

# Chapter 27

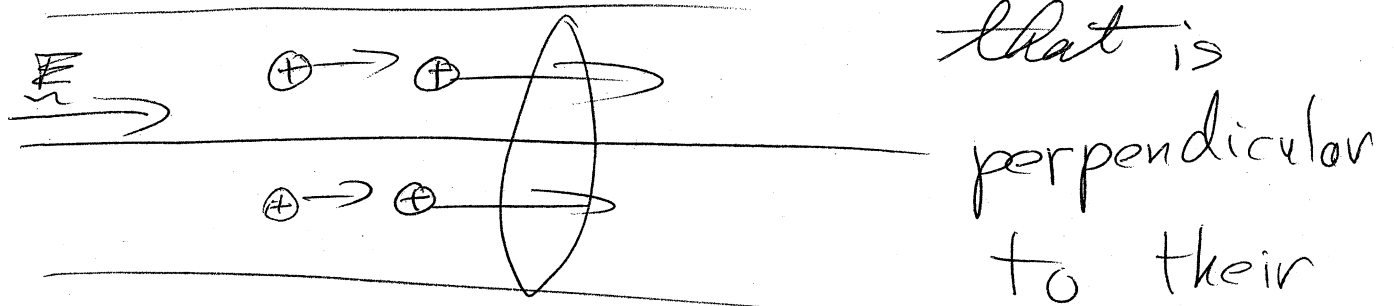
27-1

## Current & Resistance

### § 27.1 Electric Current

— Electric current is a flow of charge.

— Say one has +ve charge carriers moving thru an area  $A$



direction of motion

In time  $\Delta t$ ,  $\Delta Q$   
of charge has

27-2)

moved three A.

The average current in  $\Delta t$   
is

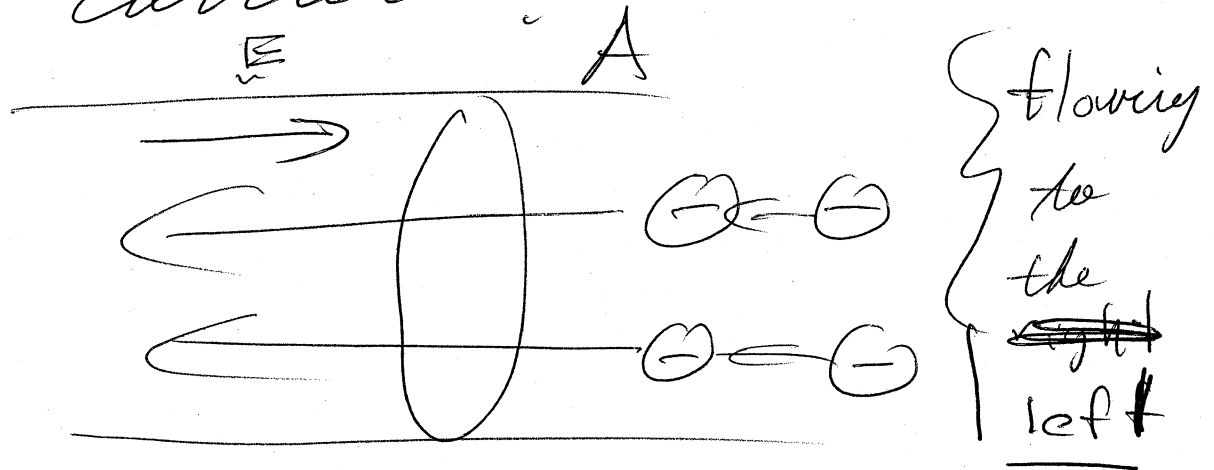
$$I_{ave} = \frac{\Delta Q}{\Delta t} \quad \left\{ \begin{array}{l} \text{to the} \\ \text{right} \\ \text{in our} \\ \text{diagram} \end{array} \right.$$

The instantaneous current  
is

$$I = \lim_{\Delta t \rightarrow 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$

Actually in our development  
we will usually be  
thinking of ~~Q~~ constant  
currents where  $I = I_{ave}$

What of negative carriers?



Well we count this as a positive current

to the right.

Most of the time we are thinking of electrons in metals — and so the carriers are ~~positive~~ really negative.

27-4)

Conventionally

But ~~conventionally~~ current is taken

as flowing from +ve to -ve

and electrons flow the other way.

A convention that traces back to Ben Franklin's bad guess.

So in conventional circuit problems we treat the current as flowing

Perhaps no matter what current is taken as flowing indicates of +ve carriers and opposite direction of -ve carriers. Usually then

Conventional current flows in the direction of the electric field. If +ve the carriers go that way. If -ve, they go

in the opposite direction.

opposite to the [27-5]  
way electrons do  
flow.

We get away with this  
because in almost all  
ordinary circuitry nothing  
distinguishes +ve carriers  
one way  
to -ve carriers the  
other

It wasn't until the Hall effect  
in 1879, that people figured  
out that metal carriers were  
negative — and by that  
time it was too late  
to change conventions.

27-6 ] We may (or  
may not) look at  
the Hall effect later.

MKS unit of

$$\text{Current is } 1 \text{ A} = \frac{1 \text{ C}}{1 \text{ s}}$$

— A for ampere.

— Now 1 C <sup>of net</sup> is a pretty  
big net charge.

— But 1 A is not such  
a big current.

The thing to remember is  
that current carrying conductors  
are mostly neutral.

e.g., in a metal,

the electrons are moving  
through positively charged  
lattice of ions that are fixed.

— Wires carrying current  
are close to neutral  
overall.  $\Rightarrow$  But not  
quite actually  
in general.

Other carriers?

— An important  
point as well  
see.

— ions in gases.

— ions in electrolytes

$\hookrightarrow$  generally fluids  
with a solute that  
yields ions.

~~of~~ one has these  
in wet cells = chemical  
batteries

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We are going to skirt mostly what actually goes on in a chemical battery and mainly deal with electrical power sources in a generic way.

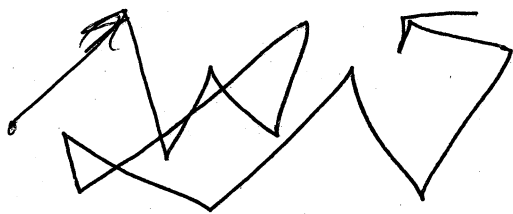
- in semiconductor one has electrons & holes / propagate  
Microscopic Model

of Conduction

→ absence of electrons + the carriers.

→ depending on doping of semiconductor

- thinking of electrons really / conduction.



One can think of the electron ~~as something~~ as like a gas



They fly around a  
collide with ions  
and impurities and defects,

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But they are not a  
classical gas — like  
molecules in the air,  
but a Fermi gas.

ideal  
gas

The whole subject of what the collisions are is complex and inherently Quantum mechanical so we'll leave just skit that subject.

Going into details is beyond  
our scope

typical velocities  $\sim 1 \times 10^6$  m/s (AM-36)  
<sup>v<sub>random</sub></sup>

— fast, very fast  
but still NOT

relativistic.

$$c = 3 \times 10^8 \text{ m/s}$$

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$$\text{So } \beta \approx \frac{1}{3} \times 10^{-2}$$

$\hookrightarrow \frac{v_r}{c}$  which is a measure of relativistic effect.

But these  $v_{\text{random}}$  are ~~not~~ random <sup>velocities</sup>, not the net velocity of ~~the~~ current.

