Chapter 23

Electric Charge & Electric Fields & the Electric Force

Intro

Electric Charge is a fundamental property of matter: ie, is a Just-So

(At least at our level. It may have an explanation in terms of something else — e.g., strings in deep, deep physics beyond us including me)

23-2

- The electric force & electric field are intimately bound up with electric charge.

→ These three things don't have meaning apart from each other — I mean, they tell you anything about the world except as a package deal.

This is not unusual in physics

e.g., Force, mass, and Newton's laws have to be taken together as a package to have ~~meaning~~ and utility — separated they seem like arbitrary definitions.

There is more to the package. Electricity and magnetism are really both manifestations of the same very rich physical realm of electromagnetism which is responsible (in part) for matter structure from atoms to the whole universe and electromagnetic Radiation (EMR)

But for some distance electricity can be treated alone.

§23.1 Electric Charge

- ordinary matter has electric charge in it.
- Since prehistoric times (since forever)

23-4

people have known that if you ~~rubb~~ rub certain substunce together, they can become <u>charged</u> ⟶ i.e., able to exert forces at distance, without macroscopic contant ⟶ What we call <u>body forces</u> or in Serway's jargon (of which I approve) field forces. (Serway-101)

For example rubber rubbed on fur causes the rubber to become negatively ~~positively~~ charged & the fur to become positively charged.

- One finds that ~~the~~

~~a~~) positive repels positive
~~b~~) negative repels negative
c) positive attracts negative & vice versa of course (by 3rd law & also by all experience).

<u>Or</u> likes repel, unlikes attract.

23-6

& of course, one ~~can~~ sense if a ~~an~~ object is charged by receiving a shock.

Our bodies actually work using electrical signaling and also allow charge flow (i.e., are conductors)

Ben Franklin ~~(1706)~~
(1706 – 1790) statesman, printer, businessman, inventor, scientist, humorist
was the foremost "electrician" of his day and coined the terms "negative" & "positive"

He was actually thinking of electricity as a single ~~fluit~~ fluid

- too much caused positiveness
- ~~too~~ little caused negativeness

(Wiki: electric charge)

Not our modern understanding, but the terminology stuck with some change in meaning.

Actually, Ben from a modern point of view got it sort of wrong & should've named

positive, negative

& negative, positive

2-38)

Since it turned out that <u>negative charge</u> was in many circumstances (particularly those of human technology) is the <u>more mobile kind</u> of charge and it would've have saved us some ~~verbal~~ ~~conventional~~ confusion inconventions if it had been called "positive", But <u>Ben</u> couldn't have known this —— it wasn't the only time he had to make a choice out of two possibilities.

Nowadays (without giving history here) we know the most common sort of fundamental matter (at least fundamental if we don't delve into quarks) is made up of

q is the general charge symbol

	mass (kg)	q or Charge
~~positon~~ proton	$1.6726\ldots \times 10^{-27}$	e
neutrons	$1.6749\ldots \times 10^{-27}$	0
electrons	$9.109\ldots \times 10^{-31}$	$-e$

$m_p \lesssim m_n$

and $m_e \approx \frac{1}{2000} m_p$ (or $\frac{1}{1836.152\ldots} m_p$)

— positrons & neutrons are mostly

~~RG~~ 23-10

bound up in atomic nuclei $\sim 10^{-5}$ to 10^{-4} ~~times~~ smaller than atoms. (in linear size

by the nuclear force — beyond our scope

the nuclei have most of the mass, but little of the atomic volume.

Atomic size scale $\sim 10^{-10}$ m (or 0.1 nm or 1 Å)

Mostly a swarm of electrons bound to the nucleus by the electric force

the outer most electrons are relatively loosely bound.

These are the Valence electrons — responsible for chemistry — ie, chemical bonding into molecules + solids + also when freed or quasi-freed electric charge flow (current) in most human technological systems of interest.

Swarm of eletrons?

— at the quantum mechanical level — which is mostly beyond our scope — the electrons

23-12

are in ~~a superp~~ a continuum superposition of places — i.e., they are everywhere at once in the region of the atom

but only partially everywhere (the terminology of QM takes some getting used to).

$$\underbrace{e}_{\text{fundamental unit of charge}} = 1.602\ldots \times 10^{-19}\ \underbrace{C}_{\text{coulomb}}$$

fundamental unit of charge (quarks can have $\pm\frac{1}{3}e$ or $\pm\frac{2}{3}e$

but we never observe them as free particles. i.e., we never ~~see~~ measure $\pm\frac{1}{3}e$ or $\pm\frac{2}{3}e$,

coulomb

a macroscopic unit of charge

So charge is quantized into 1e

just as ordinary matter is quantized into protons, neutrons, electrons.

~~In charging by rubbing~~
~~Some electron~~

Neutrality

— Most matter in the universe is overwhelmingly neutral at the macroscopic level.

23-14

That is when averaging or ~~size~~ volumes ~~of~~ above the atomic scale

i.e., positive + negative charge add up to zero.

The reasons for this are

1) unlikes attract and so are hard to separate

2) The universe as far as we ~~cat~~ can tell is neutral overall.

At the atomic scale neutralization does NOT happen.

protons & electrons (mostly) don't coalesce to form ~~a~~ neutral matter.

QM rules forbid.

…t
…oes
…appen
…ometimes
…when
…conditions
…are right
…g., neutron
stars
– mostly
neutral
neutrons

Charge is Conserved

– if the universe is to remain neutral, net charge cannot be created or destroyed.

and this seems to be so.

One or Nature can actually create and destroy positive & negative charge

e.g.,

electron + antielectron (positron) $\rightleftarrows$ γ-rays

But no net charge creation or destruction happens

— ordinary human chemistry & material science doesn't do much charge creation/destruction

23-17

but in nature
of nuclear science
it's a pretty common
process in some
contexts.

23-18

~~23-19~~

In charging by rubbing

which more formally should be called charging by conduction.

actually a complex process at the ~~atomic~~ level by chemical bonding ~~ext~~ processes causes electron flows.

- positively charged means an electron deficiency from neutral
- negatively charged means an electron excess from neutral.

L 23-19

— such processes are self-limiting in that once something develops a sufficient excess charge → the like-repelling-like prevents further excess build-up & other processes can lead back to neutralization.
(But there are tricks like the Van de Graaf generator for building up huge charges)

→ So slight charge excesses happen all the time, but they usually don't go to far.

1 Coulomb of net charge charge is

actually an immense & dangerous net charge at the human scale.

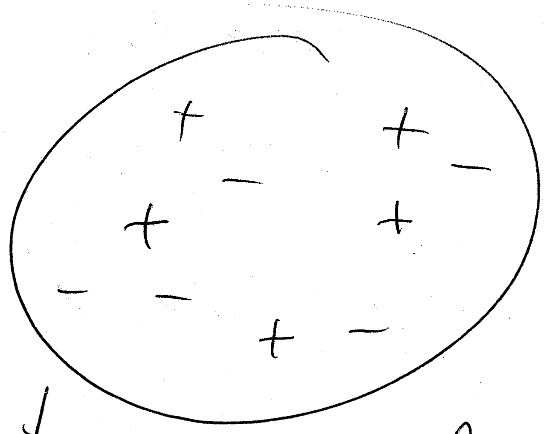
- typical charging by conduction is ~~10^{-6}~~ of order 10^{-6}C.

(Serway-646)

§23.2 Charging by Induction

& Kinds of Conductors

There are several



main categories of materials in ~~the~~ material conduction properties.

23-21

Conduction is the property of charge flow

— we won't be exhaustive or thorough

a) Conductors

— allow charge flow with vanishingly small electric forces.

metals obviously where some electrons are bound to material, but not to any particular atom.

free or quasi-free electrons — outermost atomic electrons are freed

— But also fluids with ions

non-neutral atoms or molecules where electrons have been exchanged

23-22) creating positive & negative atoms or molecules

(ions — non-neutral atom or molecule)

b) Insulators

— material where charge isn't free to move.

— It is bound tightly

Actually no sharp line between ~~inst~~ insulator & conductor.

With a big enough electric field (an E-field) an insulator will break down & conduct.

c) <u>Semi-conductors</u> [23-23

— low conductivity materials (silicon, germanium, and alloys) ~~that~~ have special electrical properties ⟶ these make almost all modern electronics feasible

⟶ but a bit beyond our elementary scope (no matter how ubiquitous in our cellphones, calculators, televisions, computers, etc.

d) <u>Super-Conductor</u>

↳ materials in a low-temperature state where

23-24

resistance (yet to be defined) to current flow vanishes.

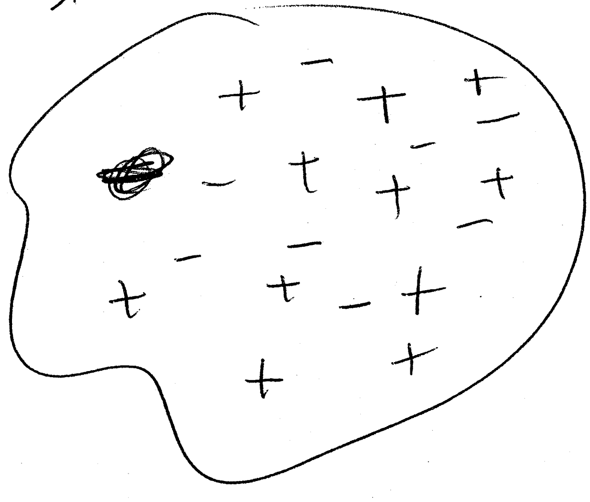
Superconductivity is ~~inhe inte~~ inherently quantum mechanical

superconductors are scientifically immensely important, but technologically they have not lived up to promise at least not yet.

