Chapter 28

Direct Current Circuits

DC circuits

I'm going to go a bit out of order from the text — I just can't bear it's order

§28.3

Kirchhoff's Laws (or rules)

— they are really easy to understand and apply (at least in simple cases)

— they apply to time invariant direct currents.

↳ but they also apply when current is varying as long

28-2

~~not too fast~~
but "too fast"
is really, ~~~~ really
fast.

→ they work ~~even up to~~
for AC (alternating current)
even up to radio frequencies
($\sim 10^6$ Hz or 1 MHz)!
(GrEM-292)

1) <u>Kirchhof Voltage Law (or loop law)</u>

1) The changes in electric
potential around any
closed path sum to
zero ~~~~ (in the physical or by any other path)

— or the sum of potential rises
and drops is zero.

$\sum \Delta V_i = 0$

— ~~this~~ law is just a manifestation of the fact ~~that~~ ~~the~~ electric force of a static charge ~~(or even quasi static~~ distribution is a conservative force. (Gr ~~E~~M-223) BO-216

$$V_{\text{closed loop}} = -\oint \underset{\sim}{E} \cdot d\underset{\sim}{s} = 0$$

Believe it or not in DC electric circuit (and in AC ones too viewed at an instant in time) ~~there~~ is ~~an~~ a distribution of ~~net charge~~ +ve and -ve charge. The sum of it

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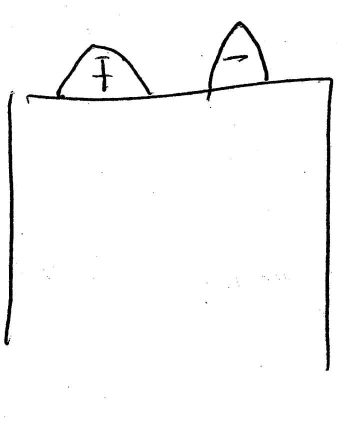
it all is usually zero
but the charge is separated.

— where this separated
static array of charge
is exactly is in general
hard to say.

→ Much must be in
the source of electric
energy

→ the emf device
(electromotive force = emf
— we'll describe in a bit)

like a battery
with +ve & -ve
~~terminals~~
terminals



— ~~probably~~ a lot of the charge is
on surfaces of
terminals.

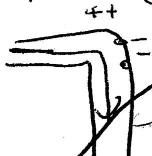
→ but maybe some
inside too

The
emf
is the
thing
that
separates
the
static
charges
and
keeps
them
separated
and
drives the
current.
— we'll
come to
it.

– but there is also some in the wires and other outer parts (B0-216)

the "guiding charge":

Not a lot, but needed to make current go around bends e.g.)



except at highest level of detailed understanding, you don't know where exactly the separated static charge is in detail

B0-217

and you don't need to know for most circuit calculations.

Q

Note the charge in the current itself is not part of this separated charge.

E-field outside wires in general are complex but usually you don't need to know anything about them [illegible]

the current is flowing (of negative electrons)

28-6)

against a background of positive charge.

~~ ~~ – in the interior of the conductors the net charge density is zero. (Ohanian-260)

— the separated charge is on the surfaces of the conductors.

— We proved this for electrostatic cases.

→ It is also true for ~~time-~~ time-independent flow cases

+ cases where the flow can be so approximated (i.e., AC cases)

But unlike electrostatic cases $\underset{\sim}{E} \neq 0$ inside conductors.

— it is just a bit beyound our scope to prove it here (but it can be done Ohanian -260)

2) Kirchhoff's current law (or the junction law) or node law

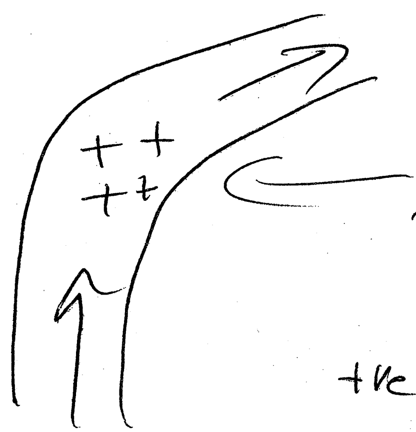
— in steady flow there is no build up charge anywhere (not counting the already established separated charge)

— thus all the current flowing into a junction must be matched by current flowing out.

— in equation form $\sum_i I_i = 0$ for any junction.

28-8

What keeps ~~the~~ any net charge from piling up —— beyond what it takes to establish the DC current (the separated electrostatic distribution.)



say there was a build up of +ve charge here — it would tend repel more +ve charge ~~from~~ entering and tend push out more charge the other way.

A steady state situation with steady inflows and outflows of energy is self-regulating. (amazingly so)

We'll show how to use Kirchhoff's laws soon but let's return to emfs

§28.1 Electromotive force emf

— a misnomer. It's not a force. It's integral of a force per unit charge over a ~~distance~~ line — or a work per unit charge