A macroscopic  $\mathbf{E}$ -field superimposes an average velocity in some direction

(we go into how in §27.2 on Ohm's law).

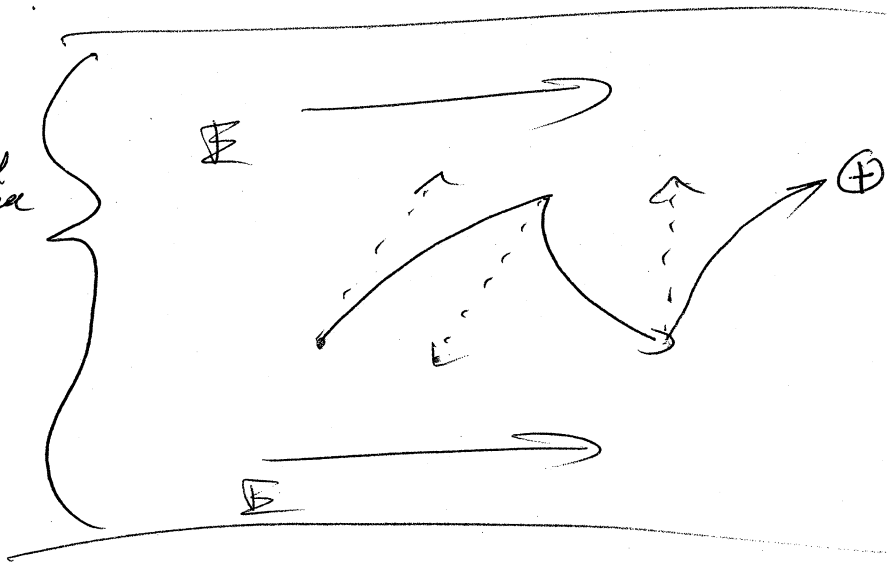
$\hookrightarrow$  The Drift velocity  $v_d$

typically

$$N_d \ll N_r$$

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Cross  
sectional  
area  
A



After each collision the charge carrier heads in a random direction, but the  $E$ -field imposes a motion in the  $E$ -field direction for a +ve charge.

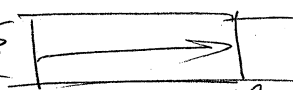
Opposite  
 $E$ -field  
direction  
for a -ve charge.

So if the charge density is  $n$ , drift velocity  $N_d$

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$$\Delta Q = n \Delta V q = n A \Delta x$$

Cross  
sectional  
area  
perpendicular  
to flow.



by drift  $v_d$

Ex 27.1

$\Delta x$  all cleared out in  $\Delta t$

$$\therefore v_d = \frac{\Delta x}{\Delta t}, \quad I = nqAv_d$$

Drift velocity in a copper wire

I prefer fiducial numbers instead of the textbook's numbers

Say  $A = 10^{-6} \text{ m}^2$

I don't want to get into wire gauge

Cross-sectional area of wire

$$d = 2\sqrt{\frac{A}{\pi}} \quad \text{is diameter}$$

$$\approx 2\sqrt{\frac{10^{-6}}{3}} \approx 1.1 \times 10^{-3} \text{ m}$$

Current  $I = 1 \text{ A}$

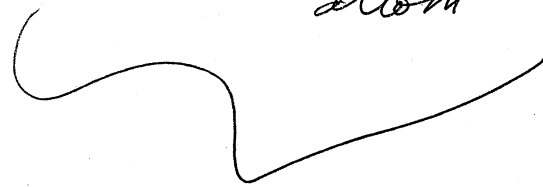
$$\therefore v_d = \frac{I}{nqA}$$

$q = -e$  for  
electrons

$n$  for copper?

$$n = \frac{\rho \text{ (density of copper)}}{M}$$

$M$   
Mass of copper  
atom



number of  
atoms per  
unit volume

$Z$  free  
Number  
of  
free  
electrons  
per  
atom  
= 1 for  
Copper  
(wik)

Atomic weight

$$M_{Cu} = A m_{amu} \quad \left\{ \begin{array}{l} \text{Atomic mass} \\ \text{unit} \end{array} \right.$$

$$\approx 63.5 \cdot 1.66 \times 10^{-27} \text{ kg}$$

$$\begin{aligned} \rho_{Cu} &= 8.96 \text{ ~~g/cm}^3 \text{ (wik at} \\ &= 8.96 \times 10^3 \text{ kg/m}^3 \text{ } \end{aligned}~~$$

(wik at  
~ room temperature)

27-14)

$$\cancel{1 \frac{\text{kg}}{\text{m}^3}} = \cancel{1 \frac{\text{kg}}{\text{m}^3}} \cdot \left( \frac{1 \text{ kg}}{1000 \text{ g}} \right) \left( \frac{100 \text{ cm}}{1 \text{ m}} \right)^3$$

$$1 \frac{\text{g}}{\text{cm}^3} = 1 \frac{\text{g}}{\text{cm}^3} \cdot \left( \frac{1 \text{ kg}}{1000 \text{ g}} \right) \left( \frac{100 \text{ cm}}{1 \text{ m}} \right)^3$$

$$= 10^3 \frac{\text{kg}}{\text{m}^3}$$

$$So \ n \approx \frac{8.96 \times 10^3}{63.5 \cdot 1.66 \times 10^{-27}} \cdot 1$$

$$= 9 \times 10^{28} \text{ electrons/m}^3$$

$$v_d = \frac{1 \cdot \text{A}}{9 \times 10^{28} \text{ m}^{-3} \cdot 1.6 \times 10^{-19} \text{ C} \cdot 10^{-6} \text{ m}^2} \left( \frac{1}{1 \text{ A}_{\text{amp}}} \right) \left( \frac{10^{-6} \text{ m}^2}{\text{A}_{\text{area}}} \right)$$

$$\approx .6 \times 10^{-4} \left( \frac{1}{1 \text{ A}} \right) \left( \frac{10^{-6} \text{ m}^2}{\text{A}} \right) \frac{\text{m}}{\text{s}}$$

$$\approx .06 \frac{\text{mm}}{\text{s}} \left( \frac{1}{1 \text{ A}_{\text{amp}}} \right) \left( \frac{10^{-6} \text{ m}^2}{\text{A}_{\text{area}}} \right)$$

- dropping the negative sign.

So typical drift

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velocities are quite small.

— less than 1 millimeter/s.

But when you turn on an electrical device (i.e., close a circuit)

the device starts working quasi-instantly.