— the state is too delicate for a lot of purposes.
(so far)

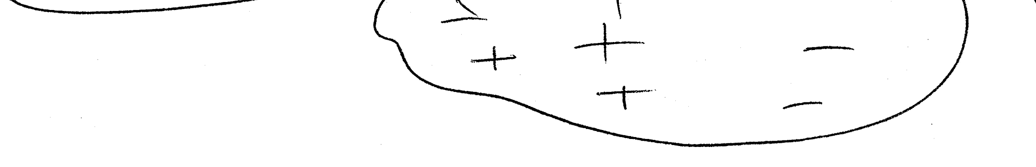
Charging by Induction

- Say we have a sample of **good conductor** — and we almost always mean a metal when we say this - e.g., copper, silver, aluminum - unless we say explicitly otherwise
  - It's electrically neutral.



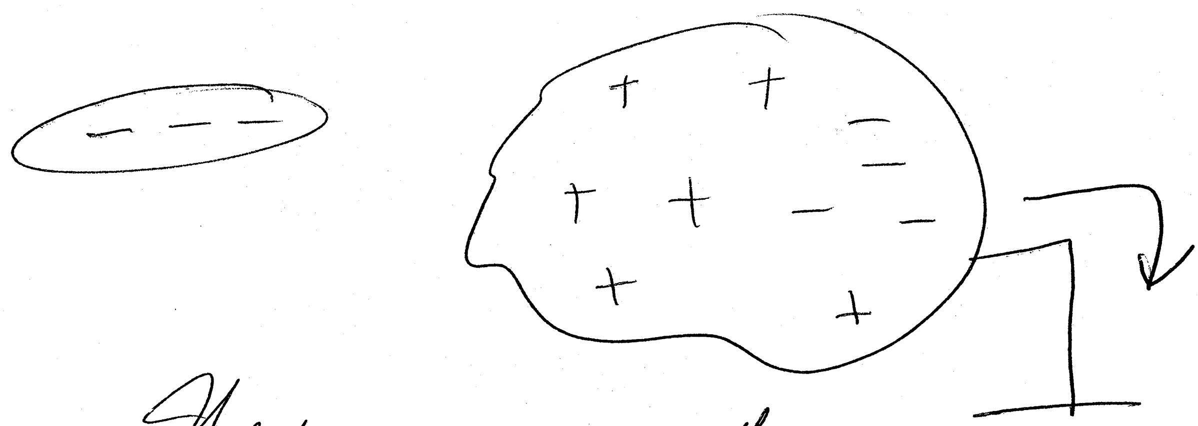
As we'll discuss in Ch 24 (p. 682) the excesses in charge are actually all on the surface of the conductor.

We bring up a charged object



our sample becomes <u>macroscopica</u>
<u>polarized</u>

It's still neutral overall, but the electrons are repelled by the negative object and tend to move away leaving a positive end (a "pole") near the negative object.



Now we <u>ground</u> the sample.

"<u>Ground</u>" means to connect ~~an~~ object to a large conducting reservoir

→ So large that it can ~~be~~ absorbed or give as much charge as you like, but be essentially unaffected itself because it's so large.

The actual "ground" is a pretty good "ground". Either because it's moist or somewhat metallic, it conducts — and its way big.

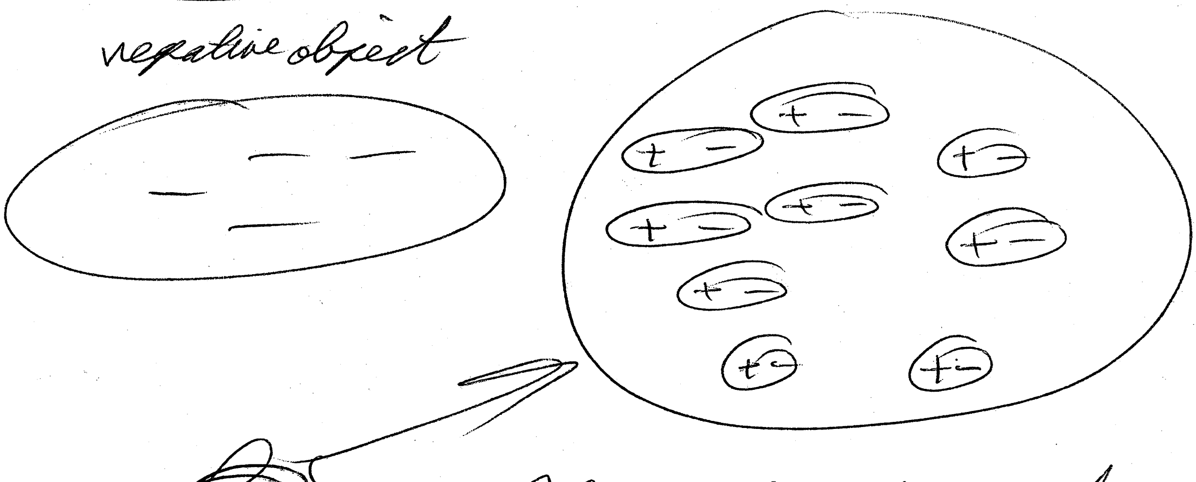
~~The~~ Some negative charge flows off our sample into the ground — then disconnect the ground and the sample

23-28

is left positively charged.

This is <u>charging by induction</u> (which doesn't seem like ~~so~~ such a hot topic to me, but all textbooks like to describe it).

One can also polarize <u>insulators</u>



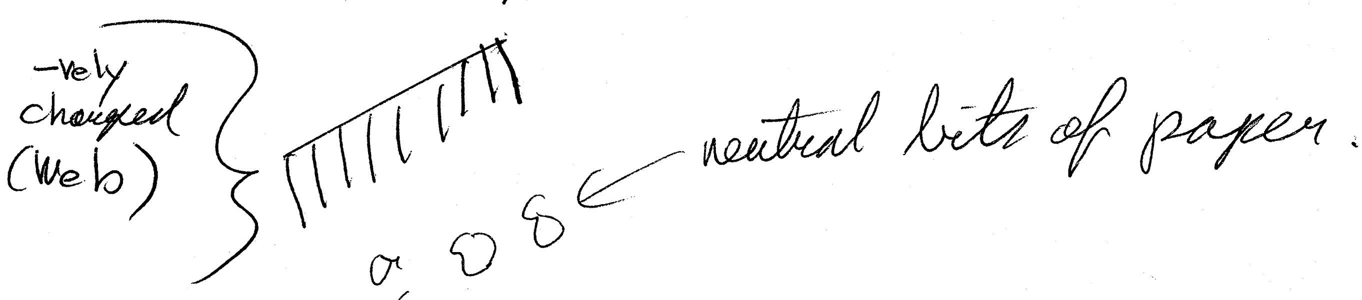
The molecules or atoms in the insulator individually polarize, but the object stays neutral overall.

So an insulator can be polarized by induction, but not charged

(well not an ideal insulator)

Actually there will be an ~~attractive~~ force between <u>negative object</u> & <u>polarized insulator</u>

<u>Demo</u> with comb & neutral bits of paper



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As we'll see in just a bit the electric force between charges depends on the distance between them. — it decreases with distance.

So even though the paper bits are neutral (overall), the +ve charge in them is on average closer to comb than the negative charge.

So the net force is attractive.

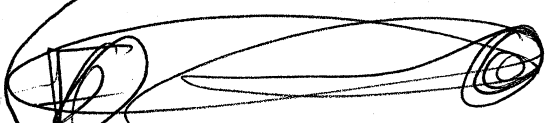
§ 23.3 Coulomb's Law

— on the Electrostatic Force law

first fully elucidated by Charles de Coulomb (1736 – 1806 — he made thru the Reign of terror unlike poor old Lavosier)

Although the law is stated for point charges, it has a general role since finite charge distributions can be built up from point ones (see p. 23-41)

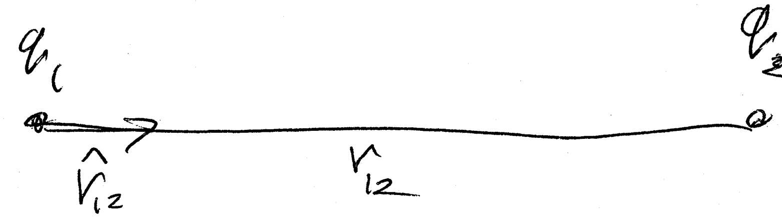
Also all localizable charge distributions are point-like from far enough away (see p. 23-69)



Also as we'll prove with Gauss's law a spherically symmetric charge distribution ~~is like~~ acts like a point charge from the outside

It's very simple and very similar — and yet very different — from the Gravity law

A law for two ideal point charges



$$\underset{\sim}{F}_{12} = \frac{k q_1 q_2}{r_{12}^2} \hat{r}_{12} \qquad \underset{\sim}{F}_{21} = -\underset{\sim}{F}_{12}$$

force of 1 on 2.

like gravity law

by 3rd law and also by the formula itself explicitly.

