Ch. 29 Magnetism

Magnetic Fields

{ Show some Images Demos

— Magnetism occurs in nature of course — all over the place, but naturally occurring magnetism in the human environment is harder to notice than electricity where one has static electricity & lightning.

— But humans familiar with naturally occurring magnetic materials know about it ⟹

But those materials

— magnetite Fe_3O_4 (lodestone) being the most magnetic — would have seen it

which attracts iron & nickel, cobalt alloys & compounds
- rare earths +

29-2

but those materials occur only in special places (in obvious amounts)

So probably only a few prehistoric humans knew of it

— The ancient Greeks (Aristotle) thought it was known at least back to ~600 BCE

Magnesia is ancient Greek city (now in Turkey) was a source for Magnetite

— the Chinese ~~knew~~ recorded it from ~~~4th BCE~~ ~ 4th century BCE

Compass

— maybe Olmecs in Mesoamerica had a primitive version (Wik) from ~1000 BCE

— but very uncertain

- the Chinese had the compass from before 1044, but how much before is very unknown. (NOT 1300 BCE or ~~or th~~ 55-808 mention)

- in Europe known from before 1190 can't be really - no records from then
  - may have been invented independently or diffused from China → either way is possible.

In 1600 William Gilbert in "De Magnete" reported a whole host of magnetic experiments and proposed that the Earth itself was a <u>giant magnet</u>.

- he created the <u>terrella</u>
  - a magnetic sphere to model the Earth.

Microscopic currents give rise to magnetic fields & forces in magnetic materials.

In 1819, Oersted showed that electric current caused a magnetic field. (earlier noticed by Romagnosi in 1802 but he didn't make anyone notice)

Later in the 19th century Faraday & Maxwell & others discovered ~~that~~ electricity & magnetism were intimately connected and could be subsumed as ~~Elect~~ one branch of physics as electromagnetism

Maxwell's equations subsume all classical electromagnetism — except for sp. rel. understanding,

in the 20th century (already becoming a long-ago historical period)

Einstein's special relativity clarified things further and showed that electric & magnetic fields could interchange their identities depending on the relative motion of the observer. (29-5

(GrEM-522)

But one can go quite a ways studying electricity & magnetism somewhat independently & that's how one begins.

But charge & current are

29-6

involved ~~early~~ on
in magnetism as we'll
see

— We don't do a
pre-Oersted Magnetism.

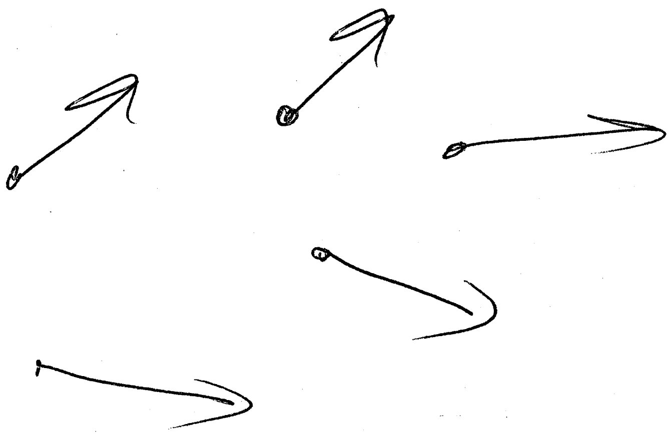
§ 29.1 Magnetic Field & Force

B-Field

It's a vector field
and has the conventional
symbol $\underset{\sim}{B}$.

— I often ~~all~~ abbreviate to
B-field.

— it's defined at
every point in space.



— of course there is a continuum of them.

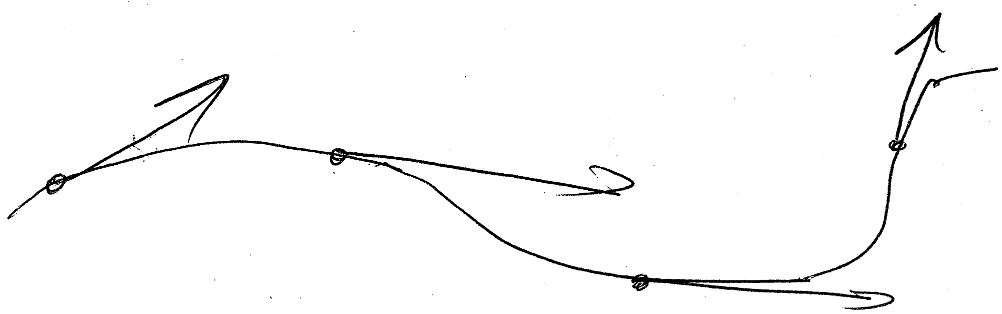
It has a <u>magnitude & direction</u>

— the direction is in space space and the extent of the vectors is in an abstract B-field space.

It's the cause of the <u>magnetic force</u>.

And one can trace it out using <u>magnetic field lines</u> defined analogously to electric field lines.

Just follow a curve that is everywhere tangent to the local magnetic field.



Empirically one can use Magnetic dipoles to trace out magnetic field lines

e.g. Iron fillings See p. 29-12a below

small Greek MU

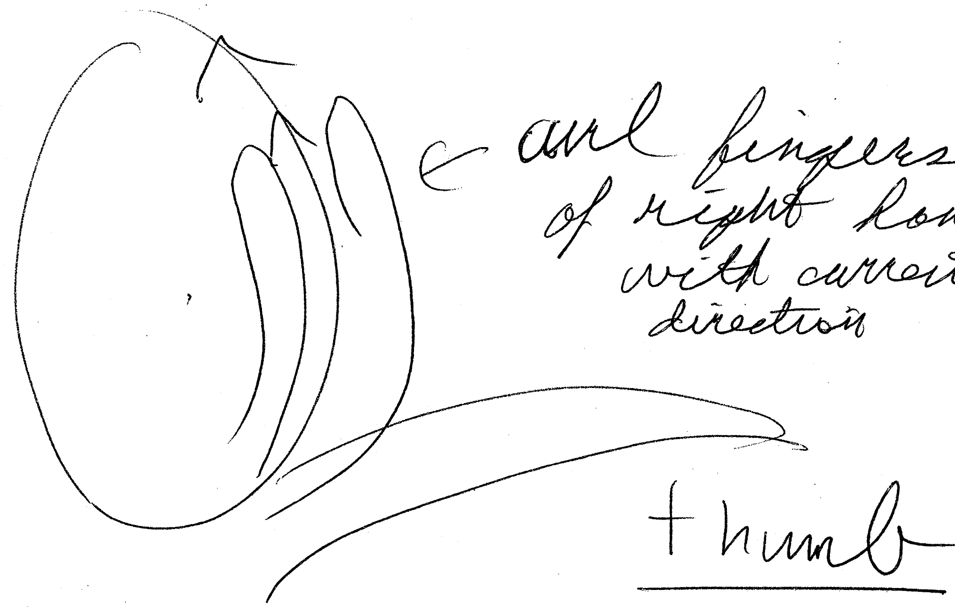
$\underset{\sim}{\mu}$ is the traditional symbol for a magnetic dipole ~~moler~~ moment

$\nearrow$

to anticipate § 29.5

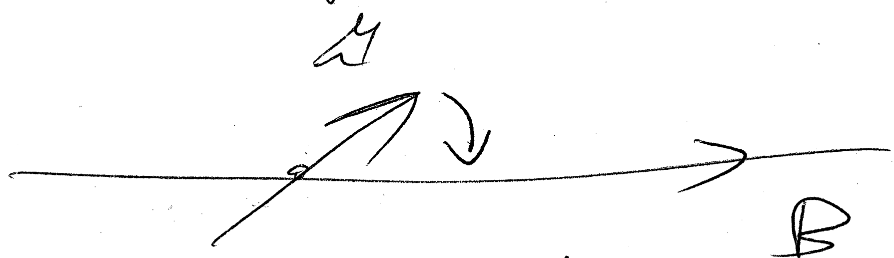
and just as an electric

29-10



The thumb points
more or less
in direction
of $\underset{\sim}{\mu}$

field ~~that~~ will ~~to~~ align an ~~ele~~ electric dipole moment, so ~~an~~ B-field will align a magnetic dipole

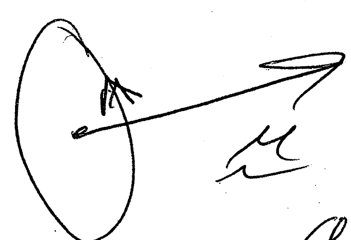


— it will torque it into alignment.

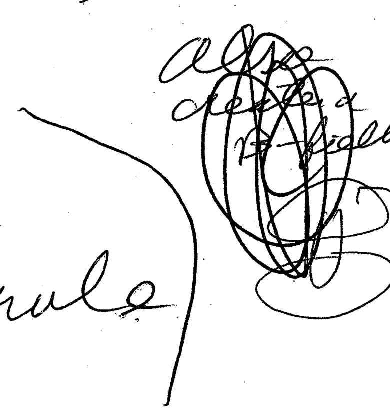
Not to be coy.

a current loop — Not necessarily circular

is a magnetic dipole



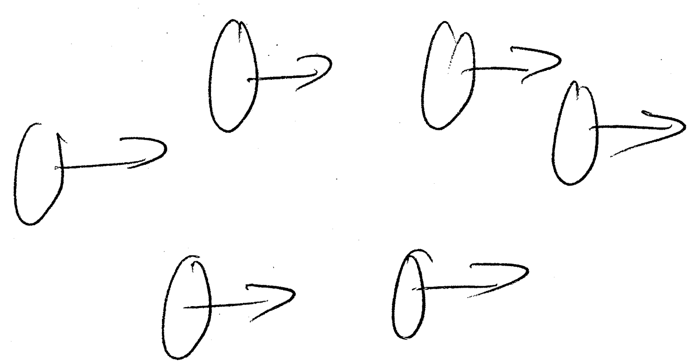
— μ's direction determined by a right-hand rule



(It's always a right-hand rule.
— It's a right-handers world.
— sorry southpaws
— but you already knew it.)

Such current loops can be made macroscopically but they exist microscopically in atoms.

~~on~~



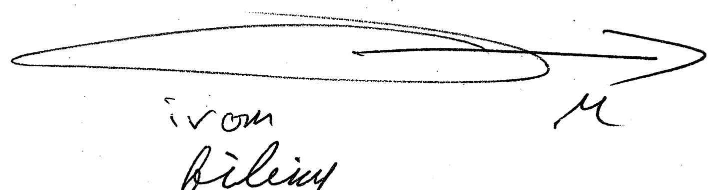
~~some~~

in magnetic materials they are or can be made to line up

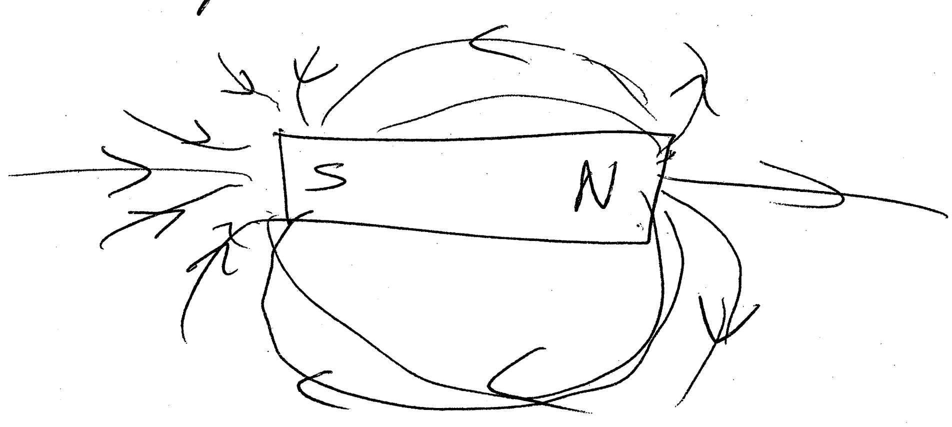
~~201~~ 29-12a

to make large dipole moments.

— for example iron filing in a magnetic field tend to have the micro dipoles aligned with the long axis of the filing.

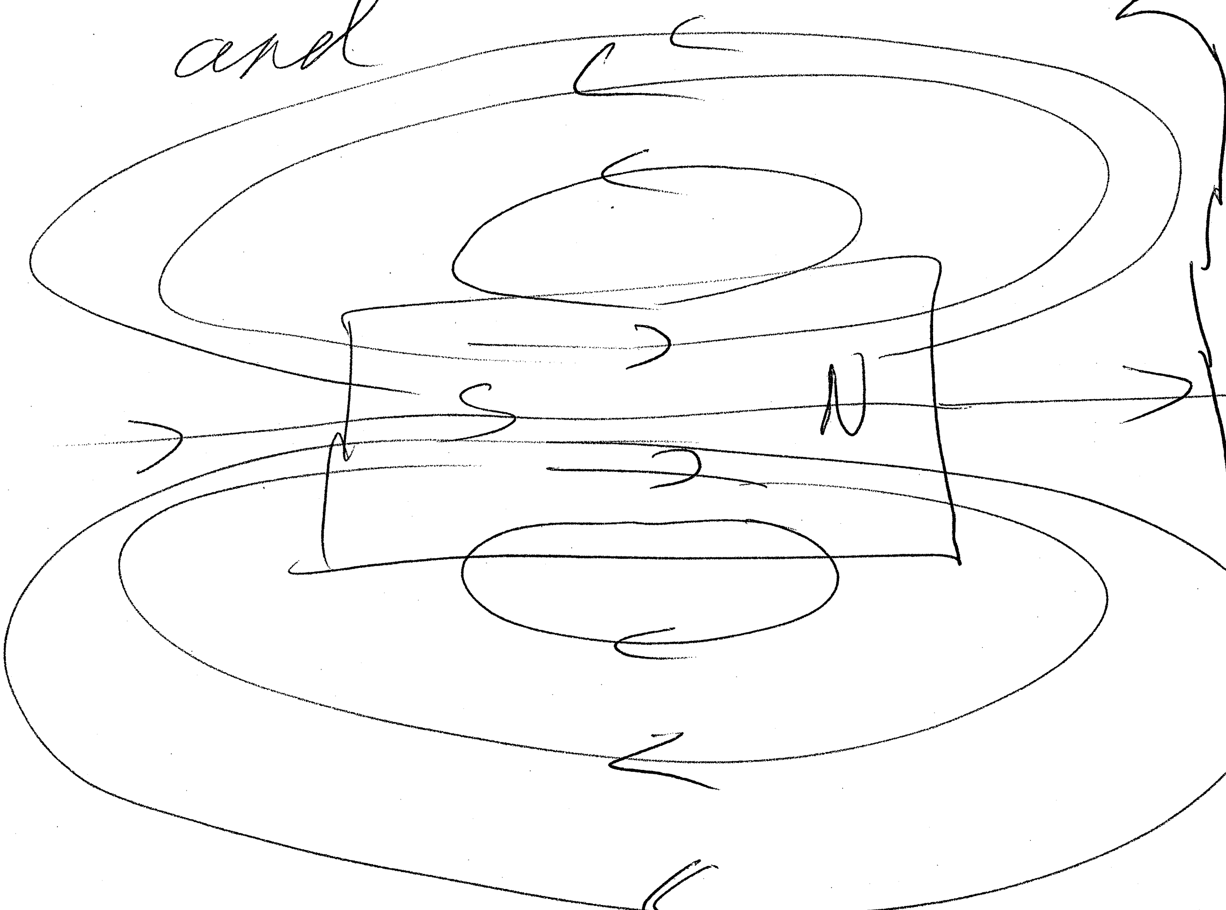


Then in a vivid and common demo will ~~line on~~ align to trace out the magnetic field of a bar magnet