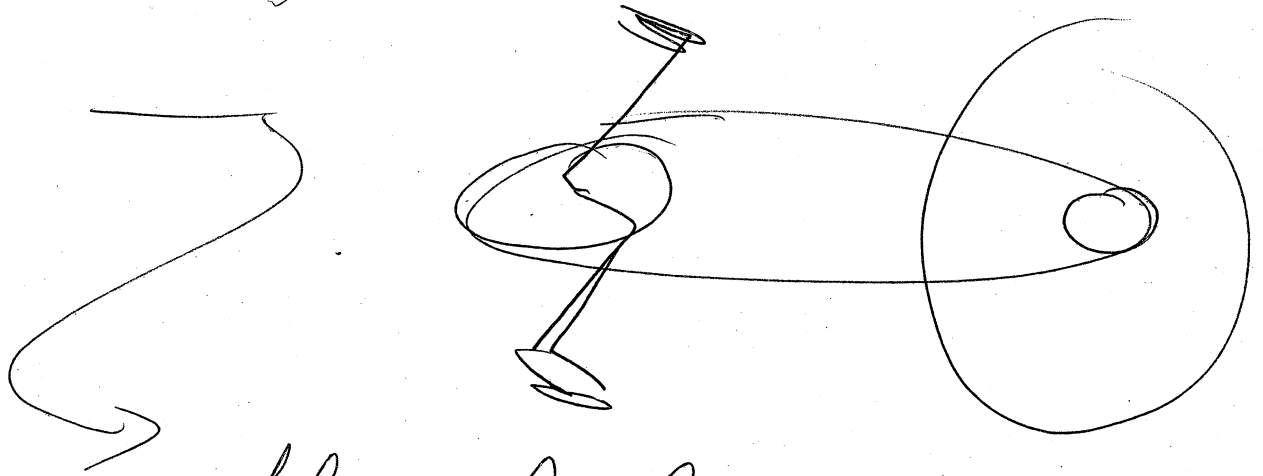
— The electrons are already in the conductors.

↳ an electric field signal traveling at near the speed of light sets them in motion.

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Couple of analogies

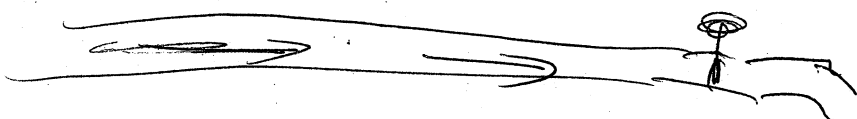
a) — starting to pedal a bike.



the wheel starts moving as soon as a tension signal in the chain reaches it.

— not the time it takes a chain link to move from pedals to wheel.

b) Opening a tap





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Once you ~~open~~  
open a tap, water  
all along the pipe  
starts in motion — not  
on the time scale it  
takes water molecules to  
move along the pipe,  
but on the time scale  
that the sound speed  
of water takes to signal  
along the pipe that  
there's been a drop  
in pressure at the open  
tap.

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## § 27.2 Resistance

### of Ohm's law

Ohm's law is NOT  
a fundamental law of  
nature.

— It's just a law that  
many materials obey — and  
those materials have many  
important technological  
uses.

→ They are called  
ohmic materials.

To Understand

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Ohm's law we need to define current density.

Recall  $I = nq v_d A$

↑  
cross sectional area perpendicular to current flow.

$$J = \frac{I}{A} = nq v_d$$

— has units of  $A/m^2$

Actually average  $J$  over  $A$  but we will not usually consider non-uniform current density

In current density form of Ohm's law is

$$J = \sigma E$$

where the direction of  $E$  and  $J$  are the same.

in vector form  $\underline{J} = \sigma \underline{E}$

27-20 } units  $[\sigma] = \left[ \frac{J}{E} \right] = \left[ \frac{(C/s)/m^2}{J/(mC)} \right]$   
 $= \frac{C^2/m}{Js}$  but

and  $\sigma$  is conductivity

(not surface charge density — we have to recycle the symbol — context has to decide which you mean)

~~units~~  
 $= \frac{1}{\Omega m}$   
 $\uparrow$   
 $\Omega m$   
 or  
 well  
 see  
 on  
 p. 27-28

and  $\sigma$  is independent of  $E$  itself.

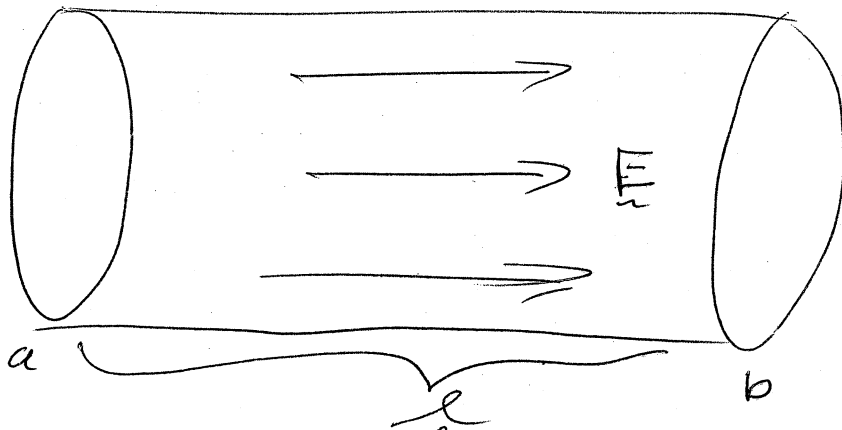
Such materials that have this property are ohmic.

Special case

say we have  $J$  constant over the cross sectional area  $A$  of a conductor and points normal to  $A$

## Important Special Case

Say we have a conductor with constant cross sectional shape & area  $A$ .



- the shape need not be circular.
- there is a uniform  $E$ -field normal to the cross section
- the material is ohmic and so

$$\underline{J} = \sigma \underline{E}$$

It is also homogeneous and so

27-22

$\sigma$  is constant.

$\underline{J}$  must be constant and perpendicular to the cross section (by Ohm's law).

Now

$$\underline{J} = \sigma \underline{E}$$

↙

$$\frac{J}{A} \hat{E} = \sigma \underline{E}$$

Since  $\underline{J}$  is uniform  $\underline{J} = \frac{J}{A} \hat{E}$

Now ~~multiply by~~ dot product with  $d\underline{\Sigma}$  where  $\underline{\Sigma}$  is a path length vector normal to the cross section and integrate from a to b.

$$\int_{S_a}^{S_b} \frac{I}{A} \hat{E} \cdot d\vec{s} = \int_{S_a}^{S_b} \sigma \vec{E} \cdot d\vec{s}$$

~~or~~ 
$$\frac{I}{A} l = \sigma (-V_{ab})$$

Recall

$$\Delta V = - \int \vec{E} \cdot d\vec{s}$$

or

$$V_{ba} = \frac{l}{\sigma A} I = \rho \frac{l}{A} I = IR$$

Where  $V_{ba} = -V_{ab}$ ,

$\rho = \frac{1}{\sigma}$ , is the resistivity

and we define

All ohmic conductors have resistance.  
 — devices designed to have resistance do so intentionally

$$R = \rho \frac{l}{A} \text{ as the } \underline{\text{resistance}}.$$

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So dropping the  
subscripts ~~to~~  $ba$ , we  
have

$$V = IR$$

which is the usual form  
of Ohm's law.

The potential drop across  
a resistance equals  
the current thru it times  
a constant that is independent  
of current and potential,  
but that depends on the  
material and geometry  
of the resistance.

Proper  
way to  
read it  
is that  
the electric  
force  
must inject  
energy per  
unit charge  $V$   
to make up for  
the energy lost to  
waste heat per unit  
charge  $IR$  and

then maintain  
a  
constant  
charge  
flow.



27-25

Our derivation was for a particular geometry, but result is actually general — but the formula for  $R$  is in general complex and beyond our scope to go into.

$$V = IR$$

for ohmic materials.

— Typical one measures it or the manufacturer tells you.

Units

$$[R] = \left[ \frac{V}{I} \right] = \frac{V}{A}$$

$$= \frac{J/C}{C/s} = \frac{J \cdot s}{C^2}$$

$$= \Omega \quad \left\{ \begin{array}{l} \text{Capital Greek} \\ \text{omega for} \\ \text{ohm} \end{array} \right.$$

27-26

So the unit of resistance does have a special name (ohm) and sign  $\Omega$ .

