Modern Physics 305: 1st Exam NAME:
2008 February 25 Monday

Instructions: There are 10 multiple-choice questions each worth 2 marks for a total of 20 marks altogether. Choose the **BEST** answer, completion, etc., and darken fully the appropriate circle on the table provided below. Read all responses carefully. **NOTE** long detailed preambles and responses won't depend on hidden keywords: keywords in such preambles and responses are bold-faced capitalized.

There are **FOUR** full answer questions each worth 10 marks for a total of 40 marks altogether. Answer them all on the paper provided. It is important that you **SHOW** (**SHOW, SHOW, SHOW**) how you got the answer.

This is a **CLOSED-BOOK** exam. **NO** cheat sheets allowed. An equation sheet is provided. Calculators are permitted for calculations. Cell phones **MUST** be turned off. The test is out of 60 marks altogether.

This a 50-minute test. Remember your name (and write it down on the exam too).

Answer Table for the Multiple-Choice Questions

	a	b	c	d	e		a	b	c	d	e
1.	O	O	O	O	O	6.	O	O	O	O	O
2.	O	O	O	O	O	7.	O	O	O	O	O
3.	O	O	O	O	O	8.	O	O	O	O	O
4.	O	O	O	O	O	9.	O	O	O	O	O
5.	O	O	O	O	O	10.	O	O	O	O	O

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038 qmult 00320 1 4 5 easy deducto-memory: relativity postulate 2

Extra keywords: physci KB-92-11, KB-94-19

1. "Let's play *Jeopardy*! For \$100, the answer is: That the laws of physics are the same for observers in all inertial frames."

   What is ____________, Alex?

   a) Einstein's vacuum speed of light postulate b) postulate of the invariance of coincidence c) $\vec{F}_{\rm net} = m\vec{a}$ d) horse sense e) Einstein's relativity postulate in special relativity

SUGGESTED ANSWER: (e)

Wrong answers:

a) That's the other postulate.
d) For a very enlightened being perhaps.

Redaction: Jeffery, 2001jan01

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038 qmult 01020 1 5 5 easy thinking: light cone Alpha Centauri

2. We know already that a tricky political situation will arise on Alpha Centauri 6 (4 light-years distant) in a election 3 years hence. In our Terracentric way, we the people of Earth are going to make our views known to the Alpha Centaurs. What is our chance of influencing their decision?

   a) It's possible: Alpha Centauri 3 years hence is **INSIDE** our light cone. A radio signal can be sent and received and the Alpha Centaurs will conclude that there is life out there—arguably intelligent.
   b) So-so.
   c) Immense: nothing is more cherished than unasked for advice from those incompetent to give it.
   d) Nil: Alpha Centaurs would rather take their own bad advice, than someone else's good advice: they're pretty normal that way—explaining the horse's hind end is another matter.
   e) Nil: Alpha Centauri 3 years hence is **OUTSIDE** of our light cone. Nothing we do now can have the slightest effect on them then. Jump up and down, scream, shout, giggle inopportunely—nothing can help.

SUGGESTED ANSWER: (e)

Wrong answers:

a) Wrong, they are outside our light cone. Of course, the message will eventually arrive and then the will conclude that there is life out there—arguably intelligent.
b) Well no.
c) Right if you recognize sarcasm.
d) Not the best answer in context. They will never have the chance to contemplate our advice—however momentarily—before going on to do whatever they will anyway.

Redaction: Jeffery, 2001jan01

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038 qmult 00800 1 1 2 easy memory: time dilation mnemonic
Extra keywords: physci

3. The mnemonic for the time dilation effect is "moving clocks:

   a) run fast." b) run slow." c) are stopped." d) are right twice a day." e) run backward."

SUGGESTED ANSWER: (b)

Wrong answers:

e) You can make a mechanical clock run backwards, of course, but you can't really make time run backward. Entropy always increases in a closed system and time hurrying chariot doesn't step into the same stream twice.

Redaction: Jeffery, 2001jan01

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001 qmult 06030 1 4 5 easy deducto-memory: superstring theory defn.
Extra keywords: Gre-17–18

4. "Let's play *Jeopardy*! For $100, the answer is: In this physical theory (circa 2004 at least), the basic element of matter is a string/filament/little-thingy which vibrates in different ways to make the fundamental particles (e.g., electron, neutrino, quark). The theory requires 9 or 10 space dimensions plus 1 time dimension and thus 10 or 11 spacetime dimensions. The higher numbers are for the version called M-theory."

   What is ____________, Alex?

   a) Aristotelian physics b) Newtonian physics
   c) Einsteinian relativistic physics d) quantum mechancis
   e) superstring theory

SUGGESTED ANSWER: (e)

Wrong answers:

a) As Lurch would say AAAARGH.

Redaction: Jeffery, 2008jan01

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002 qmult 03010 1 4 1 easy deducto-memory: Newton bucket
Extra keywords: mathematical physics

5. "Let's play *Jeopardy*! For $100, the answer is: A thought experiment (which can actually be done) that has been used for arguing for absolute space as a physically active thing."