A bar magnet
is itself a large
permanent magnetic
dipole

and



Butterfly shape in cross-section but has rotational symmetry. But maybe not perfectly

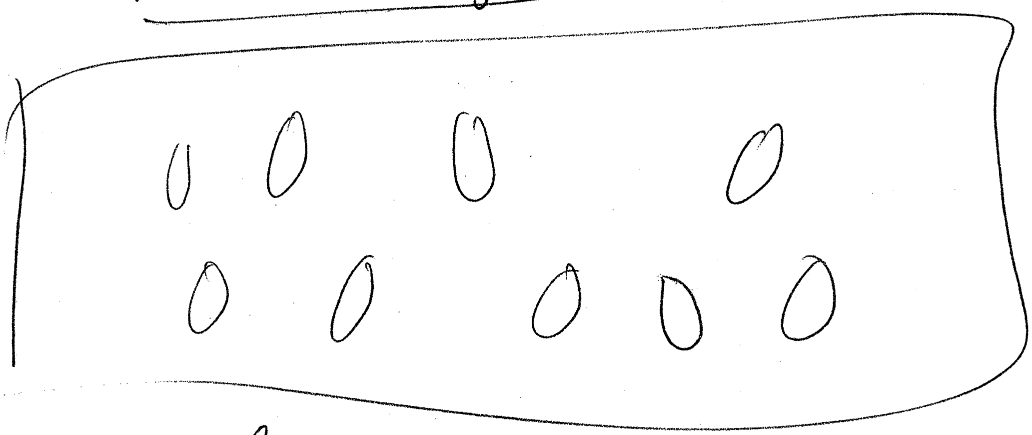
— ~~this~~ is the ~~B-field~~
dipole B-field created
by an magnetic dipole
(ψ's dipoles are affected by
and create B-fields

29-12c

29-13

A single dipole field is

So bar magnet is a collection



of ~~these~~ dipoles ~ 1 mole (6×10^{23}) of them in ~ 56 g of iron.

$$N_{\text{of atoms}} = \frac{\overbrace{m}^{\text{mass}}}{\underbrace{A}_{\text{atomic number}}\, \underbrace{m_{amu}}_{\text{Atomic mass unit}}} = \frac{m}{A\cdot 1g}\underbrace{\left(\frac{1g}{m_{am}}\right)}_{\text{Avogadro's number } 6.022\ldots\times 10^{23}}$$

Avogadro's number

29-14

(As we'll — and as you know — unlike poles attract and like poles repel.)

What are north and south magnetic poles actually?

In fact they are not precisely defined (Wik)

My own idiosyncratic definition is a north pole is any region from which you ~~you~~ take it that magnetic field lines diverge

& a south pole is any region into which the converge.

But nature

has not apparently given us any point ~~sources of~~ or sinks of magnetic field lines

— No isolated north or south poles

or in the jargon of physics magnetic monopoles

Actually in some physical theories magnetic monopoles are predicted to exist, but no one has ever

But that's a bit definitional.

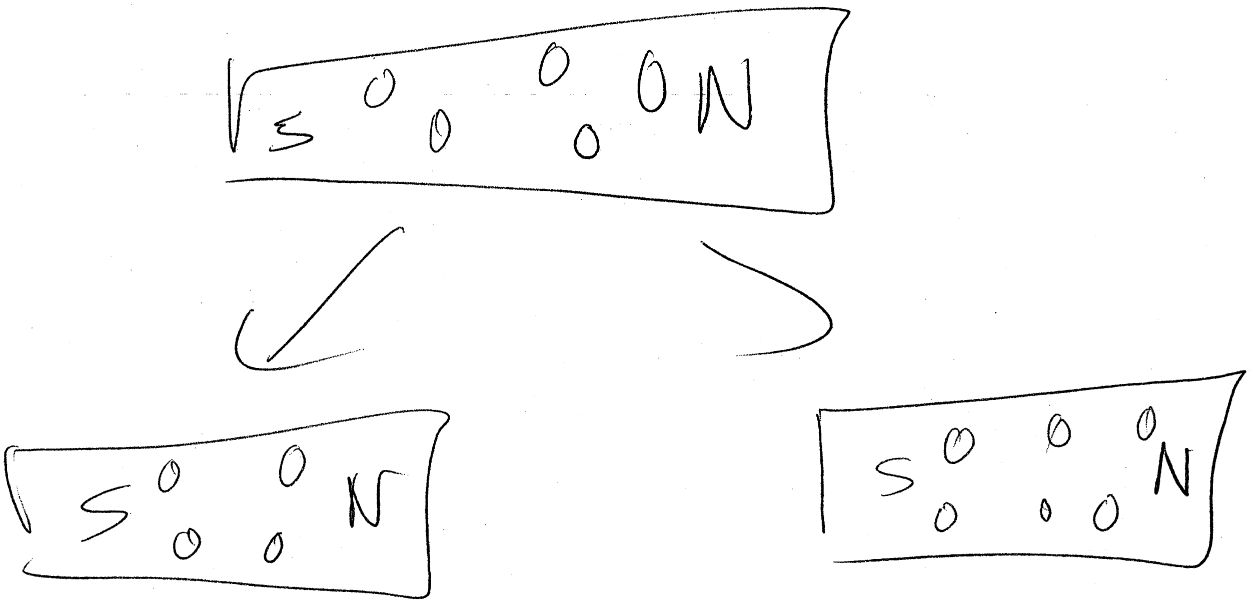
B-field lines have no non-zero ends. They either go to infinity or form closed curves. They also have no non-zero intersection since a vector B can't point two ways at once. If B = 0, you might say there are ends and crossings.

29-16

found one empirically.
Maybe one day we'll be able to create them in accelerators or detect them in the enviroment if they are some almost indetectable.
But we don't know

The fact is you can't isolate a north or south pole as people long ~~ato~~ ago discovered.

Cut a bar magnet in half



and you just get two bar magnets both with <u>North and South poles</u>.

And from the fact that the field arises from current loops you can see why.

29-18

If you try cutting a current loop, you may disrupt the current altogether, but won't create ~~a~~ monopoles.

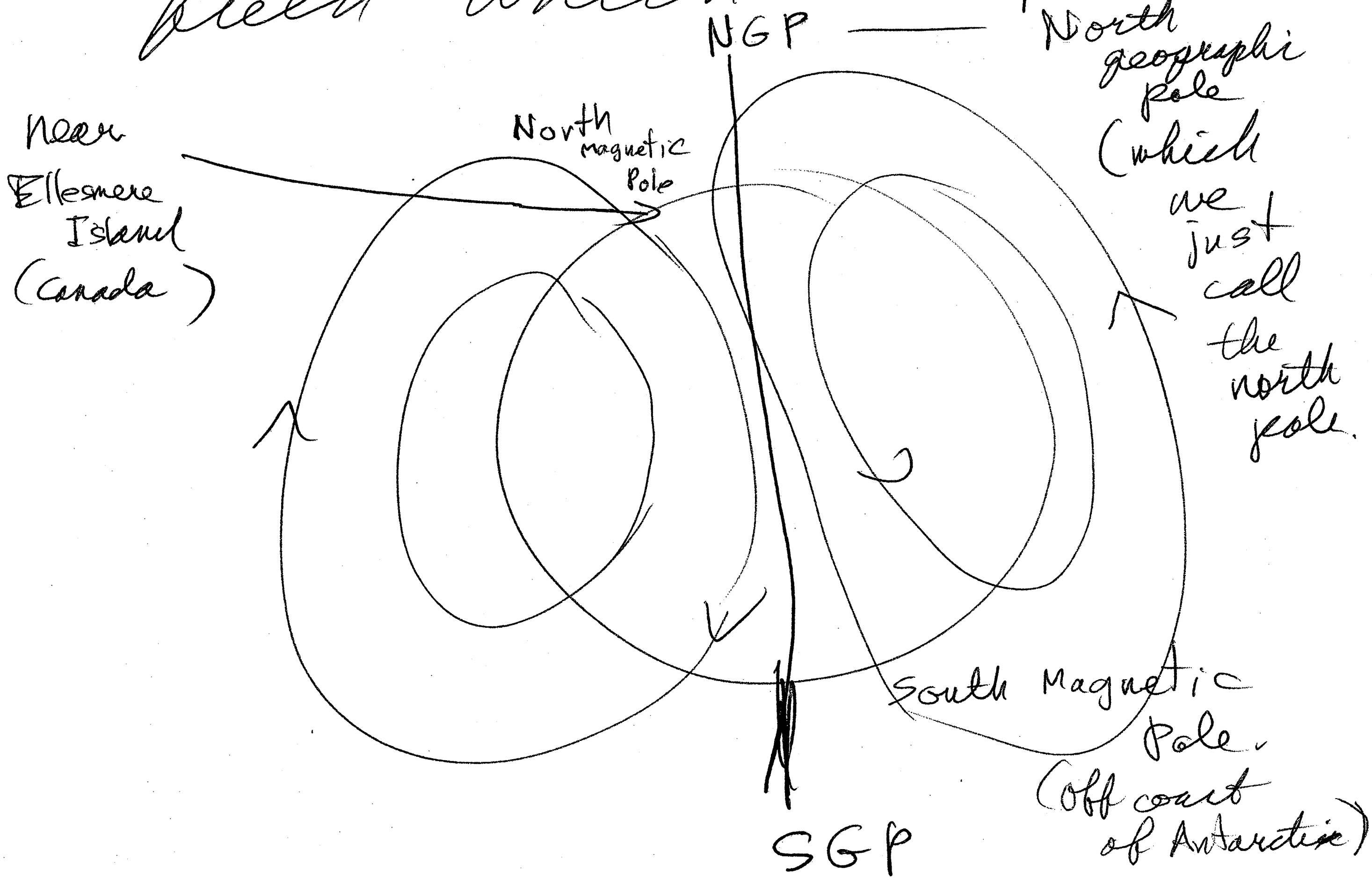
Historically

— the magnetic ~~end~~ north pole points roughly north and the south pole points roughly south

That's where the names come from.

29-19

— they are aligning with the Earth's magnetic field which is dipolar



Of course the North magnetic Pole

is the magnetic pole in the north.

— it's a magnetic south pole

& the South magnetic Pole

is the magnetic pole in the south

— it's a magnetic north pole.

29-20

Magnetic force

— it's law is a bit tricky.

— here we put ~~find~~ investigate the basic form for the force on a point charge q.

It's a law of nature. So no derivation really at our level)

(well you can define alternative starting ~~points~~ laws for Electromagnetism and derive it from them

~~but that's more of different perspective~~

~~on the physics~~

~~than a derivation~~

— that is how it is done at a more advanced level.

$$\underset{\sim}{F} = q\,\underset{\sim}{v} \times \underset{\sim}{B}$$

magnetic field where the point charge is.

charge of point charge.

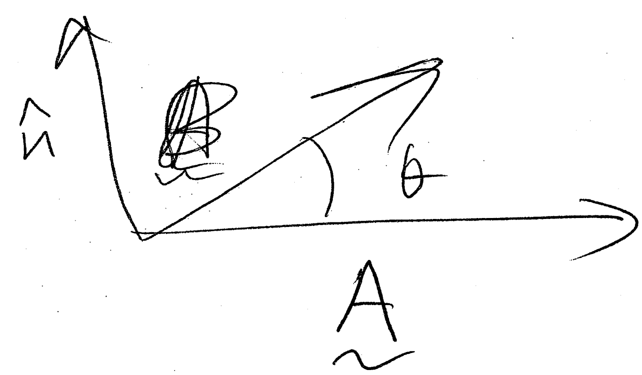
velocity vector of point charge

Vector cross product

— the magnetic force is velocity dependent force

and that cross product makes things tricky

Brief review of cross product



$\underset{\sim}{A}$ and $\underset{\sim}{B}$ general vectors

$$\underset{\sim}{A} \times \underset{\sim}{B} = AB \sin\theta \, \hat{n}$$

where $\hat{n}$ is normal to both A & B

and its direction

is determined
by a right-hand rule.

— sweep right-hand
fingers from the
first vector to the
second and the
thumb more or
less points in the
$\hat{n}$ direction.

