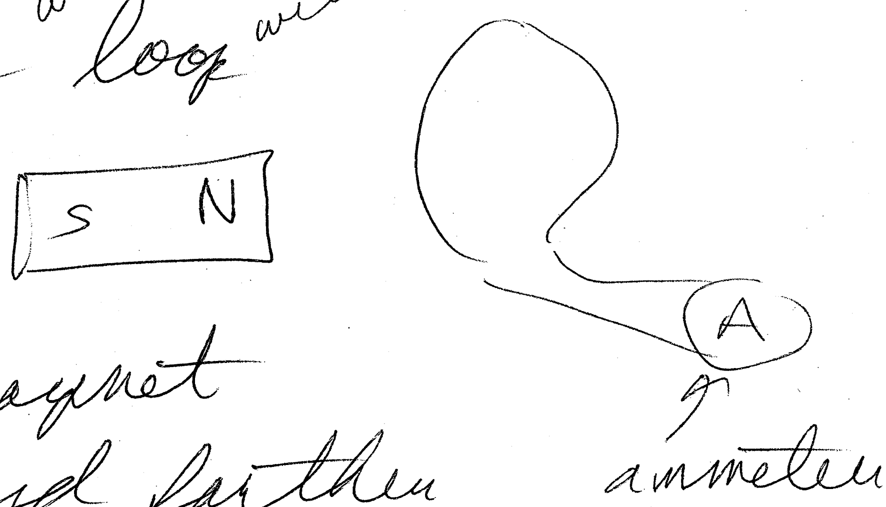
Chapter 31

Faraday's Law and induced EMF

§ Faraday's law of induction

Take a wire loop with ammeter and move a bar magnet closer and farther away.



— the ammeter will show a current.

when the bar magnet is in motion

— the current will have one sense when the magnet approaches

— the other sense when the ~~ba~~ magnet is drawn away. (I'm assuming that one isn't changing the magnet orientation that would complicated things.)

Somehow the changing magnetic field in the rest frame of the loop is inducing an emf.

This experiment is a demonstration of Faraday's law of induction

a fundamental law of classical electromagnetism

(in advanced QM ~~theories~~ it may be somehow derivable from some more basic law).

In short form

An E-field and ~~of~~ the path ~~of~~ the integral is unmoving in frame of analysis

A key point.

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$\oint \underset{\sim}{E} \cdot d\underset{\sim}{s}$

around a closed path in space

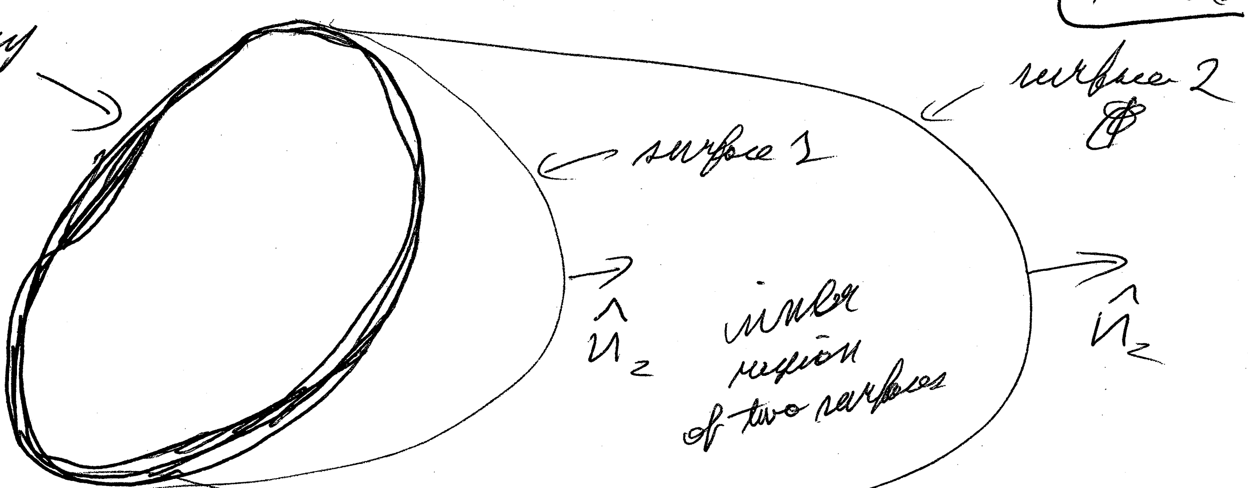
Φ_B is the magnetic flux

at one instant in time [illegible]

~~through~~ any ~~closed path~~ surface bounded by that closed path.

= "Flux linked" by the path (Wik Flux linkag) is more or less this ~~see~~ — a seldom defined term (IMHO)

bounding closed path — nothing has to be here — empty space or there could be a wire.



Recall $\Phi_B = \int \underset{\sim}{B} \cdot d\underset{\sim}{A}$ is the magnetic flux thru a surface.

Hey, we do use it after all.

<u>~~Gauss~~ Gauss law for magnetism</u>

$$\oint \underset{\sim}{B} \cdot d\underset{\sim}{A} = 0$$

$$\int_{sur\,1} \underset{\sim}{B} \cdot d\underset{\sim}{A}_1' + \int_{sur\,2} \underset{\sim}{B} \cdot d\underset{\sim}{A}_2 = 0$$

where $d\underset{\sim}{A}_1'$ is with the unit vector

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chooses the easiest one to work with.

Now $\mathcal{E} = \oint \underset{\sim}{E} \cdot d\underset{\sim}{s}$

The integrand is an E-field and we are in a frame where the path of the integral is at rest.
— We'll consider other cases in the section on motional emf
— see p. 31-39

an induced electric field.

No charge caused it.

— a changing magnetic field did.

But it's effect on charge is just like any E-field.

— in special relativity it transforms between E-field & B-field like an E-field.

It just is an electric field.

but not for E-fields due to charge ($\oint \underset{\sim}{E} \cdot d\underset{\sim}{s} = 0$

Due to charge we proved that

outward from the inner region.

If we flip the unit vector on 1 to point inward we call the differential area vector

$$d\underset{\sim}{A}_1 = -dA_1'$$

$$\therefore -\int_{surf\,1} \underset{\sim}{B} \cdot dA_1 + \int_{surf} \underset{\sim}{B} \cdot dA_2 = 0$$

$$\therefore \int_{surf\,1} \underset{\sim}{B} \cdot dA_1 = \int_{surf\,2} B \cdot dA_2$$

These are general, and ~~the~~ any surface bounded by the closed path will do, but for evaluation, one usually and a PE can be defined

~~PE~~ PE ~~...~~ can't (i.e. defined) for Faraday law induced $\mathbf{E}$-field

but the energy ~~stored~~ density in the induced $\mathbf{E}$-field is still

$$u = \frac{1}{2}\varepsilon_0 E^2$$

(for vacuum case)

SJ-733

since $\oint \mathbf{E}\cdot d\mathbf{s} \neq 0$

a the $\mathbf{E}$-field lines can have no non-zero ends – ~~they~~ must extend to infinity, form closed loops, or go to zero.

Law again

$$\mathcal{E} = \oint \mathbf{E}\cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

This minus sign – if you follow the conventions, it will give the sense of the

That go with the law.

30-8

the emf $\mathcal{E}$,

— but those conventions are tricky to remember & I don't remember them.

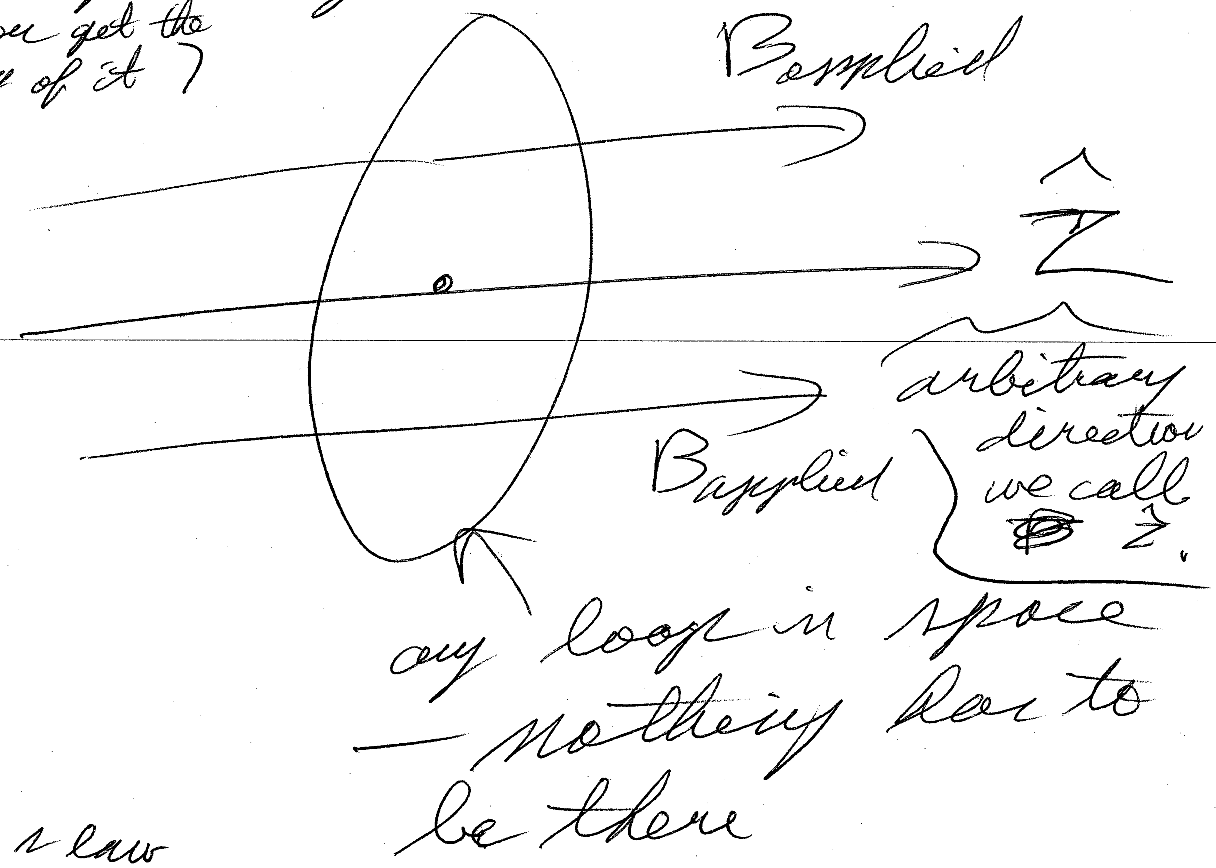
Instead one uses Lenz's law (§ 31.3 53-876)

which is not really a separate law from Faraday's law.