a line integral

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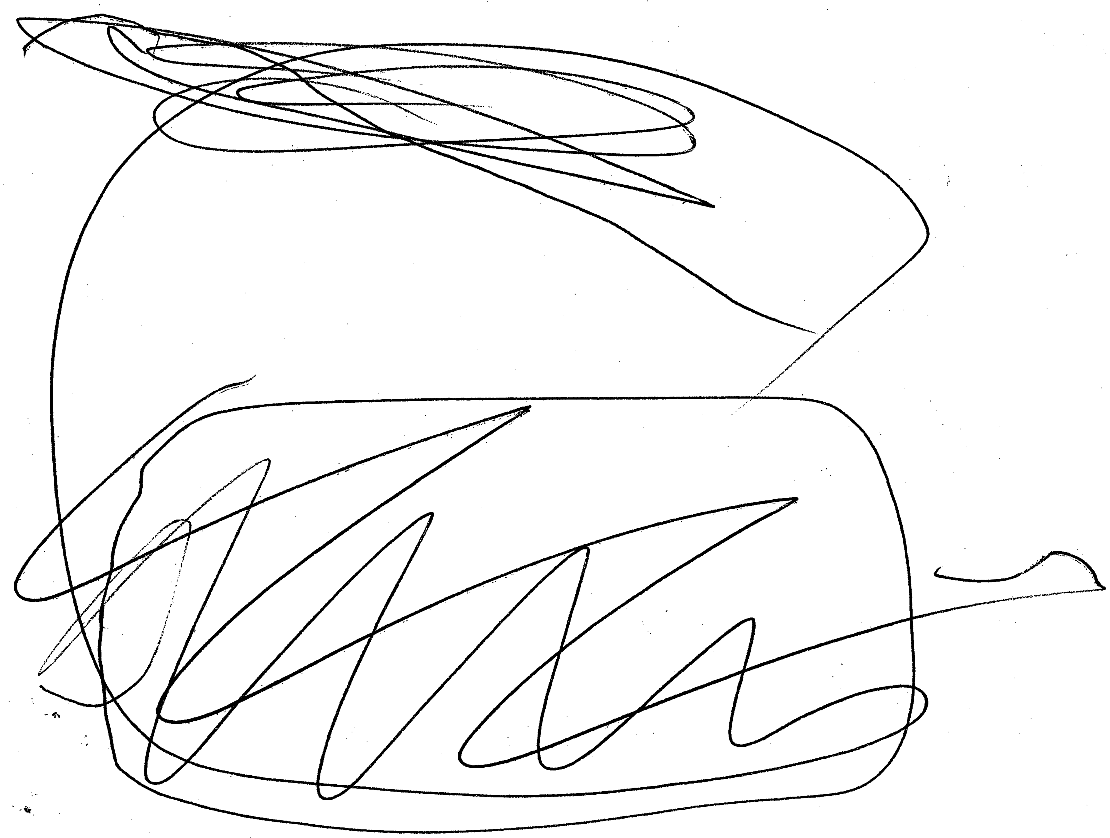
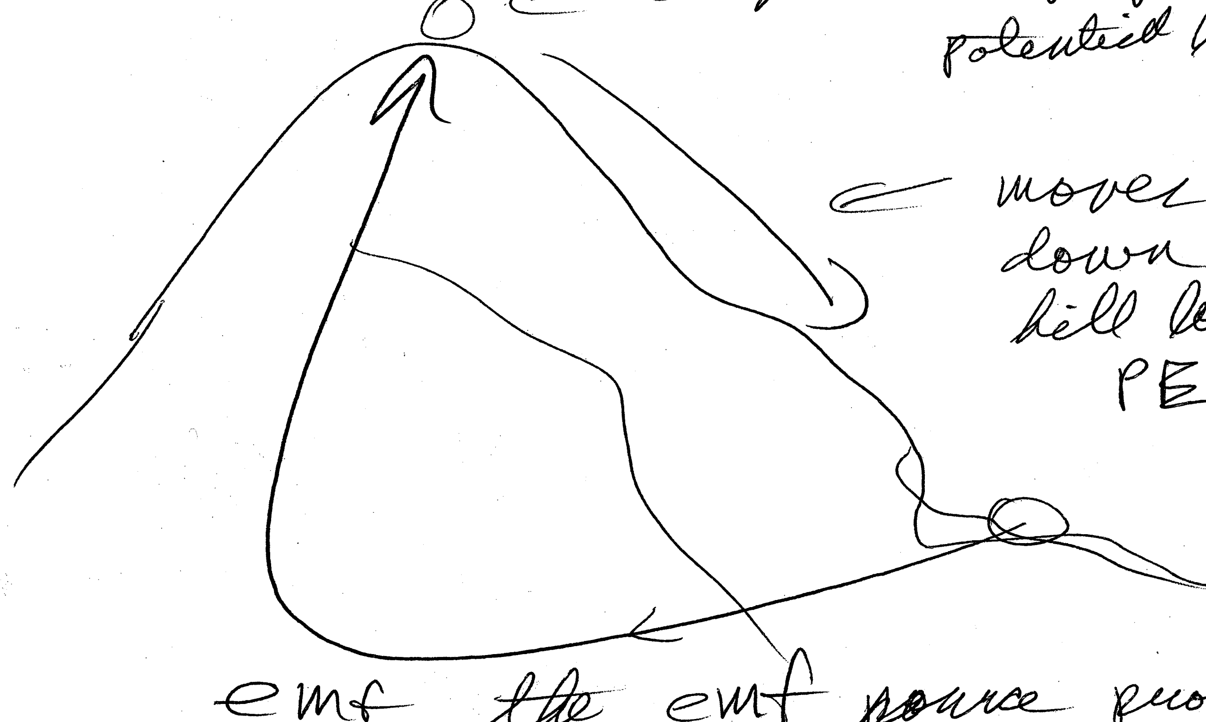




emf the emf ~~source~~ provide the energy to get the charge back up the hill.

We'll just consider a generic emf (~~though~~ a chemical battery is the obvious DC example,)

Inside in a steady state (which could ~~also~~ also be the no flow case) the charge is not accelerating

~~So $E = f_{emf}$~~

So $\underset{\sim}{f}_{emf} = -\underset{\sim}{E}_{electrostatic}$ for an ideal emf with no internal resistance.

countering emf force pushing up the potential hill

electrostatic force per unit charge pushing charge down the potential hill

Now we do a path integral through the emf device.

28-12

Emf $\mathcal{E}$ = Potential here ideally even though the two are not the same thing, but many books sort of loosely imply they are

(GrEM-293)

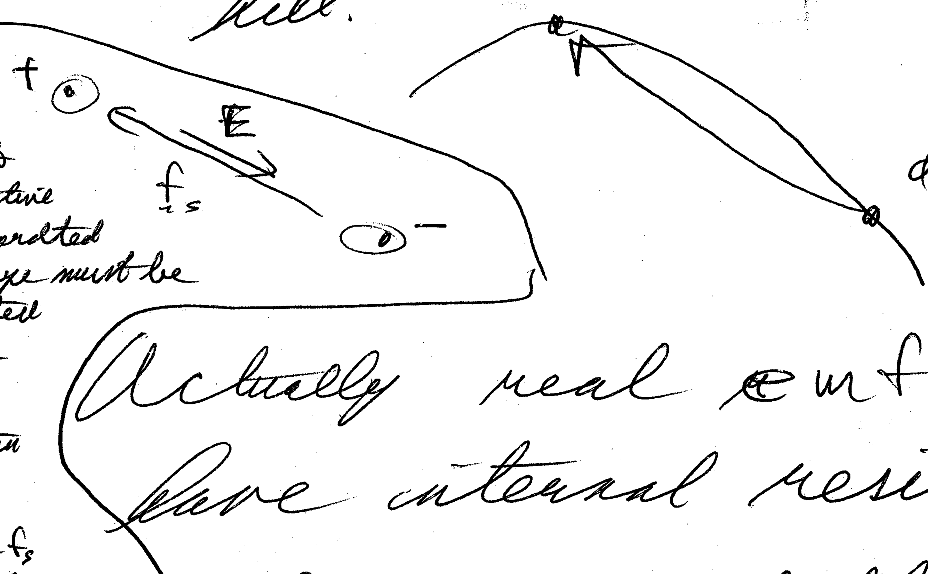
$$\mathcal{E} = \int_a^b \underset{\sim}{f_s} \cdot d\underset{\sim}{s} = -\int_a^b \underset{\sim}{E} \cdot d\underset{\sim}{s}$$

defined to be emf

the work done pushing the charge up the PE hill.

$$= V_{ab} = V_{-ve \text{ to } +ve \text{ terminals}}$$

the potential change from a to b.



the +ve & negative separated charge must be located where they can counter the emf force f_s all along but where that is maybe a complex thing to know.

An ideal emf maintains potential no matter what current flows.
In a static case the emf builds up the charge separation causing the potential until $\mathcal{E} = V_{ab}$ and then the static case is established

Actually real emf devices have internal resistance

So some of the energy coming from the emf supply goes into doing work against the resistive force.

and gets lost as waste heat.

— usually one can treat this loss as due to an ohmic resistor of resistance r say. — an internal resistance

In this case, Ir is the energy per charge lost to resistance (which get from Ohm's law).

$Ir = \int_a^b f_{\text{resistance}} \cdot d\ell$

still low + have perfect argument.

$$\mathcal{E} = V_{ab} + Ir$$ more

or $\mathcal{E} - Ir = V_{ab}$ <u>realistically</u>.

more physically

What actually provides emf?

— well there are several sources — we'll just mention, not go into ~~these~~ actual ~~details~~ specifications {GrEM -292 -293

Examples

1 — chemical battery

↳ here a chemical force pushes the charge.

28-14)

this chemical force is really an electromagnetic force ~~too~~ — but not due to macroscopic ~~E~~-fields.