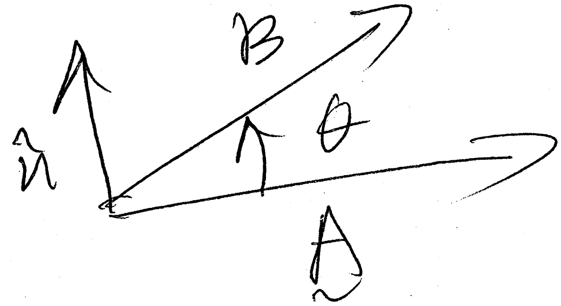
(there are other ways
of mnemonicking
the right-hand rule,
but this is the
one I use)

The cross product is
anticommutative

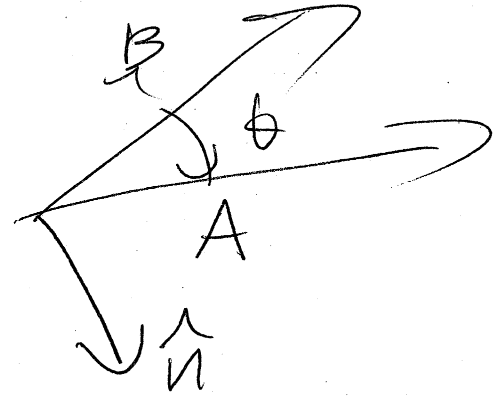
$$\underset{\sim}{B} \times \underset{\sim}{A} = -\underset{\sim}{A} \times \underset{\sim}{B}$$

29-24

because of the way it's define



$\underset{\sim}{A} \times \underset{\sim}{B}$



$\underset{\sim}{B} \times \underset{\sim}{A}$

Now $$\underset{\sim}{A} \times \underset{\sim}{B} = \begin{cases} AB\sin\theta\,\hat{n} & \text{in general} \\ 0 & \text{for } \theta = 0 \text{ or aligned vectors} \\ 0 & \text{for } \theta = 180 \text{ or anti-aligned vectors} \\ AB\,\hat{n} & \text{for } \theta = 90^\circ \end{cases}$$

One often makes use of these special cases, and so good to keep in mind.

29-25

The fact that
the magnetic force
on a point charge

$$\underset{\sim}{F} = q \underset{\sim}{v} \times \underset{\sim}{B}$$

depends on velocity
and the cross product
operation leads to
~~to~~ a couple of remarkable
features:

dot product

a) $dW = \underset{\sim}{F} \cdot d\underset{\sim}{s}$

is the differential expression
for work done by a
force.

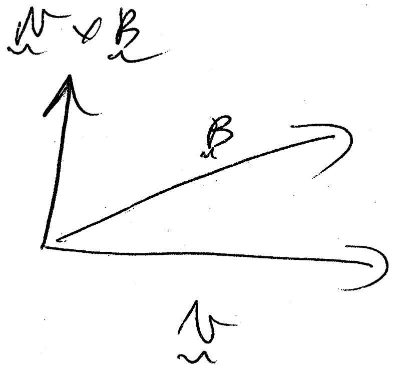
$$d\underset{\sim}{s} = \underset{\sim}{v}\, dt$$

is a differential bit
of path written

$\boxed{29-26}$

in terms of velocity $\underset{\sim}{v}$

Now $dW = (q\,\underset{\sim}{v} \times \underset{\sim}{B}) \cdot \underset{\sim}{v}\,dt$



$\underset{\sim}{v} \times \underset{\sim}{B}$ is perpendicular to $\underset{\sim}{v}$

$\therefore (\underset{\sim}{v} \times \underset{\sim}{B}) \cdot \underset{\sim}{v} = 0$

B-field acting alone. & combination of forces including B-fields can do work.

$\therefore dW = 0$ in all cases. for point charges the magnetic field can do no work on a point charge!!

This means it can't

B-field ~~They~~ can do work on currents in constraints (in wires) as we'll see and on dipoles

In a manner of speaking. The ~~joint~~ force does net work.

transfer energy to or from a point charge.

It can't change its KE and we can't define a Potential energy for the magnetic field in this context (we can define PE for a magnetic dipole as we'll see (see p. 29-29))

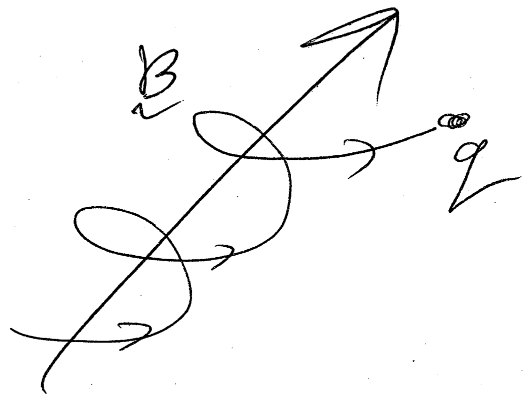
~~It does~~ The B-field does no work on a point charge

— but the B-field can change the direction of the charge motion and so can accelerate it.

As we'll investigate in §29.2.

29-~~28~~

the typical motion of a charged particle in a B-field is helical



b) Since $\underset{\sim}{F} = q\,\underset{\sim}{v} \times \underset{\sim}{B}$ depends on velocity, it is a frame dependent force.

Is ~~there~~ no force then in the frame moving with the charge?

No there
is a force,
but the there is
also an E-field
that causes it even if
B-field and E-field
there was none in the original frame
get mixed up
and transformed
when changing
frames of reference
— how this is done
is well beyond our scope
(and my knowledge),
but that it happens
shows ~~again~~ that

2930

electricity and magnetism are two manifestations of electromagnetism ~~(as joint phys~~ a single physical force. (or interaction)

Or rather shows ~~that~~ that is the best way to understand them.

§ 27.2 Motion of Charges in a Uniform B-field

— Such B-fields can be set up ~~experimentally~~ technologically and are very useful in science & technology.

— It's also easy to analyze

Units

Since $F = q \underset{\sim}{v} \times \underset{\sim}{B}$

$$[\underset{\sim}{B}] = \left[\frac{F}{q v}\right]$$

$$= \frac{N}{C \ m/s} = \frac{N}{A \ m}$$

$$\equiv 1 \text{ tesla} = 1 \ T$$

– a tesla is actually a ~~pretty big~~ biggish magnetic field.

– An older unit still in use is the gauss = Ga

$$1 \ Ga = 10^{-4} \ T$$

29-32

Table of ~~the~~ B-fields

Case	B (T)
Smallest value in Magnetically shielded room	10^{-14}
Earth's B-field	~~10~~ ~.5 × 10^{-4} (.5 Ga)
Small Bar Magnet	.01
Small NIB	.2

Strongest permanent magnets

neodymium

$Nd_2 Fe_{14} B$

rare earth magnet

but some can go up to ~1.4 T

- dangerous to your credit card and otherwise

they can break fingers.

29-33

Big electromagnet	1.5
Strong lab magnet	10
Strongest sustained lab magnet	45
Neutron star surface	10^8
Magnetar	10^{11}

---

§ 29.2 Motion of Charges in a uniform B-field

Uniform B-fields are not so hard to create technologically and have wide uses both

In solenoids in fact.

29–34

in science and technology.

— They are also easy to analyze motion in.

A couple of conventions

$\odot$

vector pointed out of page or board

head of arrow

$\otimes$

vector pointed into page or board

tail of arrow.

(Whenever one has a two-choice situation there's an awful tendency to get choices exactly wrong: in/out, [illegible] left/right)

Say we have a~~n~~ uniform B-field into page/board

$$\otimes \qquad\qquad\qquad\qquad \otimes$$

$$\otimes \qquad\qquad\qquad \otimes$$

and a point charge q with velocity $\underset{\sim}{v}$ and $\underset{\sim}{v}$ is perpendicular to $\underset{\sim}{B}$

— so q moves in plane of page/board

29-36

It experiences a magnetic force $\underset{\sim}{F} = q\,\underset{\sim}{v} \times \underset{\sim}{B}$

and let's say ~~that~~ is the only force.

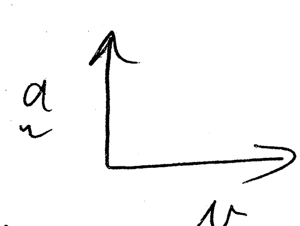
Thus we can apply Newton's 2nd law

$$\underset{\sim}{F}^{net} = m\,\underset{\sim}{a}$$

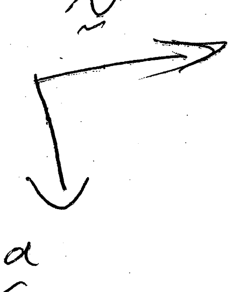
$\underset{\sim}{a}$ is perpendicular the $\underset{\sim}{v}$ and $\underset{\sim}{B}$ and in plane of page/board.

$F = q v B$
in this case.

But I don't remember these using hand rules.



for $q > 0$



for $q < 0$

~~Left-hand rule — left thumb along~~ B-field ... curl

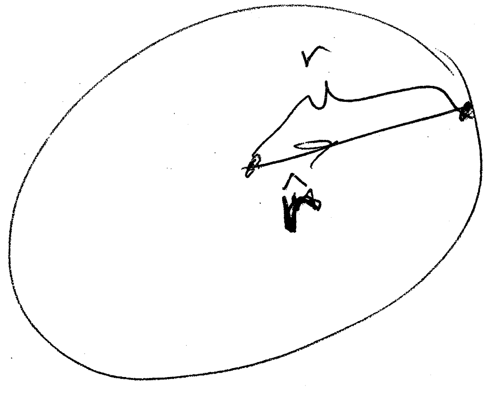
Since $\underset{\sim}{a}$ & $\underset{\sim}{v}$ are perpendicular and since the magnitude of $\underset{\sim}{v}$ (i.e., v) is constant we anticipate that charge is in uniform circular motion.

— Let's assume that.

i.e. $\underset{\sim}{F}^{net} = m\frac{v^2}{r}(-\hat{r})$

The centripetal force requirement,

Not a magic force that turns on when a particle goes into uniform circular motion, but a requirement that a real force must supply



v is tangential speed.

i.e. $qvB = \frac{mv^2}{r}$ to equate the magnitudes.

$r = \frac{mv}{qB}$

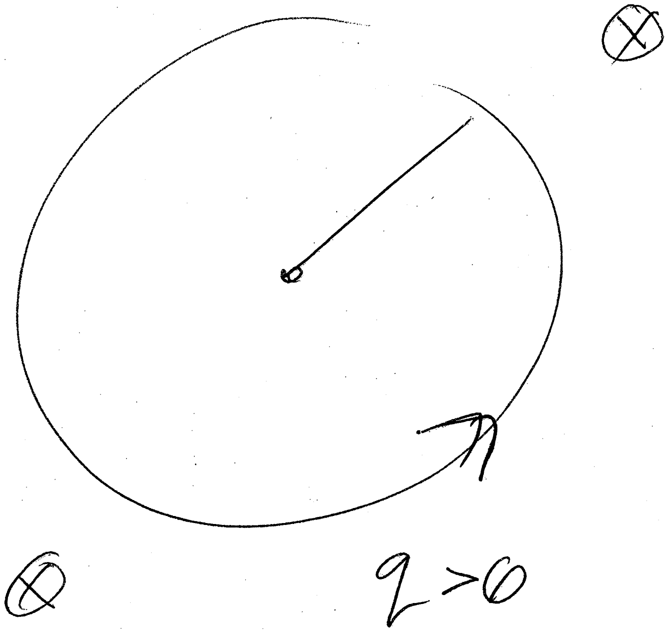
Actually all accelerated charges radiate electromagnetic radiation EMR, and no ~~loce~~ energy. But we won't worry about this effect here (ER-739)

For a single charge pretty small except at relat[illegible] ... (EK-737)

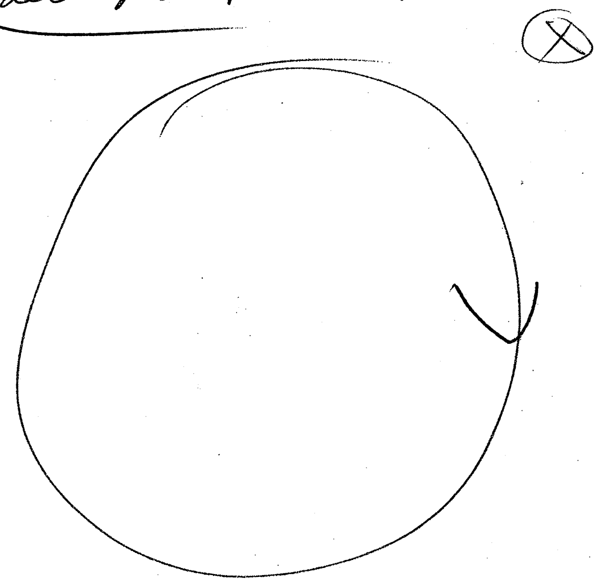
29-38

Since we can, in fact, solve for a radius of uniform circular motion, we must have uniform circular motion with that radius.

I don't think there is any easy direct way to get the result directly. One just finds that ~~at ff~~ it is the consistent description of motion.



$q > 0$



$q < 0$

A left-hand rule!!

— curl left around B-field line with thumb aligned and that gives the sense of the ~~[illegible]~~ rotation for $q > 0$ case (which is the fiducial case)

Bigger q & B smaller r, the particle is pulled into a tighter circle. On the other hand, bigger m & v then a bigger circle - the particle has more momentum and is less affected by the B-field.

$$r = \frac{mv}{qB}$$ is called the cyclotron radius

— for historical reasons.

— in cyclotron devices this formula turns up.

often one sets, v, B and the particle (which sets q & m) and all that sets r

The angular velocity is

$$\omega = \frac{v}{r} = \frac{qB}{m} \cancel{\frac{m}{qB}}$$ which is independent of v

29–40

$\omega = \frac{qB}{m}$ is called the cyclotron angular frequency (actually an angular frequency — radians per unit time)

$f = \frac{\omega}{2\pi} = \frac{qB}{2\pi m}$ is the revolution per unit time frequency

$T = \frac{1}{f} = \frac{2\pi m}{qB}$ is the cyclotron period.

What if $\underset{\sim}{v}$ is NOT perpendicular to $\underset{\sim}{B}$?

