easy math: work function of gold

5. Given that the work function of gold (Au) is 4.8 eV, what is the maximum wavelength of light that will cause the emission of a photoelectron? **HINT:** $hc = 12398.419\,\text{eV}\,\text{Å}$.

a) 2600 Å. b) 3000 Å. c) 5000 Å. d) 7000 Å. e) 10000 Å.

SUGGESTED ANSWER: (a) Well

$$w = h\nu_{\text{min}} = \frac{hc}{\lambda_{\text{max}}} ,$$

where w is the work function, ν_{min} is the minimum frequency for emission, and λ_{max} is the maximum wavelength for emission. Thus,

$$\lambda_{\text{max}} = \frac{hc}{w} = \frac{12398.419}{4.8} = 2600\,\text{Å} .$$

Wrong answers:
a) A nonsense answer.

Fortran-95 Code

```
        print*
        caa=clight*1.d+8
        hc=planckev*caa
        w=4.8d0
        xlambda=hc/w
        print*,'planckev,hc,w,xlambda'
        print*,planckev,hc,w,xlambda
!   4.13566743E-15 12398.419043102429 4.8 2583.0039673130063
```

Redaction: Jeffery, 2008jan01

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002 qmult 00430 1 1 3 easy memory: Compton equation
Extra keywords: ER-37

6. The Compton equation can be derived using the photon picture of electromagnetic radiation and:

a) the photoelectric effect. b) classical energy and momentum conservation laws.
c) relativistic energy and momentum conservation laws. d) the Planck spectrum.
e) the Einstein equation $E = mc^2$.

SUGGESTED ANSWER: (c) Actually in special relativity, the conservation of energy and momentum is subsumed under the conservation of 4-momentum: of the 4-momentum vector, three component are the momentum components and one is the energy component (La-42–43). Of course, for heuristic reasons and tradition, one often continues to speak of conservation of energy and momentum separately.

Wrong answers:
b) Well no.

Fortran-95 Code

Redaction: Jeffery, 2008jan01

002 qmult 00440 1 3 5 easy math: proton Compton wavelength

7. The standard Compton equation is

$$\Delta\lambda = \lambda_{\rm scat} - \lambda_{\rm inc} = \lambda_{\rm C}(1-\cos\theta) \ ,$$

where $\lambda_{\rm scat}$ is the wavelength of the scattered photon, $\lambda_{\rm inc}$ is the wavelength of the incident photon, θ is the scattering angle (i.e., the angle between the incident and scattering directions), and $\lambda_{\rm C}$ is the Compton wavelength. Note that

$$\lambda_{\rm C} = \frac{h}{m_e c} = 2.426310238(16)\times 10^{-12}\,{\rm m}$$

where h is Planck's constant, m_e is the electron mass, and c is the speed of light. Compton scattering by protons can occur too. What is the proton Compton wavelength?

a) $2.426310238(16)\times 10^{-15}\,{\rm m} \approx 24.3\,{\rm fm}$.
b) $2.426310238(16)\times 10^{-15}\,{\rm m} \approx 2.4\,{\rm fm}$.
c) $2.426310238(16)\times 10^{-12}\,{\rm m} \approx 0.024\,\text{Å}$.
d) $1.321409855\times 10^{-15}\,{\rm m} \approx 13.2\,{\rm fm}$.
e) $1.321409855\times 10^{-15}\,{\rm m} \approx 1.3\,{\rm fm}$.

SUGGESTED ANSWER: (e)

Wrong answers:

c) As Lurch would say AAAAaargh.

Fortran-95 Code

```
        print*
        xlmp=planck/(proton_mass*clight) ! CGS
        xlmp=xlmp*.01d0  ! MKS
        print*,'The proton Compton wavelength'
        print*,xlmp
!  1.3214098546973617E-15
```

Redaction: Jeffery, 2008jan01

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002 qmult 00750 1 4 4 easy deducto-memory: positronium

8. "Let's play *Jeopardy*! For \$100, the answer is: A bound state of matter which is usually formed by a positron on its way to annihilation with an electron. It has a mean lifetime of $1.25\times 10^{-10}\,{\rm s}$ if it forms in the singlet ground state."

What is ____________, Alex?

a) pragmatium b) plutonium c) protonium d) positronium e) protesium

SUGGESTED ANSWER: (d) See Wikipedia (positronium, 2008jan20). Positronium can live longer if it forms in other states. The Wikipedia article doesn't give an overall mean lifetime.

Wrong answers:

a) As Lurch would say AAAARGH.

Redaction: Jeffery, 2008jan01

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002 qfull 00830 1 3 0 easy math: photon probability density

Extra keywords: ER-52-8

9. You are given that the probability density for photon removal from a beam along a beam path is

$$\rho(s) = \frac{e^{-s/\ell}}{\ell} \ ,$$

where s is the path coordinate from some initial position and ℓ turns out to be the mean free path.

a) Find the probability for removal by point s.

b) Find the probability for survival to point s.

c) Find the moments of the probability distribution.

d) Find the standard deviation of the distribution.