2) Photons ejecting electrons in photovoltaic cells

3) In a Von de Graaf generator (used as an emf), one literally loads electrons on a conveyor belt to transport them (~~SS~~ SJ-710)

4) In the usual AC case ~~a non-DC~~ ~~case~~ (SJ-880)

One uses an electric generator to create an induced electric field ~~through~~ Faraday's law (or universal flux law)

this is not an electrostatic electric field and this induced field pushes the charge up the potential hill

There are others.
— I don't really (Gr EM-293)
pretend to know much
about ~~their~~ operation.

Let us do ~~a very~~ some examples

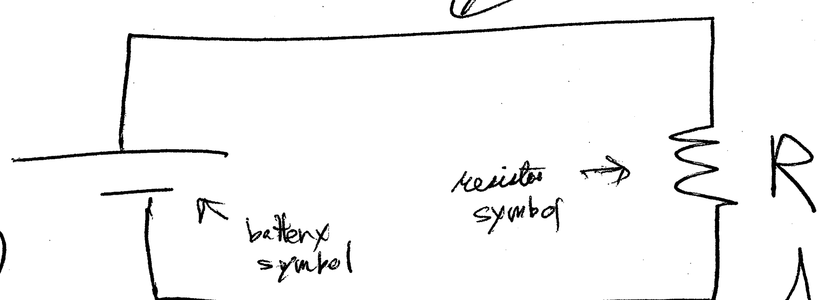
Ex 1

ideal resistanceless wires
— current flows with $E = 0$ and so the ~~wire~~ wire segments are equipotentials

$\mathcal{E}$

an ideal battery

so the potential rise across it is $\mathcal{E}$



resistor R.
— a potential drop across it to ~~push~~ give enough E-field ~~the charge down~~ to cancel the resistive forces.

$V_R = IR$ by Ohm's law.

$\boxed{28-16}$

There is only one loop and so only one constant current I by Kirchhoff current law.

The ~~potential~~ voltage law tells us $\sum V_i = 0$

here $\sum V_i = \mathcal{E} - V_R = 0$

or $\mathcal{E} = IR$

$\therefore$ Solving for I

we get $I = \frac{\mathcal{E}}{R}$

for $\mathcal{E} = 1V$ & $R = 1\Omega$

$I = 1A$

$$P_{\text{input from emf}} = I\mathcal{E}$$
$$= 1\,W$$

$$P_{\text{output as heat in resistor}} = IV_R = I^2R = \frac{\mathcal{E}^2}{R}$$
$$= I\mathcal{E} = 1\,W$$

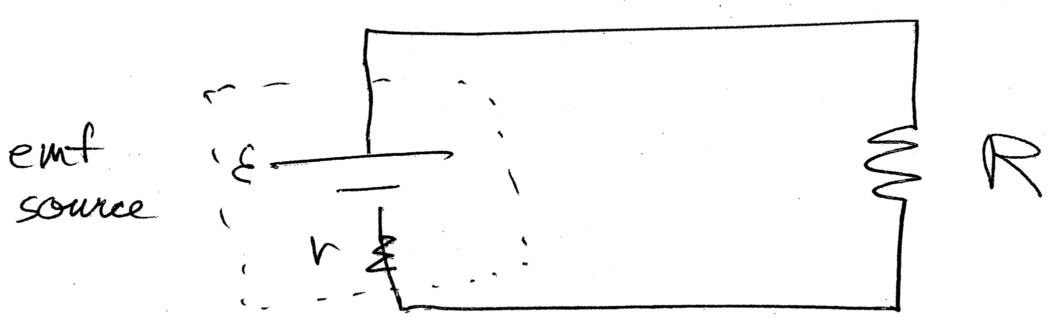
Note if $R \to 0$
$I \to \infty$
$P_{out} \to \infty$
and one has a short circuit

Ex 2 (Ex 28.2 in text)

Load Matching.

Almost the same case, but now our emf is not ideal, but has internal resistance r

This is when your wires and insulation melt down and electrical fires start. Fuses and circuit breakers cut the current off if it gets too high.



Potential rise $\mathcal{E} - Ir = IR$ Potential drops.

$$\mathcal{E} = I(R + r)$$

$$I = \frac{\mathcal{E}}{R + r}$$

28-18

$$P_{input} = I\mathcal{E} = \frac{\mathcal{E}^2}{R+r}$$

Not equal in this case since some power is heating up the internal resistance

$$P_{output \text{ in resistor}} = I V_R = I^2 R$$

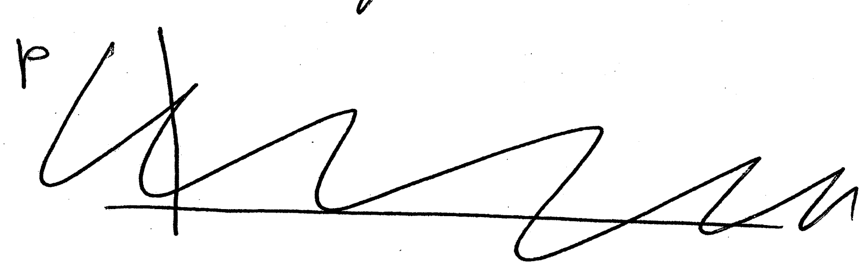
$$= \frac{\mathcal{E}^2 R}{(R+r)^2}$$

The P_{out} actually has an interesting behavior as a function of R.

If $R = 0$, no resistance no power out.

But if $R = \infty$, $I = 0$ and no power out.

So P_{out} must have a maximum (at least one) as a function of R.



$$P_{out} = \frac{\mathcal{E}^2}{r^2} \frac{R}{(1+\frac{R}{r})^2}$$

$$\simeq \frac{\mathcal{E}^2}{r^2} R \text{ for } \frac{R}{r} << 1$$

so rises linearly with R for R small.

$$\frac{1}{(1+\frac{R}{r})^2} \simeq 1 - 2\frac{R}{r}$$

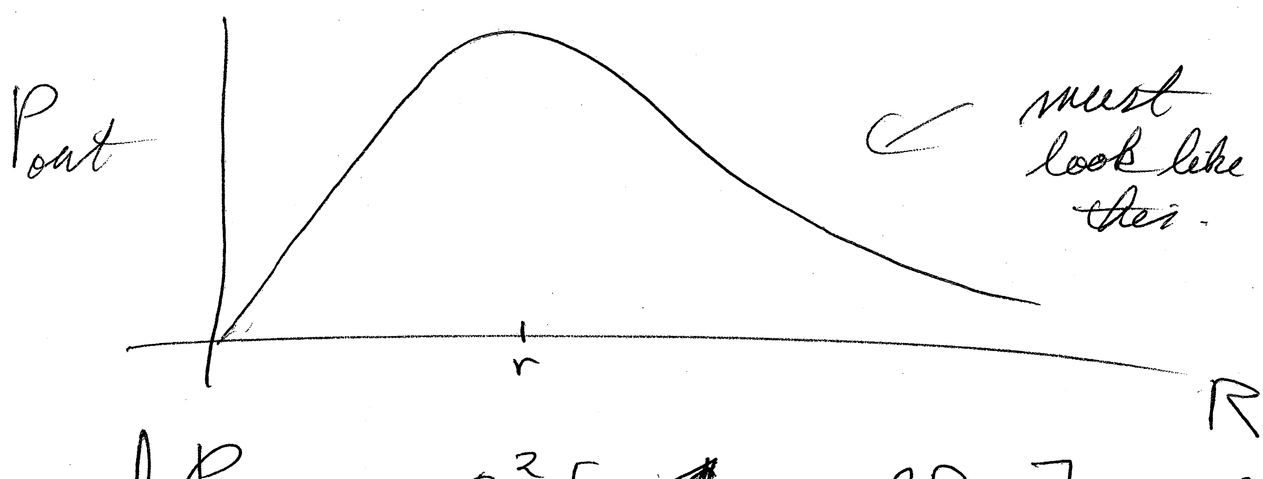
by the Taylor ~~geometric~~ series ~~series~~

$$\cancel{\frac{1}{1-x} = \sum_{l} x^l}$$

$$P_{out} = \frac{\mathcal{E}^2}{R(1+\frac{r}{R})^2}$$

$$\simeq \frac{\mathcal{E}^2}{R} \text{ for } \frac{r}{R} << 1$$

falls off as $1/R$



✓ must look like this.

$$\frac{dP_{out}}{dR} = \mathcal{E}^2 \left[\frac{1}{(R+r)^2} - \frac{2R}{(R+r)^3} \right] = 0$$

for maximum

$$R + r - 2R = 0$$

or $R = r$

28-20

So the maximum output power is for $R = r$ external resistance (the load) matched to internal resistance.