Well nothing forbids us from partitioning $\underset{\sim}{v}$ thusly

$$\underset{\sim}{v} = \underbrace{\underset{\sim}{v}_{\perp}}_{\substack{\text{component}\\ \text{perpendicular}\\ \text{to } \underset{\sim}{B}}} + \underbrace{\underset{\sim}{v}_{\parallel}}_{\substack{\text{component}\\ \text{parallel}\\ \text{to } \underset{\sim}{v}}}$$

Now
$$\underset{\sim}{F} = q\,\underset{\sim}{v} \times \underset{\sim}{B}$$
$$= q(\underset{\sim}{v}_{\perp} + \underset{\sim}{v}_{\parallel}) \times \underset{\sim}{B}$$
$$= q\,\underset{\sim}{v}_{\perp} \times B$$

since $\underset{\sim}{v}_{\parallel} \times \underset{\sim}{B} = 0$

$v_{\parallel} B \sin(0°) = 0$

So one has

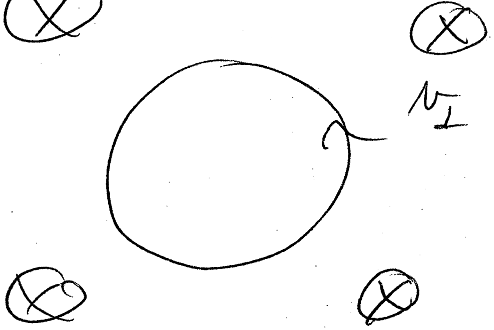
$$r = \frac{m v_{\perp}}{qB}, \quad \omega = \frac{qB}{m}, \quad f = \frac{qB}{2\pi m}, \quad T = \frac{2\pi m}{qB}$$

$\boxed{29\text{-}42}$

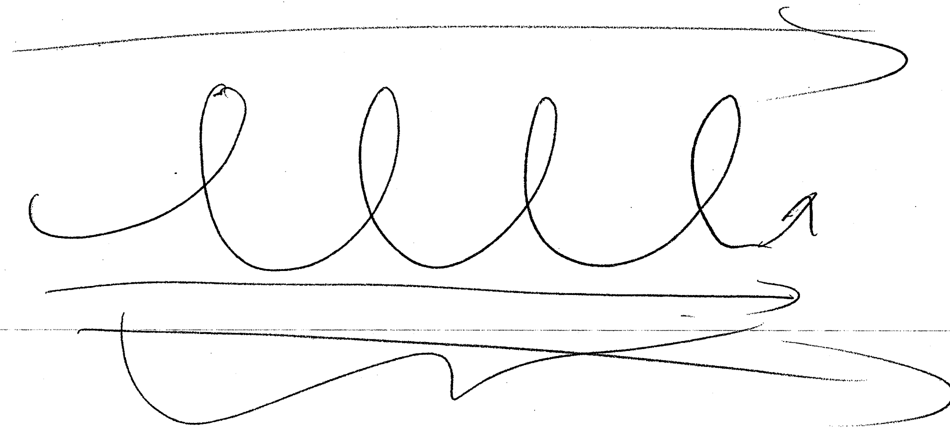
Just a change from v to $v_\perp$ in the cyclotron radius formula.

In the direction along the field line, there is no force and ~~that~~ the parallel velocity $v_\parallel$ is constant.

The result is helical motion



In projection on the plane perpendicular to B, it's circular motion at speed $v_\perp$



velocity

$v_{\parallel}$ along the field lines.

Ex 29.2

a proton has $r = .14\,m$

in $B = .35\,T$

radius of helical or circular motion.

What is $v_{\perp}$?

$$r = \frac{m v_{\perp}}{qB}$$

$$v_{\perp} = \frac{qBr}{m} \simeq \frac{1.6\times10^{-19}\cdot .35 \cdot .14}{1.7\times10^{-27}}$$

all MKS units.

29-44

$$= 10^8 * .05$$

$$= 5 \times 10^6 \text{ m/s}$$

Ans 4.7×10^6 m/s

$$\beta = \frac{v_\perp}{c} \cong \frac{5 \times 10^6}{3 \times 10^8}$$

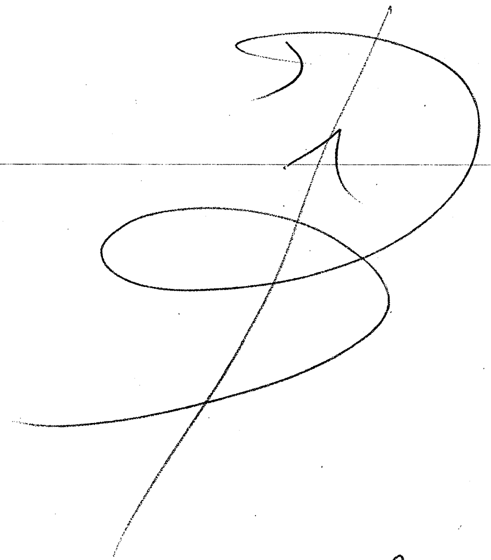
$$\cong 1.7 \times 10^{-2} << 1$$

So $v_\perp$ is fast, but not really relativistic fast.

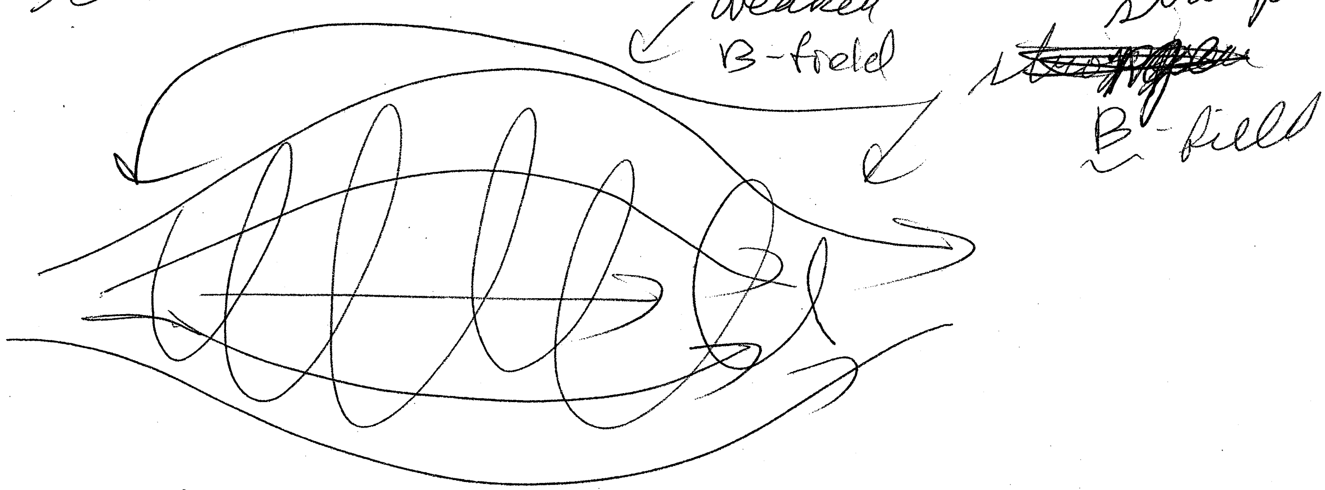
Non-Uniform B-fields & free charged particles

- in ~~general~~ detail the motion is complex, but qualitatively the charged particles

~~spera~~ spiral
around field lines



A ~~sose~~ of special interest
is the Magnetic bottle



— Where the particles
oscillate back and forth
and are sort of trapped.
— but collisions can knock

2946

the particles out of the bottle.

— Something like is what people hope to use for <u>nuclear fusion reactors</u>

the hot ionized gas would vaporize and be cooled by solid containers.

So maybe one day magnetic bottles in fusion reactors will be a vital technology.

But I've kind of lost faith.

— everything is good about <u>fusion power</u> — except it may not work.

— clean, limitless, non-proliferating, but it may not work.

In nature the magnetic bottle effect happens in the Earth's magnetosphere ← really its outer dipole field

charge particles get trapped in regions called the Van Allen belts and sometimes leak in near the poles — and elsewhere — and give rise to aurora.

29-48

§29.3 | 29-49

Applications

of charged particles
in Magnetic fields.

If you have both
an $\underset{\sim}{E}$-field and a B-field
the net force is

$$\underset{\sim}{F} = q\underset{\sim}{E} + q\,\underset{\sim}{v} \times \underset{\sim}{B}$$

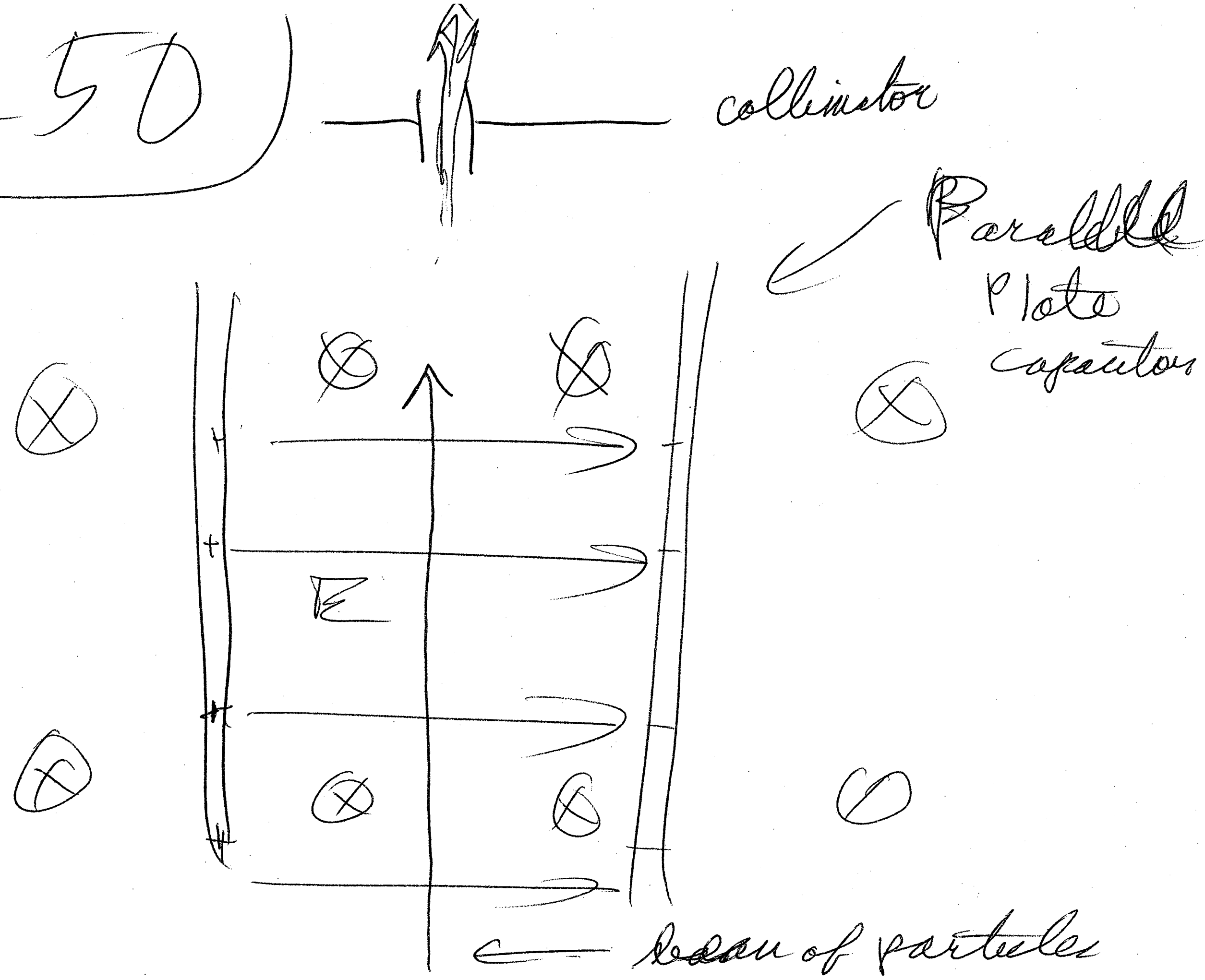
$$= q(\underset{\sim}{E} + \underset{\sim}{v} \times \underset{\sim}{B})$$

→ called the Lorentz force.

Velocity selector

You are studying charged
particles and want
one particular velocity.

29-50)



One has crossed $\underset{\sim}{E}$ & $\underset{\sim}{B}$ fields.

What condition gives no deflection to beam

$$\underset{\sim}{F}_{lorentz} = 0 = q\underset{\sim}{E} + q\,\underset{\sim}{v} \times \underset{\sim}{B}$$

$$0 = \underbrace{qE}_{\text{force to right}} \underbrace{- qvB}_{\text{force to left.}}$$

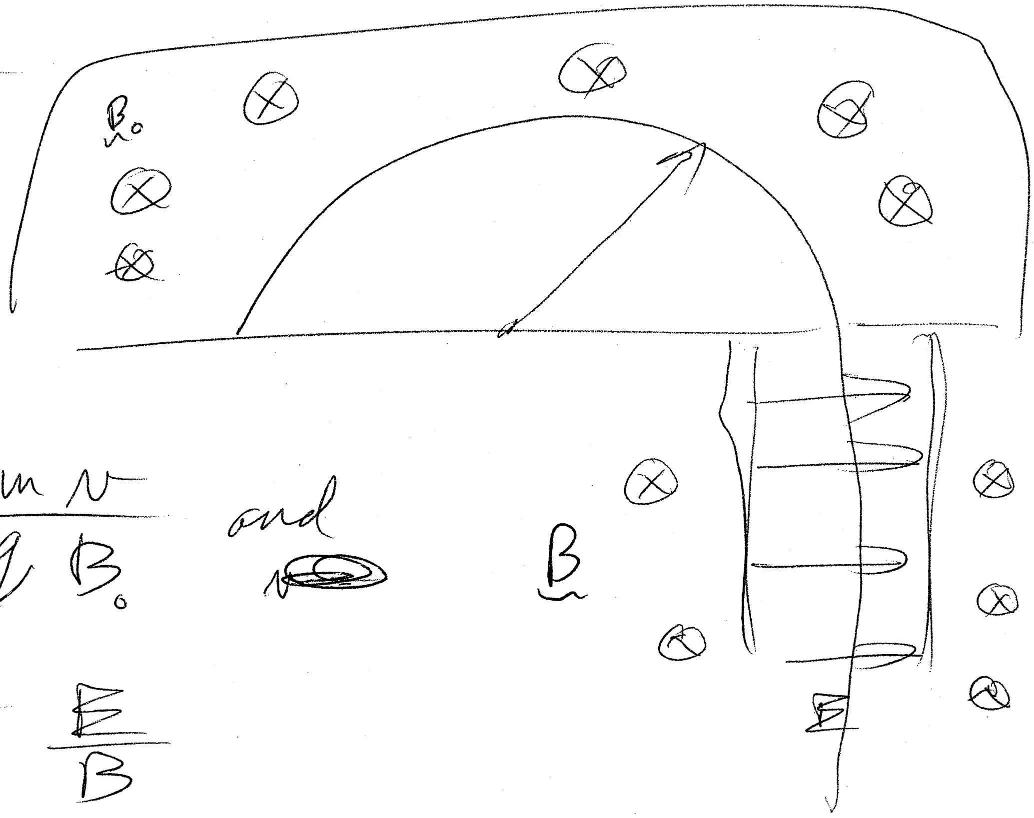
[29-5]

$$v = \frac{E}{B} \quad \left\{ \text{Independent of the particle mass} \right.$$

So adjust ~~V~~ E/B ratio and only the $v = E/B$ goes thru undeflected and into your apparatus beyond

Mass Spectrometer

with a velocity selector



$$r = \frac{mv}{qB_0} \quad \text{and}$$

$$v = \frac{E}{B}$$

$$\frac{M}{q} = \frac{rB_0}{v} = \frac{rB_0B}{E}$$

29-52

B, B_0, E are set and one measures v to get the charge-to-mass ratio.