   What is ____________, Alex?

   a) Newton's bucket experiment b) Maxwell's demon experiment
   c) Einstein's elevator experiment d) Bohr's microscope experiment
   e) Schrödinger's cat experiment

SUGGESTED ANSWER: (a)

Wrong answers:

d) See ER-67.

e) Schrödinger described this as a hellish contraption (Gri-382). Still he could have picked a dog or a rat. But no-o-o-o, he had to choose a cat.

Redaction: Jeffery, 2008jan01

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002 qmult 06010 1 4 3 easy deducto-memory: Mach principle

Extra keywords: mathematical physics

6. "Let's play *Jeopardy*! For \$100, the answer is: It is the vague hypothesis that the bulk distribution of matter in the universe determines the inertial mass of bodies."

What is ____________, Alex?

a) Zeno's paradox   b) Fermat's last theorem   c) Mach's principle
d) Poincaré's conjecture   e) the Merton thesis

SUGGESTED ANSWER: (c)

Wrong answers:

a) Zeno had a bit of a problem with the concept of limit.

b) This has actually been proven by Andrew Wiles in 1995.

Redaction: Jeffery, 2008jan01

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001 qmult 01120 2 1 2 moderate memory: density of states box quantization

7. The density of wavenumber states in wavenumber space (or k-space) per space space volume V in the continuum limit for the box-quantization system (or particle-in-a-box system) is:

a) linear $1/V$.   b) independent of V.   c) linear in V.
d) quadratic in V.   e) cubic in V.

SUGGESTED ANSWER: (b) This is actually a remarkable result. Even though the states are non-local in space space (i.e., they are spread through the whole volume), the density is still volume independent. If the states were localized in space space, it would be easy to imagine that there would be a fixed density per volume V.

Wrong answers:

c) Reasonable guess.

Redaction: Jeffery, 2008jan01

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003 qmult 13030 1 1 4 easy memory: dynamic spacetime

Extra keywords: Gre-75

8. Because spacetime responds to mass-energy in general relativity, one can say that in general relativity spacetime is:

a) static.   b) ellipsoidal.   c) hyperbolical.   d) dynamic.   e) flat.

SUGGESTED ANSWER: (d) By saying a thing is dynamic, one means in this context that the thing can be acted upon by something or responds to something.

Wrong answers:

e) At present, space seems flat, but this isn't a valid completion of the sentence.

Redaction: Jeffery, 2008jan01

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002 qmult 00320 1 3 1 easy math: work function of gold

9. Given that the work function of gold (Au) is 4.8 eV, what is the maximum wavelength of light that will cause the emission of a photoelectron? **HINT:** $hc = 12398.419\,\text{eV}\,\text{Å}$.

   a) 2600 Å. b) 3000 Å. c) 5000 Å. d) 7000 Å. e) 10000 Å.

SUGGESTED ANSWER: (a) Well

$$w = h\nu_{\text{min}} = \frac{hc}{\lambda_{\text{max}}} \ ,$$

where w is the work function, ν_{min} is the minimum frequency for emission, and λ_{max} is the maximum wavelength for emission. Thus,

$$\lambda_{\text{max}} = \frac{hc}{w} = \frac{12398.419}{4.8} = 2600\,\text{Å} \ .$$

Wrong answers:

a) A nonsense answer.

Fortran-95 Code

```
        print*
        caa=clight*1.d+8
        hc=planckev*caa
        w=4.8d0
        xlambda=hc/w
        print*,'planckev,hc,w,xlambda'
        print*,planckev,hc,w,xlambda
!  4.13566743E-15 12398.419043102429 4.8 2583.0039673130063
```

Redaction: Jeffery, 2008jan01

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002 qmult 00750 1 4 4 easy deducto-memory: positronium

10. "Let's play *Jeopardy*! For \$100, the answer is: A bound state of matter which is usually formed by a positron on its way to annihilation with an electron. It has a mean lifetime of 1.25×10^{-10} s if it forms in the singlet ground state."

    What is ____________, Alex?

    a) pragmatium b) plutonium c) protonium d) positronium e) protesium

SUGGESTED ANSWER: (d) See Wikipedia (positronium, 2008jan20). Positronium can live longer if it forms in other states. The Wikipedia article doesn't give an overall mean lifetime.

Wrong answers:

a) As Lurch would say AAAARGH.