— hence the term load matching or resistance matching

$$P_{out\,max}(R=r) = \frac{\varepsilon^2 r}{(2r)^2} = \frac{\varepsilon^2}{4r}$$

$$P_{input}(R=r) = \frac{\varepsilon^2}{2r}$$

So the maximum power you can get is $\frac{1}{2} P_{input}$. So half is wasted in the internal resistance.

But there are a few other twists in the story.

$$P_{out} = \frac{\mathcal{E}^2 R}{(R+r)^2} \quad \text{recall}$$

Clearly $P_{out} \uparrow$ if $r \downarrow$, in all cases,

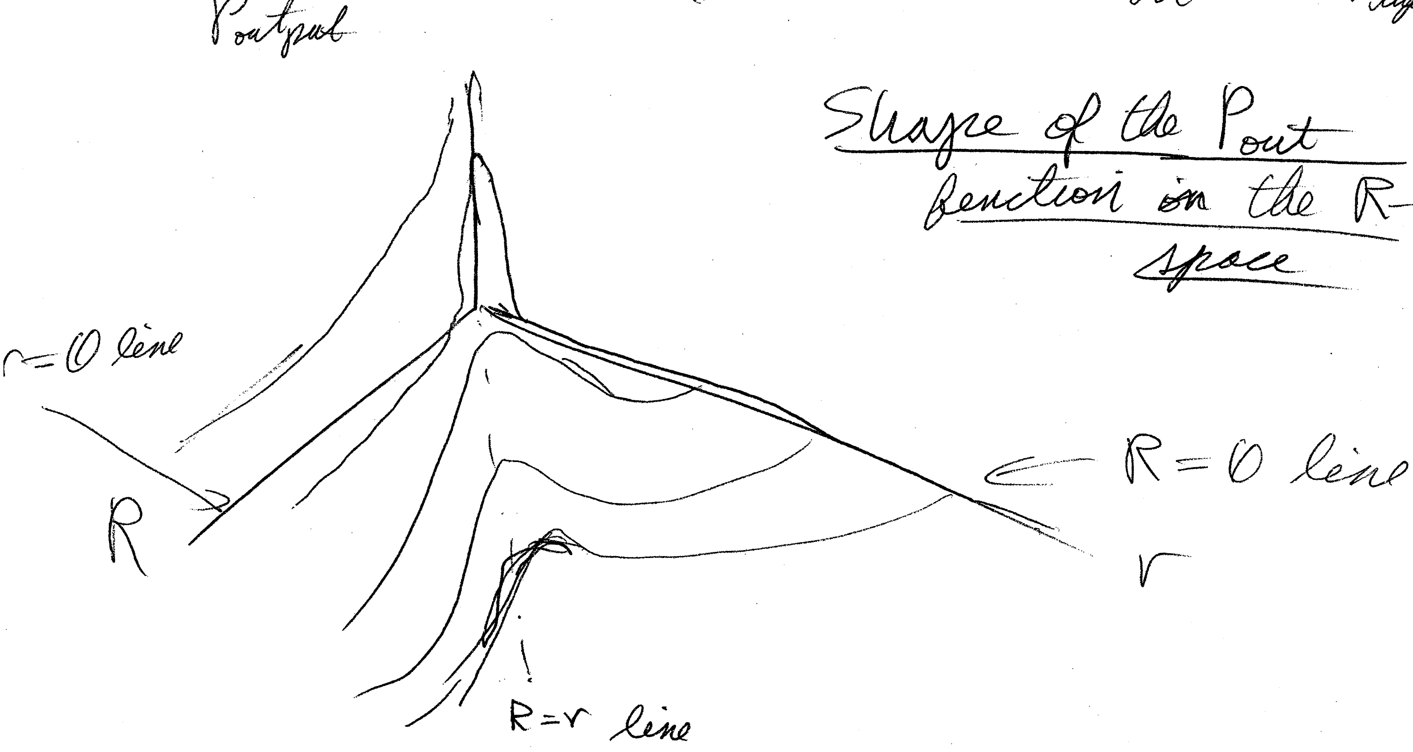
and so for $r = 0$

$$P_{out}^{(r=0)} = \frac{\mathcal{E}^2}{R}$$

which is $P_{input}(r=0) = \dfrac{\mathcal{E}^2}{R}$

In this case $P_{out} = P_{input}$.

Shape of the Pout function on the R-r space



28-22

The funniness is because of the infinity

for $\lim_{R \to 0} P_{out}(r=0) = \infty$

and $\lim_{r \to 0} P_{out}(R=0) = 0$

The $R = r = 0$ origin point is just kind of funny because of the infinity there. — a singularity where the function is undefined.

But this is just a mathematical conundrum.

Another aspect is ~~total~~ energy output.

$$\underbrace{E_{input}}_{\text{Let's hold this constant}} = P_{input} \Delta t$$

or $\Delta t = \dfrac{E_{input}}{P_{input}}$

Now $E_{out} = P_{out}\,\Delta t$

$$= \frac{P_{out}}{P_{input}} E_{input}$$

$$= \frac{\dfrac{\varepsilon^2 R}{(R+r)^2}}{\dfrac{\varepsilon^2}{(R+r)}} E_{input}$$ (see p. 28-18)

$$= \frac{R}{R+r} E_{input}$$

Thus we get all energy if $R \to \infty$ or $r = 0$

Both conditions are a ~~p~~ bit impossible.

But note $P_{out} = \dfrac{\varepsilon^2 R}{(R+r)^2}$, $P_{input} = \dfrac{\varepsilon}{R+r}$

both go to zero as $R \to \infty$.

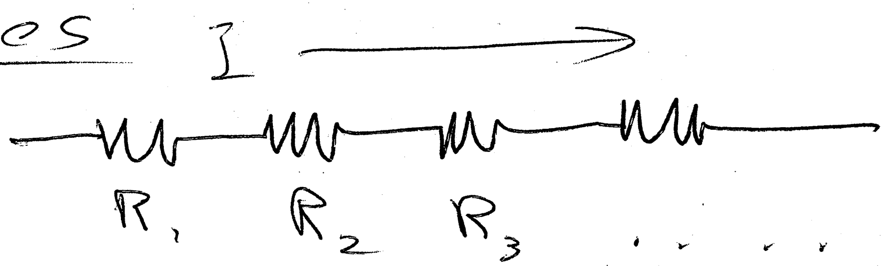
So you get all the energy for infinite R, but it takes you infinite time since your power is zero.

So there is a trade off between high power and not wasting any energy.

§28.2 Resistors in Series
Parallel ~~and~~

Nothing to these with Kirchhoff's laws in hand.