SUGGESTED ANSWER:

a) Behold:

$$P_{\text{removal}}(s) = \int_0^s \frac{e^{-s'/\ell}}{\ell}\, ds' = -e^{s'/\ell}|_0^s = 1 - e^{-s/\ell} \ .$$

b) Behold:

$$P_{\text{survival}}(s) = 1 - P_{\text{removal}}(s) = e^{-s/\ell} \ .$$

c) Behold:

$$\begin{aligned}
\langle s^m \rangle &= \int_0^\infty s^m \frac{e^{-s'/\ell}}{\ell}\, ds \\
&= \ell^m \int_0^\infty t^m e^{-t}\, dt \\
&= \begin{cases} \ell^m m! & \text{in general;} \\ 1! & \text{for } m = 0 \text{ which is just the norm;} \\ \ell & \text{for } m = 1 \text{ which is just the mean free path;} \\ 2\ell^2 & \text{for } m = 2, \end{cases}
\end{aligned}$$

where we have used the factorial function

$$z! = \int_0^\infty t^z e^{-t}\, dt \ ,$$

where z is any real number not a negative integer (Arf-453–454; WA-528–530). Actually I think z can be any complex number not a negative real integer, but the sources are ambiguous.

d) The variance is

$$\sigma^2 = \langle (s - \langle s \rangle)^2 \rangle = \langle s^2 \rangle - 2\langle s \langle s \rangle \rangle + \langle s \rangle^2 = \langle s^2 \rangle - \langle s \rangle^2 = 2\ell^2 - \ell^2 = \ell^2 \ ,$$

and thus the standard deviation is

$$\sigma = \ell$$

which is coincidentally the same as the mean free path.

Redaction: Jeffery, 2008jan01

Equation Sheet for Modern Physics

These equation sheets are intended for students writing tests or reviewing material. Therefore they are neither intended to be complete nor completely explicit. There are fewer symbols than variables, and so some symbols must be used for different things: context must distinguish.

The equations are mnemonic. Students are expected to understand how to interpret and use them.

1 Geometrical Formulae

$$C_{\rm cir} = 2\pi r \qquad A_{\rm cir} = \pi r^2 \qquad A_{\rm sph} = 4\pi r^2 \qquad V_{\rm sph} = \frac{4}{3}\pi r^3$$

2 Trigonometry

$$\frac{x}{r} = \cos\theta \qquad \frac{y}{r} = \sin\theta \qquad \frac{y}{x} = \tan\theta \qquad \cos^2\theta + \sin^2\theta = 1$$

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b) \qquad \cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$\cos^2\theta = \frac{1}{2}[1+\cos(2\theta)] \qquad \sin^2\theta = \frac{1}{2}[1-\cos(2\theta)] \qquad \sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$\cos(a)\cos(b) = \frac{1}{2}\left[\cos(a-b)+\cos(a+b)\right] \qquad \sin(a)\sin(b) = \frac{1}{2}\left[\cos(a-b)-\cos(a+b)\right]$$

$$\sin(a)\cos(b) = \frac{1}{2}\left[\sin(a-b)+\sin(a+b)\right]$$

3 Blackbody Radiation

$$B_\nu = \frac{2h\nu^3}{c^2}\frac{1}{[e^{h\nu/(kT)}-1]} \qquad B_\lambda = \frac{2hc^2}{\lambda^5}\frac{1}{[e^{hc/(kT\lambda)}-1]}$$

$$B_\lambda\,d\lambda = B_\nu\,d\nu \qquad \nu\lambda = c \qquad \frac{d\nu}{d\lambda} = -\frac{c}{\lambda^2}$$

$$k = 1.3806505(24)\times10^{-23}\,\mathrm{J/K} \qquad c = 2.99792458\times10^{8}\,\mathrm{m}$$

$$h = 6.6260693(11)\times10^{-34}\,\mathrm{J\,s} = 4.13566743(35)\times10^{-15}\,\mathrm{eV\,s}$$

$$\hbar = \frac{h}{2\pi} = 1.05457168(18)\times10^{-34}\,\mathrm{J\,s}$$

$$hc = 12398.419\,\mathrm{eV\,\AA} \approx 10^4\,\mathrm{eV\,\AA} \qquad E = h\nu = \frac{hc}{\lambda} \qquad p = \frac{h}{\lambda}$$

$$F = \sigma T^4 \qquad \sigma = \frac{2\pi^5}{15}\frac{k^4}{c^2h^3} = 5.670400(40) \times 10^{-8}\,\mathrm{W/m^2/K^4}$$

$$\lambda_{\max} T = \text{constant} = \frac{hc}{kx_{\max}} \approx \frac{1.4387751 \times 10^{-2}}{x_{\max}}$$

$$B_{\lambda,\mathrm{Wien}} = \frac{2hc^2}{\lambda^5} e^{-hc/(kT\lambda)} \qquad B_{\lambda,\mathrm{Rayleigh-Jeans}} = \frac{2ckT}{\lambda^4}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{c}\nu = \frac{\omega}{c} \qquad k_i = \frac{\pi}{L} n_i \quad \text{standing wave BCs} \qquad k_i = \frac{2\pi}{L} n_i \quad \text{periodic BCs}$$