J. J. Thomson used a similar set up to ~~mass~~ measure m/q for the electron in 1890's and ~~found~~ it much smaller than ~~that~~ ions since $m_e = \frac{m_p}{1836}$.

Thomson is usually credited as the electron discoverer, but as often the case ~~there~~ were several contributors to the discovery.

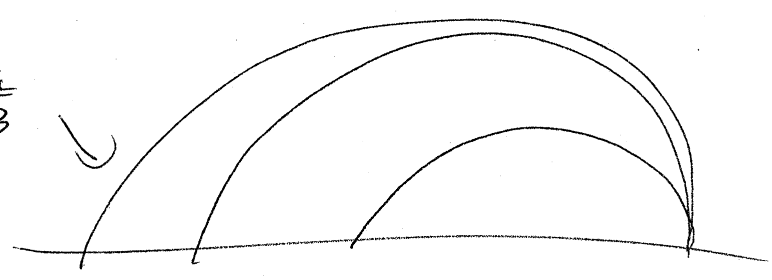
Mass spectroscopy is often used for <u>isotopic analyses</u>.

— isotopes of an element are <u>chemically</u> identical and <u>can't</u> be separated by usual chemical means

but they have different masses.

— so given a common ionization q, the masses can be determined and relative abundances determined.

$r = \frac{mv_\perp}{qB}$



— many applications like radioactive dating (less exciting than it sounds)

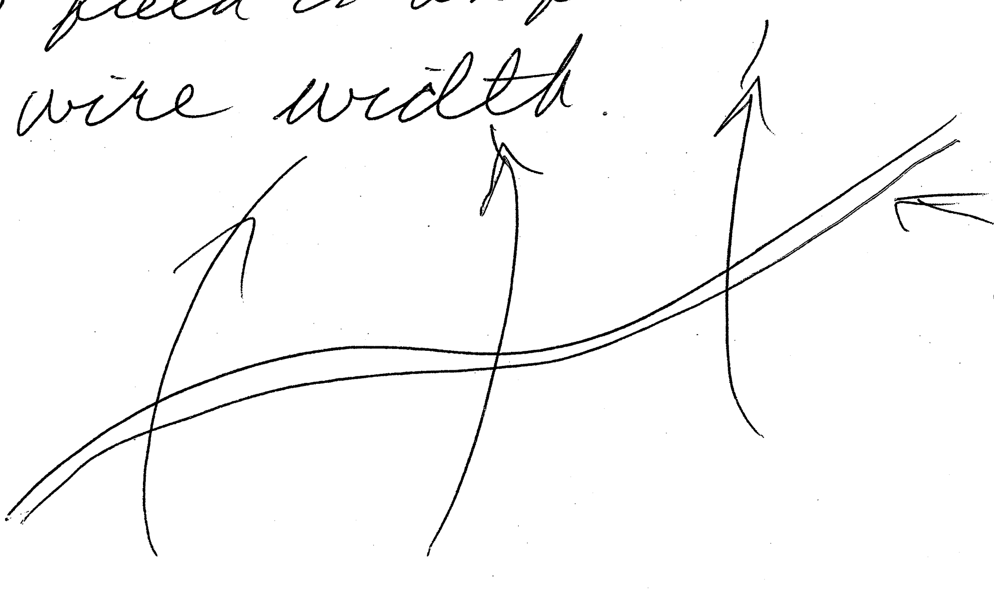
29-54)

where the ratio of (e.g.,) C^{14} to C^{12} is used to determine the age of organic material (i.e., the age since the organism died)

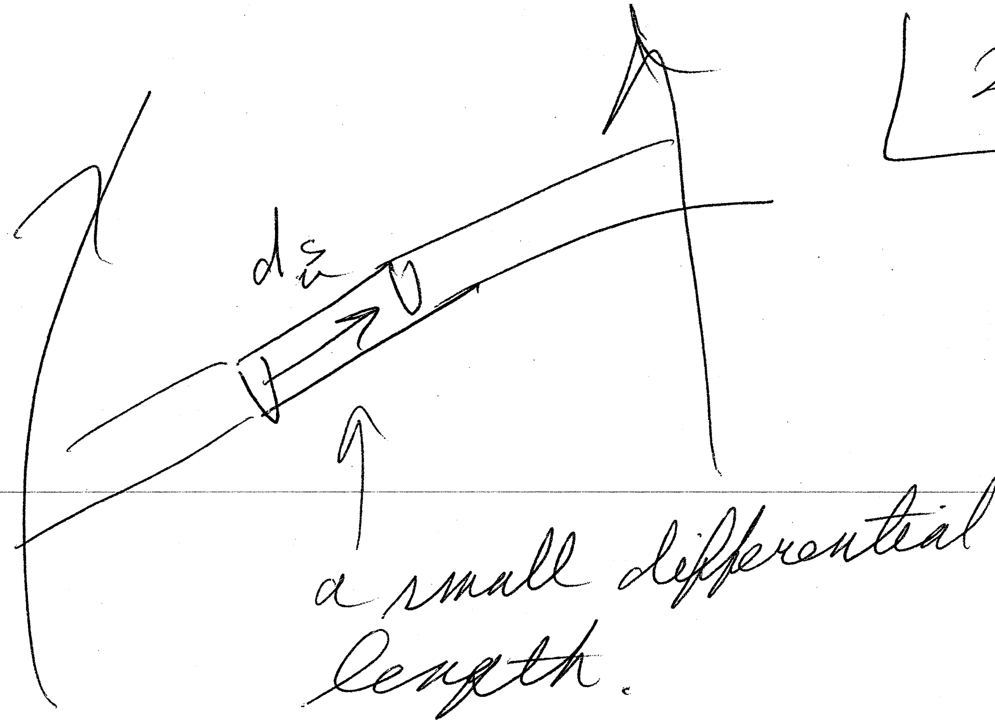
§29.4

Magnetic force on a current carrying conductor

— we'll just consider thin wires where the B-field is uniform over the wire width.



wire carrying current I in a B-field.



The ~~charge~~ moving charge in the wire element ds — where we only the ~~drift~~ ~~veloc~~ (the drift velocity) is non-random and doesn't cancel —— is

$$\underbrace{\underbrace{n}_{\text{charge density}}\;\underbrace{ds\,A}_{\text{volume}}}_{\text{Number of charge carriers.}}\;\underbrace{q}_{\text{moving charge}}$$

$\boxed{29-56}$

So the net magnetic force on all the carriers is

$$d\underset{\sim}{F} = (n\, ds\, A q)\, \underset{\sim}{v}_{drift} \times \underset{\sim}{B}$$

$\underset{\sim}{v}_{drift}$ is the average velocity and so this is the average force.

but

$$n A q v_{drift} = I$$

the electric current

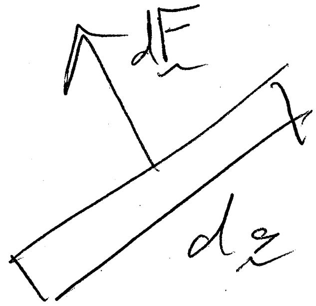
$$\underset{\sim}{v}_{drift} = v_{drift}\, \hat{s}$$

$$ds\, \hat{s} = d\underset{\sim}{s}$$

So $$d\underset{\sim}{F} = I\, d\underset{\sim}{s} \times \underset{\sim}{B}$$

Over a finite bit of wire one has $$\underset{\sim}{F} = I \int d\underset{\sim}{s} \times \underset{\sim}{B}$$

Now this is the $d\mathbf{F}$ is net force on the carriers and it's perpendicular to $d\mathbf{s}$:



But the carriers can't escape the wire (usually). They are effectively a trapped in the perpendicular direction

the net path tendency of the carriers.

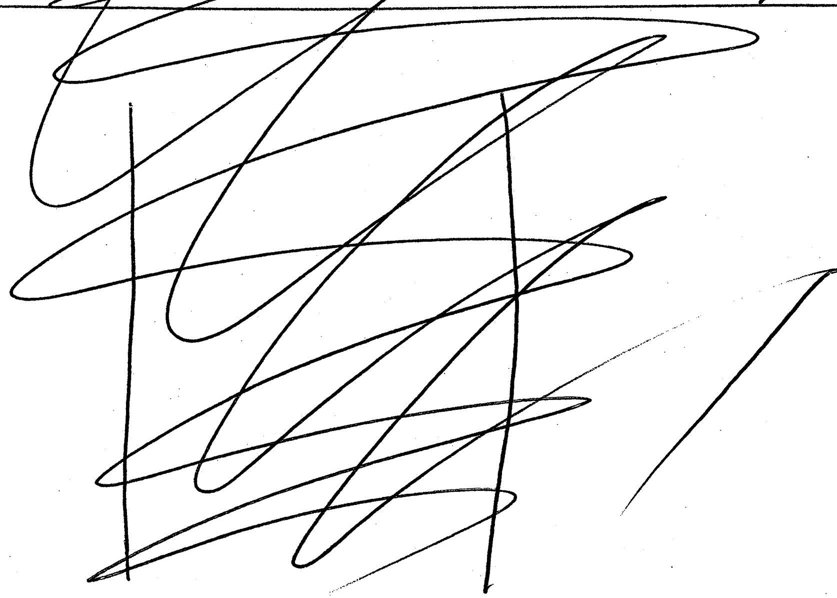
If you regard the carriers and wire as one system, the $d\mathbf{F}$ is on the ~~net~~ system and

29-58

and since the carriers are trapped rigidly, the whole wire can be treated as one rigid object subject to $d\mathbf{F}$

So $d\mathbf{F}$ and $\mathbf{F}$ are regardable as forces on the wire, and so we regard them.

~~Parallel Wire Segments~~



Force & Torque on a ~~current~~ Loop in a Uniform Magnetic Field

— This is not an esoteric subject as we'll see in Ch. 31 on Faraday's law ~~(essence of all electric motors)~~

— Current loop (or coils) ~~are essential~~ in B-fields are essential elements in electric generators and motors — which are arguably the most important of all

29-60

energy converters in technology.

Actually electric motors & generators are really the same thing run in opposite modes — few are designed to work both ways.

— ~~And~~ they convert mechanical energy to electrical energy (PE) and vice versa.

all of modern society relies on this.

— electrical energy is so flexible because of it.

The only major hold out from electrical energy is the transport sector because it's hard to store abundant ~~electrical~~ energy — but even that's changing

and sometimes heating gas furnaces and the like.

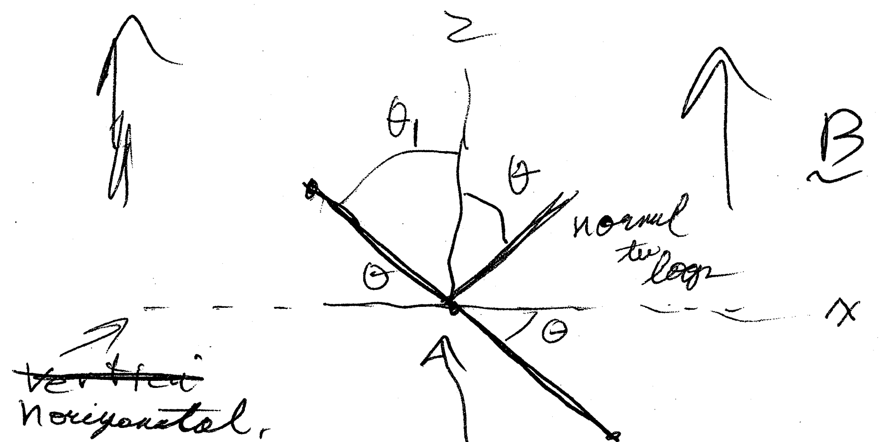
We ~~may~~ all be driving electric cars in 10 years (but more likely 20).

Consider a uniform B-field and a rigid rectangular current loop with current I. A thin loop (no width)

An idealization since the current is just flowing steadily. It's freely floating space — no gravity or electric fields.

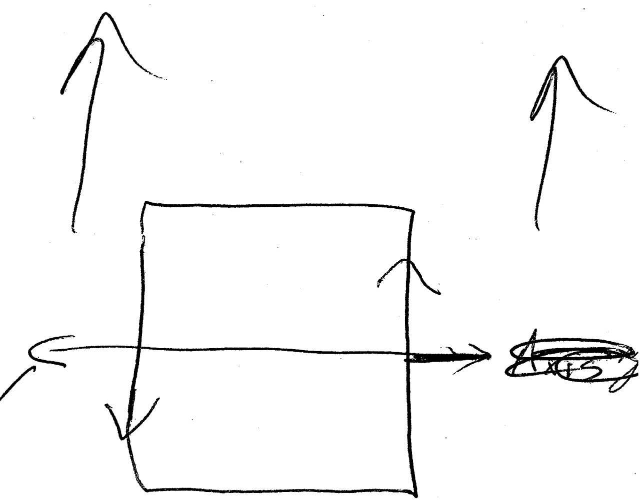
29-62

without energy input to make up for resistance losses. (of which there are none)



edge on view

axis ~~through~~ the center of mass (CM)



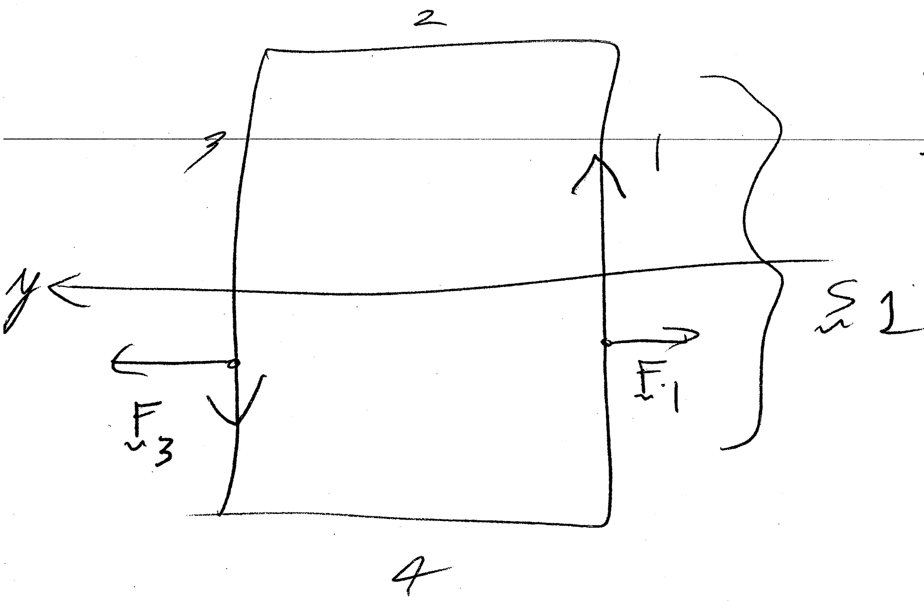
foreshortened front view.