Series



$$V = \sum_i V_i$$

drop across all   drop across the ith one of potential

but by Ohm's law

$$V_i = I R_i$$

Kirchoff's law tells us the same current I in all resistors — no ~~bran~~ junctions or nodes.

Define

$$R_{eq} = \frac{V}{I}$$

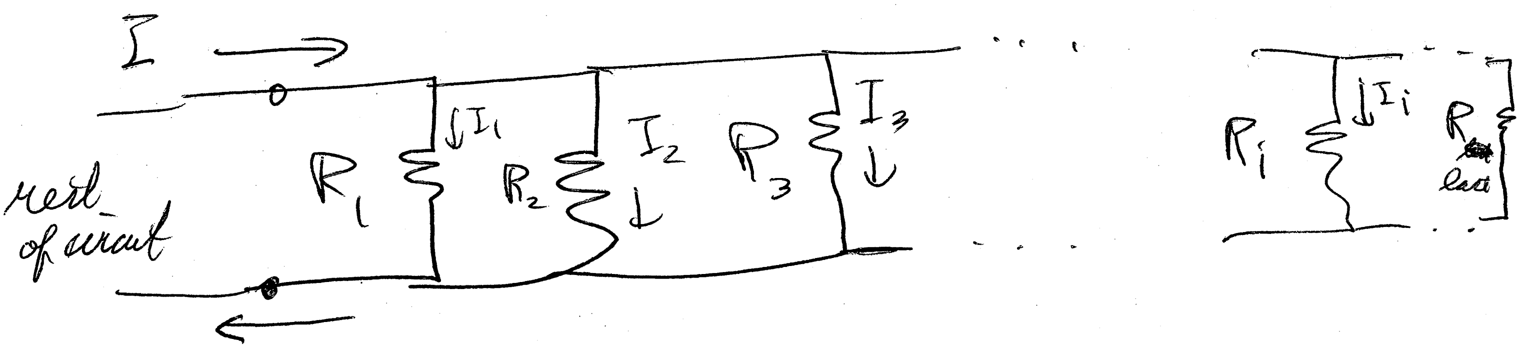
Then $I R_{eq} = \sum_i I R_i$

or $R_{eq} = \sum_i R_i$

$R_{eq} \geq R_{max}$ of course.

28-26

In Parallel



Ideal wires have no resistance

— ~~So the potential drop across~~

So $\quad V_{\text{rest of circuit}} + (-V_i) = 0$

drop and rises

same for all V_i

— by choice $V_i > 0$ although a drop.

Current ?

By Kirchhoff's current law

Let $V = V_i$

$$I = \sum_i I_i$$

Now use Ohm's law $V = I_i R_i$
and define $R_{eq} = \frac{V}{I}$

$$\therefore \quad \frac{V}{R_{eq}} = \sum_i \frac{V}{R_i}$$

or $\frac{1}{R_{eq}} = \sum_i \frac{1}{R_i}$ is the rule

implies $\frac{1}{R_{eq}} \geq \mathrm{Max}\left(\frac{1}{R_i}\right)$

or $R_{eq} \leq R_i$

So the more resistors one put in parallel the smaller the overall equivalent resistance.

Makes sense — there are more channels for current to flow thru.

A generalization of this situation is when you put too many devices on household ~~circuit~~ outlet

— They must go in parallel since they are all designed to use AC with RMS voltage 120V

Root mean square — we'll discuss in Ch. 33

They may not all be resistors,

but they all make the ~~generalize~~ effective resistence go down.

$I \cong \frac{V}{R_{eff}}$

— Thus the current goes up in the outlet

→ if it goes to high, then the wires in the wall could melt/burn

→ So the circuit breaker will break circuit.

It's not lack of power — a whole power grid (or some big fraction) stands behind the outlet — it's just the household wiring can't take it.

28-30

Special Cases

Ex 1 all in parallel resistors have some resistance R. There are N of them

In this case $\frac{1}{R_{eq}} = \sum_i \frac{1}{R}$

$$= \frac{N}{R}$$

or $R_{eq} = \frac{R}{N}$

Ex 2 Only two resistors

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

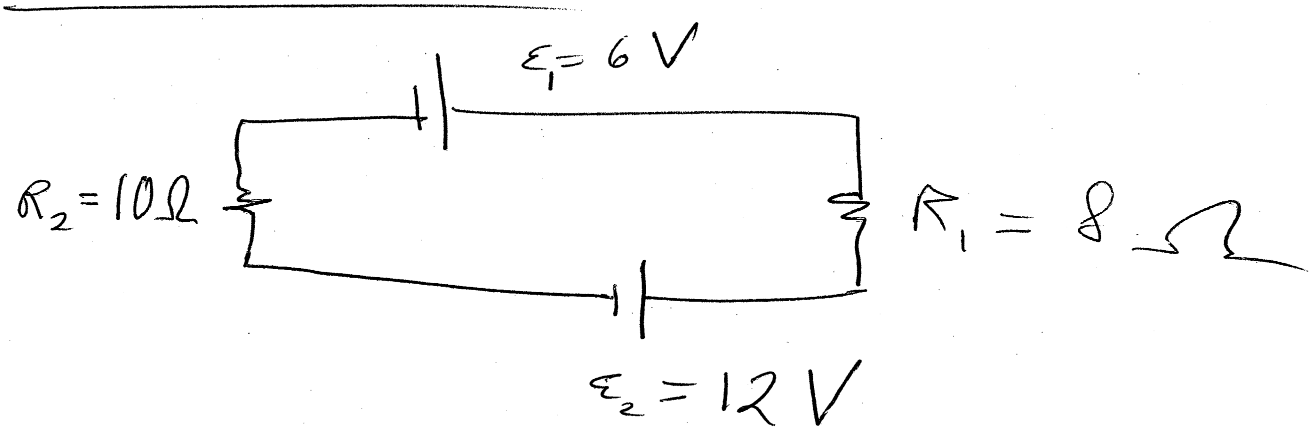
— all numerical examples are pretty easy.

§ 28.3 Again

Kirchhoff's laws

— Now we look at a few less simple circuit examples.

Ex 28.6



We assume ideal emf

— so no internal resistance.

Rules of ~~thumb~~ — that follow from Ohm's law and ideal emf

In ~~coun~~ counting up ~~at~~ potential rises and drops around a loop.

— Across a resistor - going with current, potential drops

- going against current potential rises.

28-32

— ~~for emfs — going with current there's a potential rise.~~

— go

— for emfs from -ve to +ve is always ~~at~~ a rise.

from +ve to negative is always a drop.

These rules almost going without saying — but it's still good to say them.

Now in this example we need to make a guess at which way the current flows to set up for solving.

— if we are wrong, there is no problem, we just get a negative current.

28-33

In this case, we expect the $\mathcal{E}_2 = 12\,V$ to set the current direction because its the biggest potential hill.

so we guess counterclockwise current

Just one loop and so Kirchhoff's current law trivially tells us that there just one current I.