Redaction: Jeffery, 2008jan01

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038 qfull 00500 2 3 0 moderate math: Smaug, Frodo, and time dilation

11. You and your loyal hound Frodo are traveling through space in your star ship Smaug. You are limited to going at less than the ultimate physical speed $c = 1\,\text{lyr/yr}$ or $\beta = 1$, of course. But other than that Smaug can go at any speed and you and Frodo are real tough and can stand any acceleration.

a) Say Frodo wants to travel from Earth to Sirius (distance $d = 8.7\,\text{lyr}$) and you want to make the trip in 1 year for **YOU**. At what β value would Smaug have to go? **HINTS:** You will have to do some algebra. Recall the Galaxy-frame travel time is $d/(c\beta)$.

b) Stirred by your first conquest of space, you and Frodo—well Frodo is less eager—now want to cross the bulge to the far end of the disk of Galaxy: distance $100,000\,\text{lyr}$. But you want to live to see the journey's end. What physical effect makes it possible to survive the journey?

c) How much **LESS** than the speed of light (in units of c: i.e., in terms of β) do you need to go at to make the pan-Galactic journey of $100,000\,\text{lyr}$ in 1 year for **YOU**? **HINT:** Recall the first order Taylor expansion

$$\frac{1}{\sqrt{1+x}} \approx 1 - \frac{1}{2}x$$

which is valid for $x << 1$.

d) How long are each of your the trips in dog years for Frodo? And why does Frodo want to stay at Sirius?

SUGGESTED ANSWER:

a) Ah, the old time dilation effect means you can slow down your proper clocks (e.g., wrist watch, heart beats, biorhythms) by going fast enough. In this case, the Galaxy frame time to Sirius must $t = d/(c\beta)$, where β is your velocity. From the time dilation formula

$$t' = t\sqrt{1-\beta^2} = \frac{d}{c\beta}\sqrt{1-\beta^2} \ ,$$

where t' is the proper time which you want to be 1 year. We now solve for β:

$$\beta = \frac{1}{\sqrt{1+(ct'/d)^2}} \approx 1 - \frac{1}{2}\left(\frac{ct'}{d}\right)^2 \approx 1 - \frac{1}{160} \approx 0.993 \ ,$$

to about 3-digit accuracy. In fact, accidentally, $\beta = 0.993$ is right to 3 digits as the computer calculation shows. Note that I've used the first order Taylor expansion which should be pretty accurate here since $ct'/d << 1$.

b) Again, that old time dilation effect means you can do it if you only go fast enough with respect to the Galaxy rest frame.

c) Well from the formula from the part (a) answer and using the Taylor expansion formula, we have

$$\beta \approx 1 - \frac{1}{2}\left(\frac{ct'}{d}\right)^2 ,$$

and thus you must travel slower than the speed of light by

$$\frac{1}{2}\left(\frac{ct'}{d}\right)^2 \approx 0.5 \times 10^{-10}$$

in units of the speed of light. That is pretty fast—but any old light beam will beat you.

d) That's 7 years in dog years for both.

Sirius is the Dog Star since it is the brightest star in Canis Major, the larger of Orion's dogs. To the ancient Egyptians Sirius was Anubis, the jackal-head god. In fact Sirius is the brightest star seen from Earth. As you all know, the Sun moves eastward relative to the celestial sphere of the fixed stars in the course of a year. When the Sun is very close to a star, the star is never seen: it is always lost in the daytime sky. As the Sun moves just east of star, that star can be seen rising just before sunrise before being lost in the daytime sky. This first appearance of a bright star after being lost in the Sun—the heliacal rising—was considered a good omen in ancient times—the cosmos was still working as it had. The heliacal risings often were used to mark the beginning of seasons. The heliacal rising of Sirius circa mid-July (but even dozens of websites fail to say exactly when—it's varied a bit over the millennia) marks the beginning of the Nile floods and the long, hot, dry days of summer—the Dog days. Sirius from the Greek Seirios means scorcher. "By the Dog" was Socrates's favorite oath. Hesiod (circa 700 BC) tells us:

Oh when thistle bursts and cicada,
hid in his tree, shrill and timeless,
sings his song—timeless,
then summer swoons and goat is fat
and wine is good, and maids are riggish,
but burnt are streams and men—burnt dry
by Sirius teaming with the Sun—but I
in the Dog days love a rocky shade
and Biblos from the vine.

Fortran-95 Code

```
print*
ctp=1.d0
d=8.7d0
```

```
        first=.5d0*(ctp/d)**2
        beta=1.d0/sqrt(1.d0+first)
        beta1=1.d0-first
        print*,'beta,beta1,first'
        print*,beta,beta1,first
!  0.9967133283869132 0.9933941075439292 6.605892456070817E-3
        d=1.d5
        first=.5d0*(ctp/d)**2
        beta=1.d0/sqrt(1.d0+first)
        beta1=1.d0-first
        print*,'beta,beta1,first'
        print*,beta,beta1,first
 0.999999999975 0.99999999995 5.000000000000001E-11
```

Redaction: Jeffery, 2001jan01

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001 qfull 01020 1 3 0 easy math: wave equation

12. The standard wave equation in 1 dimension is

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \ ,$$

where y is the oscillating quantity, x is the 1 space dimension, t is time, and $|v|$ is the constant phase speed of wave propagation (WA-710). Because this differential equation has more than one independent variable (it has x and t as independent variables), it is a partial differential equation.

a) Verify that $f(x - vt)$ is a general traveling wave solution of the wave equation where $f(x)$ is any function. What is the initial condition of the solution at time zero? What is the direction of propagation of the solution? Consider the wave system as nonrelativistic and note that v can be positive or negative.

b) In quantum mechanics, it is traditional to write the argument of a 1-dimensional wave as $kx - \omega t$ (rather than $x - vt$), where k is the wavenumber and ω is the angular frequency. The ω is always taken as positive and the sign of k determines the direction of a traveling wave: $k > 0$ gives travel in the positive direction and $k < 0$ gives travel in the negative direction.