It's a mnemonic device to remember ~~the~~ ~~see~~ how to find the sense of the induced emf

Really easy

(when you get the hang of it)



Lenz's law

The *sense* of the induced emf $\mathcal{E}$ in any loop is such that ~~it oppos~~

if a current could flow in the ~~emf~~ loop due to the emf,

it would create an induced *B-field*

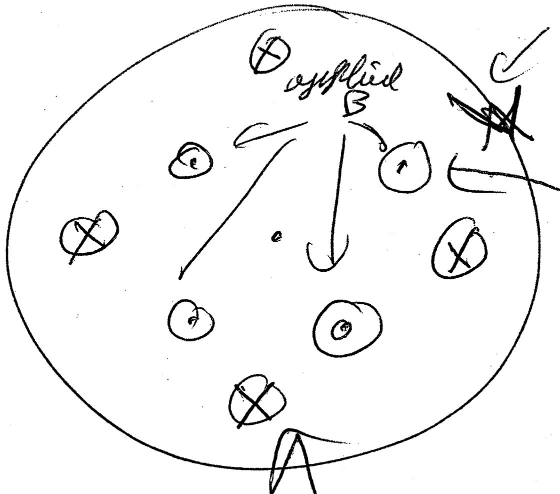
There doesn't have to be any current. — Just if there were one.

~~that~~ would oppose ~~the~~ ~~change~~ in ~~the~~ applied B-field magnetic flux.

Example

Looking in +ve $\hat{z}$ direction $\otimes$

induced emf



Say the applied B field flux was increasing, then the induced emf would be ~~counter~~clockwise in this view so as to create an induced B-field opposing the applied field change.

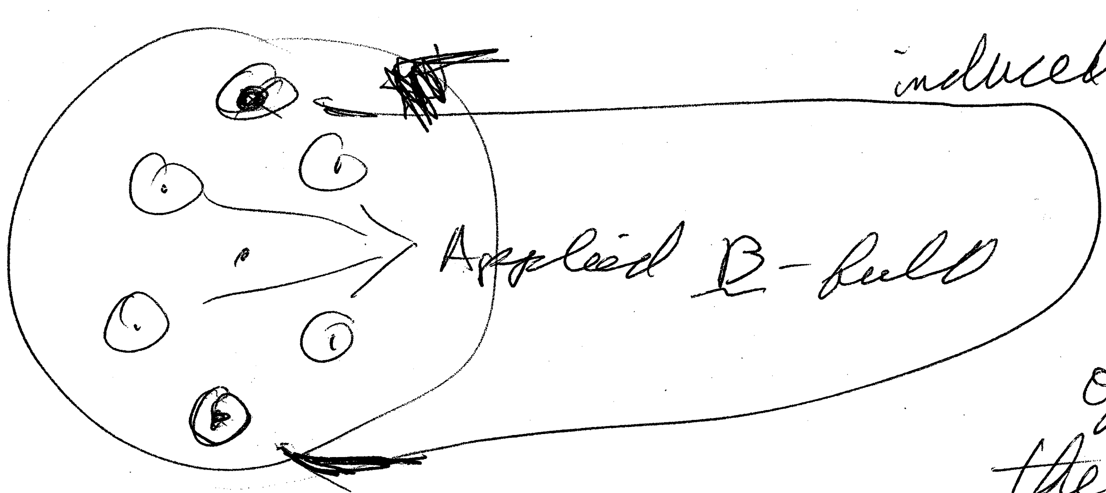
The right hand rule — thumb in +ve z direction for induced emf (It is a magnetic dipole

B-field

— so fingers curl ~~counter~~ clockwise.

On other hand

if $\bar{\Phi}_B$ applied were decreasing.



Another remarks

① — there is a differential form of Faraday's law → and that is useful at a higher level than ours for solving for the induced E-field everywhere.

② Faraday's law is another of Maxwell's equations. Ampere's law with some more.
We've seen them all now (Gauss's law for E & M, ... [illegible], Faraday's law

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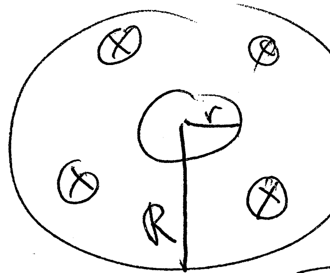


Need so that we can use of ~~for~~ $\underline{E}$ tangent to circle, — No reason to spiral in or out. Consist

cylindrical symmetry

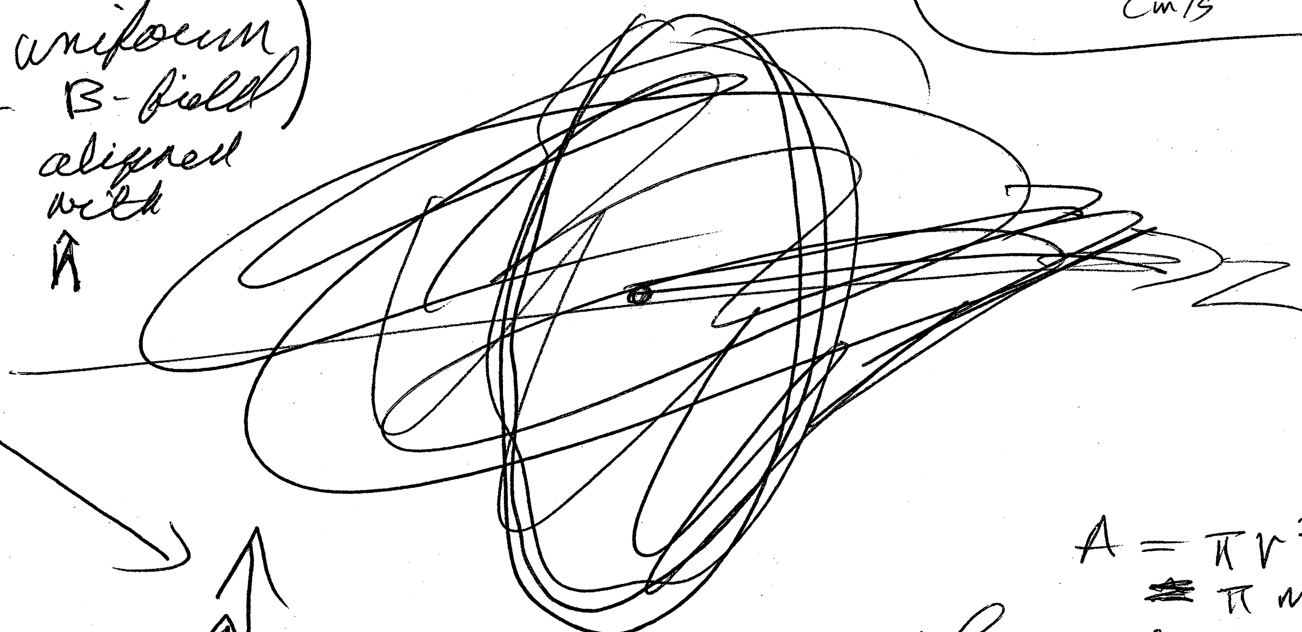
Example

Fiducial case

SJ-812

$$1\,T = \frac{N}{A \cdot m} = \frac{N}{Cm/s}$$

a uniform B-field aligned with $\hat{k}$



$$A = \pi r^2 = \pi\, m^2$$

planar area ~~A~~ $1\,m^2$

$$\frac{d\Phi}{dt} = \frac{dB}{dt} A \cos\theta$$

$\theta = 0°$

$$= 1\,\tfrac{T}{s} \cdot m^2$$

$$\mathcal{E} = \oint \underline{E} \cdot d\underline{s}$$

$$= E \cdot 2\pi r$$

by symmetry — the E-field

$$E = \frac{1\,T/s\ \pi\, m^2}{2\pi\ 1m}$$

$$= \frac{1}{2}\ \frac{T}{s}\, m = \frac{1}{2}\ \frac{N}{C}$$

$$= \frac{1}{2}\ V/$$

Note the induced emf $\mathcal{E}$ has units of volts but it's not actually a potential.

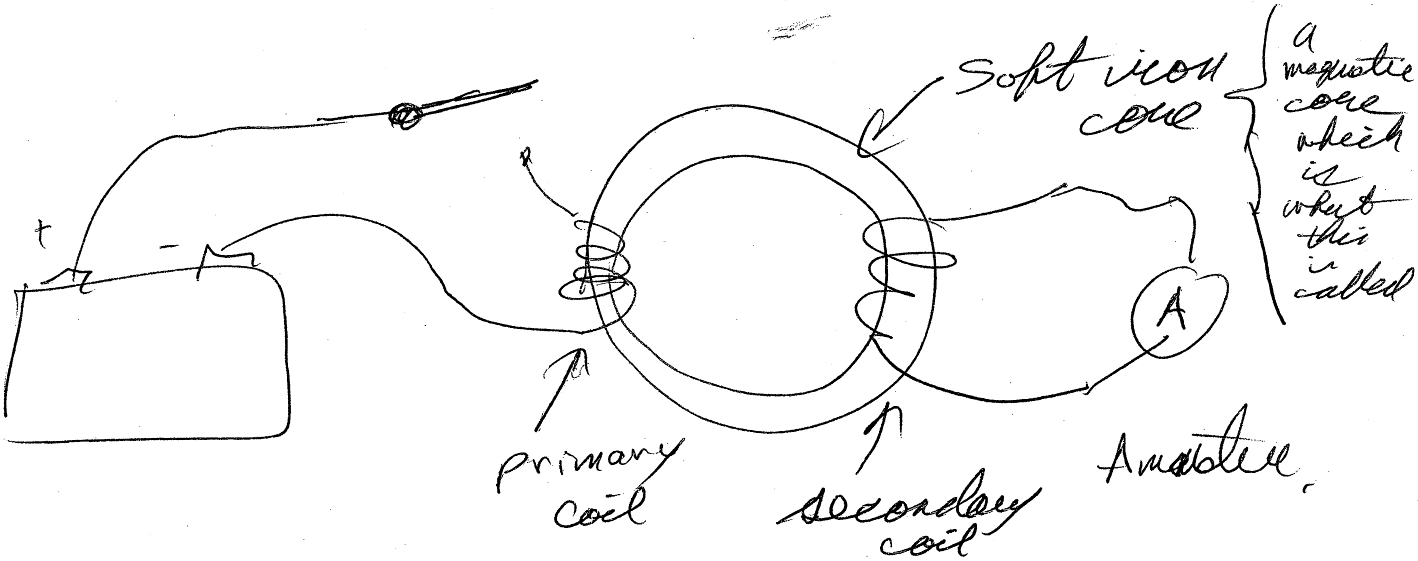
But like any emf, it can in circuit create a charge separation that creates a potential that then can drive a current in an external circuit.

→ This is the basis of electric generators.

3H14

Omit?

Example from SJ-868



— in this case if you close the switch

→ a somewhat complex chain of events follows that is the essence actually of electrical transformers

— a current starts flowing, but ~~instan~~ not practically instantaneously like in previous circuits

→ the coils create a ~~solenoid~~ solenoi-like B-field

→ but as it grows it creates an induced emf that opposes the current and it takes a while to grow

The primary acts as an ~~[illegible]~~ inductor in terminology we'll use later.