— one can imagine a sheet of paper's ~~outlining~~ edges as forming the loop.

29-63

Let's find the forces on the ~~sides~~ sides

(Left-hand system z, y, x?)



$$\underset{\sim}{F}_1 = I\,\underset{\sim}{S}_1 \times \underset{\sim}{B}$$

$$= I\,S_1 B \sin\theta_1(-\hat{y})$$

$$\underset{\sim}{F}_3 = I\,S_3 B \sin(\pi-\theta_1)(+\hat{y})$$

$$= I S_3 B \sin\theta_1(+\hat{y})$$

$$\left(\sin(\pi-\theta_1) = \sin\pi\cos\theta_1 - \cos\pi\sin\theta_1 = \sin\theta_1\right)$$

$$\underset{\sim}{F}_1 + \underset{\sim}{F}_3 = 0$$

since $S_1 = S_3$

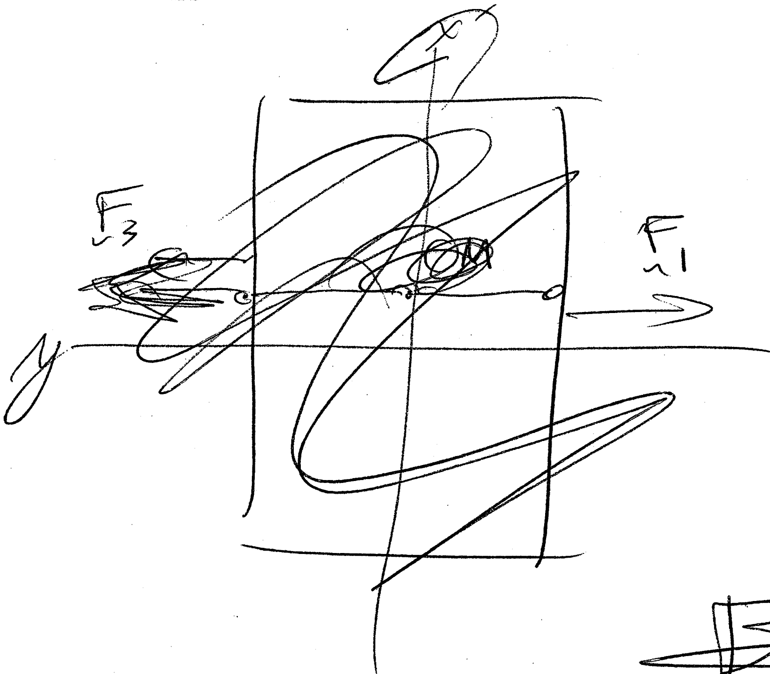
They exert no net force.
They cancel.

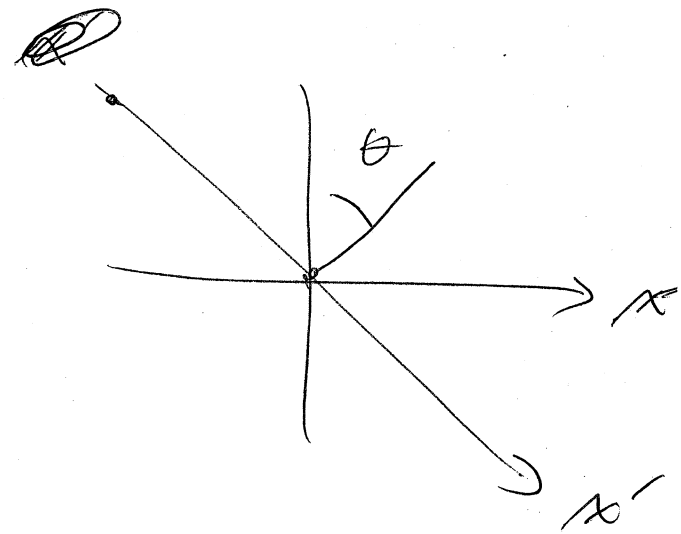
~~But~~ So there is NO acceleration ~~of the~~ of the CM due to these forces.

$\tau_i = rF$

But what of torques about
Recall $\underset{\sim}{\tau} = \underset{\sim}{r} \times \underset{\sim}{F}$ (where $\underset{\sim}{r}$ is from some origin)

2264) the x' axes?





~~By inspection~~

By inspection one would say ~~so~~ zero.

F_1 ~~&~~ and F_3 are just pulling apart and are <u>Not</u> trying to rotate the loop.

More formally.

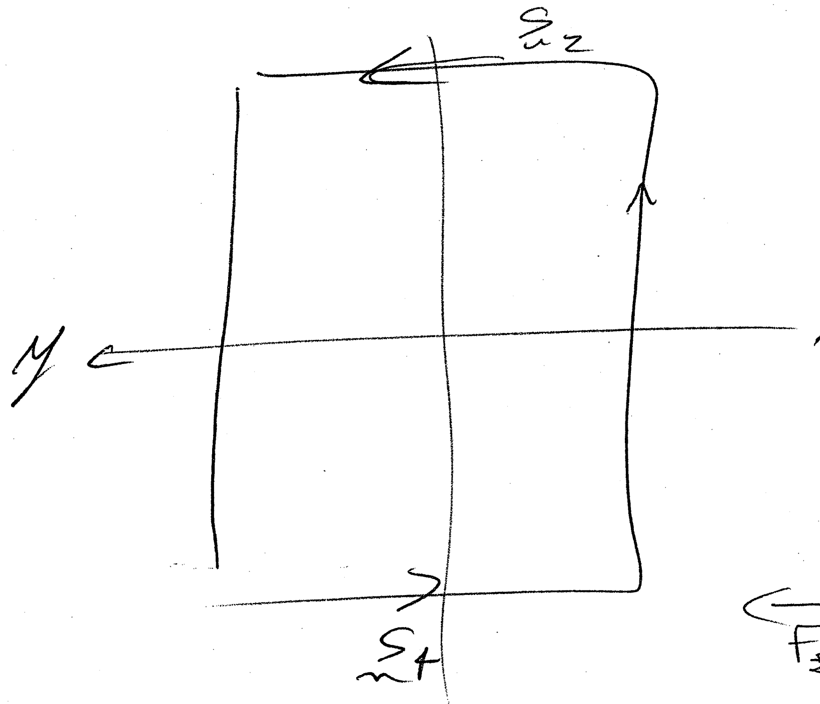
$$\left.\begin{array}{l} r_3 \times F_3 = 0 \\ r_1 \times F_1 = 0 \end{array}\right| \text{about the axis.}$$

$$\left.\begin{array}{l} r_3' \times dF_3 \\ + r_1' \times dF_1 = 0 \end{array}\right\} \text{about normal axes}$$

since the radius vector from the CM

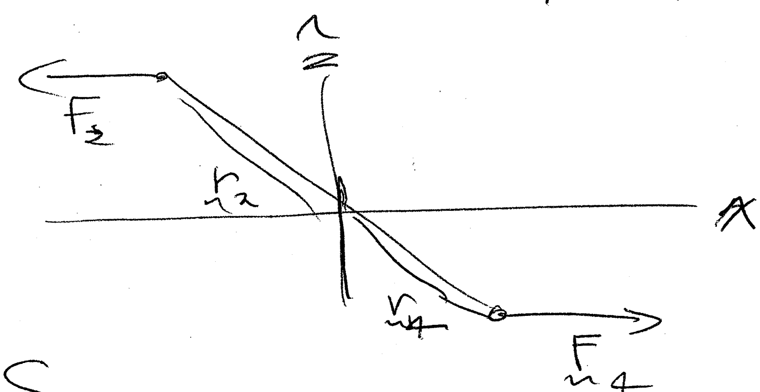
are aligned with $\mathbf{F}_3$ and $\mathbf{F}_1$

Now what of sides 2 and 4



$$\mathbf{F}_2 = I\,\mathbf{s}_2 \times \mathbf{B}$$
$$= I s_2 B \cdot 1 \, (-\hat{x})$$

$$\mathbf{F}_4 = I\,\mathbf{s}_4 \times \mathbf{B}$$
$$= I s_4 B \, \hat{x}$$



Since $s_2 = s_4$

$$\mathbf{F}_2 + \mathbf{F}_4 = 0$$

So these forces cancel.

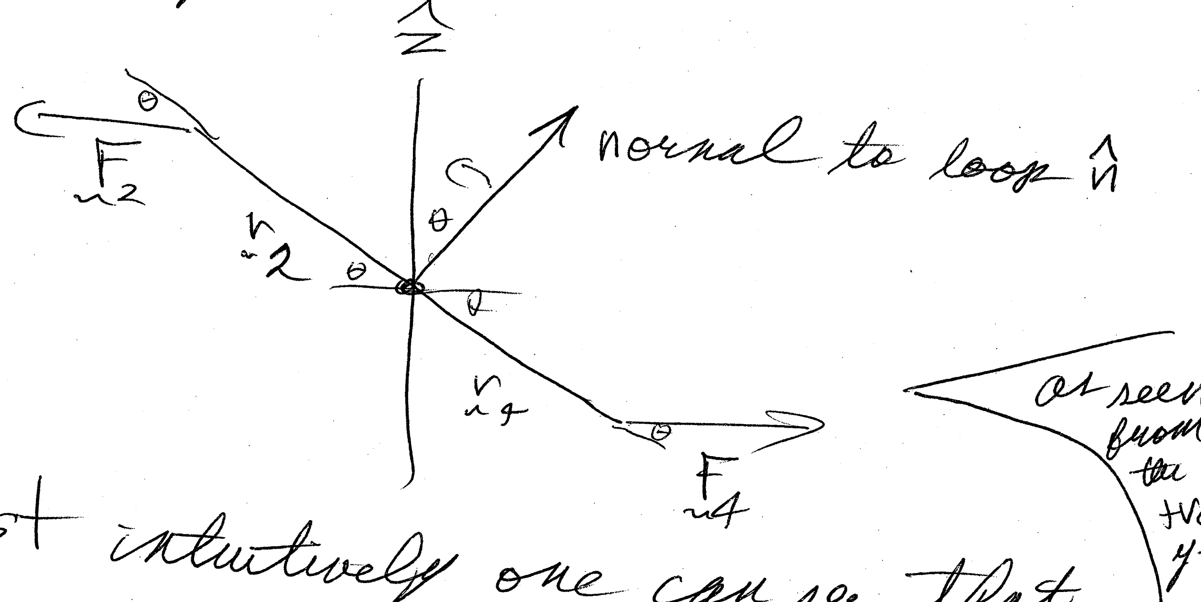
So in fact $\mathbf{F}_{net,\,external} = 0$

29-66

and the CM is not accelerated.

Since we assume the loop is rigid, the forces can't deform it.

But the F_2 & F_4 forces can torque it.



Just intuitively one can see that the torques are going to try to rotate the loop counterclockwise
→ try to align the normal with the z-axis as we'll see.

$$\underset{\sim}{\tau}_2 = \underset{\sim}{r}_2 \times \underset{\sim}{F}_2$$

$$= r_2 F_2 \sin\theta \, \hat{y}$$

$$= r_2 I s_2 B \sin\theta \, \hat{y}$$

$$\underset{\sim}{\tau}_4 = r_4 I s_4 B \sin\theta \, \hat{y}$$

The Torques have the same direction — both try to rotate ~~clockwise~~ counterclockwise as seen from the +ve y-direction

$$\underset{\sim}{\tau} = (r_2 s_2 + r_4 s_4) I B \sin\theta \, \hat{y}$$

$$s_2 = s_4$$

$$r_2 + r_4 = s_1 = s_3$$

$$= s_1 s_2 \, I B \sin\theta \, \hat{y}$$

$$\cancel{= A I B \sin}$$

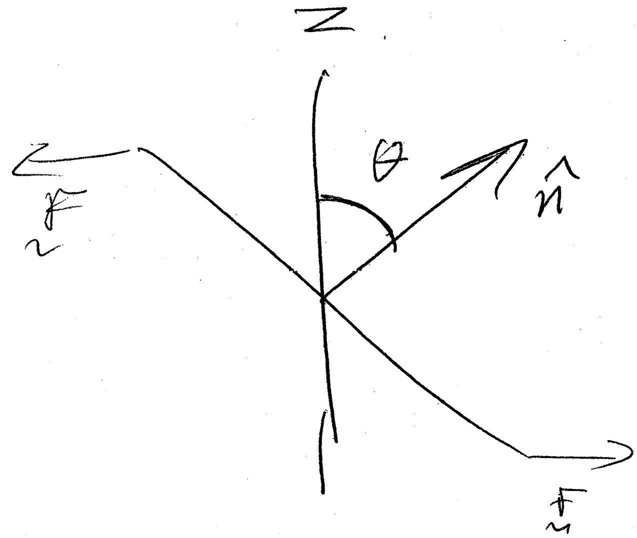
$$= I A B \sin\theta \, \hat{y}$$

where A is the area of the rectangle.

By the by the symmetry of the forces shows there is no torque for rotation about $\hat{n}$. We can skip that formal analysis.