Since the wave equation is a linear equation, any two solutions can be added to give another solution. You are given two traveling wave solutions $A\sin(kx-\omega t)$ and $A\sin(-kx-\omega t)$, where A is a constant amplitude, k is a positive wave number, ω is angular frequency, and $\omega/|k| = v$, the phase speed. What is the superposition of the waves (i.e., what is their sum) and what does this superposition amount to physically. **HINT:** The trivial answer is not an answer.

SUGGESTED ANSWER:

a) Let $z = x - vt$ and note that

$$\frac{\partial z}{\partial x} = 1 \qquad \text{and} \qquad \frac{\partial z}{\partial t} = -v \ .$$

Now

$$\frac{\partial^2 f(x-vt)}{\partial x^2} = \frac{\partial^2 f(z)}{\partial z^2}\left(\frac{\partial z}{\partial x}\right)^2 = \frac{\partial^2 f(z)}{\partial z^2}$$

and

$$\frac{\partial^2 f(x-vt)}{\partial t^2} = \frac{\partial^2 f(z)}{\partial z^2}\left(\frac{\partial z}{\partial t}\right)^2 = \frac{\partial^2 f(z)}{\partial z^2}v^2 \ .$$

It follows that

$$\frac{\partial^2 f(x-vt)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f(x-vt)}{\partial t^2} \ ,$$

and thus $f(x-vt)$ is a general solution. The initial condition for the solution is clearly $f(x)$: i.e., $f(x-vt)$ with $t=0$.

The solution is a traveling wave solution. Let $x' = x - vt$ and consider $f(x')$. The function $f(x')$ is just a function shape at rest relative to the x'-axis which defines a primed frame. Now if an observer's position x changes as vt just so as to keep x' constant at some value, then the observer observes the rest function shape $f(x')$. Clearly, then the solution is a traveling solution with the phase velocity being v. The phase velocity is the velocity at which the shape moves. If $v > 0$, the wave travels in the positive direction. If $v < 0$, the wave travels in the negative direction. If $v = 0$, the wave is static.

b) The superposition gives

$$y = A[\sin(kx-\omega t) + \sin(-kx-\omega t)] = -2A\cos(kx)\sin(\omega t) \ ,$$

where we have used the trignometric identity

$$\sin(A)\cos(B) = \frac{1}{2}[\sin(A-B) + \sin(A+B)] \ .$$

The superposition is a standing wave since there is no moving waveform, but just an up and down oscillation of the medium at each point x. The oscillation is, in fact, simple harmonic motion with angular frequency ω and amplitude $|2A\cos(kx)|$ for each point x.

Redaction: Jeffery, 2008jan01

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001 qfull 02210 2 5 0 moderate thinking: Earth's effective temperature

Extra keywords: suggested by ER-23-10

13. The solar constant $S = 1366\,\mathrm{W/m^2}$ on average (Wikipedia: Solar radiation, 2008feb18). In fact, it's not exactly constant, due to varying Earth-Sun distance, sunspots, and the 11-year solar cycle. But averaged over those variations, it is really very constant which is good for life on Earth. The solar constant is the power per unit area (or energy flux) on a sphere surrounding the Sun at the Earth's distance from the Sun.

a) What is the **AVERAGE** power per unit area on the Earth? **HINT:** Remember the Earth's a rotating sphere. Think of its cross-sectional area in respect to the solar light flux and its surface area.

b) The Earth's average albedo is $A = 0.30$. The albedo is the fraction of just light **REFLECTED**. What is the average power per unit area **ABSORBED** by the Earth?

c) Assuming the Earth is a perfect blackbody radiator and is in thermal energy steady state (i.e., emits all the energy it receives and maintains a steady state), solve for the Earth's mean temperature. **HINT:** Power per unit area is flux. Blackbody flux is given by the Stefan-Boltzmann law.

SUGGESTED ANSWER:

a) Since the Earth is a sphere, the average power per unit area is

$$P_{\rm avg} = \frac{\pi r^2 S}{4\pi r^2} = \frac{S}{4} = 341.5\,{\rm W/m^2} \ ,$$

where r is the Earth's radius.

b) Behold:

$$P_{\rm avg,absorbed} = \frac{S}{4}(1-A) = 239\,{\rm W/m^2} \ .$$

c) The equating F from the Stefan-Boltzmann law $F = \sigma T^4$ to $P_{\rm avg,absorbed}$ and solving for T gives

$$T = \left[\frac{(S/4)(1-A)}{\sigma}\right]^{1/4} = 255\,{\rm K} \ .$$

The kind of temperature just calculated is called an effective temperature. It is the temperature the Earth would have in the absence of the greenhouse effect. The greenhouse effect raises the Earth's mean to 288 K. The greenhouse effect is good. The Earth would be mighty nippy without it.