— the B-field is greatly enhanced by soft iron core

→ but this B-field is almost ~~[illegible]~~ entirely

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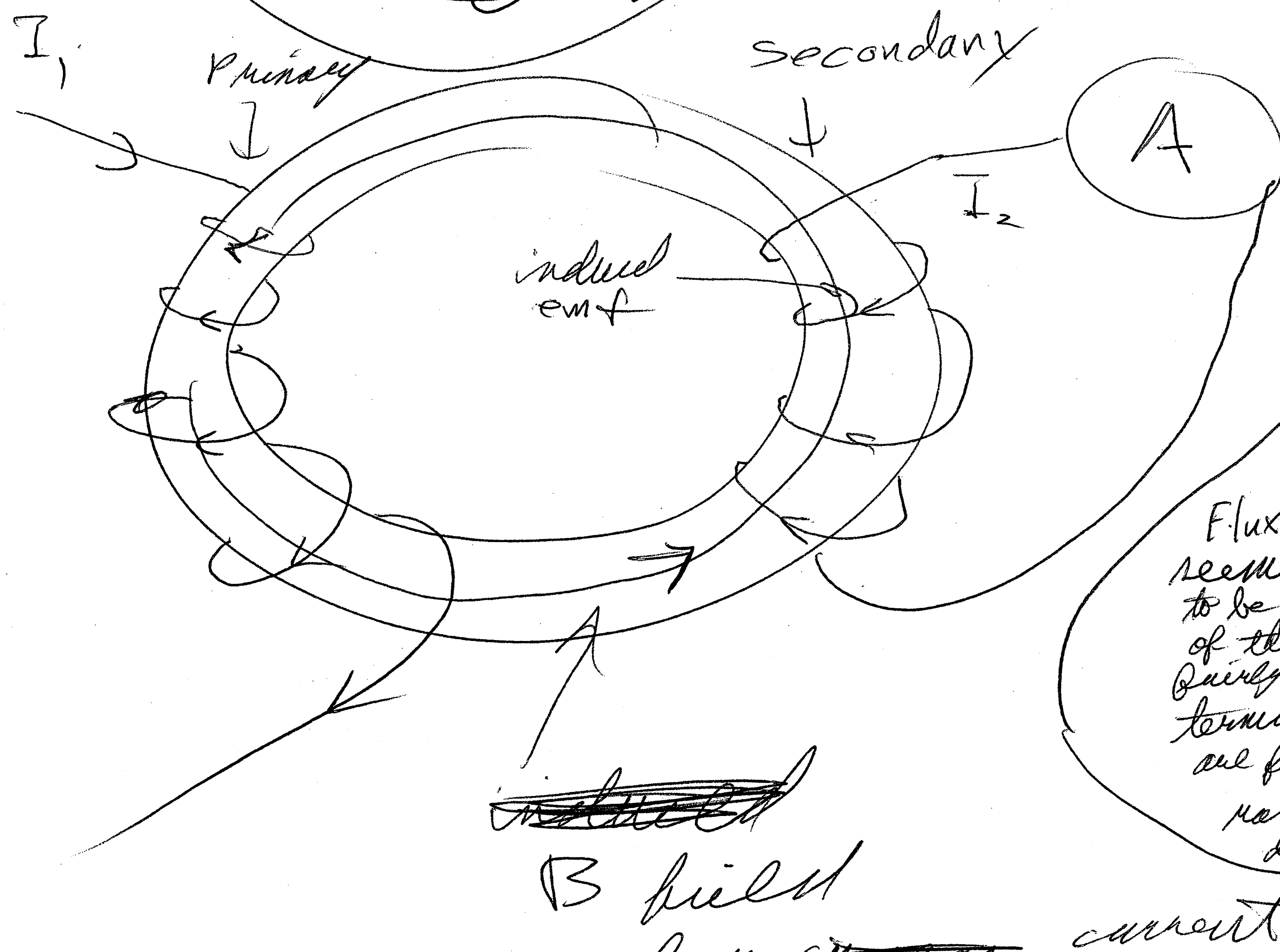
TM-1004
for linkage. (Wik

in the iron core itself

ideally all the ~~[illegible]~~ magnetic flux "linked" by the primary is "linked" by the secondary



Flux linkage seems to be one of those fairly common terms that are fairly rarely defined

~~[illegible]~~

B field

for ~~[illegible]~~ current

— it grows as current is increased

— there is an induced emf in the secondary that causes a current

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to flow in the second loop.

— once I_1 becomes constant the B-field in the core becomes constant.

— no more flux change

— no more induced emf in either coil

— no current in the second loop.

The B-field by the way has stored energy — the energy the ~~emf~~ battery expended to create it.

<u>But</u> if you then open the 1st circuit

then the current
will try to stop,
but the primary
coil induced emf
will try to stop but
nevertheless the
B-field in the
core will try
to decrease
causing an induced
emf in the secondary
in the current.

the current in the
primary actually might

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lead to a bit of induced oscillation of the current

current builds up a charge separation that creates a potential that reverses the ~~cu~~ current flow

this goes on until resistance in the wires and battery dissipates stored energy to heat. We'll look quantitatively at oscillating circuits in Ch. 32

Called LC and RLC circuits

inductor - capacitor

resistor inductor capacitor)

back to p. 31-14

§ 3~~1~~2.2 Motional EMF

("emotional emf")

The analysis leads to the flux rule $\mathcal{E} = -\frac{d\Phi_B}{dt}$ which looks like Faraday's law and some folks call it that & others don't (cv EM-302)

~~sort of Faraday's law with a twist~~

— The following analysis is a bit tricky — but I think it doesn't leave the logical holes of ~~the~~ SJ-871

Say one has a uniform B-field into the page

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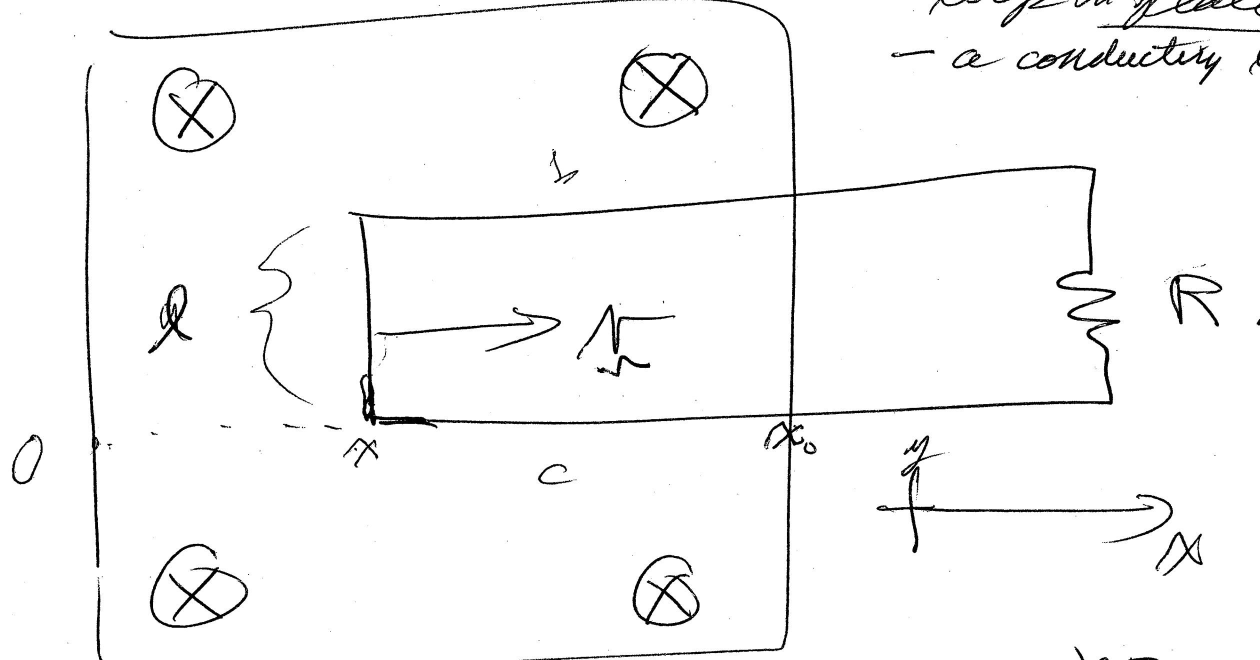
in a finite region

— We put a rectangular loop in place.

— a conducting loop of ~~steel~~ ideal wire.



a resistor for the heck of it.

— no infinite currents for us.

$\Phi_B = (x_0 - x)\ell B$

$\frac{d\Phi_B}{dt} = -\frac{dx}{dt}\ell B = -v\ell B$

The system has come to a steady state, — so any current is steady.

Pull the loop to the right at constant velocity $\underset{\sim}{v}$

The charge carriers have charge $q > 0$ (+ve just for discussion simplicity)

The charge carriers in b and c ~~c feel no magnetic force since~~

experience

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$$\underset{\sim}{F}^{\text{magnetic}}_{b \text{ or } c} = q \underset{\sim}{v}_{b \text{ or } c} \times \underset{\sim}{B}$$

But the carriers in b & c are constrained to move along the x-direction only.

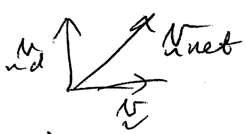
— the wire force so constrains them.

And thus $\underset{\sim}{F}_{b \times c}$ is always perpendicular to b and c and the ~~ex~~ x direction and is canceled by the wire force of constraint.

— So they is no consequence for current flow for magnetic force on the carriers in the a + b segments.

31-34 } p. 24-33 got omitted!

Now in the l segment that has length l the carriers experience



$$\mathbf{F}_B = q(\overbrace{\mathbf{v} + \mathbf{v}_d}^{\mathbf{v}_{net}}) \times \mathbf{B}$$

imposed velocity of the wire in the +ve x-direction

drift velocity of carriers in the y-direction

we can take this as being in the +ve y direction.

this is because the $q\,\mathbf{v} \times \mathbf{B}$ part of the force is in the +ve y direction and that would have set the condition for current flow

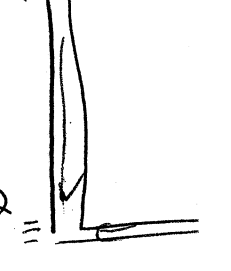
also everything works out consistently with this assumption

$$\underset{\sim}{F}_B = q(\underset{\sim}{v} + \underset{\sim}{v}_d) \times B$$

$$= q v B \hat{y} + q v_d B(-\hat{x})$$

- Note for free particles a magnetic force would cause circling
- But these particles are <u>NOT</u> free.
- The wire force constrains them to have NO acceleration in ~~x~~-direction.
- Since we've assumed steady state there must be an assembly of separated charge that creates an electrostatic force

Probably something like this



But in detail hard to know,

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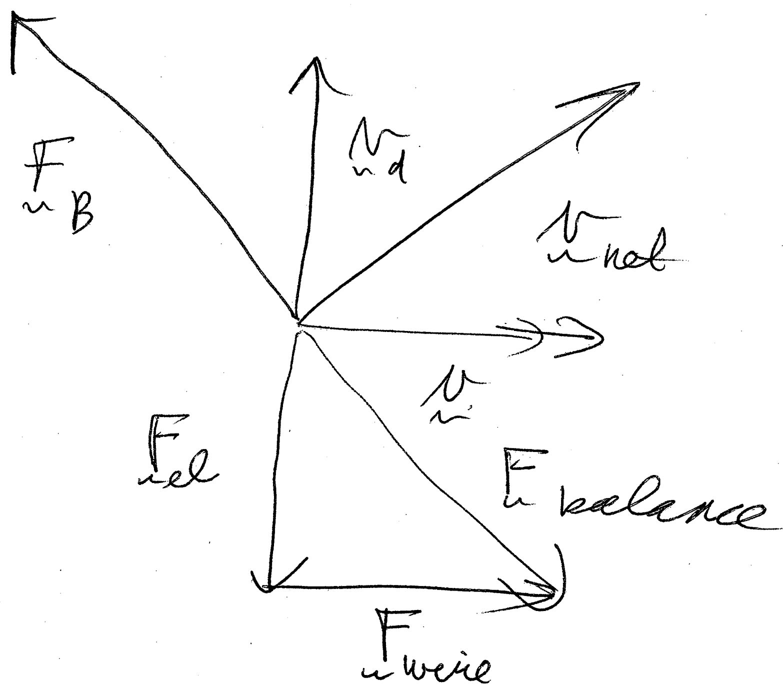
~~that~~ prevents acceleration in the y-direction.