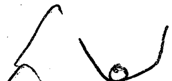
29-68) Cases

$\underset{\sim}{\tau} = IAB\sin\theta\,\hat{y}$



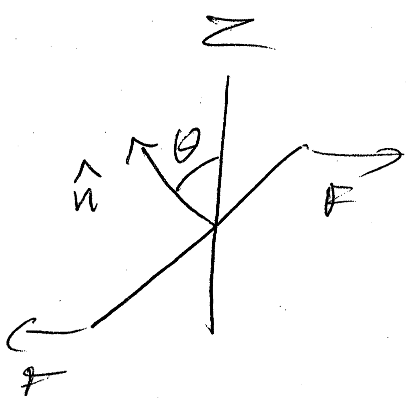
$\theta > 0$

$\underset{\sim}{\tau}$ rotates counterclockwise



$\theta = 0$

$\underset{\sim}{\tau} = 0$ } a stable equilibrium since this the torque tries to move the loop to this case



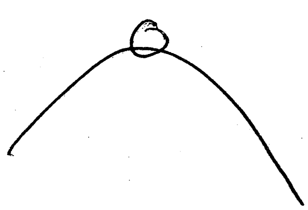
$\theta < 0$

$\underset{\sim}{\tau} < 0$ and tries to rotate clockwise.

$\theta = 180°$

$\underset{\sim}{\tau} = 0$ again

but this is an <u>unstable equilibrium</u>



since any perturbation will cause the

torque to try to rotate the loop to the $\theta = 0$ location.

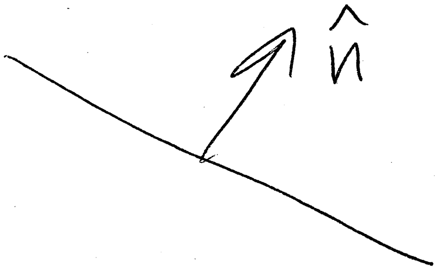
Now for simplifying ~~to~~ ~~notation~~ definitions.

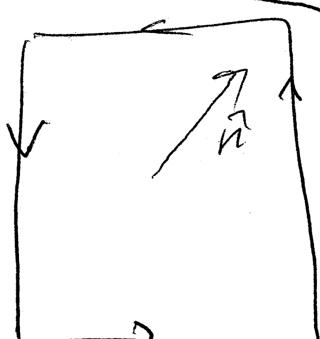
We'll see why in Ch30 we call this a magnetic dipole moment & the loop a magnetic dipole - similar to electric dipole case

We define the <u>magnetic dipole moment</u> to be

$$\underset{\sim}{\mu} = I A \hat{n}$$

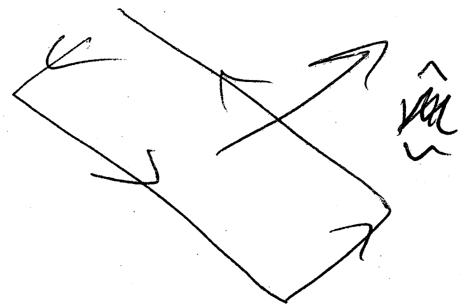
μ is the small Greek mu — the standard symbol for magnetic dipole moment



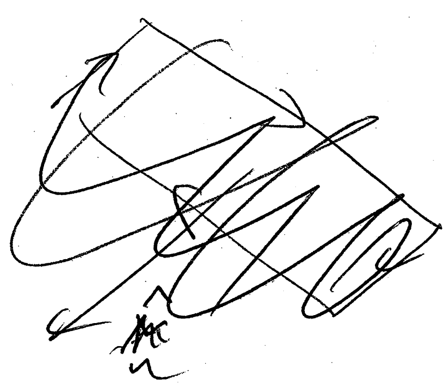


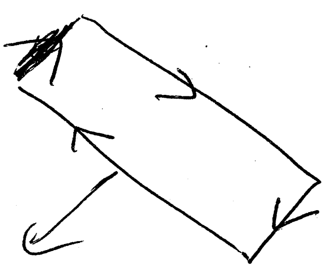
- the sense of $\hat{n}$ or $\underset{\sim}{\mu}$ is by a right-hand rule.
- curl fingers of ~~the~~ right hand with current and the thumb

27-70) is more or less in the direction of $\hat{\underset{\sim}{m}}$

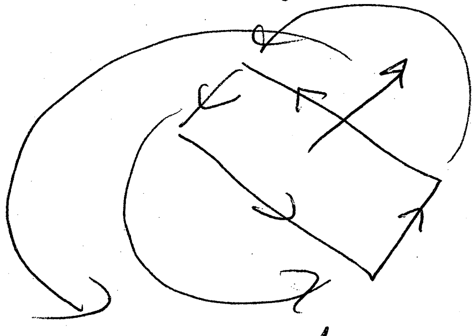


If we reversed the current (which happens all the time in electric motors and generators) then $\hat{\underset{\sim}{m}}$ reverses.





But notice this is just like rotating $\hat{\underset{\sim}{m}}$.



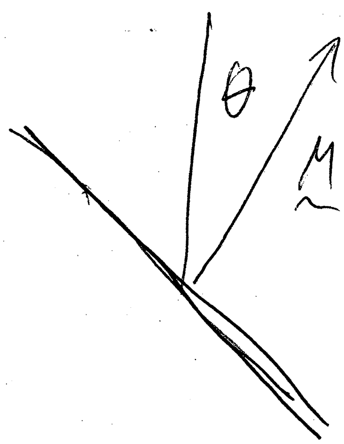
So the reversed current really isn't a new case and is treated by the formalism we've developed.

With $\underset{\sim}{\mu} = I A \hat{n}$

$$\underset{\sim}{\tau} = I A B \sin\theta \, \hat{y}$$

$$= \underset{\sim}{\mu} \times \underset{\sim}{B}$$

There is no longer any reference to any coordinate system — particular coordinate systems are just description.



Conventionally $\underset{\sim}{B}$ is often ~~chosen~~ to ~~be~~ define the $\hat{z}$ direction, but that's not physics.

29-72

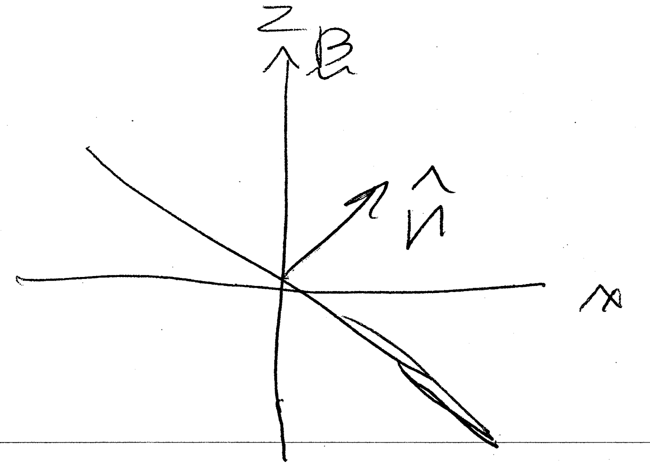
Our result is more general than our derivation

— for any current distribution $\underset{\sim}{\mu}$ can be defined by a more general rule than $\underset{\sim}{\mu} = IA\hat{n}$

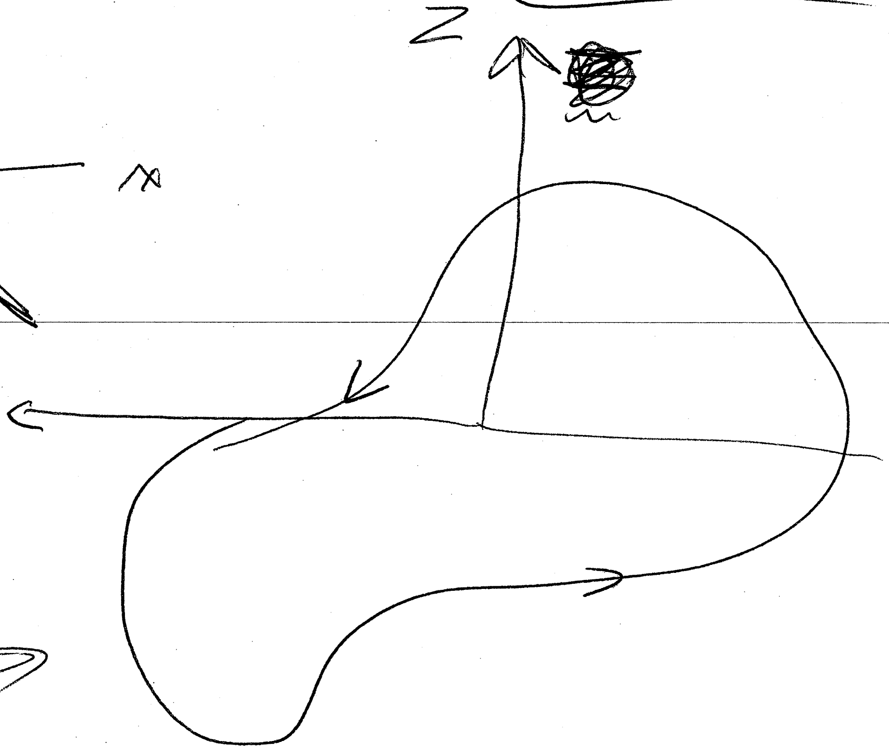
and one still gets $\underset{\sim}{\tau} = \underset{\sim}{\mu} \times \underset{\sim}{B}$

But that generalization is beyond our scope.

We can though generalize from a rectangular loop to a planar loop of any shape.



edge on view



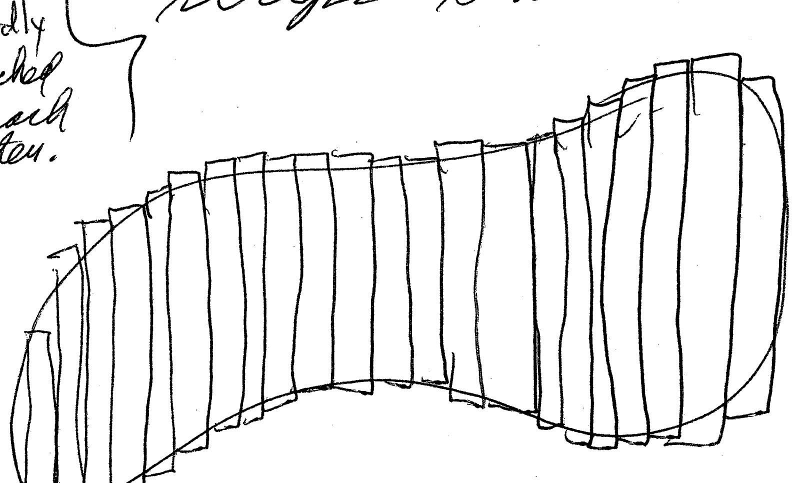
general
planar loop of current I

view from +ve x direction of a general loop.

Divide it into strips that cover the loop.

rigidly attached to each other.

Need overlap bits to ensure the sides below is the same as before



— each strip defines a rectangular loop of current I

all rigid and rigidly attached to each other

$$\underset{\sim}{\tau}_i = \underset{\sim}{\mu}_i \times \underset{\sim}{B}$$

where $\underset{\sim}{\mu}_i = I A_i \hat{n}$

where A_i is a rectangular area

$\boxed{29-74}$

We've proven

$$\underset{\sim}{\tau}_i = \underset{\sim}{\mu}_i \times \underset{\sim}{B}$$

for the rectangular loops & so this is valid.

Now we sum up the torques on the strips.

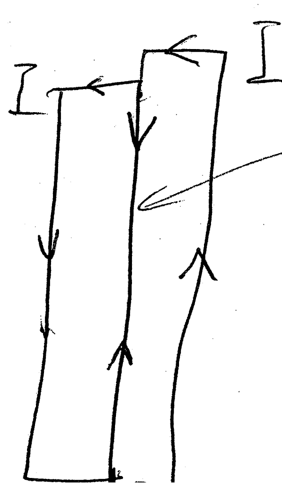
$$\underset{\sim}{\tau}_{strips} = \sum_i \underset{\sim}{\tau}_i = \sum_i I A_i \hat{n} \times \underset{\sim}{B}$$

$$= I\left(\sum_i A_i\right) \hat{n} \times \underset{\sim}{B}$$

$$= I A_{strips} \hat{n} \times B$$

$$A_{strips} = \sum_i A_i$$

Now consider ~~the~~ where the strips join for example



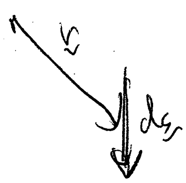
here the two currents are overlapped

Because the currents
~~Ia~~ are overlapped
all forces on the overlap
sections cancel out on each bit $d\underline{s}$

$$d\underline{F} = I\, d\underline{s} \times \underline{B}$$

changes direction with I.

— Any torques cancel out
too since from any origin
to $d\underline{s}$ has the same $\underline{r}$
for either current.

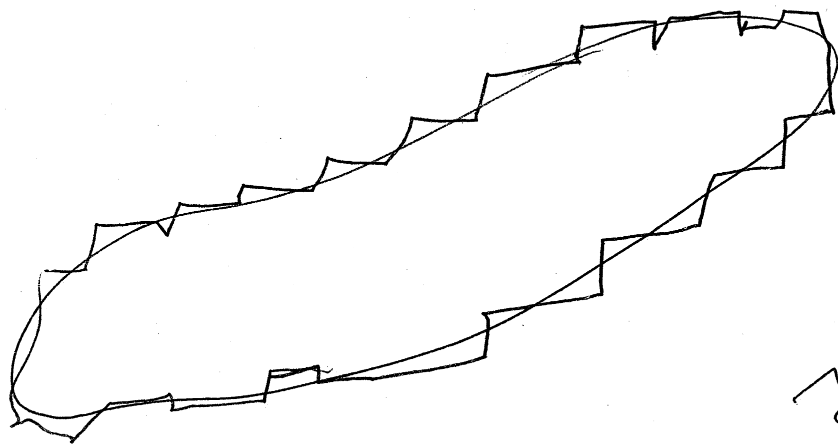


So everything is actually the
same as if the overlapped
parts of the strips were
NOT there.

— So we we can mentally

29-76)

remove them.



In which case we see $\underset{\sim}{\tau}$ stripes $\hat{n}$ $\underset{\sim}{\tau}$ of the smooth loop.

~~If~~

Now we do the old calculus trick of ~~the~~ letting the strips become infinitely many and infinitely fine ~~which~~ while have the same net area

$$\underset{\sim}{\tau} = \lim_{\text{strip}\to 0} I\left(\sum A_i\right) \hat{u} \times \underset{\sim}{B}$$

$$= I A \hat{n} \times B$$

where A is the area of the original loop.

∴ We can define

$$\underset{\sim}{\mu} = I A \hat{n}$$

for any planar loop of thin line of current I.

We can do one more generalization.
Say we had N identical loops (or <u>turns</u> in the jargon) and could overlap them exactly.
Then clearly $\underset{\sim}{\mu} = N I A \hat{n}$.