Actually the solar power recieved at the ground is about $P_{\rm grd} = 170\,{\rm W/m^2}$ (Smil 2006, p. 26). Which is really quite a lot and in my view (just following my guru Smil) is the best energy resource for the future. The whole energy production of land biomass is about 1800 EJ per year or about $0.5\,{\rm W/m^2}$ of ice-free land (Smil 2005, p. 264). (An exajoule is 10^{18} J.) This is small compared to the of order 10 % or more of the $P_{\rm grd}$ which is what photovoltaic cells can yield in theory (Smil 2005, p. 286–288). Humanity already expropriates about 1/3 of the land biomass production effectively through harvest or land use (Smil 2005, p. 265). It seems unlikely that we could harvest much more without severe ecological consequences. The world energy consumption is about 500 EJ per year of which about 86 % is from fossil fuels (Wikipedia: World energy resources and consumption, 2008feb18). This rate is likely to increase—or at least people want to increase it. Just of the face of it, it seems very unlikely that we can add globally significant amounts of energy production from biomass fuels. But there is abundant solar power.

Fortran-95 Code

```
          print*
          solcon=1366.d0   ! Wikipedia 2008feb18
          albedo=.3d0
          favg=solcon*.25d0
          fabs=favg*(1.d0-albedo)
          fgrd=favg*.5d0
          tem=(   solcon*.25d0*(1.d0-albedo)
&
         &      /(stefan_boltzmann*1.d-3)   )**.25d0
          print*,'solcon,favg,fabs,fgrd,tem'
          print*,solcon,favg,fabs,fgrd,tem
   ! 1366.0 341.5 239.04999999999998 170.75 254.81134663956905
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Redaction: Jeffery, 2008jan01

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002 qfull 00390 2 3 0 moderate math: gold foil photoelectric effect

Extra keywords: ER-52

14. X-rays eject photoelectrons from a particular thin gold foil.

a) The X-rays have wavelength 0.710 Å. What is the energy of an individual photons in units of joules, Kilo-electron-volts (KeV), and $m_e c^2 = 510.998910(13)$ KeV?

b) The electrons are directed into a region of uniform magnetic field $\vec{B}$ and go into **UNIFORM CIRCULAR MOTION** with radius r as determined by the magnetic force. The observed **MAXIMUM** value of rB (sometimes called the magnetic rigidity [Go-318]) is 1.88×10^{-4} T m where T m are tesla-meters. Find the maximum kinetic energy of the photoelectrons in KeV. **HINT:** Recall the Lorentz force (which includes the magnetic force) is given by

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \ ,$$

where $\vec{E} = 0$ in our case, and the centripetal force magnitude by

$$F = m\frac{v^2}{r} \ .$$

c) What is the minimum work in KeV done by the X-rays in ejecting the photoelectrons? **HINT:** It is not the work function of thick gold sample. That is 4.8 eV (ER-408).

SUGGESTED ANSWER:

a) Behold:

$$E = \frac{hc}{\lambda} = 2.80 \times 10^{-15}\,\text{J} = 17.5\,\text{KeV} = 0.0342 \times m_e c^2 \ .$$

b) The photoelectrons must be moving perpendicular to the B-field in order not to be moving in spiral. Thus $|\vec{v} \times \vec{B}| = vB$ The magnitude of the magnetic force is then qvB and this supplies the centripetal force. Thus we have

$$m\frac{v^2}{r} = qvB \ , \qquad v = \frac{qrB}{m} \ , \qquad KE = \frac{1}{2}mv^2 = \frac{(qrB)^2}{2m} \ .$$

Note we have done a non-relativistic calculation. This is valid since the energy of the photoelectron must be less than that of the X-ray photons and that energy in units of $m_e c^2$ is much less than 1. The maximum kinetic energy of the photoelectrons occurs for the maximum value of rB and is

$$KE = 4.98 \times 10^{-16}\,\mathrm{J} = 3.11\,\mathrm{KeV} \ .$$

c) Since the photons have constant energy, the minimum work done would be for the maximum kinetic energy photoelectrons: there is the least energy lost from the photon input in ejecting the photoelectrons. Thus the minimum work done is 14.4 KeV.

We note that the minimum work done in this case is much larger than the gold work function $w = 4.8\,\mathrm{eV} = 0.0048\,\mathrm{KeV}$ (ER-408). For some reason, it takes vastly more energy to eject a photoelectron from a thin gold foil than from a bulk gold sample. Maybe the problem is unrealistic somehow. The numbers for this question were just taken from ER-52.

Fortran-95 Code