As well as ohms, kilo-ohms ( $k\Omega$ ) are pretty commonly in use too.

$$\begin{aligned} \frac{\text{Units}}{\text{of Resistivity}} \quad [\rho] &= \left[ \frac{A}{l} \Omega \right] \\ &= \Omega \cdot m \end{aligned}$$

Some examples  
from the text

# Resistivity Table

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Material  $\rho$  ( $\Omega\text{-m}$ )

Temperature  
Coef.  $\alpha$

$^{\circ}\text{C}^{-1}$   
(inverse  
Celsius  
degree)  
discuss  
in  
atit

- bit expensive	Silver	$1.59 \times 10^{-8}$	$3.8 \times 10^{-3}$
- we're running out of cheap copper	Copper (a black market in CA)	$1.7 \times 10^{-8}$	$3.9 \times 10^{-3}$
- expensive but corrosion free	Gold	$2.44 \times 10^{-8}$	$3.7 \times 10^{-3}$
not as good as copper but lots of it (more of it - very easy <sup>intensive</sup> to produce)	Aluminum	$2.82 \times 10^{-8}$	$3.9 \times 10^{-3}$
- <del>Transit</del> Tungsten = 3422°C and so used as filament in incandescent lights	Tungsten	$5.6 \times 10^{-8}$	
	Iron	$10 \times 10^{-8}$	$5.0 \times 10^{-3}$
It's magnetic properties make it useful			
	Nichrome	$1.5 \times 10^{-6}$	$0.4 \times 10^{-3}$
Poor conductor so good in heating elements. (Nickel(-chromium alloy))			

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Carbon  $3.5 \times 10^{-5}$

$-.5 \times 10^{-3}$

used  
in resistors  
(cheaper than pure resistors)

Silicon  $2.3 \times 10^3$

$-75 \times 10^{-3}$

Not  
a great  
conductor  
- semi-conductor

Glass  $10^{10}$  to  $10^{14}$

an insulator

- but a some  
upon low level  
~~there~~ a current will flow  
even without break down

best ~~resistor of~~ <sup>18</sup>  
18 orders of mag. higher resistances  
than copper at leads

- in some cases like wires

one wants good conductors

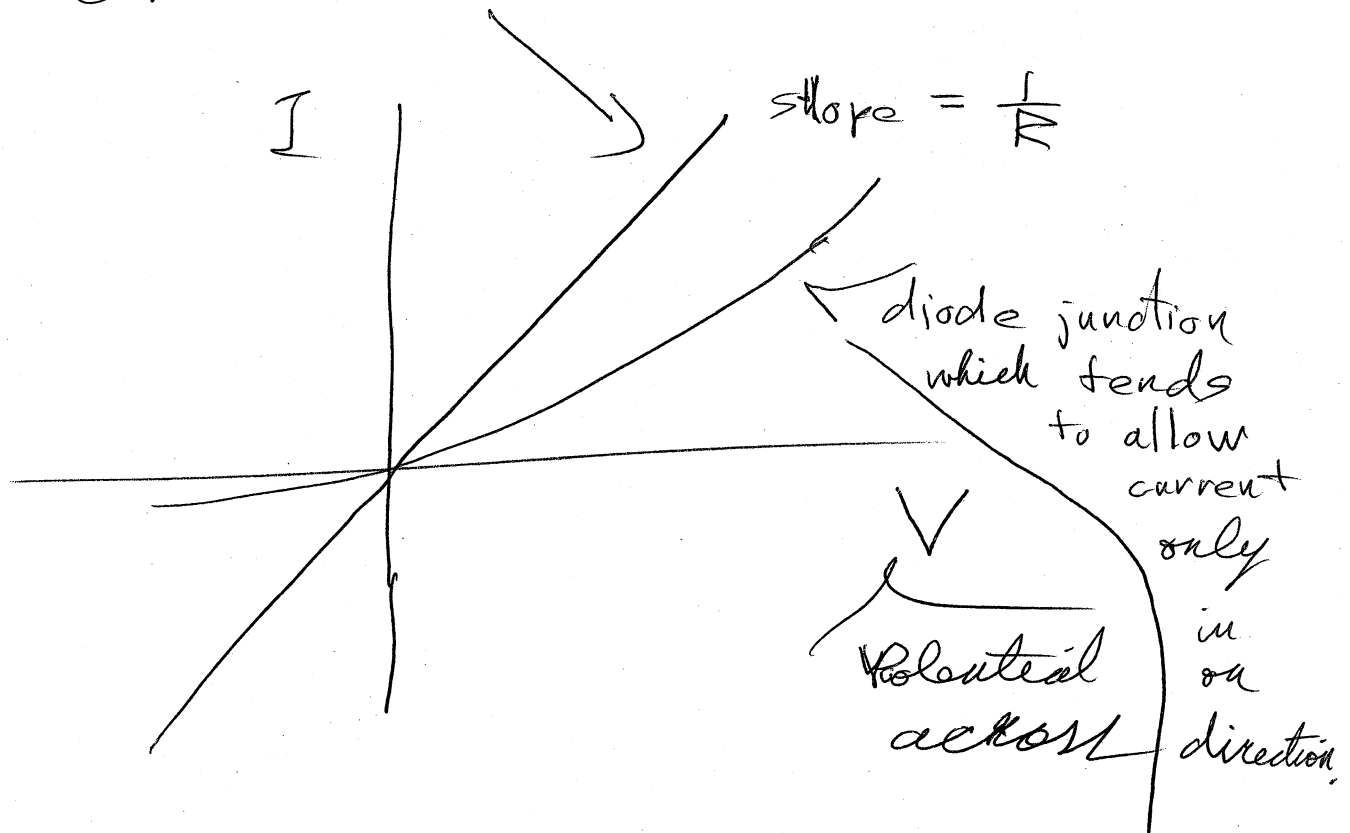
- low resistivity

- Cu, Al.

- Semi conductors have  
special properties useful in electronics

27-30

Ohmic material



## § 27.3 Model of Conduction

to Prove Ohm's Law

— This is the classical  
Drude model from 1900  
that uses classical  
physics. — some things  
are right

— some things wrong. [27-31]

— to go beyond you need Quantum Mechanical ideas.

— But the Drude model gives some insight.

Recall from p. 27-19

We ~~Proving~~ Ohm's law in the Drude model.

and

Ohm's law for current density is

$$J = n q v_d$$

both  $J$  &  $v_d$  in the same direction for  $q > 0$

$$J = \sigma E$$

both  $J$  &  $E$  in same direction for  $q > 0$

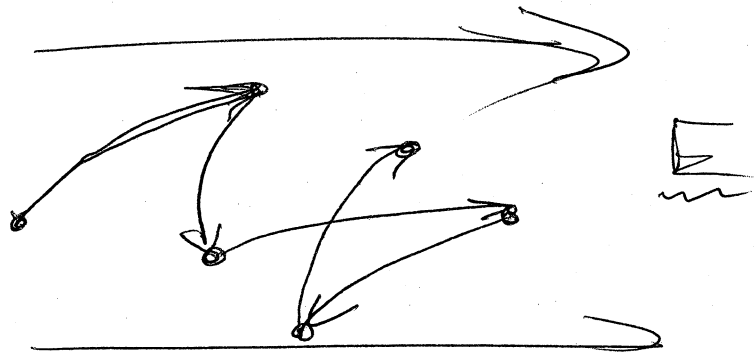
27-32

Let the  
charge carrier  
be  
+ve or  
-ve.