— under squiggle is the vector

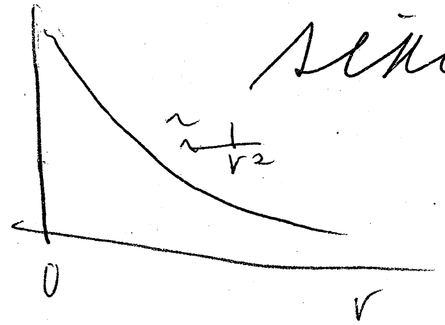
23-32

Coulomb's law is an inverse-square law force

Such forces have very interesting and special properties

Somewhat obviously ~~...~~ since they give the macroscopic & atomic structure of the world.

- Coulomb's law force is a field force since it acts at a distance (and thru a field as we'll see)
- It's actually considered a very long-range force since it falls off as slowly as $\frac{1}{r^2}$



What happens at $r \to 0$
- sort of an embarrassment

~~for~~ apparent protons ~~ar~~ point ~~charges~~ no problem

smear charge of charge

But electrons seem really point-like

[illegible]

$k = 8.9876\ldots \times 10^9 \ \frac{N\,m^2}{C^2}$ 23-33

$\simeq 9 \times 10^9$

$\cong 10^{10}$ is Coulomb's constant (or electrostatic constant, although that name is not actually used all that often — I often just call it "k").

q_1, q_2 are point charge in Coulombs (C)

— a macroscopic unit of charge actually defined using ~~currents~~ (which is a later subject & magnetic forces

r_{12} is the distance in meters between the point charges.

The force of 1 on 2

is repulsive if q_1 & q_2 are like and attractive if q_1 & q_2 are unlike.

23-34

The magnitude force is ~~$F = k|q_1||q_2|$~~

$$F_{12} = \frac{k\,|q_1 q_2|}{r_{12}^2}$$

Ex 2 A fiducial case

$$|q_1| = 1\,C$$

$$|q_2| = 1\,C$$

$$r_{12} = 1\,m$$

$$k = 8.9876\ldots \times 10^9$$

$$\simeq 10^{10}$$

$$F_{12} \simeq \frac{10^{10}\cdot 1\cdot 1}{1^2} = 10^{10}\,N$$

$\simeq 2\times 10^9$ pounds.

which is an enormous force

— and the fact that we don't see forces like this too often tells us that net charges of 1 C are pretty rare in the human environment.

we can build them in the lab but it's rather dangerous and not easy

I can't locate records — the Web draws a blank for once

thunder clouds can have ~25 C (? tens? hundreds?) of net charge

Can be spread over kilometers — so not all that concentrated

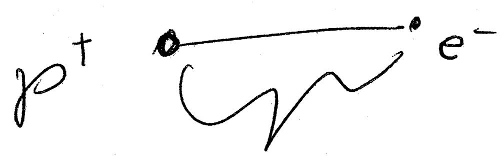
and lightning bolts transfer ~5 C (Wiki: lightning)

~~at up~~ and up to 900C (Tipler-791)

23-36

Ex 2 Hydrogen Atom

– Really QM objects, but in classical approximation one can ask what is the attraction between the proton and electron in the simplest atom



$$r_{ch} = .529 \times 10^{-10}\,m$$

characteristic size from QM.

$$F = \frac{ke^2}{r^2} \simeq \frac{10^{+10}\cdot(1.6\times10^{-19})^2}{(.529\times10^{-10})^2}$$

$$= 10\ \frac{10^{+10}\cdot10^{-38}}{10^{-20}} = \cancel{10^{-37}}N \quad 10^{-7}N$$

What keeps the H-atom from collapsing?

The system has K.E. and Angular Mom., it can't ever get rid of. — something like planet around Sun

— well QM rules the analog to Newton's law ~~F=ma~~ in QM (Schrödinger's equation) forbids.

But it's one fundamental, not ~~less~~ initial condition, determined parameters

$\boxed{Ex 3}$ Ratio of ~~the~~ Gravitational force to ~~and~~ Coulomb force for a proton & electron.

$$F_G = \frac{\frac{G m_1 m_2}{r^2}}{\frac{K|q_1 q_2|}{r^2}}$$

The distance cancels out.

$$= \frac{G}{K} \frac{m_1 m_2}{|q_1 q_2|}$$

$$\cong \frac{7 \times 10^{-11}}{10^{10}} \frac{1.7 \times 10^{-27} \cdot 10^{-30}}{2.5 \times 10^{-38}} \cong 4 \times 10^{-40}$$

23-38

Super-minute

— in virtually all cases the gravitational force between atomic scale objects is unmeasurably minute. — in ordinary physics

which is a problem if one wants to study quantum grav. — which is why we know little about it.

compared to the Coulomb force and is virtually always neglected.

— even between ~~max~~ Macroscopic bodies of human size scale gravity is a very weak force and usually negligible — but it can be measured in this case

For example

$$F = G\frac{m_1 m_2}{r} = \frac{7\times10^{-11}\cdot(1kg)^2}{(1m)^2}$$

$$\simeq 7\times10^{-11}\,N$$

But gravity's "charge" (i.e., mass) has only one flavor and like attracts like for gravity.

There's no cancellation as for the Coulomb force.

So mass tends to aggregate in huge clumps

asteroids to clusters of galaxies to maybe universe as whole

in which gravity is a dominant determiner of structure.

E&M provides pressure force & KE & Ang. Mom. too.

23-40

At smaller ~~scalle~~ scales the electromagnetic force (of which the Coulomb force is actually a ~~an~~ special case) determines structure down to atomic level and has influences even at nuclear scale

→ The electromagnetic force is actually an immensely rich force ⟹ all chemistry arises from its richness and thus ~~life~~ as we know it.

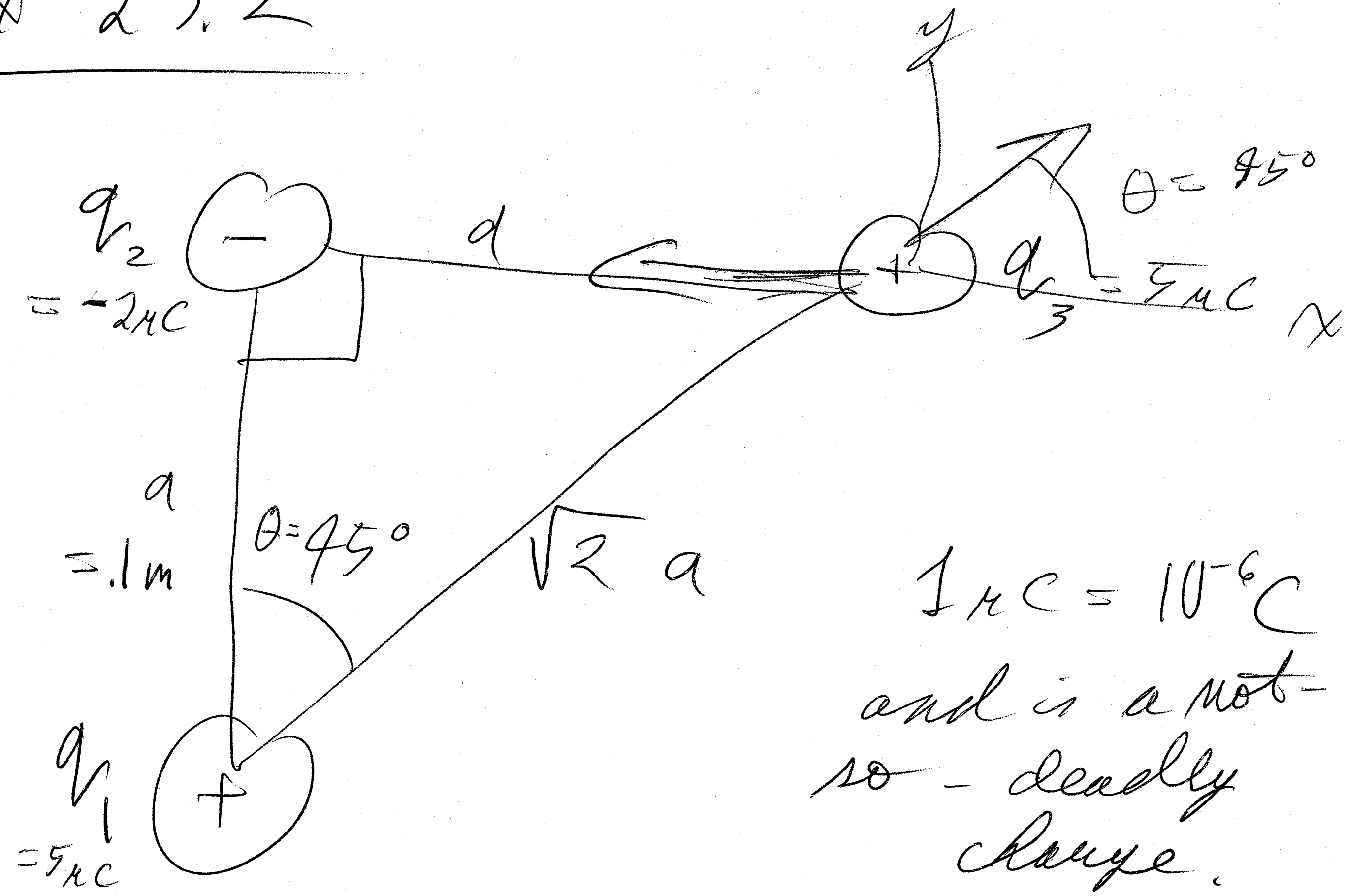
Vector force & Superposition principle

- The Coulomb force is of course a vector (as all forces are)
- and the superposition principle applies.
- the net force ~~of~~ on a point charge equals the vector sum of forces of all other charges.