If takes a detailed analysis that is beyond our scope (and ~~technique~~) to determine what ~~that~~ separated charge distribution is.
But the establishment of the steady state means it's there somewhere guiding the current (BO-216).

Since our system is idealized, thin wires — Not determined anyway?



$$F_{el} = -\,q\,v\,B\,\hat{y}$$

$$F_{wire} = -\,q\,v_d\,B\,(-\hat{x}) = q\,v_d\,B\,\hat{x}$$

Now the magnetic force ~~acts the~~ emf in this case — it's the force that pushes the charge up the potential hill of the separated charge distribution

$$\mathcal{E} = \frac{1}{q}\oint \underset{\sim}{F}_B \cdot d\underset{\sim}{s}$$

following loop.

all around loop but only the l-side contributes and gives

The $\hat{x}$ of $\underset{\sim}{F}_B$ gives zero (31-34)

Remember emf is force per charge in a line integral along a path at one instant in time GrEM-294

$$= \frac{1}{q} q\, v B \hat{y} \cdot l \hat{y}$$

$$= v B l$$

$$= -\frac{d\Phi_B}{dt}$$

(see p. 31-22)

So $\mathcal{E} = -\dfrac{d\Phi_B}{dt}$

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This looks like Faraday's law of induction

and Lenz's law holds too in ~~that~~ the emf tries to create a induced current ~~that~~ will create an induced B-field ~~that~~ will oppose the reduction in <u>magnetic flux</u> due our pulling the wire.

But the emf force is a <u>magnetic force</u> not an induced <u>electric force</u>. (GrEM-302

So there is a distinction and some folks *refuse* to call $\mathcal{E} = -\frac{d\Phi_B}{dt}$ Faraday's law in this case but call it the *flux rule for motional emf* (GrEM-296)

$\mathcal{E} = -\frac{d\Phi_B}{dt}$

also holds for non-rectangular and non-planar loops and loops moving in arbitrary directions and shape changing loops (GrEM-296)

When fails one has to go back to magnetic force (Lorentz force)

But there are cases where it fails (so it is unlike Faraday's law which in the

(GrEM-298)

31-40

realm of classical physics is an inviolate law)

So

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

is a sort of "universal flux rule" (GrEM-302)

Faraday's law

$\mathcal{E} = \oint \underset{\sim}{E} \cdot d\underset{\sim}{s}$

(never fails)

and

then is motional emf flux rule where the emf is at least partially due to magnetic force

Some folks may no distinction, but GrEM-302-303 does and do see his point.

When $\mathcal{E} \neq \oint \underset{\sim}{E} \cdot d\underset{\sim}{s}$ ~~Fara~~ the flux can't be converted the differential form of Faraday's law

But can the two rules be reconciled?

Sort of.

Say you were in a frame of reference moving with the

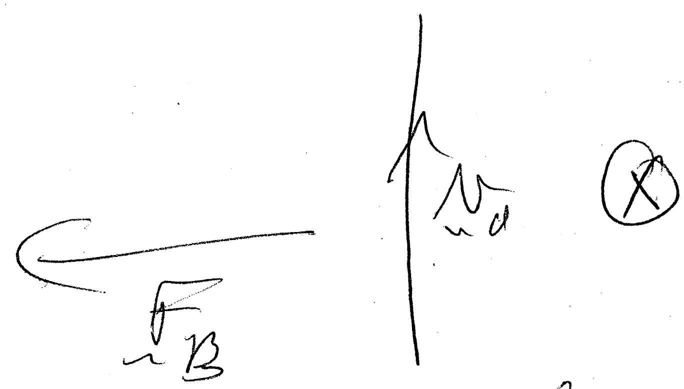
~~pulling to~~

pulled loop.

In that frame $\underset{\sim}{v} = 0$,

but there is still an emf & current.

but $F_B = q\, \underset{\sim}{v}_d \times \underset{\sim}{B}$



is perpendicular to the direction of the current and can cause no emf.

In this frame the <u>emf</u>

is ~~the~~ Faraday's law induced electric field.

~~The~~ E-field and B-fields get their identities mixed in frame transformations — This is understood in special relativity but beyond our scope to go into more deeply here.

There is some murkiness ~~though~~ since GrEM-298 hints ~~that~~ motional emf can't always be changed into Faraday's law electric field emf exactly by a frame transition.

Actually often electric generators and motors rely on motional emf and so a purist like GrEM-302 would say that is the universal flux rule at work and not Faraday's law ⟶ But most would not make the distinction I think.

as we'll see.

$\boxed{31-44}$

Power transfer for our loop?

The force on a charge to pull it along is

$$\underset{\text{on a charge}}{\underset{\sim}{F}} = -q\,\underset{\sim}{v}_d \times \underset{\sim}{B} = q v_d B \hat{x}$$

$$\therefore \; P_{\substack{\cancel{\text{on a ch}} \\ \text{input by pulling} \\ \text{on a charge}}} = q v_d B v \quad \left\{ \; v = \frac{\Delta x}{\Delta t} \right.$$

$$P_{\substack{\text{input} \\ \text{Total}}} = \underbrace{n A \ell}\, q v_d B v = \underbrace{I}\, \ell v B$$

n is charge carrier density
$A\ell =$ ~~is the~~ volume of the wire

$nAqv_d$ is current.

The emf $\mathcal{E} = vB\ell$ (see p. 31-37)

$$\therefore \; P_{\substack{\cancel{\text{out}} \\ \text{put into current.}}} = \mathcal{E} I = I \ell v B = P_{\substack{\text{input} \\ \text{from} \\ \text{outside}}}$$

So energy is conserved.

The energy put into the current gets dissipated as ~~waste~~ heat in the resistor

From Kirchhoff's ~~cur~~ voltage law $\mathcal{E} = IR$

$\therefore \quad I = \mathcal{E}/R$

and $P_{out} = \dfrac{\mathcal{E}^2}{R} = \dfrac{v^2 \ell^2 B^2}{R}$

§ 31.3 Lenz's law

— already covered

§ 31.4 Induced EMF and Induced electric fields

— we've already covered,

$\boxed{31-46}$

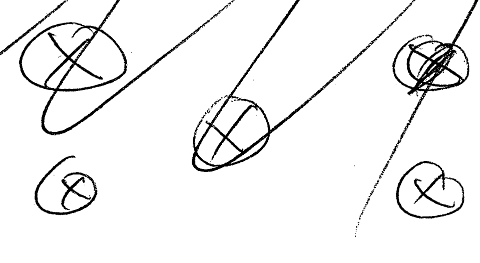
but a few examples can be done

$$\mathcal{E} = \oint \underline{E} \cdot d\underline{s} = -\frac{d\Phi_B}{dt}$$

Recall Faraday's law of induction.

integral at one instant (GrEM-294) in time over a path unmoving in the frame of the calculation. (so the emf is all electric field and no muddled with B-fields.

Ex. Say B-field uniform into page

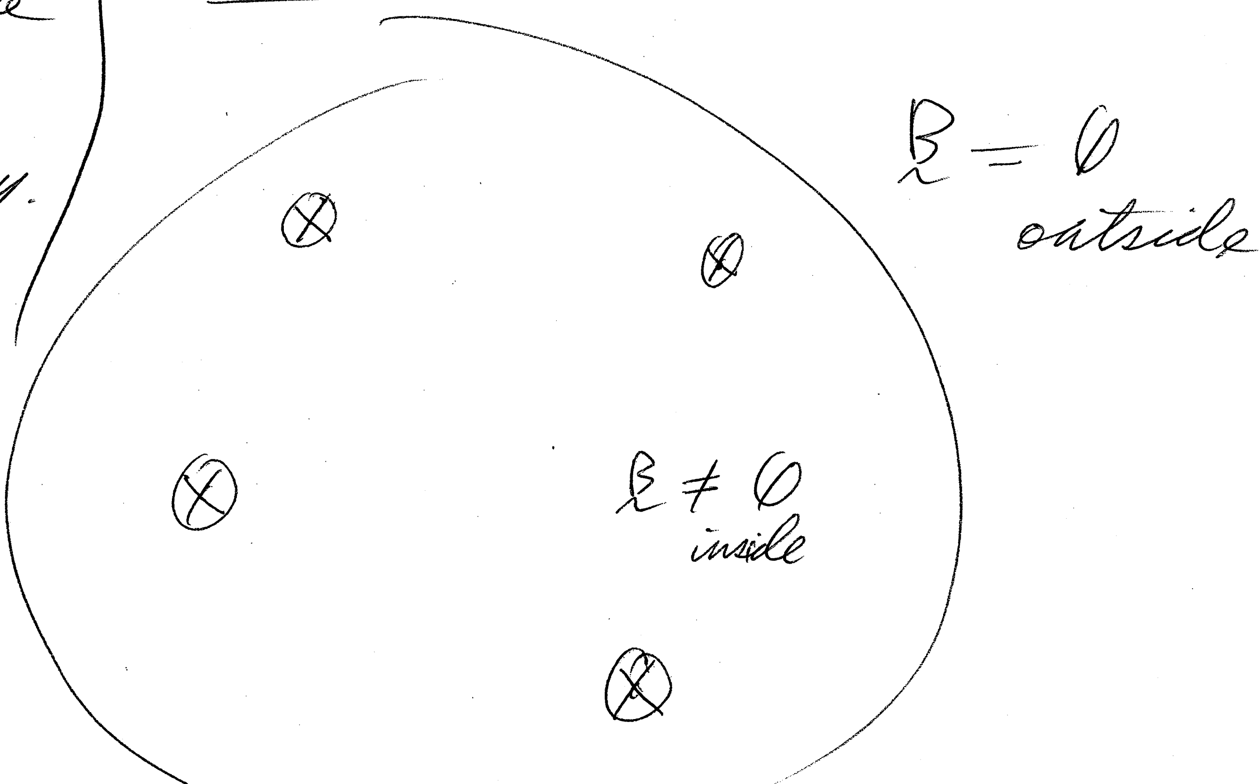


~~Say now B is varying with time~~

~~There must be an induced E-field.~~

Ex Consider a Uniform B-field in a circularly cylindrical volume

uniform in space but time varying.