```
          print*
          en=planck*1.d-7*clight*1.d-2/(.71d-10) ! MKS
          enkev=en/(echarge*1.d3)
          enred=en/(emass*1.d-3*(clight*1.d-2)**2)
          teslam=1.88d-4
          xke=(echarge*teslam)**2/(2.d0*emass*1.d-3)

xkekev=(echarge*teslam)**2/(2.d0*emass*1.d-3)/(echarge*1.d3)
          work=enkev-xkekev
          print*,en,enkev,enred,xke,xkekev,work
    !  2.7978107075004783E-15 17.462562053012213
0.03417338339184284
    !  4.979864097416916E-16 3.1081868971185815 14.354375155893631
```

Redaction: Jeffery, 2008jan01

Equation Sheet for Modern Physics

These equation sheets are intended for students writing tests or reviewing material. Therefore they are neither intended to be complete nor completely explicit. There are fewer symbols than variables, and so some symbols must be used for different things: context must distinguish.

The equations are mnemonic. Students are expected to understand how to interpret and use them.

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1 Geometrical Formulae

$$C_{\rm cir} = 2\pi r \qquad A_{\rm cir} = \pi r^2 \qquad A_{\rm sph} = 4\pi r^2 \qquad V_{\rm sph} = \frac{4}{3}\pi r^3$$

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2 Trigonometry

$$\frac{x}{r} = \cos\theta \qquad \frac{y}{r} = \sin\theta \qquad \frac{y}{x} = \tan\theta \qquad \cos^2\theta + \sin^2\theta = 1$$

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b) \qquad \cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$\cos^2\theta = \frac{1}{2}[1+\cos(2\theta)] \qquad \sin^2\theta = \frac{1}{2}[1-\cos(2\theta)] \qquad \sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$\cos(a)\cos(b) = \frac{1}{2}\left[\cos(a-b) + \cos(a+b)\right] \qquad \sin(a)\sin(b) = \frac{1}{2}\left[\cos(a-b) - \cos(a+b)\right]$$

$$\sin(a)\cos(b) = \frac{1}{2}\left[\sin(a-b) + \sin(a+b)\right]$$

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3 Blackbody Radiation

$$B_\nu = \frac{2h\nu^3}{c^2}\frac{1}{[e^{h\nu/(kT)} - 1]} \qquad B_\lambda = \frac{2hc^2}{\lambda^5}\frac{1}{[e^{hc/(kT\lambda)} - 1]}$$

$$B_\lambda\, d\lambda = B_\nu\, d\nu \qquad \nu\lambda = c \qquad \frac{d\nu}{d\lambda} = -\frac{c}{\lambda^2}$$

$$k = 1.3806505(24) \times 10^{-23}\,\mathrm{J/K} \qquad c = 2.99792458 \times 10^{8}\,\mathrm{m}$$

$$h = 6.6260693(11) \times 10^{-34}\,\mathrm{J\,s} = 4.13566743(35) \times 10^{-15}\,\mathrm{eV\,s}$$

$$\hbar = \frac{h}{2\pi} = 1.05457168(18) \times 10^{-34}\,\mathrm{J\,s}$$

$$hc = 12398.419\,\mathrm{eV\,\AA} \approx 10^{4}\,\mathrm{eV\,\AA} \qquad E = h\nu = \frac{hc}{\lambda} \qquad p = \frac{h}{\lambda}$$

$$F = \sigma T^{4} \qquad \sigma = \frac{2\pi^{5}}{15}\frac{k^{4}}{c^{2}h^{3}} = 5.670400(40) \times 10^{-8}\,\mathrm{W/m^{2}/K^{4}}$$

$$\lambda_{\mathrm{max}} T = \mathrm{constant} = \frac{hc}{k x_{\mathrm{max}}} \approx \frac{1.4387751 \times 10^{-2}}{x_{\mathrm{max}}}$$

$$B_{\lambda,\mathrm{Wien}} = \frac{2hc^{2}}{\lambda^{5}} e^{-hc/(kT\lambda)} \qquad B_{\lambda,\mathrm{Rayleigh-Jeans}} = \frac{2ckT}{\lambda^{4}}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{c}\nu = \frac{\omega}{c} \qquad k_i = \frac{\pi}{L} n_i \quad \text{standing wave BCs} \qquad k_i = \frac{2\pi}{L} n_i \quad \text{periodic BCs}$$

$$n(k)\,dk = \frac{k^{2}}{\pi^{2}}\,dk = \pi \left(\frac{2}{c}\right) \nu^{2}\,d\nu = n(\nu)\,d\nu$$