Doing vector sums for point charges is a bit tedious, but we can do an example

23-42 for "the fun of it".

Ex 23.2



$1\mu C = 10^{-6} C$ and is a not-so-deadly charge.

$$\underset{\sim}{F}_3 = \underset{\sim}{F}_{13} + \underset{\sim}{F}_{23}$$

$\cos\theta = \cos 45^\circ = \frac{1}{\sqrt{2}}$

$$= k q_3 \left[\frac{q_1}{(\sqrt{2}a)^2} \cos\theta\, \hat{x} + \frac{q_1}{(\sqrt{2}a)^2} \sin\theta\, \hat{y} + \frac{q_2}{a^2} (+1)\, \hat{x} \right]$$ using component

$$= k(5 \times 10^{-12}) \left[\frac{5}{2a^2} \frac{1}{\sqrt{2}} \hat{x} + \frac{5}{2a^2 \sqrt{2}} \hat{y} - \frac{2}{a^2} \hat{x} \right]$$

$\frac{5}{2.8} \simeq 1.8$

$$= \frac{k(5\times 10^{-12})}{10^{-2}}\left[\left(\frac{5}{2\sqrt{2}} - 2\right)\hat{x} + \frac{5}{2\sqrt{2}}\hat{y}\right]$$

$$\simeq 5\left[-.2\,\hat{x} + 1.8\,\hat{y}\right]$$

$$= -1\hat{x} + 9\hat{y}$$ (Ans. $-1.1\hat{x} + 7.9\hat{y}$)

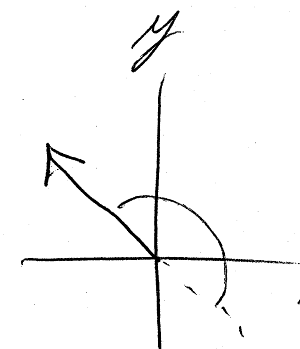
$$|F| = \sqrt{1^2 + 9^2}$$

$$\simeq \sqrt{82} \simeq 9\,N \approx 2\,lb$$

$$\theta = \tan^{-1}\left(\frac{1.8}{-.2}\right) + 180^\circ$$

$$\simeq -84 + 180$$

$$= 96^\circ$$



needed to correct for the inverse tangent ambiguity

23-44

§2.3.4 The Electric field

- often abbreviated to E-field
- given symbol $\underset{\sim}{E}$ (at least I do & I must've got it from somewhere)
  which is not to be confused for E used for energy
- $\underset{\sim}{E}$ is a vector (energy is not)

Say one has a Coulomb force on a point charge q at $(\underset{\sim}{r})$

$$\underset{\sim}{F}(\underset{\sim}{r})$$

The electric field at $\underset{\sim}{r}$

is $\underset{\sim}{E}(r) = \dfrac{F(\underset{\sim}{r})}{q}$

The E-field
is ~~the~~ force per
unit charge
and has MKS units of N/C

= $\frac{\text{newtons}}{\text{coulomb}}$.

The electric field
is not just an auxiliary quantity.

In modern physics it is
considered a real thing
and the cause of the
electric force.

It is there pervading space
when there is no charge
there to feel a force.

There are several lines of
argument that leads to
this conclusion.

23-46

a) If a hypothetical charge appear instantly at $\underset{\sim}{r}_1$, an other charge at $\underset{\sim}{r}_2$ wouldn't feel the electric force instantly → it takes a finite time for the electric field to propagate to $\underset{\sim}{r}_2$ which it can only do at the vacuum speed of light at fastest ~~for~~ as we know from special relativity.

We can't actually make charges magically appear, but the finite propagation speed is somehow undetectable.

b) light is an traveling electromagnetic wave

electromagnetic radiation (EMR)

a combination of self-propagating E-fields & B-fields (magnetic fields)

It's created by accelerated charge

and absorbed by
charge.

— EMR can travel across
the universe and arrive
at Earth long after
it's source charge has
ceased to exist even

→ So EMR is independent
of its source.

By the way since we see
EMR, we see electromagnetic
fields — it's all we
do see.

But our eyes ~~are~~ are only sensitive
to ~~high frequency EM~~
time varying EMR in a narrow
frequency window = the visible.

23-48

6) In the not so distant future, we'll introduce <u>electrical potential energy</u>.

You may ask where ~~this~~ energy is or what it is. (but probably not)

It's the energy of electric field as it turns out — we'll see later on.

Although not without paradoxes

So the <u>E-field</u> is a real thing — not just an auxiliary concept to the electric force.

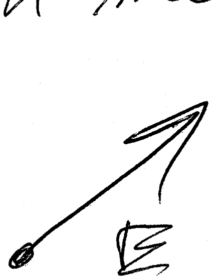
[23-49

E-field:

In mathematical physics a field is a quantity defined everywhere in space.

— So the E-field must have a value everywhere (which may be zero)

— Of course we often (almost always) consider idealized E-fields ~~that~~ where a certain set of charges create the E-field and ignore all other ones that would exist in messy reality.



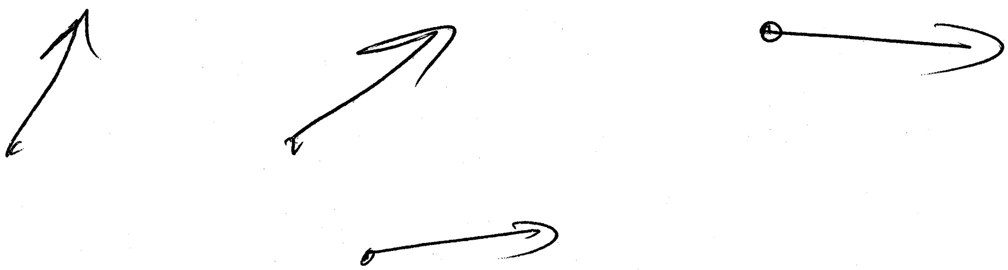
E

The E-field is a vector field. Thus it not only has a magnitude but a direction.

23-50

— the direction is in space-space, but the extent is in an abstract $\underset{\sim}{E}$-field space.

I sort of think of little arrows



throughout space.

— there's a <u>continuum of them</u>

but, of course, one can only draw a finite illustrative set.

The <u>Coulomb force</u> on charge q

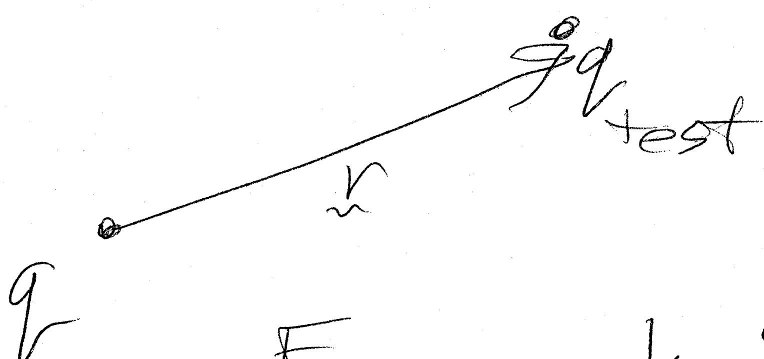
is $\underset{\sim}{F} = q\underset{\sim}{E}$ of course.

$q > 0$
$\underset{\sim}{F} \parallel \underset{\sim}{E}$
$q < 0$
$\underset{\sim}{F}$ anti-parallel to $\underset{\sim}{E}$

The sign of q affects the sign of $\underset{\sim}{F}$ of course

Point Charge

— "the" prototype $\underset{\sim}{E}$-field is that of a point charge.



at origin

$$\underset{\sim}{F}_{\text{on test}} = k \frac{q\, q_{test}}{r^2} \hat{r}$$

$$\therefore \underset{\sim}{E} = \frac{kq}{r^2} \hat{r}$$

is the $\underset{\sim}{E}$-field formula for any point in space for q at the origin

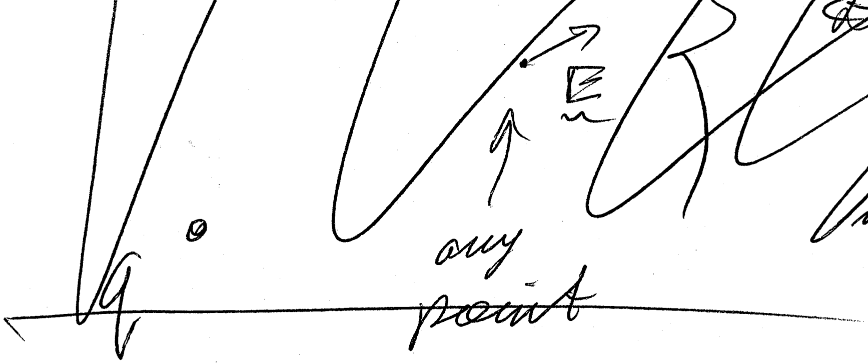
— a lone charge (a lonely charge).