Let us think of +ve  
charge for definiteness  
even though in metals  
 $q = -e$

Recall our drift velocity picture

randomizing  
collisional  
impacts.



between impacts and  
superimposed on the  
random thermal velocity  
(really high  $\sim 10^6$  m/s),  
there is the velocity  
caused by acceleration  
due to the



macroscopic electric  
field.

27-33

We assume the field is uniform  
and points to the right

Both  
 $F$   
and  
 $a$   
are  
constants.

$$F = qE$$

Force  
on charge  
carrier

$$a = \frac{qE}{m}$$

is the acceleration  
of the charge  
carrier  
divided  
by mass  $m$

If  $q > 0$ ,

$a > 0$  and

the charge carrier accelerates  
to the right.

If  $q < 0$ ,  $a < 0$  and the  
carrier accelerates to the  
left.

After a collision the carrier's  
initial non-random velocity = 0

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Let  $\tau$  be the mean time between collisions

$\therefore$  the mean velocity at the end of between collisions flight or step is

$$v = a \tau$$

Serway - 761 asserts

without proof (SI just use "because" like a parent)

that  $v = a \tau$  is also the average ~~car~~ charge-carrier velocity.

$$\therefore v_{\text{drift}} = v = a \tau$$

$$\therefore J = nq v_d = nq a \tau = \frac{nq^2 E \tau}{m}$$

See  
p. 27-37  
for  
proof.

Note the  $q^2$ ,

27-35

This means that  $J$  always points in the direction of  $E$  no matter what the <sup>charge</sup> carrier sign. (In this model of conduction)

Reason

If  $q > 0$ , the  $E$ -field gives  $v_d > 0$  and the charge flow right — we call this a positive current.

If  $q < 0$ , the  $E$ -field gives  $v_d < 0$  and charge flows to the left — but it's negative charge and so we call this a positive current.

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As we've mentioned earlier, physically for many (but not all) purposes, it doesn't matter ~~which way~~ the what the sign of the carriers — we take current as flowing in the  $E$ -field direction.

P. 27-36-72-95.

Optional Reading

{ Beyond the scope of the ~~class~~ course and so only for your interest.

Proof that  $N_d = a \tau$

where  $a$  is the constant non-random acceleration caused by ~~an~~ uniform  $E$ -field.

And  $\tau$  is the mean time

between collisions.

Assume collisions happen randomly, but with a constant probability of happening per unit time

$\rightarrow \frac{1}{\tau}$  where  $\tau$  is a time constant (that turns out to be the mean time between collisions)

Say ~~there is a collision~~ we observe the carrier at ~~and that sets~~  $t = 0$  and it is not colliding

What is the probability that it has not collided up to time  $t$ ?

at that instant.

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Say it is  $P(t)$

Now in  $dt$ ,  $P(t)$  decreases

$$\text{by } dP = -P(t) \frac{dt}{\tau}$$

probability  
of not  
colliding  
to  $t$

Probability  
of  
colliding  
in  $dt$

You could  
think of this  
as  $n$  survivors  
out of  $N$

$$dn = -n \frac{dt}{\tau}$$

$$n(t + \epsilon)$$

$$= n(t)$$

$$- dn$$

$$= n(t) - n(t) \frac{dt}{\tau}$$

the  
number  
surviving after  $t + \tau$ .

This is a  
trivial differential  
equation

$$\frac{dP}{P} = - \frac{dt}{\tau}$$

$$\ln P = -\frac{t}{\tau} + C$$

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$$P(t) = e^{-t/\tau} \quad \left\{ \begin{array}{l} \text{survival} \\ \text{probability} \\ \text{to } t \end{array} \right.$$

The coefficient is 1, since at  $t=0$  the probability of surviving since  $t=0$  is 1.

$$P_d = 1 - P = 1 - e^{-t/\tau}$$

is the probability of having had a collision by  $t$

$$P = \frac{dP_d}{dt} = \frac{e^{-t/\tau}}{\tau}$$

is the probability density of a collision at  $t$ .

We can now find the moments of distribution  $P$ .

27-40

$$\langle t^l \rangle = \int_0^{\infty} t^l P(t) dt$$

definition of a moment  
of  $P(t)$

$$\begin{aligned} \langle t^l \rangle &= \int_0^{\infty} t^l \frac{e^{-t/\tau}}{\tau} dt \\ &= \tau^l \int_0^{\infty} x^l e^{-x} dx \end{aligned}$$

$l!$

It is the factorial  
function (Art - 453).

$= \left\{ \begin{array}{l} l! \tau^l \end{array} \right.$  in general

1 for  $l=0$  which  
just show  $P(t)$  is  
normalized.

$\tau$  for  $l=1$  which is  
the mean time between  
collisions.

$2\tau^2$  is the mean squared  
time between collisions.



$$\sigma \equiv \sqrt{\langle (t - \langle t \rangle)^2 \rangle}$$

is the standard deviation

$$= \sqrt{\langle t^2 - 2t\langle t \rangle + \langle t \rangle^2 \rangle}$$

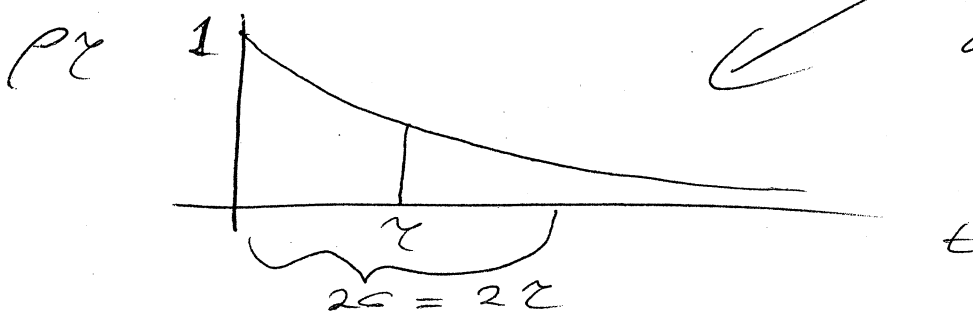
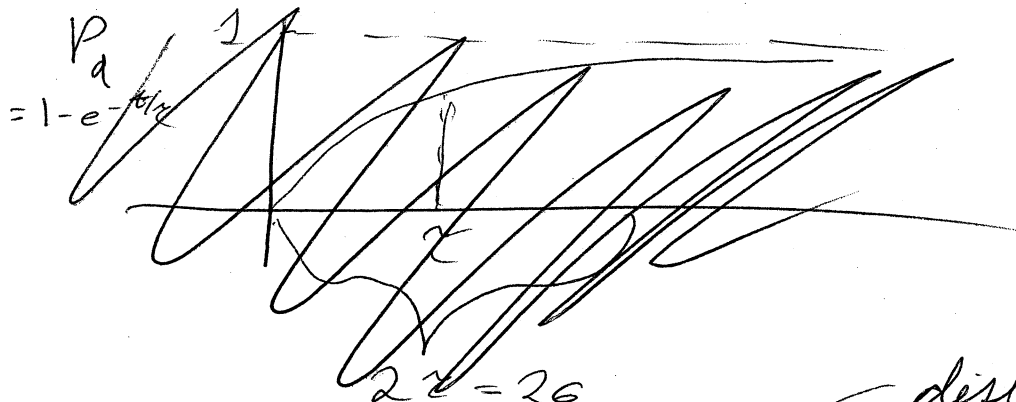
by definition

$$= \sqrt{\langle t^2 \rangle - 2\langle t \rangle^2 + \langle t \rangle^2}$$

$$= \sqrt{\langle t^2 \rangle - \langle t \rangle^2}$$

$$= \sqrt{2\tau^2 - \tau^2} = \tau$$

is the standard deviation which we don't need for the proof, but is an interesting result in it's own right.




distribution and 2\sigma width.