$$\ln(z!) \approx \left(z + \frac{1}{2}\right) \ln(z) - z + \frac{1}{2}\ln(2\pi) + \frac{1}{12z} - \frac{1}{360z^{3}} + \frac{1}{1260z^{5}} - \ldots$$

$$\ln(N!) \approx N \ln(N) - N$$

$$\rho(E)\,dE = \frac{e^{-E/(kT)}}{kT}\,dE \qquad P(n) = (1 - e^{-\alpha})e^{-n\alpha} \qquad \alpha = \frac{h\nu}{kT}$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \qquad f(x - vt) \qquad f(kx - \omega t)$$

4 **Photons**

$$KE = h\nu - w \qquad \Delta\lambda = \lambda_{\rm scat} - \lambda_{\rm inc} = \lambda_{\rm C}(1 - \cos\theta)$$

$$\lambda_{\rm C} = \frac{h}{m_e c} = 2.426310238(16) \times 10^{-12}\,{\rm m} \qquad e = 1.602176487(40) \times 10^{-19}\,{\rm C}$$

$$m_e = 9.1093826(16) \times 10^{-31}\,{\rm kg} = 0.510998918(44)\,{\rm MeV}$$

$$m_p = 1.67262171(29) \times 10^{-27}\,{\rm kg} = 938.272029(80)\,{\rm MeV}$$

$$\ell = \frac{1}{n\sigma} \qquad \rho = \frac{e^{-s/\ell}}{\ell} \qquad \langle s^m \rangle = \ell^m m!$$

5 **Matter Waves**

$$\lambda = \frac{h}{p} \qquad p = \hbar k \qquad \Delta x \Delta p \geq \frac{\hbar}{2} \qquad \Delta E \Delta t \geq \frac{\hbar}{2}$$

$$\Psi(x,t) = \int_{-\infty}^{\infty} \phi(k)\Psi_k(x,t)\,dk \qquad \phi(k) = \int_{-\infty}^{\infty} \Psi(x,0)\frac{e^{-ikx}}{\sqrt{2\pi}}\,dk$$

$$v_{\rm g} = \left.\frac{d\omega}{dk}\right|_{k_0} = \frac{\hbar k_0}{m} = \frac{p_0}{m} = v_{{\rm clas},0}$$

6 **Non-Relativistic Quantum Mechanics**

$$H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V \qquad T = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} \qquad H\Psi = -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V\Psi = i\hbar\frac{\partial \Psi}{\partial t}$$

$$\rho = \Psi^* \Psi \qquad \rho\, dx = \Psi^* \Psi\, dx$$

$$A\phi_i = a_i \phi_i \qquad f(x) = \sum_i c_i \phi_i \qquad \int_a^b \phi_i^* \phi_j\, dx = \delta_{ij} \qquad c_j = \int_a^b \phi_j^* f(x)\, dx$$

$$[A, B] = AB - BA$$

$$P_i = |c_i|^2 \qquad \langle A \rangle = \int_{-\infty}^{\infty} \Psi^* A \Psi\, dx = \sum_i |c_i|^2 a_i \qquad H\psi = E\psi \qquad \Psi(x,t) = \psi(x) e^{-i\omega t}$$

$$p_{\mathrm{op}} \phi = \frac{\hbar}{i} \frac{\partial \phi}{\partial x} = p\phi \qquad \phi = \frac{e^{ikx}}{\sqrt{2\pi}} \qquad \frac{\partial^2 \psi}{\partial x^2} = \frac{2m}{\hbar^2}(V - E)\psi$$

$$|\Psi\rangle \qquad \langle \Psi| \qquad \langle x|\Psi\rangle = \Psi(x) \qquad \langle \vec{r}|\Psi\rangle = \Psi(\vec{r}) \qquad \langle k|\Psi\rangle = \Psi(k) \qquad \langle \Psi_i|\Psi_j\rangle = \langle \Psi_j|\Psi_i\rangle^*$$

$$\langle \phi_i|\Psi\rangle = c_i \qquad 1_{\mathrm{op}} = \sum_i |\phi_i\rangle\langle \phi_i| \qquad |\Psi\rangle = \sum_i |\phi_i\rangle\langle \phi_i|\Psi\rangle = \sum_i c_i |\phi_i\rangle$$

$$1_{\mathrm{op}} = \int_{-\infty}^{\infty} dx\, |x\rangle\langle x| \qquad \langle \Psi_i|\Psi_j\rangle = \int_{-\infty}^{\infty} dx\, \langle \Psi|x\rangle\langle x|\Psi\rangle \qquad A_{ij} = \langle \phi_i|A|\phi_j\rangle$$

$$Pf(x) = f(-x) \qquad P\frac{df(x)}{dx} = \frac{df(-x)}{d(-x)} = -\frac{df(-x)}{dx} \qquad Pf_{\mathrm{e/o}}(x) = \pm f_{\mathrm{e/o}}(x)$$

$$P\frac{df_{\mathrm{e/o}}(x)}{dx} = \mp \frac{df_{\mathrm{e/o}}(x)}{dx}$$

7 Spherical Harmonics

$$Y_{0,0} = \frac{1}{\sqrt{4\pi}} \qquad Y_{1,0} = \left(\frac{3}{4\pi}\right)^{1/2} \cos(\theta) \qquad Y_{1,\pm 1} = \mp \left(\frac{3}{8\pi}\right)^{1/2} \sin(\theta) e^{\pm i\phi}$$