23-50)

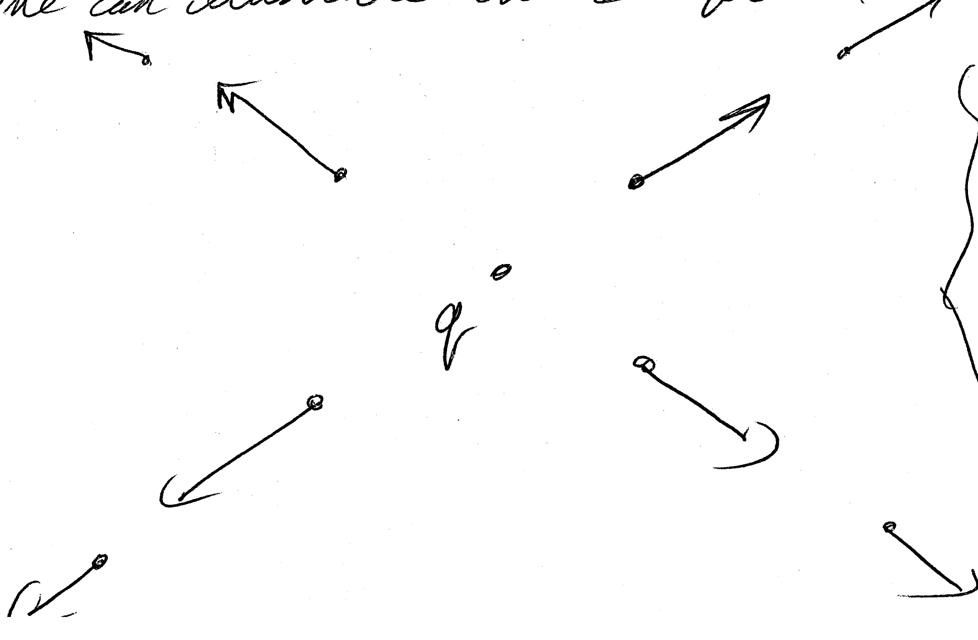
Mathematically (really as Wikipedia tells me in mathematical physics) a field is a quantity defined at every point in space.

$\underset{\sim}{E} = \frac{kq}{r^2}\hat{r}$ does that

~~the~~ The $\underset{\sim}{E}$-field is a vector field



One can illustrate the $\underset{\sim}{E}$-field of a point charge

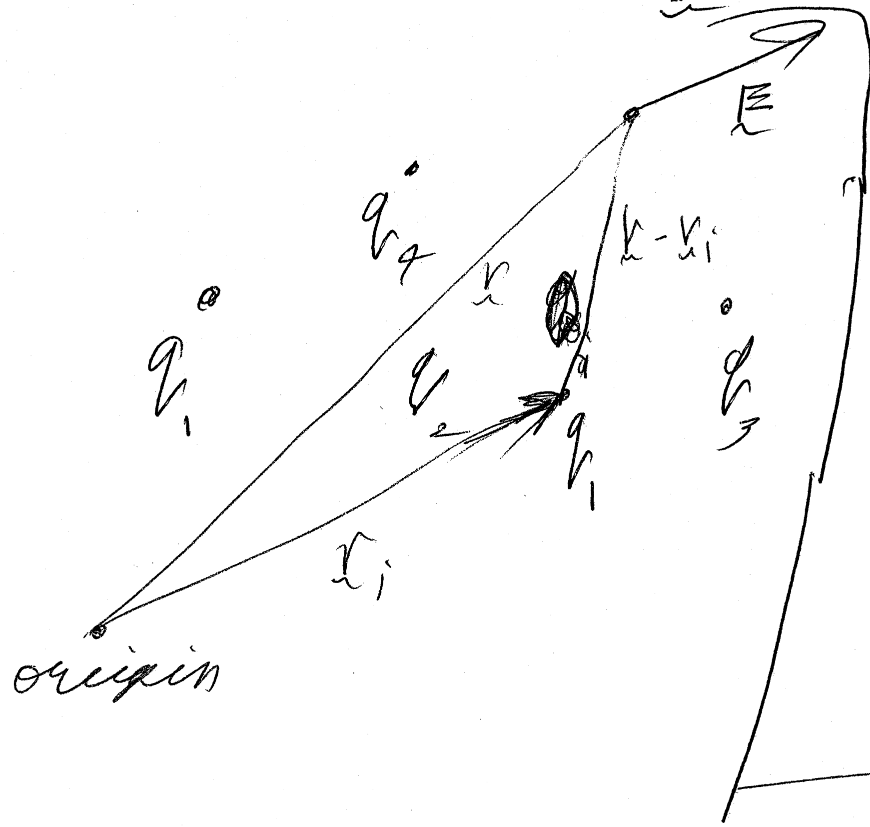


a finite set of arrows for $q > 0$

The length of the arrow qualitatively gives the size of $\underset{\sim}{E}$.

Superposition principle applies

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 + \dots$$



$$= \sum_i \mathbf{E}_i$$

The net E-field is the sum of E-fields of each charge.

$$\mathbf{E} = \sum_i \frac{k\, q_i}{|\mathbf{r} - \mathbf{r}_i|^3} (\mathbf{r} - \mathbf{r}_i)$$

Note $\dfrac{\mathbf{r} - \mathbf{r}_i}{|\mathbf{r} - \mathbf{r}_i|}$ is the unit vector pointing from $\mathbf{r}_i$ to $\mathbf{r}$.

23-52

Now in actual fact we believe charge comes in discrete clumps

— protons with e

which are actually not point charges in modern theory

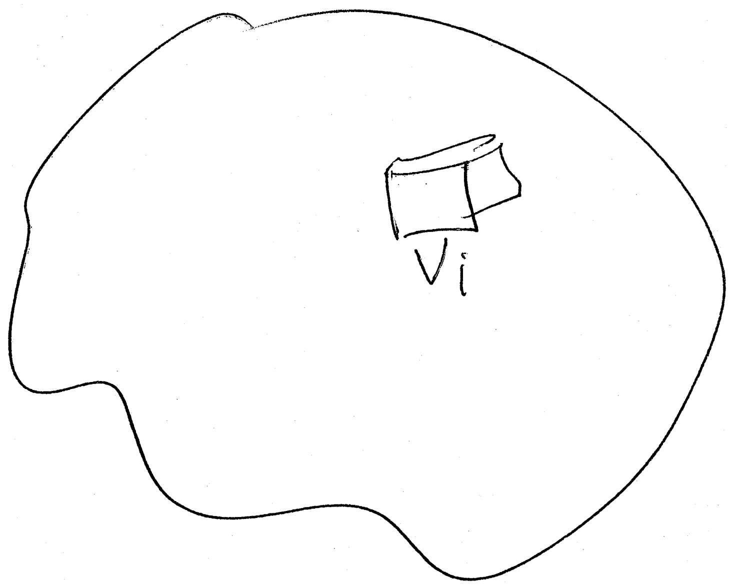
— electrons with $-e$

which are point charges as far as we know (but maybe not in advanced strong theory).

protons & nuclei are held together against Coulomb repulsion by the nuclear force

electrons by who knows (a Just so story)

But at the macroscopic level charge can often be approximated as a continuum e.g., the graininess of an excess of electrons is not noticeable.



In this case

$$dq_i = \rho_i \, dV_i$$

bit of charge in volume dV_i

charge density in dV_i

— The expression is differential and so is exact in the infinitesimal limit.

~~#~~3-54

So $\underset{\sim}{E} = \lim_{\Delta V_i \to 0} \sum \frac{k q_i}{|\underset{\sim}{r} - \underset{\sim}{r}_i|^3} |\underset{\sim}{r} - r_i|$

Miracle of ~~Fundamental Theorem of~~ calculus

$= \cancel{\int k \rho(\underset{\sim}{r}') \, dV}$

Recall the Fundamental Thm of Calculus tells us the antiderivative of the integrand can be use to evaluate the integral (if you know it)

$$= \underbrace{\int_V \frac{k \, \rho(\underset{\sim}{r}') \, (\underset{\sim}{r} - \underset{\sim}{r}')}{|\underset{\sim}{r} - \underset{\sim}{r}'|^3} \, dV'}_{\text{integral over the whole volume of charge.}}$$

In general such integral are tedious, but some special cases are easy enough

They must usually be done numerically.

~~Wee~~'ll do some examples of calculating the electric field for discrete & continuum charge after we've introduced electric field lines

§23.6 Electric Field Lines

— I prefer to introduce these before doing calculations of ~~[illegible]~~ E-fields.

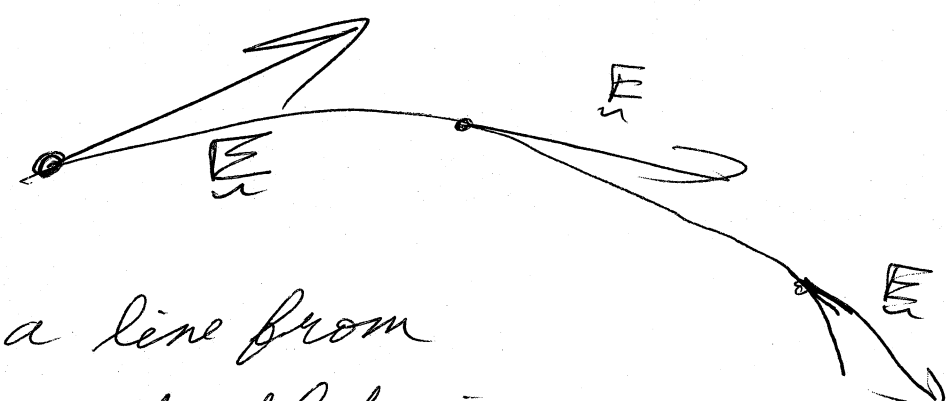
— E-field lines are a visualization tool for

23-56

understanding $\underline{E}$-fields and making qualitative predictions of how ~~they~~ their structure and how they affect charges.