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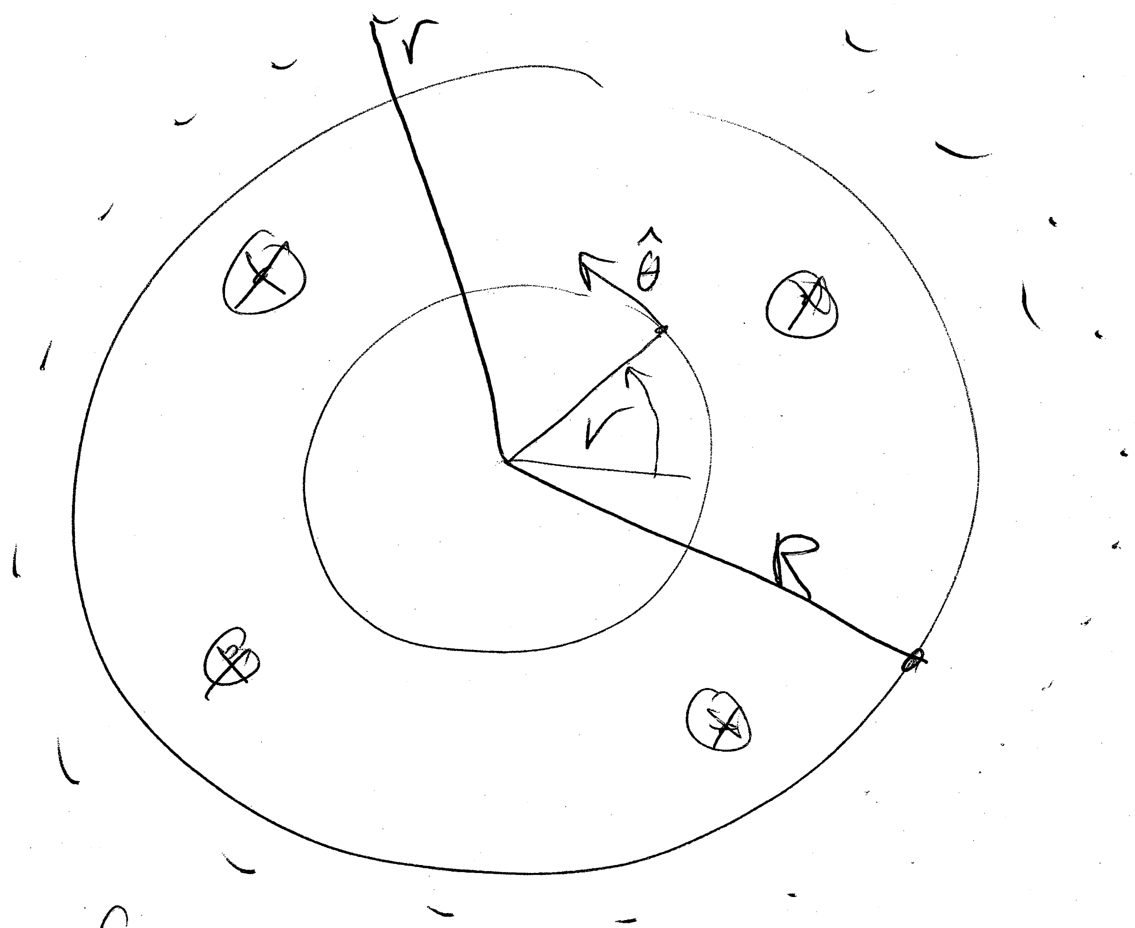
Consider a circular path about the center of symmetry — of radius r



$$\oint \underline{E} \cdot d\underline{\ell} = -\frac{d\Phi_B}{dt}$$

The sign we have to det from Lenz's law

$$E\, 2\pi r = \begin{cases} \pi r^2 \dfrac{dB}{dt} \\ \pi R^2 \dfrac{dB}{dt} \end{cases}$$

by symmetry in magnitude

$\underset{\sim}{B}$ is spatial uniform.

$$\underset{\sim}{E} = \frac{r}{2}\frac{dB}{dt}\hat{\theta} \quad r < R, \quad \frac{R^2}{2r}\frac{dB}{dt}\hat{\theta}$$

The induced E-field exists where there is no B-field!!

There must be ~~some~~ delay time effect ~~when the~~ $\frac{d^2B}{dt^2}$ is non-zero ~~that~~ E-field propagates outward — light speed in the finite signaling speed

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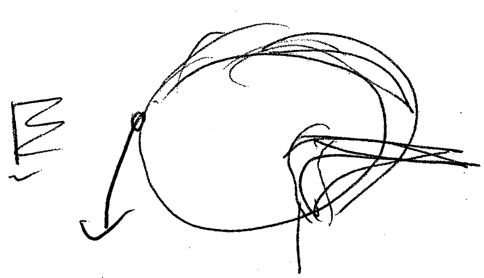
Lenz's law

tells us if B increases (in the direction into page) then to the ~~counteractivity~~

emf must be counterclockwise, So ~~that~~ the induced B-field points out of the page.



~~Wh~~ Note the E-field lines in this highly symmetrical case form closed loops.

— HRW-772 say ~~the~~ or imply E-field lines are always closed loops for induced E-fields, but I'm not sure this is true. — they could extend to infinity and not

31-50

close in general.

— maybe not even often without high symmetry.

?

What of the region of the B-field is not circular?

— well then we don't have circular symmetry and can find a simple solution.

What of an infinite region of B-field?

— this seems to be an indeterminate case

our ~~o~~ circular symmetry result

doesn't extrapolate since in ~~for~~ an infinite extent there is no center of symmetry to be define.

The induced $\underline{E}$-field at a point doesn't just depend on the local $\underline{B}$-field at that point, but on an extended region. The boundary conditions of the region affect the $\underline{E}$-field everywhere it seems.
— there are probably time-delay and relativistic complications ~~too~~.

<u>Ex 31.7</u> (SJ-889)

One can actually create regions of uniform B-field

$\boxed{31\text{-}52}$

using ~~sol~~ solenoids.

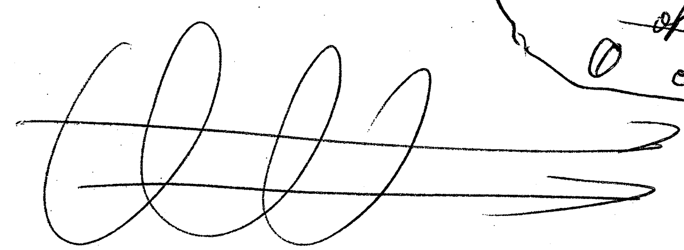
Say we have a circularly symmetric one.

Recall ~~for~~ an ideal so

$$B = \begin{cases} \mu_0 \, n \, I & \text{inside} \\ 0 & \text{outside} \end{cases}$$

number of turns per unit length

current in wire



$\underset{\sim}{B}$ direction determined by the right-hand rule.

So

$$\cancel{B}\,E = \begin{cases} \dfrac{r}{2}\dfrac{dB}{dt} = \dfrac{r}{2}\mu_0 n \dfrac{dI}{dt} & \text{inside} \\[2ex] \dfrac{R^2}{2r}\mu_0 n \dfrac{dI}{dt} & \text{outside} \end{cases}$$

so outside falls as $\sim \frac{1}{r}$

But this can only hold for

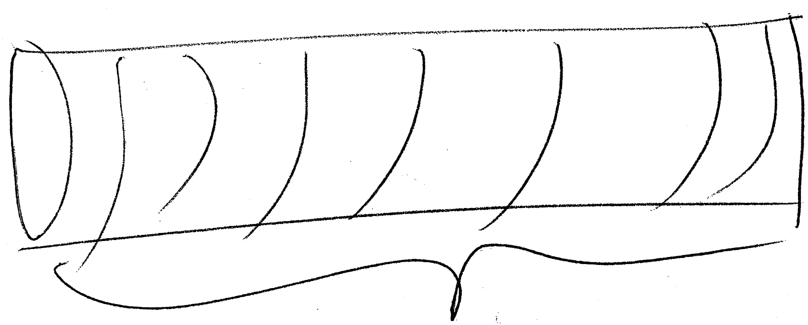
~~Actuall~~

a finite distance

for $r << l$

the length of the solenoid.

Remember we've made the infinite ideal solenoid approximation



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and so our results hold only close to and far from the ends.

Actually we can calculate the induced emf in the whole solenoid ~~and~~ and get an self-inductance formula in the infinite solenoid approximation.

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$$\mathcal{E}_{\substack{\text{around}\\\text{one loop}\\\text{at } r=R}} = -\frac{d\Phi_B}{dt} = \pi R^2 \frac{dB}{dt}$$

$$= \pi R^2 \mu_0 n \frac{dI}{dt}$$

Say the solenoid has N ~~turns~~

and $n = \frac{N}{l}$

Then $\mathcal{E}_{solenoid} = N \mathcal{E}_{one\ loop}$

$$= -\underbrace{\pi R^2 \frac{N^2}{l} \mu_0}_{L} \frac{dI}{dt}$$

The -ve sign is conventional. - It really tells us the induced emf ~~ove~~ tries to oppose any change in B-field (which means any in current by Lenz's law.)

$L = \pi R^2 \frac{N^2}{l} \mu_0 = \mu_0 n^2 V_{vol}$ is the inductance. ~~The~~ This is in the infinite solenoid ~~tm~~ approximation

(Wiki: inductance)

and
- no soft magnetic core in the solenoid to enhance the magnetic field

- a $\mu_{relative}$ factor can be included then in some approximation (Wik: Inductance)

- We will encounter self inductance & mutual inductance in Ch. 32

31-56

Electric
§ 31.5 Generator & Motor

— There are in many ways the essence of the modern world where almost all commercial energy passes thru an <u>electrical form</u> at some point in it's conversion from primary source

[coal, natural gas, oil, nuclear fission fuel, hydropower, wind, solar]

— except the transport sector ([illegible])

31-57

Electrical energy is the most convenient and flexible of all energy types.

Only the transport sector has resisted electrification — because of the problem with storing electrical PE

→ we can do it in batteries and capacitors, but ~~in the~~ they seemed inadequate except for small or special applications.

→ New storages might help.

- better batteries
- storage as hydrogen gas after hydrolysis from water
- in compressed-air storage.

31-58

this last could be the wave of the future.

solar & wind are the renewables

— solar is quasi-infinite

but it is dispersed & intermittant

— compressed air energy storage may do the job for us.

But electric motors & generators

~~moto~~ generators

convert ~~mechanical~~ kinetic ~~(mainly~~ energy to electrical potential energy.

motors do the reverse

~~and~~

generators and motors are essentially the same ~~them~~ device run in different directions