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Now if  $v = at$  at the time of collision

The mean final velocity of ~~the particle~~ is at the time of a collision is


$$\begin{aligned} \langle v \rangle &= \int_0^{\infty} v P(t) dt \\ &= \int_0^{\infty} at \frac{e^{-t/\tau}}{\tau} dt \\ &= a\tau \end{aligned}$$

But what is the mean velocity itself? The drift velocity?

Well one might be tempted

to say well  $v_{ave} = \frac{1}{2}(0 + at)$   
 $= \frac{1}{2}at$

is the mean velocity from  $t=0$  to  $t$  the collision.

$\therefore \langle v_{ave} \rangle_{false} = \frac{1}{2} a \tau$

But this  
is WRONG,

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It is wrong because it  
doesn't take into account  
how long ~~it~~ <sup>the carrier is</sup> moving  
at average velocity  $\frac{1}{2}at$ .

~~Now the~~  $\frac{1}{2}at$  is a sort of average velocity  
but it ~~does~~ is NOT the average that

The real way to find the  
average velocity is to divide  
total distance traveled by  
total time.  $\bar{v}$

tells  
you  
how  
far  
the  
carrier  
has  
gone in  
a long  
time  $t_{long}$ .

~~$v_d = \frac{1}{2}at$~~

From the constant acceleration  
kinematic equations in time  $t$   
the carrier travels

$$x = \frac{1}{2}at^2$$

27-44

Now let's sum distances and times over a large number of steps  $i$ .

$$N_d \approx \frac{\sum_i \frac{1}{2} a t_i^2}{\sum_i t_i} = \frac{\text{total displacement}}{\text{total time}}$$

But the times  $t_i$  of each step  $i$  occur with probability  $P(t_i) dt_i$

$$\begin{aligned} \therefore N_d &= \frac{\int_0^{\infty} \frac{1}{2} a t^2 \frac{e^{-t/\tau}}{\tau} dt}{\int_0^{\infty} t e^{-t/\tau} dt} \\ &= \frac{\frac{1}{2} a (2\tau^2)}{\tau} = a\tau = \langle N \rangle \end{aligned}$$

A sort of curiosity that the drift velocity, the true average velocity, is the same as

mean final velocity

27-45

of a step.

It is a peculiarity of the particular probability distribution of step times.

Since it has taken 9 pages to prove this, one can understand — but not excuse — SJ-76 I for a proof by beaver.

What if the time between collisions was a constant  $\tau$ .

$$\text{Then } N_{\text{drift}} = \frac{\frac{1}{2} a \tau^2}{\tau} = \frac{1}{2} a \tau,$$

but random times to collisions is the physically right assumption

27-46

$$\text{OK } J = \frac{nq^2 E}{m} \tau$$
$$= \frac{nq^2 \tau}{m} E$$

$$\sigma = \frac{nq^2 \tau}{m}$$

which is independent  
of  $E$  if  $\tau$  is independent  
of  $E$  which it is.

— <sup>mean</sup> the time between collisions  $\tau$   
depends on the thermal  
velocity of the carriers  
and other properties of the  
material.  
→ and these actually  
must be treated using

quantum mechanical [27-47]  
ideas (AM-9, ~~50~~ 49-52)

$$\rho = \frac{1}{\sigma} = \frac{m_e}{n q^2 \tau}$$

in the Drude model

## § 27.4 Resistance & Temperature

---

—  $\rho(T)$

depends on  $T$

and in general in some more or less complex way.

But one can Taylor expand any smooth function and nature usually allows us

But in some cases for metals is pretty linear  
e.g., Copper  
TM - 846

27-48)

to assume smooth  
behavior — but not  
always)

So if one takes a reference  
or fiducial temperature  $T_0$ ,  
one can Taylor expand around  
 $T_0$

$$P \approx \underbrace{P_0}_{P(T_0)} + (T - T_0) \left. \frac{dP}{dT} \right|_{T_0} + \text{higher order terms}$$

we can

drop  
these

if  $T - T_0$   
is small  
enough.

$$= P_0 + (T - T_0) \left. \frac{dP}{dT} \right|_{T_0}$$

$$= P_0 \left[ 1 + (T - T_0) \frac{1}{P_0} \left. \frac{dP}{dT} \right|_{T_0} \right]$$



We define

$$\alpha \equiv \left( \frac{1}{\rho} \frac{d\rho}{dT} \right) \Big|_{T_0}$$

$$= \left( \frac{\rho \ln \rho}{\rho T} \right) \Big|_{T_0}$$

Units of  $\alpha$   
 $[\alpha] = \frac{1}{^\circ\text{C}}$   
 ↑  
 a Celsius degree

as the temperature coefficient of resistivity.

Say we set  $\rho = 0$

and solved for  $\Delta T = T - T_0$

$$0 = 1 + \Delta T \alpha$$

$$\Delta T = -\frac{1}{\alpha}$$

So if our linear expression for  $\rho$  were exact (which it is not)

27-50

then  $-\frac{1}{\alpha}$  is the temperature change from  $T_0$  needed to take resistivity to zero.

Consider the  $\alpha$  for <sup>about  $T_{000} = 300$</sup>  metals on p. 27-27  
 $\Delta T \cong -\frac{1}{3 \times 10^{-3}} = -\frac{10^3}{3} \text{ K}$