Going around the loop counterclockwise

$$\mathcal{E}_2 - IR_1 - \mathcal{E}_1 - IR_2 = 0$$

Solve for I which is our unknown.

$$I = \frac{\mathcal{E}_2 - \mathcal{E}_1}{R_1 + R_2} = \frac{12-6}{8+10}$$

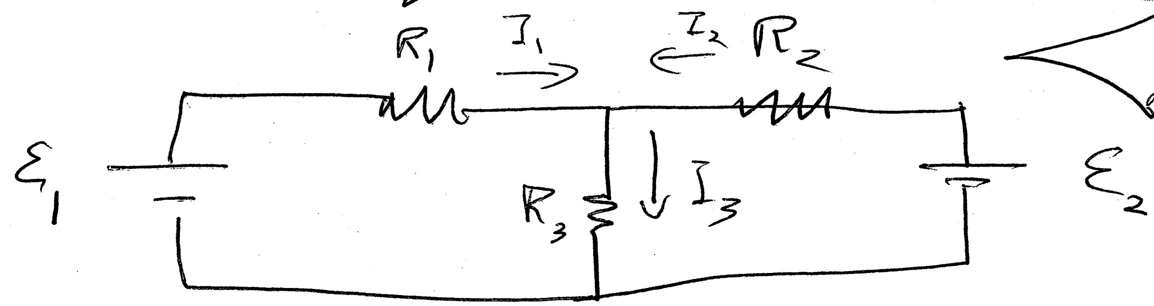
$$= \frac{6}{18} = \frac{1}{3}\,A$$

SJ-787 got $-\frac{1}{3}\,A$, but they

28-34

defined the other direction around the loop as positive.

E 28.7 generalized



I've made a current direction choice which seems obviously right unless an emf is reversed

We are given $\mathcal{E}_1$, $\mathcal{E}_2$, R_1, R_2, R_3 which ~~in practice is the usual case~~ may or not be the case in practice.

— after all in designing a circuit you may know the current you want and may need to find the emf's or resistors to give it.

But finding the current ~~and resistors~~ is sort of the straightforward case.

3 unknowns I_1, I_2, I_3.

— we have ~~4~~ 5 possible linear equations in the 3 unknowns. { linear because ~~no~~ I always occurs linearly

3 ~~loops~~ equations ~~or~~
and 2 node equations

but actually only 3 independent equations

$I_1 + I_2 = I_3$ and $I_3 = I_1 + I_2$ are trivially the same.

— proving the 3rd loop equation is ~~the same~~ redundant gives no new constraint is a bit

28-36

trickier and I won't bother to show that.

I think it's true for any linear circuit problem, given all devices one can solve for all currents in all distinct branches and one has just enough independent equations to do that → but the general proof is beyond our scope and my knowledge.

In solving such problems is best to take the most symmetrical equations. — usually that leads to simple expressions.

So

$$I_1 + I_2 = I_3$$

$$\mathcal{E}_1 = I_1 R_1 + I_3 R_3$$

$$\mathcal{E}_2 = I_2 R_2 + I_3 R_3$$

Experience (but also insight) says let's solve for I_3 first. ~~since~~ The insight is ~~that~~ it appears "unsymmetrically"

$$I_1 = \frac{\mathcal{E}_1 - I_3 R_3}{R_1}$$

$$I_2 = \frac{\mathcal{E}_2 - I_3 R_3}{R_2}$$

$$\therefore \quad \frac{\mathcal{E}_1}{R_1} + \frac{\mathcal{E}_2}{R_2} = I_3 \left(1 + \frac{R_3}{R_1} + \frac{R_3}{R_2} \right)$$

$$I_3 = \frac{\mathcal{E}_1/R_1 + \mathcal{E}_2/R_2}{1 + R_3/R_1 + R_3/R_2} = \frac{\mathcal{E}_1 R_2 + \mathcal{E}_2 R_1}{R_1 R_2 + R_3 R_2 + R_3 R_1}$$

28-38

$$I_1 = \frac{\varepsilon_1(R_1R_2 + R_1R_3 + R_2R_3) - \varepsilon_1 R_2 R_3 - \varepsilon_2 R_1 R_3}{R_1(R_1R_2 + R_1R_3 + R_2R_3)}$$

$$= \frac{\varepsilon_1(R_2 + R_3) - \varepsilon_2 R_3}{R_1R_2 + R_1R_3 + R_2R_3}$$

by symmetry, just ~~in~~interchanging 1 & 2 labels

$$I_2 = \frac{\varepsilon_2(R_1 + R_3) - \varepsilon_1 R_3}{R_1R_2 + R_1R_3 + R_2R_3}$$

~~B~~Special Cases

a) — the actual SJ-788 case

~~$\varepsilon_1 = 10V,\ R_1 = 6$~~,

$\varepsilon_1 = -10$, $R_1 = 6$

$\varepsilon_2 = 14$, $R_2 = 4$

$R_3 = 2$

$$I_1 = \frac{-10(6) - 14 \cdot 2}{24 + 12 + 8}$$

$$= \frac{-60 - 28}{44} = -\frac{88}{44} = -2\,A$$

but that okay since
my fiducial current direction
was opposite SJ-788's
($I_1 = 2A$)

$$I_2 = \frac{14(8) - (-10) \cdot 2}{44}$$

$$= \frac{112 + 20}{44} = 3\,A$$

SJ-788 got $I_2 = -3\,A$
but they set
their fiducial
current direction
opposite

$$I_3 = I_1 + I_2$$
$$= -2 + 3 = 1\,A$$

SJ-788 got +1A of course

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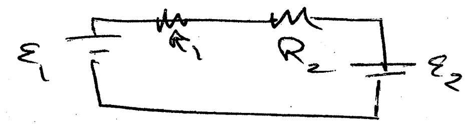
b) Say $R_3 \to \infty$

So ~~that~~ our circuit reduces to 1 loop really.



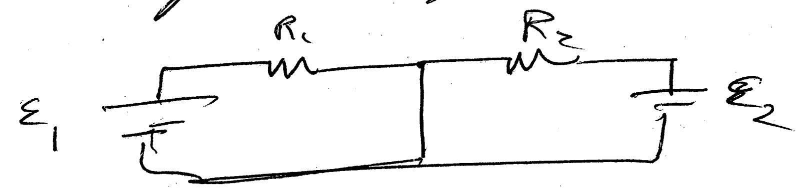
$$I_1 = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1 + R_2}$$

Just ~~the~~ result of p. 28-33

$$I_2 = \frac{\mathcal{E}_2 - \mathcal{E}_1}{R_1 + R_2} = -I_1$$

$$I_3 = 0$$

c) Say $R_3 \to 0$



In this case it's like 2 independent loops almost except $I_1 + I_2 = I_3$

$$I_1 = \frac{\mathcal{E}_1 R_2}{R_1 R_2} = \frac{\mathcal{E}_1}{R_1}$$

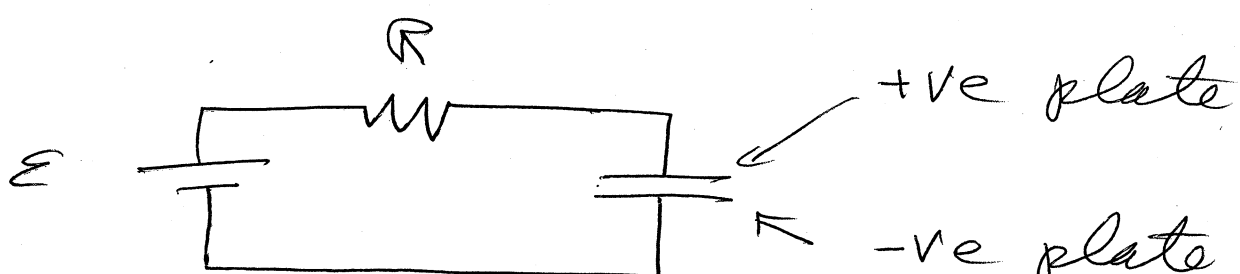
$$I_2 = \frac{\mathcal{E}_2}{R_2}$$

§28.4 RC Circuits

Now we put a capacitor in a circuit.