$$L^2 Y_{\ell m} = \ell(\ell+1)\hbar^2 Y_{\ell m} \qquad L_z Y_{\ell m} = m\hbar Y_{\ell m} \qquad |m| \leq \ell \qquad m = -\ell, -\ell+1, \ldots, \ell-1, \ell$$

0	1	2	3	4	5	6 ...
s	p	d	f	g	h	i ...

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8 Hydrogenic Atom

$$\psi_{n\ell m} = R_{n\ell}(r) Y_{\ell m}(\theta, \phi) \qquad \ell \leq n-1 \qquad \ell = 0, 1, 2, \ldots, n-1$$

$$a_z = \frac{a}{Z}\left(\frac{m_e}{m_{\rm reduced}}\right) \qquad a_0 = \frac{\hbar}{m_e c\alpha} = \frac{\lambda_{\rm C}}{2\pi\alpha} \qquad \alpha = \frac{e^2}{\hbar c}$$

$$R_{10} = 2a_Z^{-3/2} e^{-r/a_Z} \qquad R_{20} = \frac{1}{\sqrt{2}} a_Z^{-3/2}\left(1 - \frac{1}{2}\frac{r}{a_Z}\right) e^{-r/(2a_Z)}$$

$$R_{21} = \frac{1}{\sqrt{24}} a_Z^{-3/2} \frac{r}{a_Z} e^{-r/(2a_Z)}$$

$$R_{n\ell} = -\left\{\left(\frac{2}{na_Z}\right)^3 \frac{(n-\ell-1)!}{2n[(n+\ell)!]^3}\right\}^{1/2} e^{-\rho/2} \rho^\ell L_{n+\ell}^{2\ell+1}(\rho) \qquad \rho = \frac{2r}{nr_Z}$$

$$L_q(x) = e^x \left(\frac{d}{dx}\right)^q \left(e^{-x} x^q\right) \quad \text{Rodrigues' formula for the Laguerre polynomials}$$

$$L_q^j(x) = \left(\frac{d}{dx}\right)^j L_q(x) \qquad \text{Associated Laguerre polynomials}$$

$$\langle r \rangle_{n\ell m} = \frac{a_Z}{2}\left[3n^2 - \ell(\ell+1)\right]$$

$$\text{Nodes} = (n-1) - \ell \qquad \text{not counting zero or infinity}$$

$$E_n = -\frac{1}{2} m_e c^2 \alpha^2 \frac{Z^2}{n^2} \frac{m_{\text{reduced}}}{m_e} = -E_{\text{Ryd}} \frac{Z^2}{n^2} \frac{m_{\text{reduced}}}{m_e} = -13.606 \frac{Z^2}{n^2} \frac{m_{\text{reduced}}}{m_e} \text{ eV}$$

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9 Special Relativity

$$c = 2.99792458 \times 10^8\,\text{m/s} \approx 2.998 \times 10^8\,\text{m/s} \approx 3 \times 10^8\,\text{m/s} \approx 1\,\text{lyr/yr} \approx 1\,\text{ft/ns}$$

$$\beta = \frac{v}{c} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}} \qquad \gamma(\beta << 1) = 1 + \frac{1}{2}\beta^2 \qquad \tau = ct$$

Galilean Transformations	Lorentz Transformations
$x' = x - \beta\tau$	$x' = \gamma(x - \beta\tau)$
$y' = y$	$y' = y$
$z' = z$	$z' = z$
$\tau' = \tau$	$\tau' = \gamma(\tau - \beta x)$
$\beta'_{\text{obj}} = \beta_{\text{obj}} - \beta$	$\beta'_{\text{obj}} = \dfrac{\beta_{\text{obj}} - \beta}{1 - \beta\beta_{\text{obj}}}$

$$\ell = \ell_{\text{proper}} \sqrt{1-\beta^2} \qquad \Delta\tau_{\text{proper}} = \Delta\tau \sqrt{1-\beta^2}$$

$$m = \gamma m_0 \qquad p = mv = \gamma m_0 c \beta \qquad E_0 = m_0 c^2 \qquad E = \gamma E_0 = \gamma m_0 c^2 = mc^2$$

$$E = mc^2 \qquad E = \sqrt{(pc)^2 + (m_0 c^2)^2}$$

$$KE = E - E_0 = \sqrt{(pc)^2 + (m_0 c^2)^2} - m_0 c^2 = (\gamma - 1) m_0 c^2$$

$$f = f_{\text{proper}} \sqrt{\frac{1-\beta}{1+\beta}} \qquad \text{for source and detector separating}$$

$$f(\beta << 1) = f_{\text{proper}}\left(1 - \beta + \frac{1}{2}\beta^2\right)$$

$$f_{\text{trans}} = f_{\text{proper}}\sqrt{1-\beta^2} \qquad f_{\text{trans}}(\beta << 1) = f_{\text{proper}}\left(1 - \frac{1}{2}\beta^2\right)$$

$$\tau = \beta x + \gamma^{-1}\tau' \quad \text{for lines of constant } \tau'$$

$$\tau = \frac{x - \gamma^{-1}x'}{\beta} \quad \text{for lines of constant } x'$$

$$x' = \frac{x_{\text{intersection}}}{\gamma} = x'_{x\text{ scale}}\sqrt{\frac{1-\beta^2}{1+\beta^2}} \qquad \tau' = \frac{\tau_{\text{intersection}}}{\gamma} = \tau'_{\tau\text{ scale}}\sqrt{\frac{1-\beta^2}{1+\beta^2}}$$

$$\theta_{\text{Mink}} = \tan^{-1}(\beta)$$