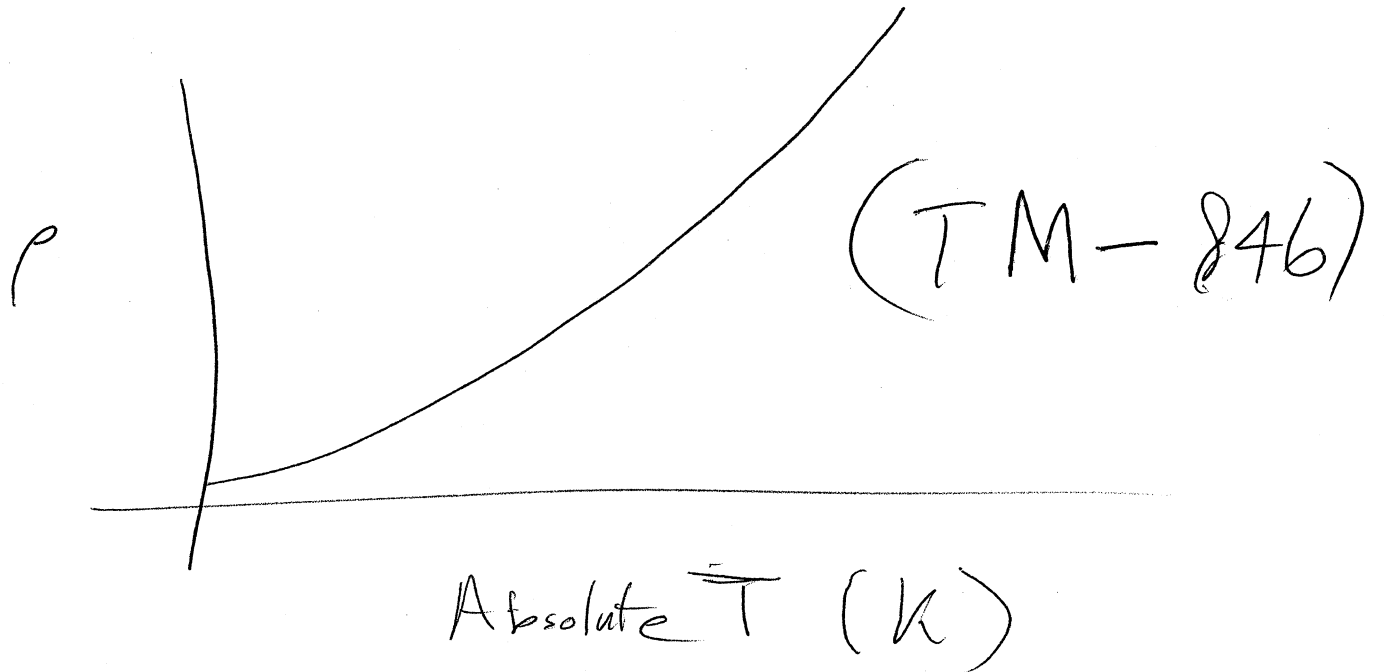
$$\rho = \rho_0 [1 + (T - T_0) \alpha] \cong -300 \text{ K}$$

actually applies over a limited range since our Taylor's expansion does. Which is about right  $\rho$  should go to zero at about  $T = 0 \text{ K}$

Since Resistance for any ~~conductor~~ ~~and~~ ohmic conductor is proportional to resistivity  $\rho$ ,

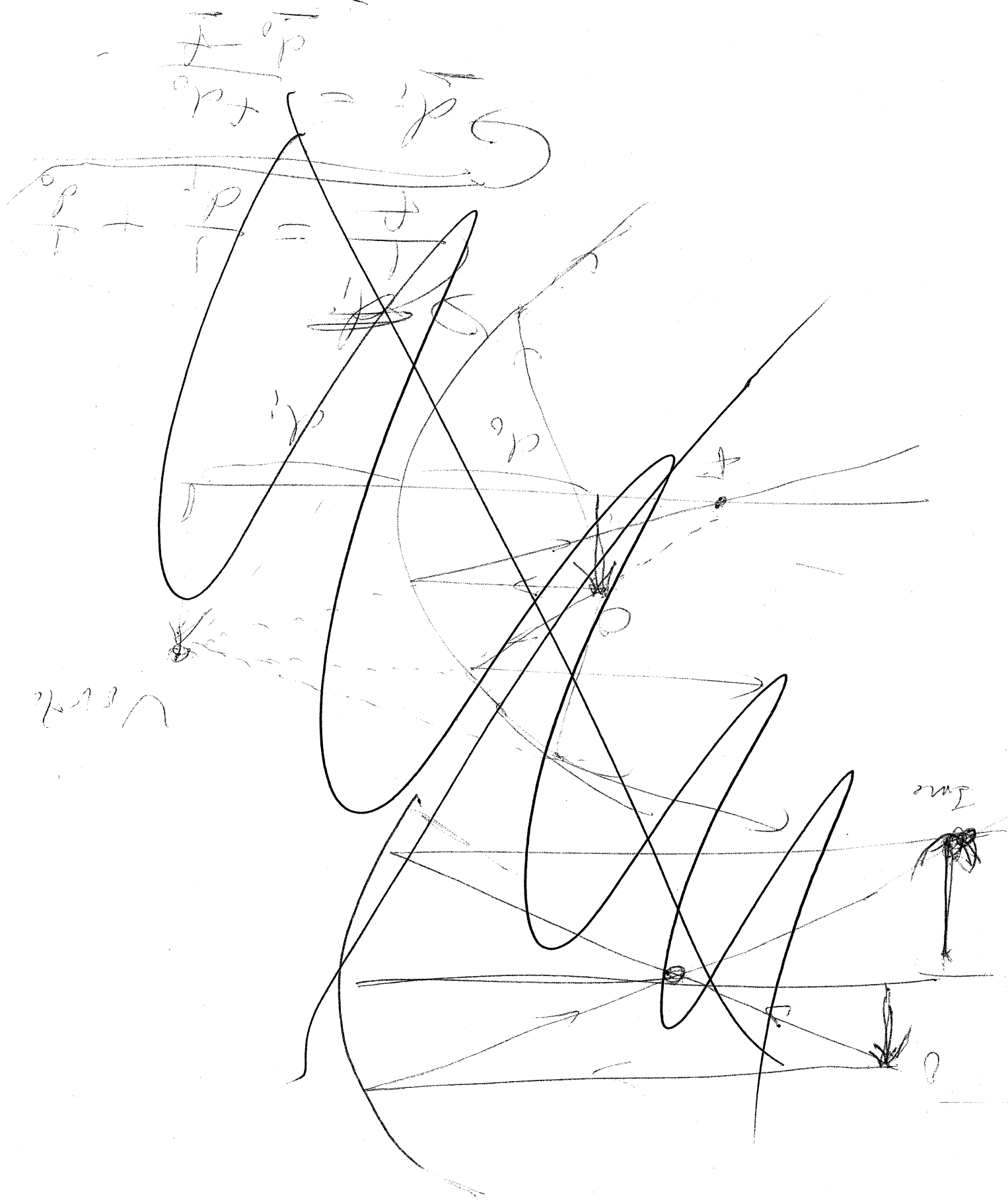
$$R = R_0 [1 + \alpha(T - T_0)]$$

For metals  $\alpha > 0$  (27-51)  
and often  $\rho$  is rather  
linear



- the higher the temperature the more interactions are possible it turns out in Quantum mechanically.
- as  $T \rightarrow 0$ ,  $\rho \rightarrow \rho_{\text{residual}}$   
~~which~~ where  $\rho_{\text{residual}}$  is due to defects and impurities (wiki)

27-52



27-53

if impurities and defects didn't exist

$\rho \rightarrow 0$  ~~near~~ one

queness.

— no resistance and the electrons would be able to move without a driving force

→ But this is not superconductivity

→ that is  $\rho \rightarrow 0$

for finite although small  $T$  (on the absolute scale)

27-54)

semi-conductors have  $\alpha < 0$ .  
(ST-758)

This means resistivity  
decreases as temperature  
increases.

The reason for this is higher T,  
creates more charge carriers

$$\rho = \frac{m}{nq^2\tau}$$

as  $n \uparrow$ ,  $\rho \downarrow$ .

— the ability of semiconductors  
to change their resistivity  
greatly thru the ~~release~~ freeing  
of charge carriers and by  
means other than temperature increase

is an aspect  
of their electronic properties  
that make them so useful  
in electronics.

## §27.9 Superconductors

— left as a reading

## §27.6 Electrical Power

If you have a  
Potential change  $V$   
and a charge  $\Delta q$  goes  
thru that change then

$\Delta E = \Delta q V$  is the  
energy ~~converted~~  
transformed

27-56)

Which way end into  
what?