29-78

Now practically one can't do that ~~@~~ — overlap the turns exactly.

But nevertheless

$$\mu = NIA\hat{n}$$

is a good approximation often for thin coils to the exact dipole moment ~~formula for a co~~ for a coil of N turns

The ~~exact~~ general formula for ~~beyond our~~

magnet dipole moment.

<u>Can</u> a PE be defined for a magnetic dipole in a B-field.

(Remember we cannot define one for point charge in a B-field.
See p. 29-27)

Well yes.

$$\underset{\sim}{\tau} = \underset{\sim}{\mu} \times \underset{\sim}{B}$$

is exactly analogous

to $\underset{\sim}{\tau} = \underset{\sim}{p} \times \underset{\sim}{E}$ that

we found for an <u>electric dipole</u>

$\underset{\sim}{p}$ is electric dipole moment.

29–80

in Ch. 26.

— there is, of course, part of the reason a current loop is called a magnetic dipole and $\underset{\sim}{\mu}$ is called a magnetic dipole moment.

Now $PE = -\underset{\sim}{p} \cdot \underset{\sim}{E}$

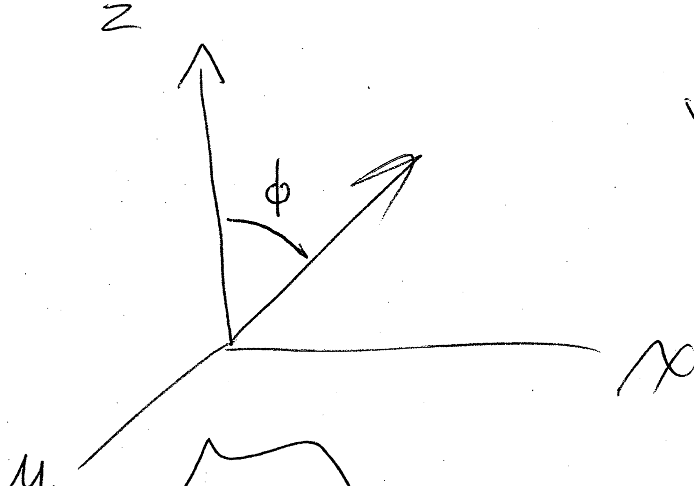
and so by an identical derivation

$PE = -\underset{\sim}{\mu} \cdot \underset{\sim}{B}$

We can give the derivation of course.

Recall from Rotational dynamics

$$\cancel{W(\theta_a,\theta_b) = \int_{\theta_a}^{\theta_b} \tau \cdot \hat{z}}$$



$$W(\phi_a,\phi_b) = \int_{\phi_a}^{\phi_b} \tau \cdot \hat{y}\, d\phi$$

is the work done by an external torque τ on a rigid body where τ is aligned with the $\hat{y}$ direction.

(only allow rotation around y-axis.)

A crazy left-hand system — in the Looking Glass world.

In our case

$\mu \times B$ defines $\hat{y}$ direction

$$W(\phi_a,\phi_b) = \int_{\phi_a}^{\phi_b} (\mu \times B)\cdot\hat{y}\, d\phi$$

29-82

$$= \int_{\phi_a}^{\phi_b} \mu B \sin\phi \, d\phi$$

$$= \mu B (\cos\phi_b - \cos\phi_a)$$

This work depends only on the end points and so a PE can be defined by

$$\therefore \Delta PE = -W$$ ~~as always~~ as usual

$$\Delta PE_{ab} = -\mu B(\cos\phi_b - \cos\phi_a)$$

By convention the zero of PE is at $\phi = 90°$

$$\therefore PE = -\mu B \cos\phi$$

$$= -\underset{\sim}{\mu} \cdot \underset{\sim}{B}$$

is the general formula without reference to peculiar coordinate systems.

29-83

§ 29.6 Hall ~~Effect~~

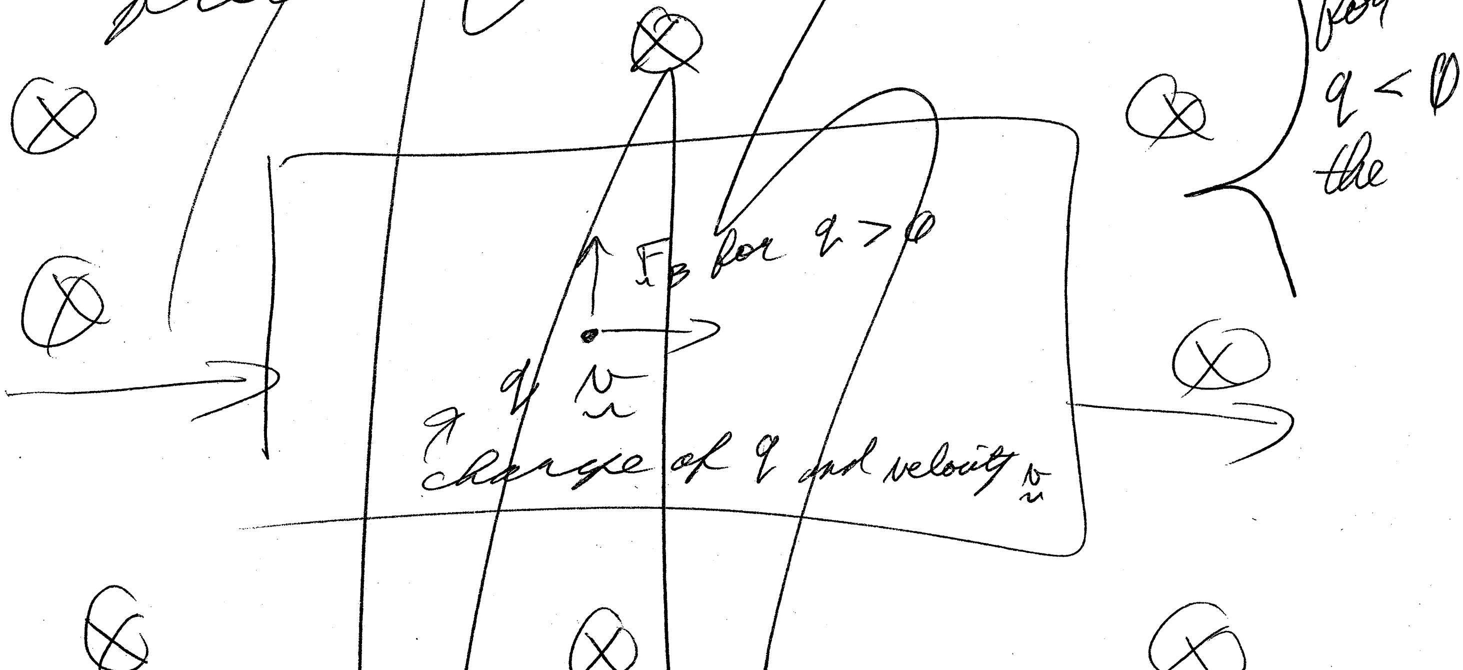
— discovered by Edwin Hall
in 1879

(who was actually an American
which in the 19th century
was unusual for physicist
— but there were others
Joseph Henry
Albert Michelson
& many who could be
called applied physicists.
T. Edison, N. ~~Tesla~~ Tesla.

— really easy to understand
and it finally showed
the sign of the charge carriers
in metals.

29-84

Take a slab of conductor and put it in a uniform $\mathbf{B}$ field



— the $\mathbf{B}$-field does penetrate unlike an $\mathbf{E}$-field.

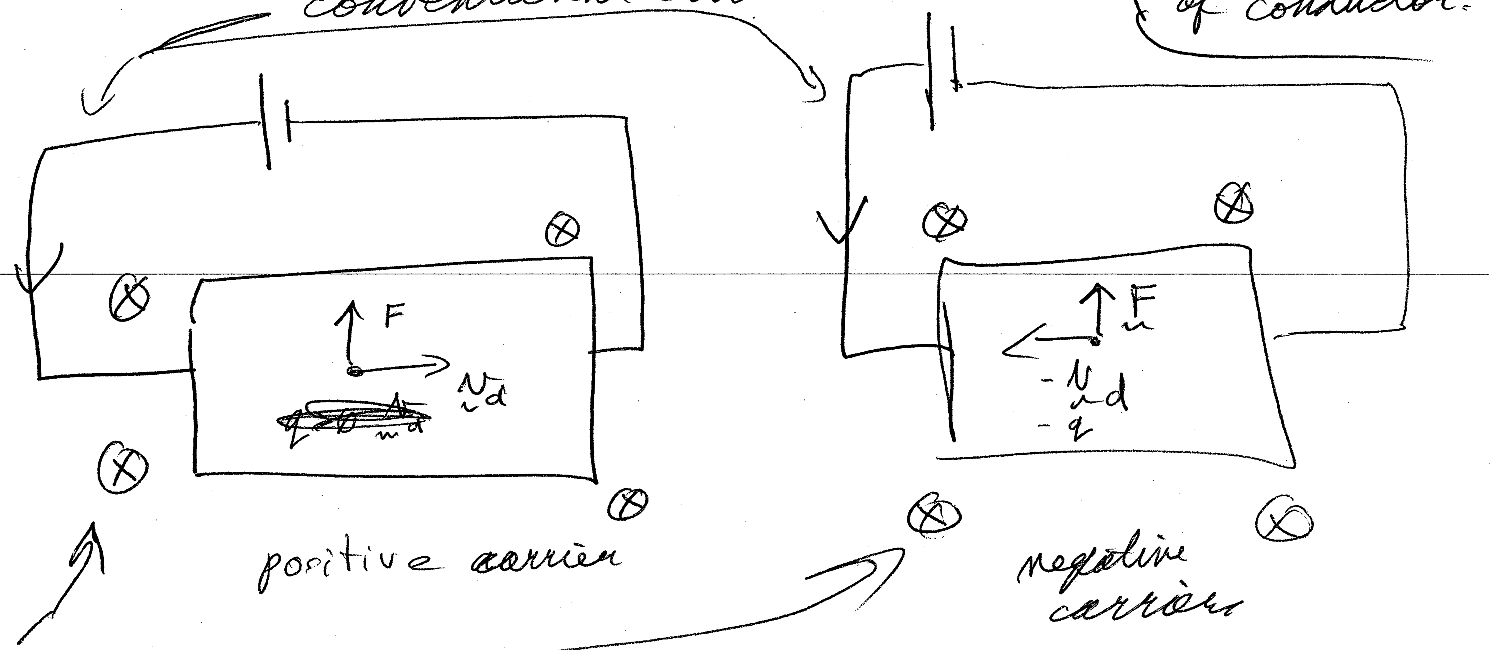
— run a ~~current~~ steady current thru it.

Each charge feels a magnetic force $\mathbf{F} = q\, \mathbf{v}_d \times \mathbf{B}$

29-85

Consider two cases with slabs of conductor.

conventional current direction



uniform B-field

$$\mathbf{F} = q\,\mathbf{v}_d \times \mathbf{B}$$

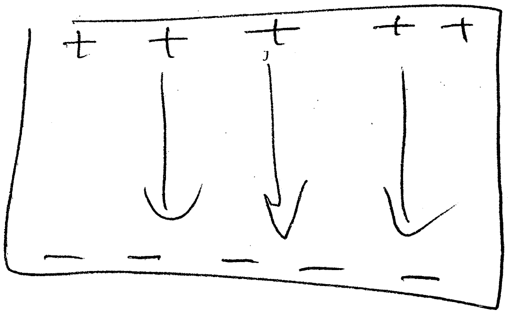
$$\mathbf{F} = (-q)(-\mathbf{v}_d) \times \mathbf{B} = q\,\mathbf{v}_d \times \mathbf{B}$$

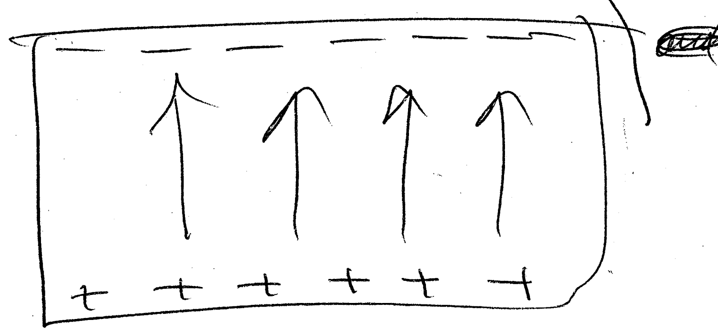
In both cases the magnetic force is up.

What happens very quickly when the circuits are set up is ~~that~~ ~~there~~ is a charge separation

29-86

and this separation accumulates till it ~~just cancels the~~ creates an E-field force to just cancel the B-field force.





with net charges on the upper and lower edges. (Apparently just as with no B field, there can be no net charge in interior of conductor in steady-state (Ohanian 260)

$F_E = qE(-\hat{y})$ $\qquad$ $F_F = (-q)(-E)(-\hat{y})$

$= qE(-\hat{y})$

in both cases an electric force points down.

- In steady state it cancels the magnetic force.

29-87

A simple potential measurement from top to bottom gives the Hall potential V_H as it is called (TM-905)

$V_T - V_B = V_H > 0$

~~top to bottom~~

~~V_T~~

This tells us the carriers are ~~negative~~ positive

$V_T - V_B = V_H < 0$

~~top to~~

This tells us the carriers are negative.

For metals, Hall discovered the carriers are negative in 1879 (Wik)

29-88

— actually in 1879
it was not clear that
charge came in discrete
units and so one
might have said the
mobile electric fluid
is negative.

although the idea of
a discrete unit of charge
was proposed in 1874 &
named "electron" in 1894
by G.J. Stoney

— ~ 1897, the electron was
accepted as a particle after J. J. Thomson's
experiments
with cathode rays (beams of electrons)

So Ben Franklin sort of got it wrong since it would have been better to reverse the names

and ~~let current~~ have electrons flow in the direction of ~~current~~ conventional current.

But actually there can be positive charge carrier

— positive ions in gases & electrolytes (solutions with ions) or any substance with free ions.

Also in semi-conductors (TM-905)

↳ mobile absences of electrons

↳ called holes are sometimes

29-90)

more abandont
than electrons

the holes are then
the dominant charge
carrier — and ~~their~~ are positive.

A lot more can be done
with the Hall effect
and ~~the~~ more with its cousin
the quantum Hall effect
& the fractional quantum Hall effect.