Michael Faraday invented them in the 19th century.

Consider a point in space



draw a line from ~~that~~ point that is everywhere tangent to the local $\underline{E}$-field.

— That line is an $\underline{E}$-field line.

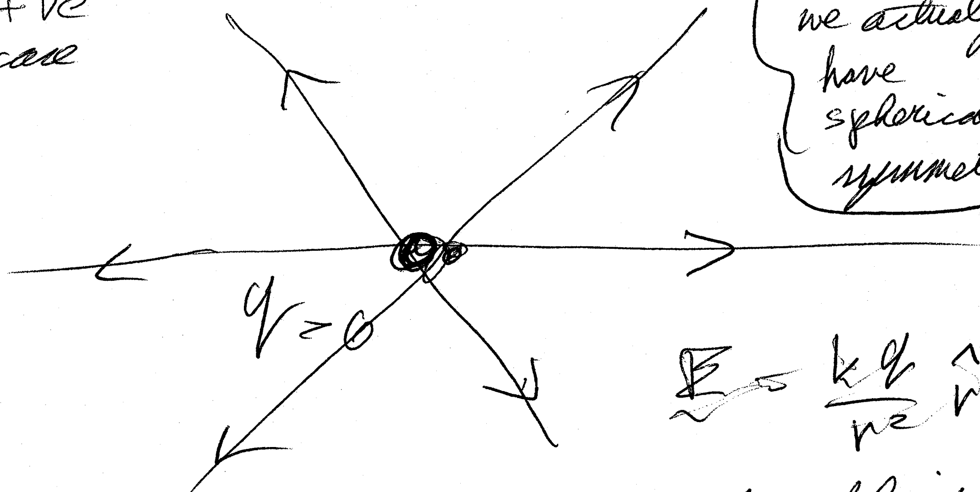
— direction of line is $\underline{E}$-field direction

Examples

Consider +ve & -ve point charges.

23 - 57

+ve case

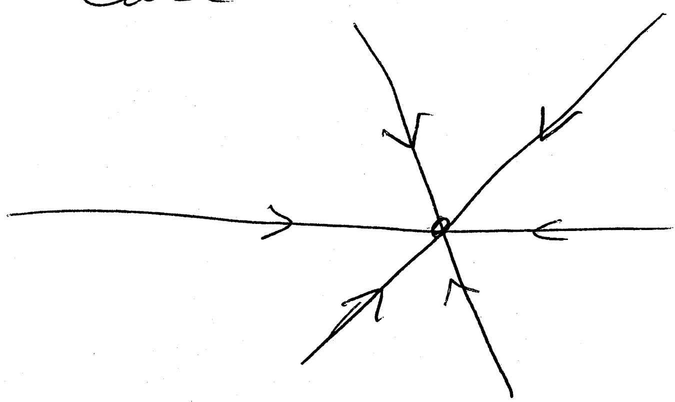


Remember we actually have spherical symmetry.

$$\underset{\sim}{E} = \frac{kq}{r^2}\hat{r}$$

— so their structure is easy to find.

-ve Case



Of course there is actually a continuum of such lines, but one can only draw a finite number.

~~Facts~~ Rule

1) — The E-field lines have limited physical ~~existence~~ meaning.

They are **NOT** in general the path of ~~particle~~ charged particle moving under the electric force alone.

23-58

— such a particle is accelerated by the local E-field force and usually won't be moving in the E-field direction after a bit even if it is a one point (and it needn't be so at any point.

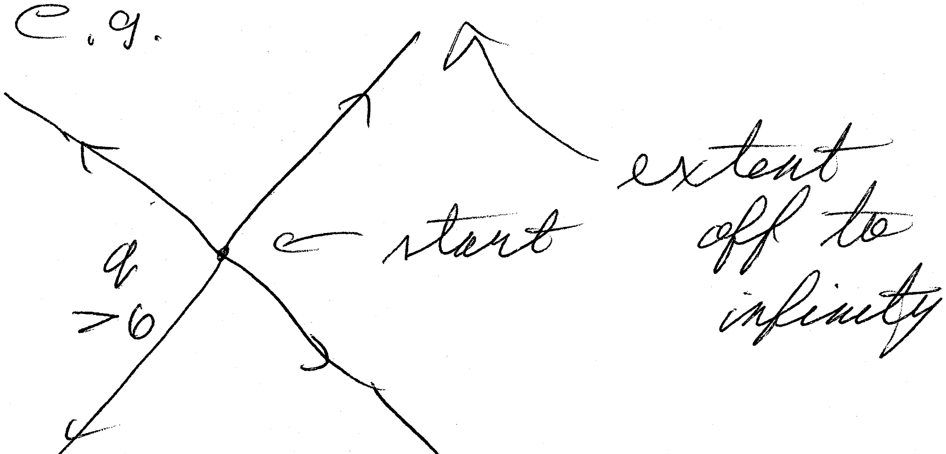
TWO special cases where a charged particle does follow an E-field line when acted on by ~~the~~ the electric force alone.

a) If the E-field lines are straight and the particle is moving along one of them at ~~the initial instant~~. one point in time

b) When the particle starts from rest, but only for an infinitesimal time in ~~the~~ general.

2) E-field lines only ~~end on~~ start on positive charge or infinity and only end on negative charge or infinity.

e.g.



Actually one can have E-fields + E-field lines without charge — and they can form loops

But not in electrostatic cases

Faraday's law chapter.

~~Other Examples~~

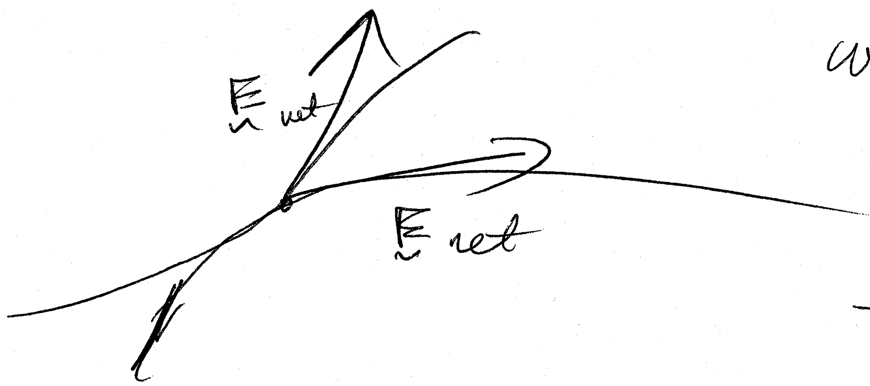
~~a) Two equal positive charges~~

3) E-field lines never cross

In order for the E-field lines

$\boxed{23-60}$

to cross the net E-field at point would have to point in 2 directions — this never happens



<u>Except</u> one can have cases where $\underset{\sim}{E} = 0$ and its direction is undefined and so E-field lines can <u>sort of</u> <u>cross</u> there

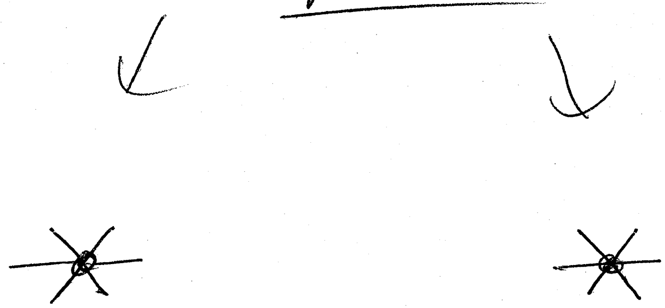
or you could sort of considering them as ending there which is sort of an exception to rule 2.

Example

2 equal positive charges

Note we actually have axial symmetry about the line joining the charges.