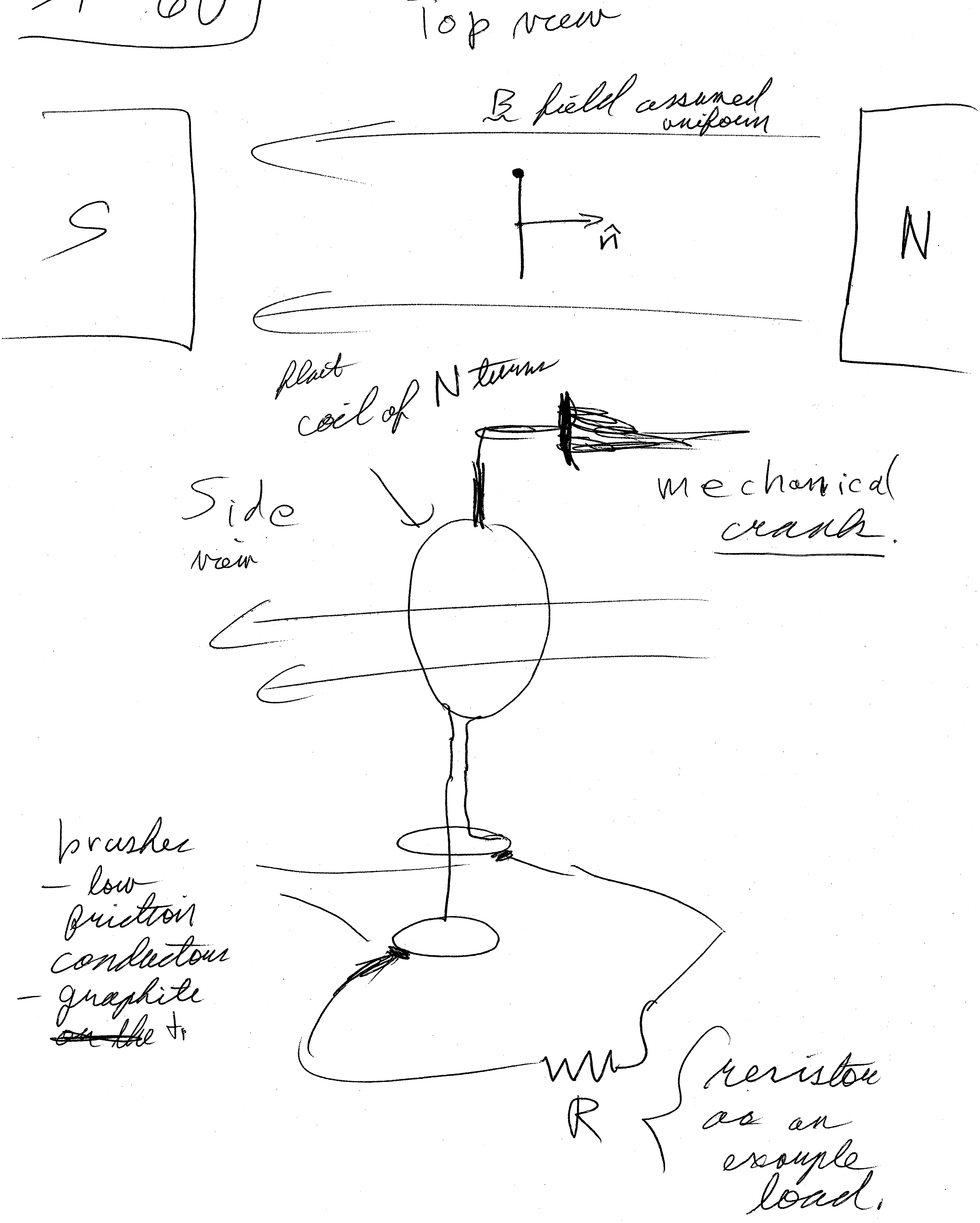
But few (any?) practical generators or motors are designed to run in reverse.

Here we just look at the essence of generators/motors

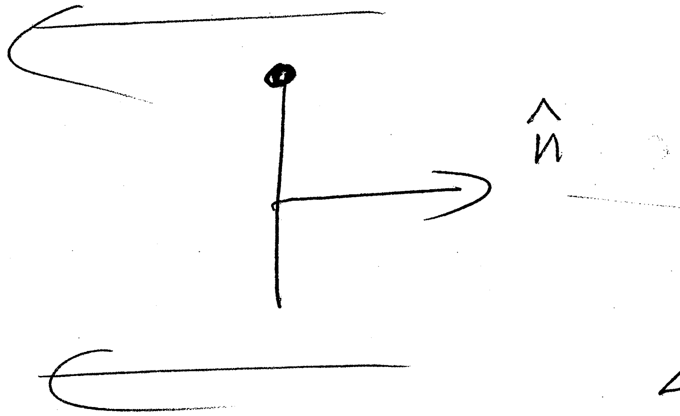
— practical designs are beyond our scope & my knowledge (but go back a long way to 1870s & 1880s)

The essence is a rotating coil in a permanent magnet's magnetic field.

31-60


Top view
B field assumed uniform
S
N
n̂
flat coil of N turns
Side view
mechanical crank.
brushes
- low friction conductors
- graphite ~~on the~~
R
resistor as an example load.

A ~~the~~ start position



unlike textbooks I can't gloss over it.

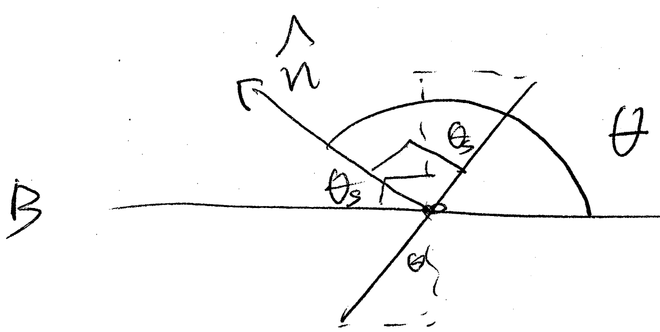
Getting the signs to work out right is a major headache in the analysis

But really at the end one can guess by energy conserv & Lenz's law

$$\Phi_{B_{\text{one turn}}} = -AB$$ thru one turn of the coil.

$\theta = 0$

In general positn



~~$A_{\text{area}} = A$~~

$\cos(\theta) = \cos(\pi - \theta_s)$

$= -\cos\theta_s$

< 0

and so $\Phi > 0$

$$\Phi_{B_{\text{one turn}}} = -AB\cos\theta$$

$$\Phi_B = -NAB\cos\theta$$ for N turns

Say we cranked the ~~the~~ coil at constant angular velocity ω.

31-62

[actually cranking it at constant ω is ~~itself a nontrivial to do~~ takes a bit of discourse to explain, but for the moment we just say we do it]

$$\mathcal{E} = -\frac{d(\Phi_B)}{dt} = +NAB\frac{d\cos\omega t}{dt}$$

$$= -NAB\omega\sin\omega t$$

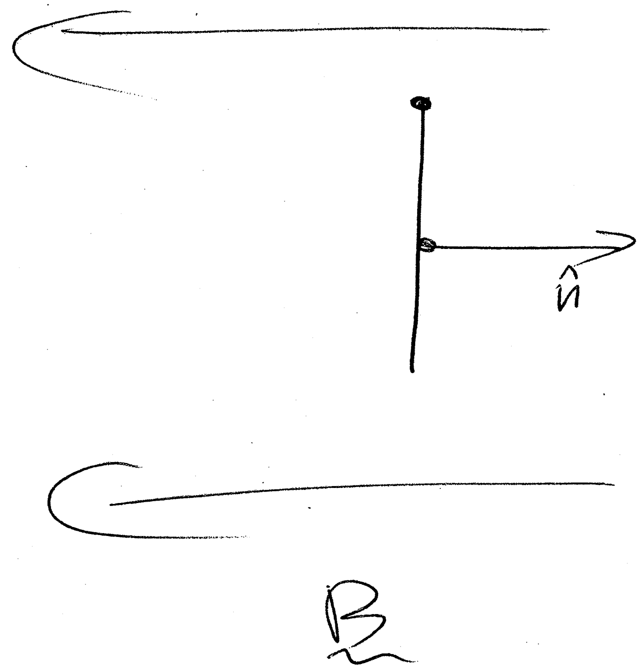
$\mathcal{E}_{MAX} = NAB\omega$ is ~~the~~ amplitude

$\underset{\sim}{\mu} = NAI\hat{n}$ is the magnetic dipole moment recall.

(Faraday's law or universal flux rule if one prefers GrEM-302)

If one is in the frame of the coil the emf is caused by an induced electric field — in the frame of device, it's magnetic force that acts.)

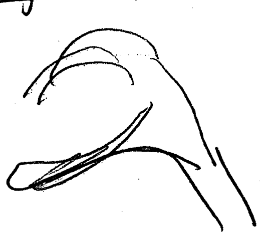
Sequence of snapshots

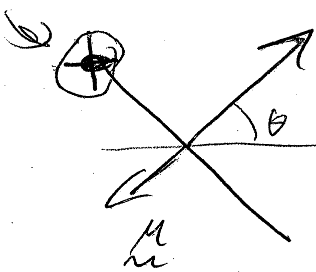


$\mathcal{E} = 0$

$\Phi_B = -NAB$

Right-hand





Φ_B increasing

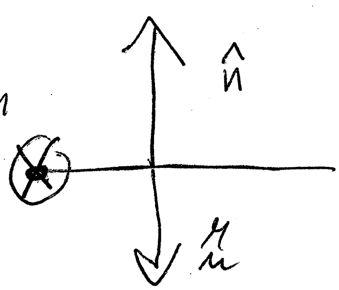
but $\Phi_B < 0$

$\mathcal{E}$ decreasing

$|\mathcal{E}|$ increases

The current must act to stop the change in flux by Lenz's law

Magnetic moment always being moved toward anti-alignment.

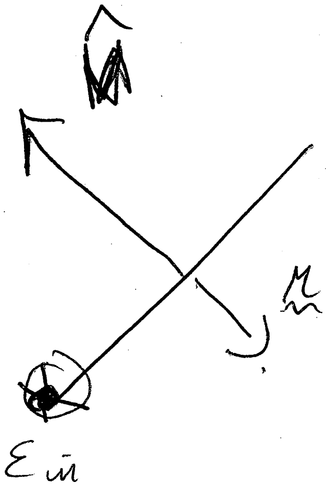


$\Phi_B = 0$ and increasing

$|\mathcal{E}|$ at max

$\mathcal{E}$ at min.

31-64

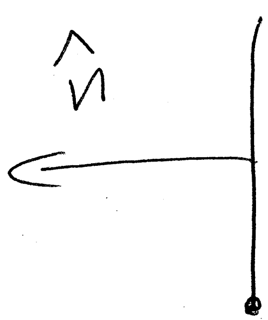


$\Phi_B > 0$

$|\varepsilon|$ now ~~decreasing~~

ε increasing

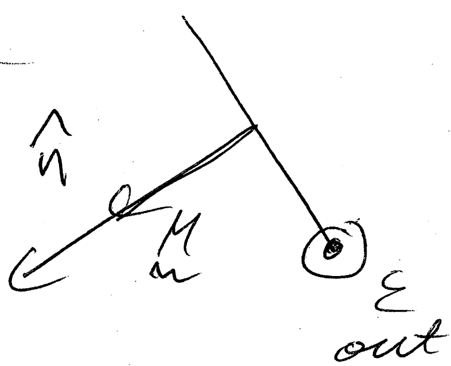
The μ has flipped



$\Phi_B = NAB$ at max

$\varepsilon = 0$

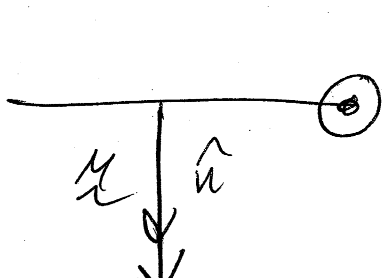
$\mu = 0$



Φ_B decreasing

ε decreasing

~~$|\varepsilon|$ increasing~~

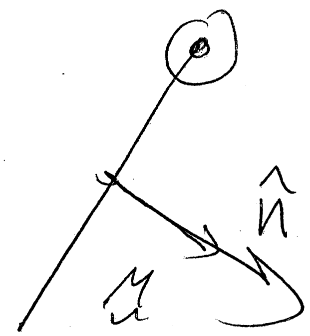


$\Phi_B = 0$

ε at ~~minimum~~ max

~~$|\varepsilon|$ at max~~

31-65



Always pulling μ out of alignment.

$\Phi_B < 0$ and decreasing

$\mathcal{E}$ decreasing

~~$|\mathcal{E}|$ decreasing~~

$\Phi_B < 0$ at minimum

$\mathcal{E} = 0$

and so on.