Well it depends.

If  $\Delta Q > 0$  and it  
goes down a potential  
but hill, energy  
has come out of the  
PE form.

— in a vacuum it could  
go into kinetic energy

— in a resistance into  
thermal energy of material.

↳ the collision events slow  
down the carriers and  
turn their KE into heat.



27-57

In a <sup>electric</sup> motor the  
 $\gamma E$  gets transformed  
to the macroscopic  
mechanical energy of  
the motor which does  
some kind of work.

If  $\Delta q > 0$  and the  
charge goes up a potential  
hill, then energy from  
somewhere ~~pushing the force~~ ~~charge~~  
~~against the electrostatic force~~  
is being transferred to electrical  
 $\gamma E$  thru some force.

If  $\Delta q < 0$ , it's *mutatis mutandis*.

27-58

What if the  $\Delta q$  is transferred thru the potential in  $\Delta t$ .

Then

$$P_{\text{average}} = \frac{\Delta q}{\Delta t} V$$

average power.

assume a continuous process of charge movement. So  $\Delta q$  goes every  $\Delta t$  in a continuous stream.

$$P = \lim_{\Delta t \rightarrow 0} \frac{\Delta q}{\Delta t} V = IV$$

$$\text{So } P = IV \quad [P] = \left[ \frac{C}{s} \frac{J}{C} \right] = \frac{J}{s} = \text{watt} = W$$

This is a very general formula. Which way the energy is being transferred depends on the case — but it's to or from electrical PE.

# Resistors

— devices designed to have resistance

— unlike wires which ideally should have zero resistance.

→ Why do you want them.

— electric space heaters  
(electrical  $\rightarrow$  into heat)

— incandescent light bulbs.

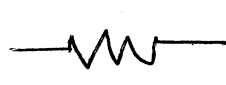
→ the tungsten filaments heat up from energy deposited in it and glow.

→ unfortunately not very efficiently. Most energy comes out as IR rather than visible light.

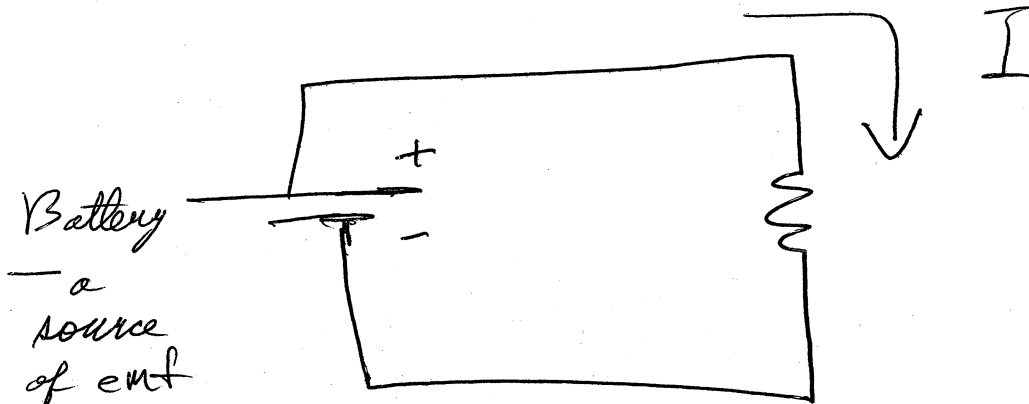
only  
~7 to 20%  
efficient

27-60)

— and of course standard circuit devices with many uses.

Resistor symbol 

So a simple circuit



electromotive force to be defined in Ch 28

$\mathcal{E}$

There is a potential rise  $V = \mathcal{E}$  across the battery where the emf pushes charge up a potential hill of  $V$

$$P_{in} = I V = I \mathcal{E}$$

where we assume

27-61

I constant ~~for near~~

which is normal for reasons  
we'll discuss in Ch. 28. Constant  
in time  
and around

— Now potential is actually <sup>the</sup> ~~the~~ circuit.  
due to ~~to~~ charge  
separation that the EMF  
originally causes.

~~no~~ poles

— remember <sup>the</sup> electric ~~field~~  
force is conservative.

— If you go around a  
closed loop  $\Delta V = 0$

∴  $V_{\oplus}$  is a rise across  
battery from negative to

27-62

positive end.

- there must be a drop by any path from +ve to -ve end.
- So ~~all~~ drop going around the wire.
- Ideal wires have  $\Delta V = 0$
- So the whole drop is across the ~~battery~~ resistor

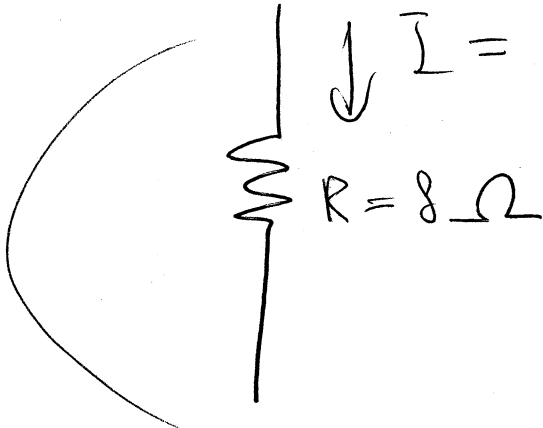
$$\begin{array}{l} \therefore V_{\text{drop}} = I R \quad \left( \begin{array}{l} \sum V_{\text{rise}} = \sum V_{\text{drop}} \\ \therefore \epsilon = IR \\ \text{in this case} \end{array} \right) \\ \begin{array}{l} \nearrow \\ \text{drop across} \\ \text{resistor} \end{array} \quad \begin{array}{l} \uparrow \\ \text{current} \\ \text{thru resistor} \end{array} \end{array}$$
$$\therefore I = V/R$$

27-64

— DC = direct current.

so  $V = 120\text{ V}$  is constant  
and so is the current

$V = 120$   
is  
potential  
drop


$$I = \frac{V}{R} = \frac{120}{8} = 15\text{ A}$$

$$P = IV$$
$$= 15 \cdot 120$$
$$= 1800\text{ W}$$
$$= 1.8\text{ kW}$$

If you run the heater  
for 1 hour you energy

$P_{\text{into PE}}$

$$= IV$$

going thru battery

27-63

$P_{\text{out to heat in resistor}}$

$$= IV = \frac{V^2}{R}$$

$$= I^2 R$$

These formula only apply to the resistor.

Ex 27.4 say  $V = 120V$

across a Nichrome space heater.

~~Let~~ We'll assume DC for the moment.



consumption from  
the ~~grid~~ source

27-65

is  $E = 1.8 \text{ kW} \cdot 1 \text{ h}$   
 $= 1.8 \text{ kW} \cdot \text{h}$

a kilowatt hour  
— a unit of energy  
— a stupid unit.

$$= 1.8 \text{ kW} \cdot \text{h} \left( \frac{3600 \text{ s}}{1 \text{ h}} \right)$$

$$\cong 7000 \text{ kJ}$$

$$\cong 7 \text{ MJ}$$

The electric companies should  
bill us in megajoules.

— I think part of the problem  
with public understanding of

27-66

energy is the insistence  
on using odd energy  
units in different contexts.

— Why not bill electric  
~~power~~ energy use in MJ's  
and report energy  
content of food in MJ's.