$$n(k)\,dk = \frac{k^2}{\pi^2}\,dk = \pi\left(\frac{2}{c}\right)\nu^2\,d\nu = n(\nu)\,d\nu$$

$$\ln(z!) \approx \left(z + \frac{1}{2}\right)\ln(z) - z + \frac{1}{2}\ln(2\pi) + \frac{1}{12z} - \frac{1}{360z^3} + \frac{1}{1260z^5} - \ldots$$

$$\ln(N!) \approx N\ln(N) - N$$

$$\rho(E)\,dE = \frac{e^{-E/(kT)}}{kT}\,dE \qquad P(n) = (1 - e^{-\alpha})e^{-n\alpha} \qquad \alpha = \frac{h\nu}{kT}$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \qquad f(x - vt) \qquad f(kx - \omega t)$$

4 Photons

$$KE = h\nu - w \qquad \Delta\lambda = \lambda_{\mathrm{scat}} - \lambda_{\mathrm{inc}} = \lambda_{\mathrm{C}}(1 - \cos\theta)$$

$$\lambda_{\mathrm{C}} = \frac{h}{m_e c} = 2.426310238(16) \times 10^{-12}\,\mathrm{m} \qquad e = 1.602176487(40) \times 10^{-19}\,\mathrm{C}$$

$$m_e = 9.1093826(16) \times 10^{-31}\,\mathrm{kg} = 0.510998918(44)\,\mathrm{MeV}$$

$$m_p = 1.67262171(29) \times 10^{-27}\,\mathrm{kg} = 938.272029(80)\,\mathrm{MeV}$$

$$\ell = \frac{1}{n\sigma} \qquad \rho = \frac{e^{-s/\ell}}{\ell} \qquad \langle s^m \rangle = \ell^m m!$$

5 Special Relativity

$$c = 2.99792458 \times 10^8\,\mathrm{m/s} \approx 2.998 \times 10^8\,\mathrm{m/s} \approx 3 \times 10^8\,\mathrm{m/s} \approx 1\,\mathrm{lyr/yr} \approx 1\,\mathrm{ft/ns}$$

$$\beta = \frac{v}{c} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}} \qquad \gamma(\beta << 1) = 1 + \frac{1}{2}\beta^2 \qquad \tau = ct$$

Galilean Transformations	Lorentz Transformations
$x' = x - \beta\tau$	$x' = \gamma(x - \beta\tau)$
$y' = y$	$y' = y$
$z' = z$	$z' = z$
$\tau' = \tau$	$\tau' = \gamma(\tau - \beta x)$
$\beta'_{\mathrm{obj}} = \beta_{\mathrm{obj}} - \beta$	$\beta'_{\mathrm{obj}} = \dfrac{\beta_{\mathrm{obj}} - \beta}{1 - \beta\beta_{\mathrm{obj}}}$

$$\ell = \ell_{\mathrm{proper}}\sqrt{1-\beta^2} \qquad \Delta\tau_{\mathrm{proper}} = \Delta\tau\sqrt{1-\beta^2}$$

$$m = \gamma m_0 \qquad p = mv = \gamma m_0 c\beta \qquad E_0 = m_0c^2 \qquad E = \gamma E_0 = \gamma m_0 c^2 = mc^2$$

$$E = mc^2 \qquad E = \sqrt{(pc)^2 + (m_0c^2)^2}$$

$$KE = E - E_0 = \sqrt{(pc)^2 + (m_0c^2)^2} - m_0c^2 = (\gamma - 1)m_0c^2$$

$$f = f_{\mathrm{proper}}\sqrt{\frac{1-\beta}{1+\beta}} \quad \text{for source and detector separating}$$

$$f(\beta << 1) = f_{\mathrm{proper}}\left(1 - \beta + \frac{1}{2}\beta^2\right)$$

$$f_{\mathrm{trans}} = f_{\mathrm{proper}}\sqrt{1-\beta^2} \qquad f_{\mathrm{trans}}(\beta << 1) = f_{\mathrm{proper}}\left(1 - \frac{1}{2}\beta^2\right)$$

$$\tau = \beta x + \gamma^{-1}\tau' \quad \text{for lines of constant } \tau'$$

$$\tau = \frac{x - \gamma^{-1}x'}{\beta} \quad \text{for lines of constant } x'$$

$$x' = \frac{x_{\mathrm{intersection}}}{\gamma} = x'_{x\ \mathrm{scale}}\sqrt{\frac{1-\beta^2}{1+\beta^2}} \qquad \tau' = \frac{\tau_{\mathrm{intersection}}}{\gamma} = \tau'_{\tau\ \mathrm{scale}}\sqrt{\frac{1-\beta^2}{1+\beta^2}}$$

$$\theta_{\mathrm{Mink}} = \tan^{-1}(\beta)$$