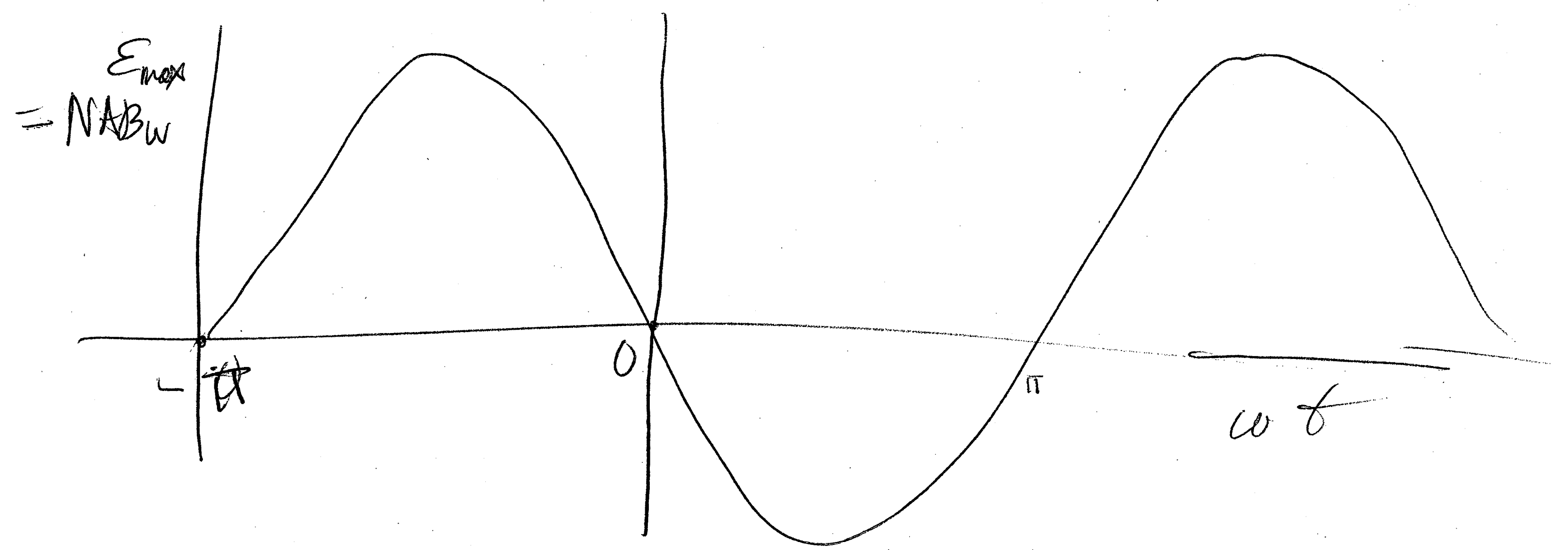
Anyway the sign of

$$\mathcal{E} = -NAB\omega \sin \omega t$$

isn't so important since we aren't interested in time zero or where ~~it is~~ that is.

31-66

The important thing is that the time dependence is sinusoidal



— this is the "natural" AC (alternating current output)

But when you think about it, not all that trivial to get as we'll see in a moment.

Now what sets $\mathcal{E} = -NAB\omega \sin\omega t$?

31-67

N number of turns.
— increase for more

A area of coil
increase for more

B magnetic field
— increase for more
actually a soft magnetic core (soft iron~~e~~) put in the coil enhances B — sometimes a lot. — factor of ~~thousands~~ (TM-945)
But there are complications (e.g. eddy currents in the iron TM-1004) — which cause heating loss of energy.

w → the angular velocity of cranking the loop.

31-68

But you get nothing for free.

The cranking takes work, force, power, ~~torque~~ — how much?

First let's solve our AC circuit

$\mathcal{E}$ is the emf

— time varying

— it creates a time varying electrostatic distribution somewhere — we know it's there, but where exactly takes detail ~~at~~ analysis.

at instant by instant we can regard as static unless the $\mathcal{E}$ varies too too fast which ~~[illegible]~~ we can assume not usually

Anyway thru the loop

the potential rise $V_{rise} = \mathcal{E}$

Going around the circuit and using Kirchhoff voltage law (31-69)

$$\mathcal{E} = IR = V_R$$

$$\therefore \quad I = \frac{\mathcal{E}}{R} = -\frac{NAB\omega \sin\omega t}{R}$$

$$P_{out} = V_R I = I^2 R$$

$$= \frac{\mathcal{E}^2}{R}$$

$$= \frac{(NAB\omega)^2 \sin^2\omega t}{R}$$

$$\therefore \quad \underset{\sim}{\mu} = NAI\hat{n}$$

$$= -\frac{(NA)^2 B\omega \sin\omega t}{R}\,\hat{n}$$

$$\text{Recall} \quad \underset{\sim}{\tau} = \underset{\sim}{\mu} \times \underset{\sim}{B}$$

$$= -\frac{(NA)^2 B^2 \omega \sin\omega t}{R}\,\hat{z}$$

This is torque of the magnetic field on dipole.

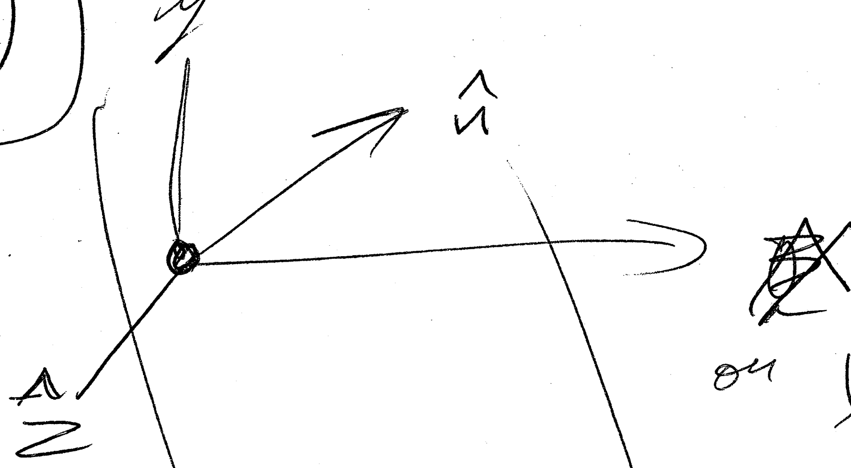
$$\hat{n} \times \underset{\sim}{B}$$

$$= B\sin\theta$$

$$= -B\sin\theta$$

$$= -B\sin\omega t$$

31-70)



on $\underline{B}$ direction

$$\tau_z = \underset{\sim}{\tau} \cdot \hat{z} = -\frac{(NAB)^2 \omega \sin \omega t}{R}$$

The magnetic torque ~~opposes / always the~~ way you are turning ~~the coil~~

always tries to align dipole with B-field recall.

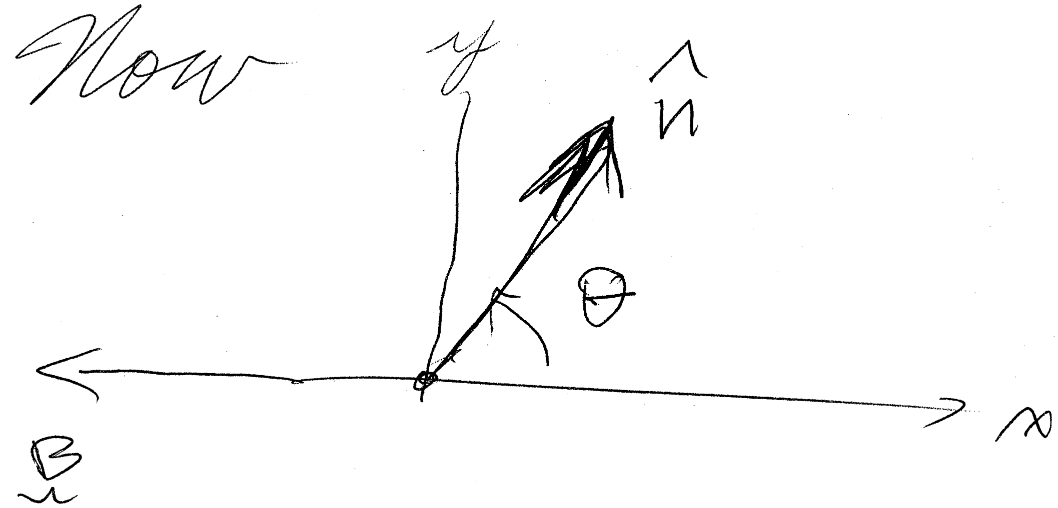
$\omega t \in [0, \pi]$

it is a negative torque & so tries to align with $\hat{x}$

for $\omega t \in [\pi, 2\pi]$ the torque is positive

Recall $\underset{\sim}{\tau} = \underset{\sim}{\mu} \times \underset{\sim}{B}$

Now



So $\underset{\sim}{B}$ points in $-\hat{x}$

Now $\hat{n} \times \hat{x} = -\sin\theta\,\hat{z}$

$\therefore\ \underset{\sim}{\tau} = \underset{\sim}{\mu} \times \underset{\sim}{B}$

$$= -\frac{(NAB)^2\,\omega \sin\omega t}{R}(-1)(-\sin\underbrace{\theta}_{\omega t}\,\hat{z})$$

$$= -\frac{(NAB)^2\,\omega \sin^2\omega t}{R}\,\hat{z}$$

a $\hat{z}$ ~~torque~~ always tries to ~~torque~~ counterclockwise

but ~~that~~ minus sign means ~~the~~ magnetic torque is

31-72

always clockwise.

which is opposite the direction of motion.

<u>But also</u> the ω is constant

and τ_{net} (in z direction) $= I\alpha$

~~Rotatation~~ Rotational 2^{nd} law for a rigid body about a fixed axis.

So the applied torque by whatever is turning the crank is $\boxed{33-73}$

Always counterclockwise

$$\tau_{app} = -\tau_{magnetic}$$

$$= \underline{(NAB)\,\omega \sin^2\omega t}$$

~~Work~~ Work done by the input source is

$$dW = \tau\, d\theta$$

$$P_{input} = \tau\omega$$

$$= \underline{(NAB\omega)^2 \sin^2\omega t}$$

but this is the same as the power output

Which is good.

— energy is conserved as it had to be.

31-74

Actually one can use the energy conservation principle to skip some of the sign conundrums.

What is time-averaged power

Well $= 2\pi = \omega T$

so $T = \frac{2\pi}{\omega}$ which is the ~~repeat~~ period.

and $\frac{1}{T}\int_{t}^{t+T} \begin{Bmatrix} \sin^2 \omega t \\ \cos^2 \omega t \end{Bmatrix} dt$

$= \frac{1}{T}\int_{t}^{t+T} \frac{1}{2}\begin{Bmatrix} 1 - \cos 2\omega t \\ 1 + \cos 2\omega t \end{Bmatrix} dt$ by trig.