Up close both ~~are~~ have ~~like~~ point-charge-like E-fields

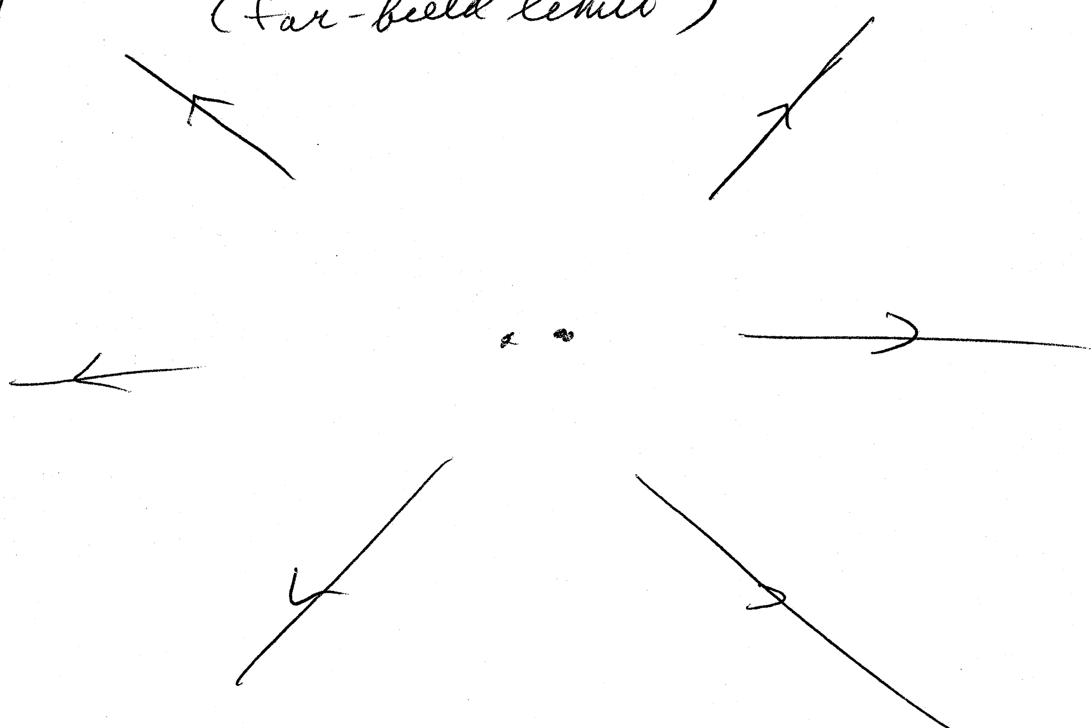


since

$$\underset{\sim}{E} = \frac{kq}{r^2}\hat{r}$$

because arbitrarily huge as $r \to 0$

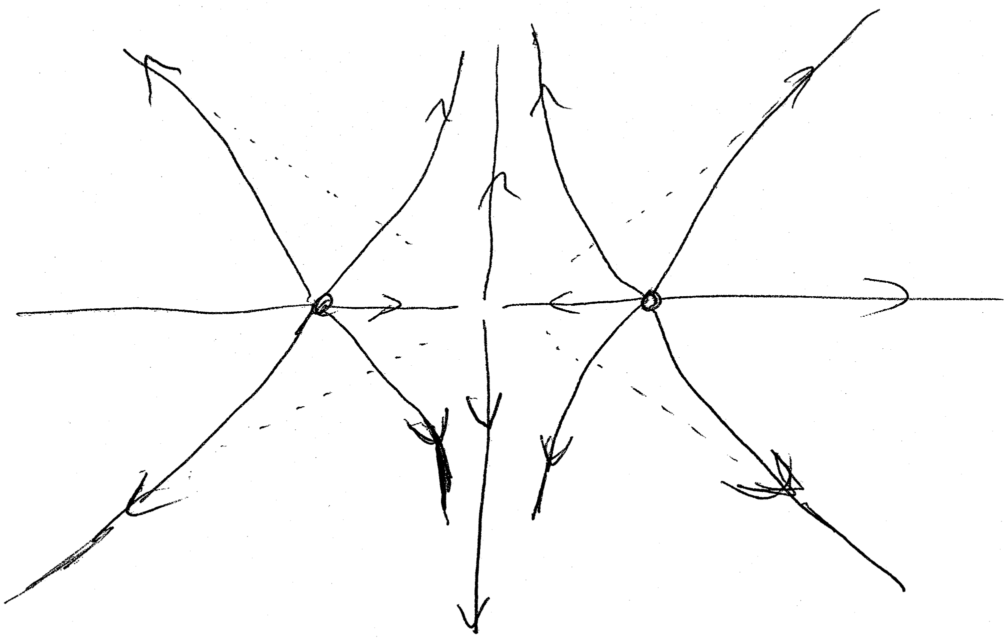
Far away
(far-field limit)



They must look a single point charge.
(this actually requires a bit of a mathematical proof which we'll give in a moment)

23-62

So one can interpolate qualitatively assuming continuity



asymptotes to the lines must point back to the midpoint to get the far field limit.

— the midpoint by symmetry must have $\underset{\sim}{E} = 0$.

— So field lines can cross there — or end there depending on how you want to think of it.

23-~~2~~4

Now we need to Taylor expand

Law of cosines

$$\frac{1}{|\underset{\sim}{v} - \underset{\sim}{r}_i|^3} = \frac{1}{(r^2 + r_i^2 - 2 r r_i \cos\theta_i)^{3/2}}$$

about $r_i/r = 0$

$$= \frac{1}{r^3 \left(1 - 2\frac{r_i}{r}\cos\theta_i + \left(\frac{r_i}{r}\right)^2\right)^{3/2}}$$

$$= \frac{1}{r^3}\left[1 - \frac{3}{2}\left(-2\frac{r_i}{r}\cos\theta_i\right) + \text{higher order terms in } \frac{r_i}{r}\right]$$

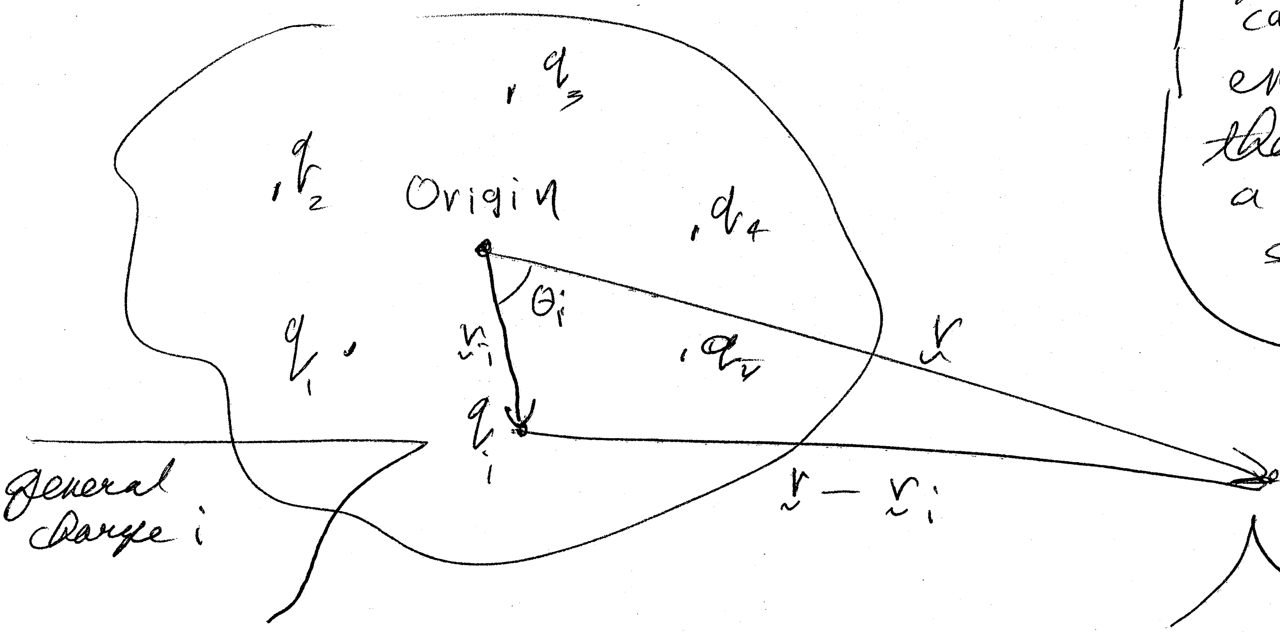
The Taylor series can be assumed to converge for r_i/r small enough and we assume we are far enough from the origin for that.

$$\simeq \frac{1}{r^3}\left(1 + 3\frac{r_i}{r}\cos\theta\right)$$

Example

E-field in Far-field limit of a system of localizable Charges

localizable in this context means you can enclose ~~them~~ in a closed surface



$$\underset{\sim}{E}(\underset{\sim}{r}) = \sum_i \frac{k q_i}{|\underset{\sim}{r} - \underset{\sim}{r}_i|^3}(\underset{\sim}{r} - \underset{\sim}{r}_i)$$

Now we will consider the far-field case where $|\underset{\sim}{r}| >> |\underset{\sim}{r}_i|$.

$$\underset{\sim}{r} - \underset{\sim}{r}_i = \underbrace{r}_{\text{magnitude of } \underset{\sim}{r}}(\hat{r} - \frac{\underset{\sim}{r}_i}{r})$$

$\left|\frac{\underset{\sim}{r}_i}{r}\right| << 1$

unit vector magnitude 1

Zeroth order in $\frac{r_i}{r}$

1^{st} order in $\left|\frac{\underset{\sim}{r}_i}{r}\right| = \frac{r_i}{r}$

to 1^{st} order in $\frac{r_i}{r}$.

$$\therefore \frac{\underset{\sim}{r} - \underset{\sim}{r}_i}{|\underset{\sim}{r} - \underset{\sim}{r}_i|^3} \simeq r\left(\hat{r} - \frac{\underset{\sim}{r}_i}{r}\right)\frac{1}{r^3}\left(1 + 3\frac{r_i}{r}\cos\theta_i\right)$$

$$= \frac{1}{r^2}\left(\hat{r} - \frac{\underset{\sim}{r}_i}{r} + 3\frac{r_i}{r}\cos\theta_i\hat{r}\right)$$

1^{st} order good expression

where we've ~~droplet~~ dropped the other term since it is of order $\left(\frac{r_i}{r}\right)^2$ and our expression is only accurate to 1^{st} order anyway as we dropped 2^{nd} and higher order terms on p. 23-64

$$\triangleq \frac{1}{r^2}\hat{r}$$

to Zeroth order

If $\left(\frac{r_i}{r}\right)$ is small, $\left(\frac{r_i}{r}\right)^2$ is smaller a lot

$$\therefore \underset{\sim}{E}(\underset{\sim}{r}) = \sum_i \frac{k q_i}{|\underset{\sim}{r} - \underset{\sim}{r}_i|^3}(\underset{\sim}{r} - \underset{\sim}{r}_i)$$

23-66

$$\cong \sum_i \frac{k q_i}{r^2}\left(\hat{r} - \frac{\vec{r}_i}{r} + 3\frac{r_i}{r}\cos\theta_i\,\hat{r}\right)$$

to 1st order in $\frac{r_i}{r}$

$$\cong \sum_i \frac{k q_i}{r^2}\hat{r}$$ to Zeroth order in $\frac{r_i}{r}$

valid whenever $\frac{r_i}{r} << 1$ and so negligible compared to 1.

$$= \frac{k}{r^2}\left(\sum_i q_i\right)\hat{r}$$

$$= \frac{kq}{r^2}\hat{r}$$ where $\sum_i q_i = q$ is the net charge of the system.