— This will give us <u>NOT</u> a linear equation for current.
but a <u>differential equation</u>
(a DE)

<u>~~Case~~ 1</u> <u>Charging & Discharging a Capacitor</u>



Using Kirchhoff voltage law

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Recall $C = \frac{q}{V}$

$\therefore \quad V = \frac{q}{C}$

$$\underbrace{\mathcal{E}}_{\substack{\text{potential rise} \\ \text{across emf}}} = \underbrace{IR}_{\substack{\text{potential drop} \\ \text{across resistor}}} + \underbrace{\frac{q}{C}}_{\substack{\text{potential drop} \\ \text{across the capacitor}}}$$

potential rise ~~acoss~~ across emf

potential drop across resistor.

potential drop across the capacitor

Note $C \to \infty$ is the no capacitor case. No capacitor is infinite capacitance

q will rise as current flows into capacitor.

— +ve in one side

— -ve out the other.

— Kirchhoff's voltage law can still be used since the time variation turns out to be slow enough.

(at any one instant a quasi electrostatic potential ~~loop exists~~ distribution exists.

~~So~~ If one just
has an open circuit
and and uncharged
capacitor $q = 0$

Close the circuit at $t=0$, $q=0$

$I = \frac{\mathcal{E}}{R}$

we assume the current starts instantaneously. Nearly true since the E-field changes at nearly the speed of light times.

and $q = \int_0^t I\,dt'$

as time passes.

and when $\frac{q}{C} = \mathcal{E}$ then $I = 0$

for the Kirchhoff voltage law to hold.

So we know
endpoints and
just need to find
how I and q
vary in time.

I prefer to differential
equation $\mathcal{E} = IR + \frac{q}{C}$

28-44

This equation is valid for $\mathcal{E} \neq 0$ ~~and~~ which would charging and $\mathcal{E} = 0$ which is discharging

$$\therefore \quad 0 = \frac{d\cancel{I}}{dt} R + \frac{I}{C}$$

$$\frac{dq}{dt} = \int_0^t I\,dt' = \frac{d\left(q(t) - q(0)\right)}{dt} = I$$

This is a famous DE with a simple solution for I

$$\therefore \quad \frac{dI}{dt} = -\frac{I}{RC} = -\frac{I}{\tau}$$

We define $\tau = RC \Rightarrow \frac{V}{A}\frac{C}{V} = s$

τ is the time constant or I prefer to call it the e-folding time for reasons soon to be seen.

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Case 1 In this case

at $t = 0$, $q = 0$

and Kirchhoff's voltage law gives $\mathcal{E} = I_0 R$

or $I_0 = \frac{\mathcal{E}}{R}$

Now at $t = \infty$, the capacitor should be fully charged and $I = 0$

$\therefore$ Kirchhoff's voltage law gives $\mathcal{E} = \frac{q_\infty}{C}$

or $q_\infty = C\mathcal{E}$

So we have $I = \frac{\mathcal{E}}{R} e^{-t/\tau}$

$$q = -\frac{\mathcal{E}}{R} RC e^{-t/\tau} + C\mathcal{E}$$

$$= C\mathcal{E}(1 - e^{-t/\tau})$$

$$\int \frac{1}{I}\frac{dI}{dt}\,dt = -\int \frac{1}{\tau}\,dt$$

indefinite integral

$$\ln I = -\frac{t}{\tau} + \text{Constant}$$

$$e^{\ln I} = e^{-t/\tau + \text{Constant}}$$

$$I = I_0 e^{-t/\tau}$$

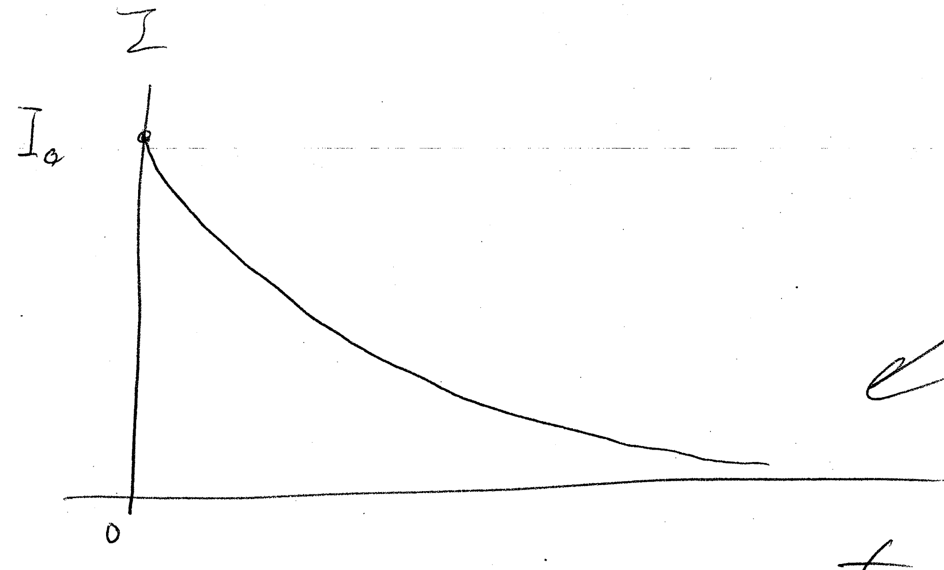
where I_0 is set by the initial condition at $t = 0$.

To get the charge ~~accumulation~~ on the capacitor we need to integrate again indefinitely.

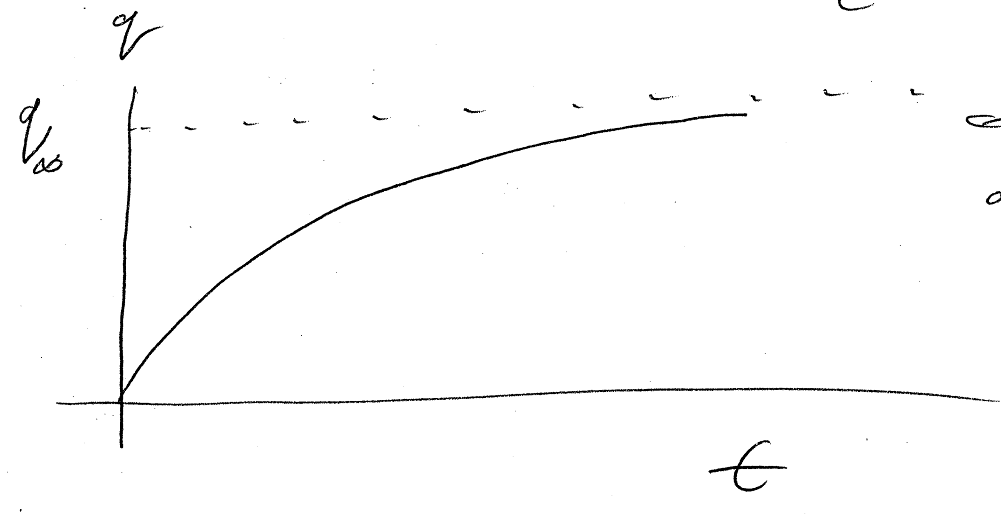
$$q = -I_0 \tau e^{-t/\tau} + q_\infty$$

where q_∞ is $q(t=\infty)$.

$$V_c = \frac{q}{C} = \mathcal{E}\left(1 - e^{-t/\tau}\right)$$



asymptotically approaches zero



asymptotically approaches q_∞

Formally the ~~cap cha~~ capacitor never gets to q_∞ and the current never drops to zero.