$= \frac{1}{T}\int_{\theta}^{\theta+2\pi} \frac{1}{2}\begin{Bmatrix} 1 - \cos 2\theta \\ 1 + \cos 2\theta \end{Bmatrix} \frac{d\theta}{\omega}$

$= \frac{1}{2\pi}\left(\frac{1}{2}\right)\Big\{ \theta - \frac{\sin\theta}{2} \Big\}\Big|_{\theta}^{\theta+2\pi}$

$$= \frac{1}{2}$$ which is a result we'll use in Ch. 32 & 33

$$\therefore P_{\substack{input \\ ave}} = P_{\substack{output \\ ave}}$$

$$= \frac{(NAB\omega)^2}{2R}$$

Now what about that applied torque

$$\tau = \frac{NAB\omega}{R} \sin^2 \omega t$$

Note it depended on the load R and it's time dependent.

3 3-76

WiK (Alternator)

~~Plan~~ I think that practical generators actually produce sinusoidal ~~signal for~~ emf for constant energy input from turbines (I'm not sure.)

I think there are tricks (worked out long ago) to get sinusoidal emf from turbines. Although modern innovations are ongoing.

Any way beyond our scope understanding.

In fact WiK seems to say constant speed is essential.

Maybe when rotation fast enough the speed variation is small in a period $\Delta\tau = I\Delta\omega$ is tiny for [illegible] $= \frac{2\pi}{\omega}$? and since the time average is constant since power input is constant from turbine

Actually power plant seem to use rotating magnet and stationary coils. Maybe the trick is in that.

Maybe some self-regulation

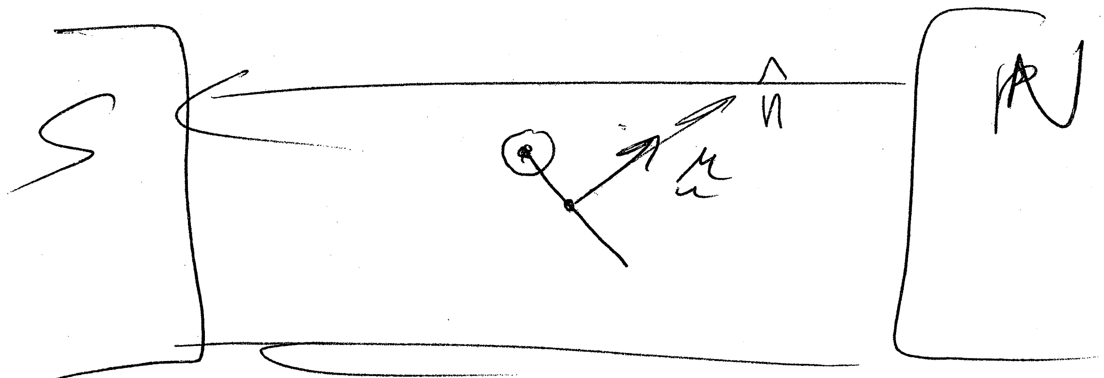
Electric Motor

- well not in detail
- but essentially all the foregoing in reverse.
- Now one attaches the coil to an emf source.
- So there is a sinusoidal potential driving a current in the coil

which is opposed by the coil emf acting

38-78

as an effective resistance.

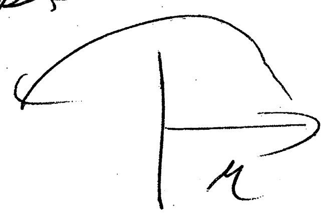


Current is 180° out of phase with generator case, — Here the magnetic moment is always moving to alignment — in the generator case it was always being moved to anti-alignment.

The magnetic torque on the dipole moment created by the current

causes a turning motion that tries to align $\hat{\mu}$ & $\hat{B}$

but then the current flips



and the alignment repeats

Actually to keep a constant ω the countertorque of what you are driving should equal the magnetic torque.

Now at this point you may wonder what decides the direction of motion if the torques are always equal?

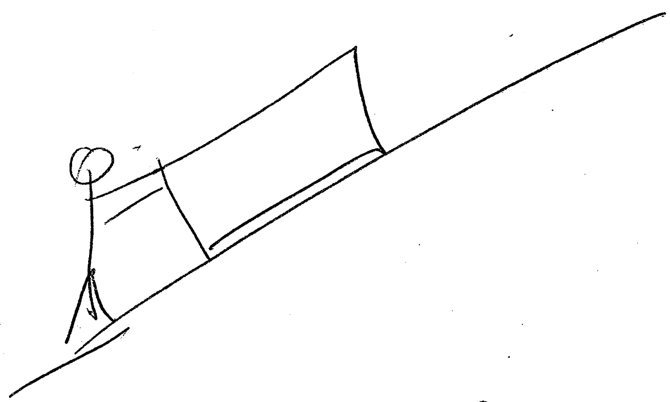
Magnet & applied are

Well how things are initially set up at turn on time ~~and the that that~~ That establishes a momentum in the rotating coil that can't easily be changed

sudden flips in direction can't happen

31-80) Also the driver of either generator or motor must make up for energy losses in the conversion process to keep the motion ongoing

— An analogous case



— Pushing a block up a frictionless slope. at constant __velocity__.

— the up force by you and gravity down force ideally balance whether the block is going up or down.

But if goin up, the __momentum__ is up. If going down, the __momentum__ is down

up and down and even though the forces nearly balance

NOT equally balanced between goin

~~at a constant speed.~~ [3-8]

the initial acceleration phase decided whether

it goes up

+ your chemical energy goes into gravitational PE

or if it

is coming down

and grav. PE

is getting ~~turned~~ into waste heat in your body.

Less ideally either way

the slope has resistance (friction).

so going up some

Ch. energy → goes into slope waste heat

~~31~~ 3B-82

and the fact that your body does that means $\underset{\sim}{F}_{you} > \underset{\sim}{F}_{grav}$ always actually.

Going down the reverse case

$\underset{\sim}{F}_{you} < \underset{\sim}{F}_{grav}$

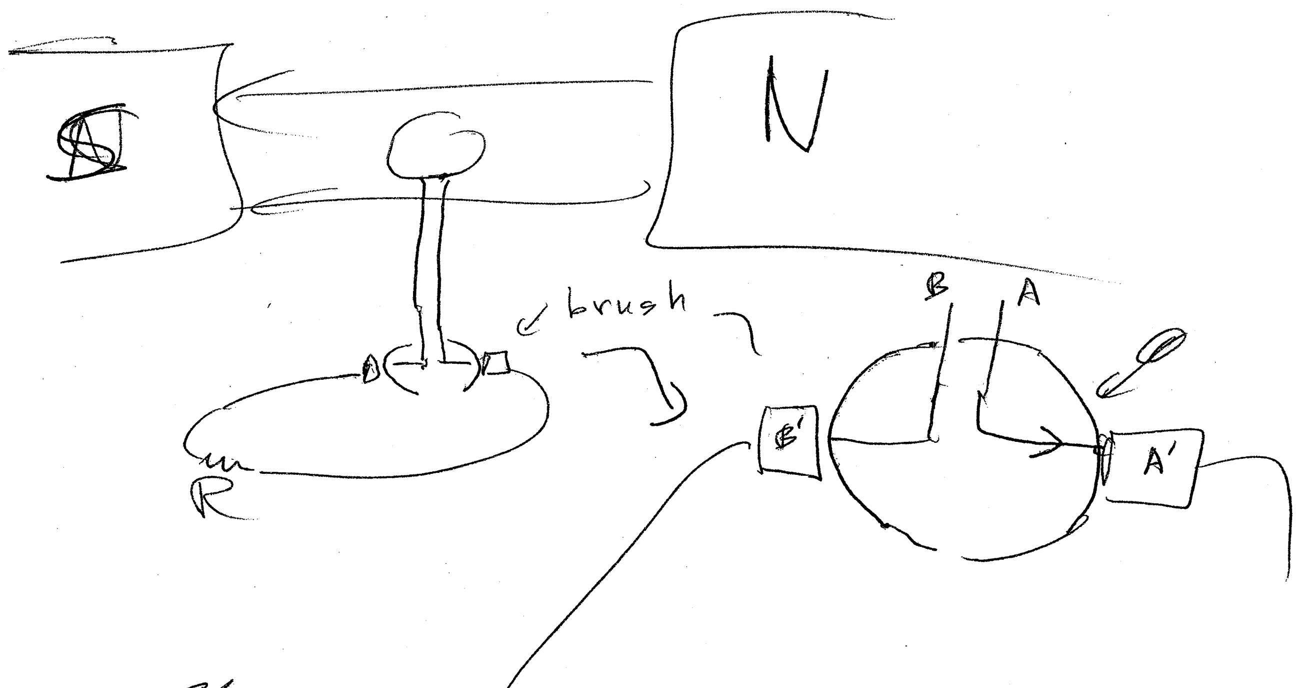
since the slope resistance helps balance the force.

The same is true for an electrical generator/motor set up. (with constant ω)

there is an initial acceleration phase in which the driver gets things moving its way and then the driver is ~~has~~ able to make up for losses and this keeps the motion going and the energy transfer in the right ~~path~~ direction at a steady rate.

DC Electric generator

– essentially the same as AC with a commutator



When A is +ve it feeds current into A'
When B is -ve it gets current from

3 ~~P~~-84

But after half a cycle

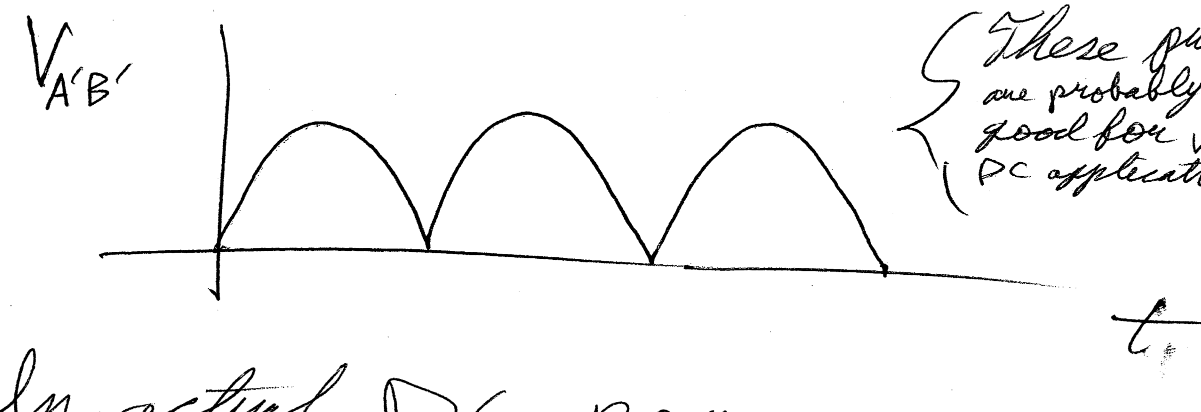
A is now ~~moving~~ -ve
and attached to B'
and so pulling current

and

B is now +ve and
attached to A'
and outputting
current.

Either way current
flows from A' to B'

But not constant

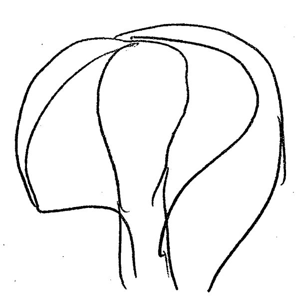


These pulses are probably not good for many DC applications

In actual DC power sources
many offset coils are used

33-86

to create a nearly constant emf



So turning AC into DC isn't so hard.

Which also I think means the ~~counter~~ Magnetic torque is nearly constant and the applied torque can be constant

(which off the top of my head seems a simpler setup for driving the array of coils)

As Wikipedia shows actual electric generator and motor design is not something to go into lightly