28–48

But practically in a finite time, the system ~~approaches~~ is closer to the final values than one's measuring error or than the size of perturbation affecting the system.

– generally a few e-foldings times.

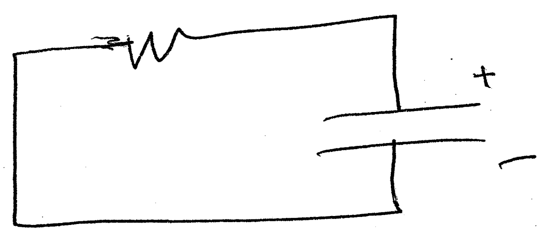
Note $e^{-t/\tau} = 10^{(\log e)(-t/\tau)}$

$$\cong 10^{.434(-t/\tau)} \begin{cases} \sim 10^{-4.3} & \text{for } t = 10\tau \\ 10^{-43} & \text{for } t = 100\tau \end{cases}$$

– relative differences of 10^{-43} are immeasurably small in almost any context one can imagine.

Case 2 Discharging capacitor

from ~~q_0~~ $q_0 = C\mathcal{E}$ ~~of the charging phase~~



The emf has be removed from the circuit.

In this case ~~the~~ Kirchoff's voltage law gives

$$0 = IR + \frac{q}{C}$$

and so $I < 0$ which

28-50

our conventions means the current flows out of the capacitor.

$$I_0 = \frac{-q_0}{RC} = -\frac{q_0}{\tau}$$

$$\therefore \quad I = -\frac{q_0}{\tau} e^{-t/\tau}$$

$$P = I^2 R = \frac{q_0^2}{\tau^2} R e^{-2t/\tau}$$

Power output to resistor.

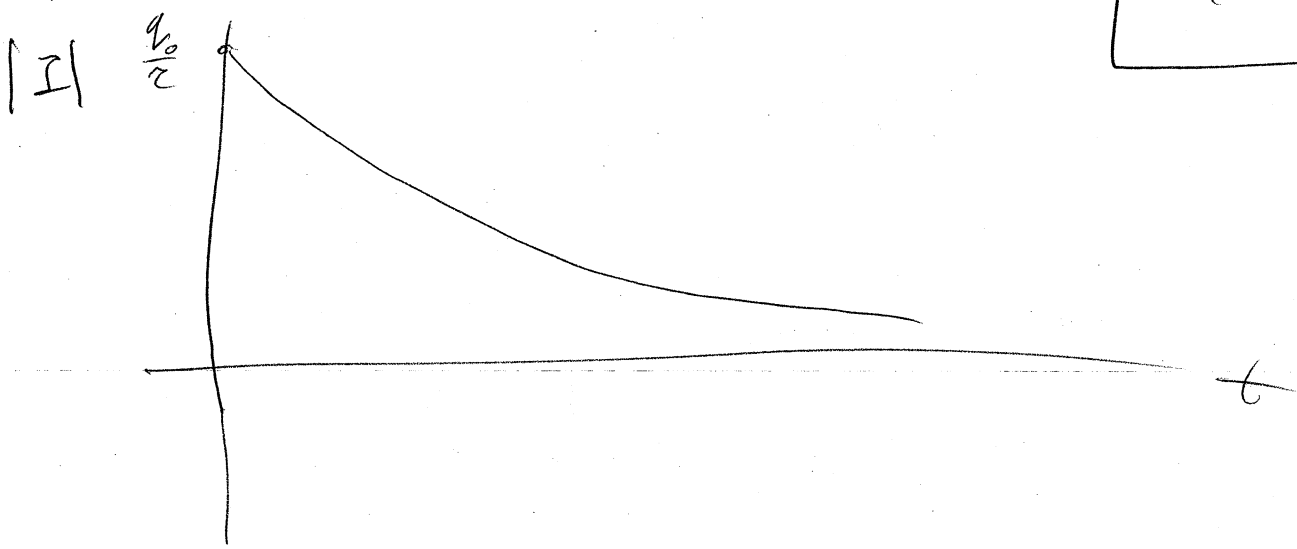
See p. 28-45

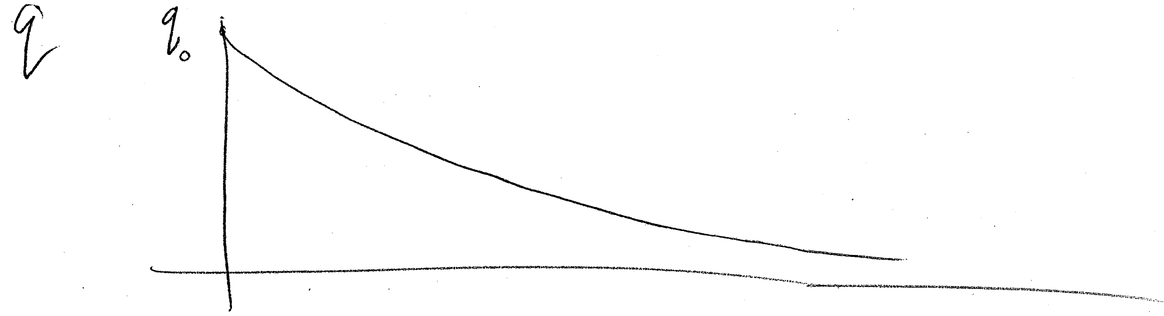
$$q_\infty = 0$$ when capacitor is fully discharged which must happen because of the current

$$\therefore \quad q = -I_0 \tau e^{-t/\tau}$$ from p. 28-45

$$= q_0 e^{-t/\tau}$$

If we assume we charged up using emf $\mathcal{E}$, then $q_0 = C\mathcal{E}$.





Again formally the capacitor is never fully discharged and current never goes to zero.

But practically in a few e-folding times the final values are reached effectively.

Our RC circuits are used for

28-52

for charging up capacitors and using them — Not necessarily with the same circuit they were charged with.

In fact a charged capacitor can be used as a source of electrical energy.

It stores $PE = \frac{1}{2}CV^2$

$= \frac{1}{2}\frac{q^2}{C}$

recall.

~~It is usual~~

Capacitors are not usually considered emf's because they do not

Maintain a constant potential across their terminal as they discharge

$$V = \frac{q}{c} \text{ recall.}$$

But you can store a lot of energy in capacitors and if the time ~~of the~~ ~~the~~ distribution of the release is not important or exponential release ~~is time~~ of energy is OK then capacitors can be used.

e.g., 1) ~~Some~~ camera flashes are powered by capacitors. (WP-685 -626)

2) Super powerful experimental lasers.

the now defunct NOVA laser

28-54) at LLNL

~~use~~ stored energy in a $C = 2.3$ F capacitor bank for emitting ~~150 kJ~~ 15 kJ pulses in .1 ns

(WP-626)

$$P_{ave} = \frac{15 \times 10^3}{10^{-9}}$$

$$= 15 \times 10^{12} \text{ W}$$

$$= 15 \text{ TW}$$

which is just about the average commercial power consumption of the whole world (Wik)

A capacitor ~~can~~ among other things is like a toilet cistern, ~~you~~ where you store up water slowly and then release it in burst (flush) of unsteady flow

(WP-625)

25 – 85

Of course, capacitors have lots of other uses — e.g., in tuning for electronic signal reception but I'm not terribly knowledgeable.

§ 28.5 Electrical Meters

— optional reading

§ 28.6 Electrical Safety

— optional reading.

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