This can be called the monopole ~~term of~~ approximation to the system.

Our derivation verifies mathematically that in the far-field limit

where $\frac{r_i}{r} << 1$,

a system of charges approximates a point charge.

— The approximation gets better as $\frac{r_i}{r}$ gets smaller.

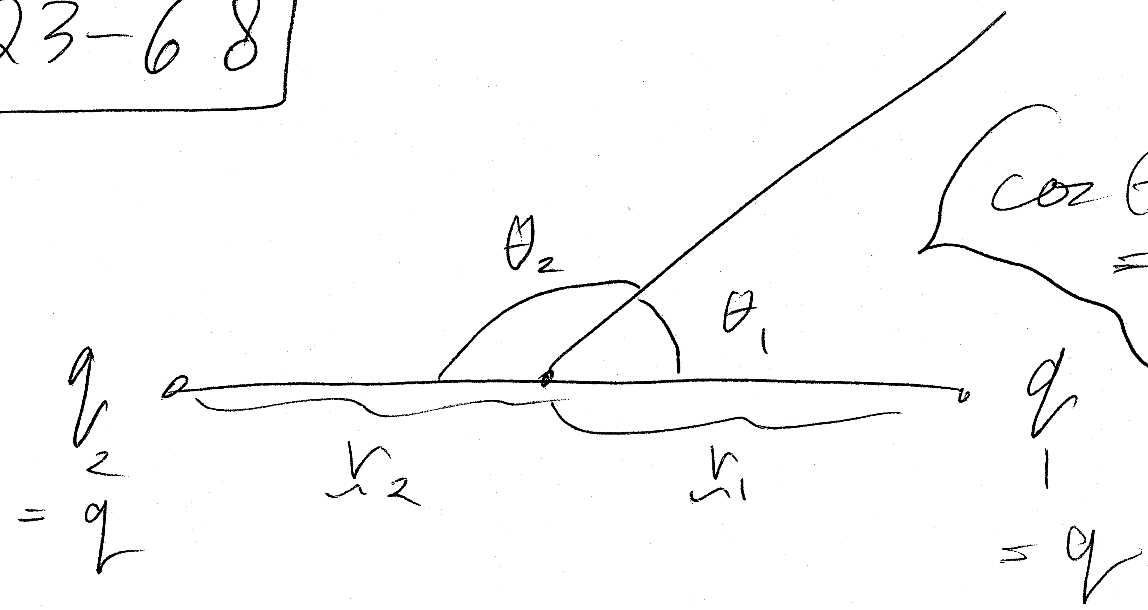
a) <u>Special case</u>

— our two equal positive charges q on a line in far field limit

In this case

$$\underset{\sim}{E} = \underbrace{\frac{k(2q)}{r^2}\hat{r}}_{\text{Zeroth term}} + \sum_i \frac{kq}{r^2}\left(-\frac{\vec{r}_i}{r} + \frac{3r_i}{r}\cos\theta_i\,\hat{r}\right)$$

23-68



$\cos\theta_2 = \cos(\pi - \theta_1) = -\cos\theta_1$

$$\underset{\sim}{r}_1 + \underset{\sim}{r}_2 = 0$$

$$+3\frac{r_1}{r}\cos\theta_1 + 3\frac{r_2}{r}\cos\theta_2$$

$$= 0$$

So in ~~this~~ case because of our symmetrical choice of origin, the 1st order term is 0

and $\underset{\sim}{E}(\underset{\sim}{r}) = \frac{k(2q)}{r^2}\hat{r}$ is actually accurate to 1st order in $\frac{r_i}{r}$.

Special case b

$$q = \sum_i q_i = 0$$

The net charge is zero.

Then $\mathbf{E}(\underset{\sim}{r})_{\text{far field}} = \sum_i \frac{kq_i}{r^2}\left(-\frac{\underset{\sim}{r_i}}{r} + 3\frac{r_i}{r}\cos\theta_i\,\hat{r}\right)$

There is no 0^{th} order term and the leading term is the 1^{st} order term which actually decreases with distance from the origin as $\frac{1}{r^3}$ rather than as $\frac{1}{r^2}$

The 1^{st} order term in this case is called the dipole term and the system is an electric dipole to 1^{st} order in $\frac{r_i}{r}$

If the 2^{nd} order term is zero too, $\mathbf{E} \sim \frac{1}{r^5}$

Actually the first order term can be zero ~~too~~. Then $\mathbf{E} \sim \frac{1}{r^4}$ etc.

23-70)

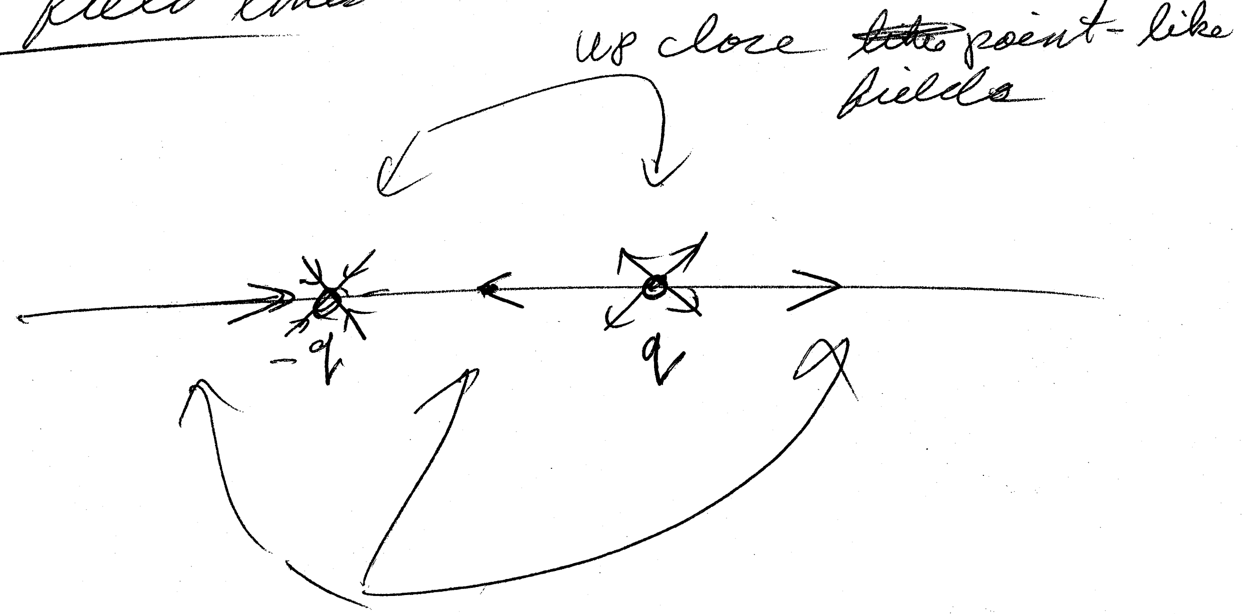
Example: The Electric Dipole

— the simplest case

A pure dipole consisting of 2 point charges

— two point charges on a line of equal magnitude and opposite sign.

E-field lines

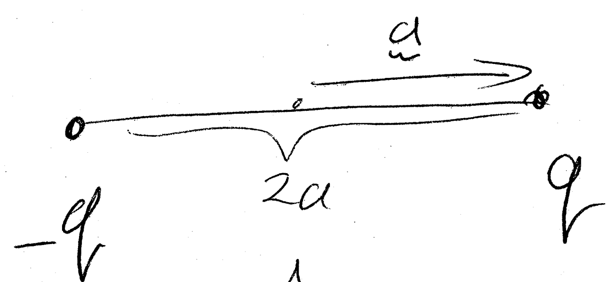


by axial symmetry

What about on the plane ~~thru~~ the ~~center~~ perpendicular to the axis?

Example : The Electric Dipole

— the simplest case is two point charges of opposite sign and equal magnitude



separation distance often defined as $2a$

— when one says electric dipole without qualification this is often what one means.

(GrEM-146 call this a physical dipole)

23-70b

Eventually we need a parameter describing a dipole called the <u>dipole moment</u>

$$\underset{\sim}{p} \equiv 2q\underset{\sim}{a} = qd\,\hat{p}$$

— a vector with units C·m

but this definition is only for a physical dipole.

— any charge distribution can have a <u>dipole moment</u>.

The general definition for the record is for a continuum of charge

$$\underset{\sim}{p} = \int_{\substack{\text{All}\\\text{space}}} \underset{\sim}{r}\,\rho(\underset{\sim}{r})\,dV$$

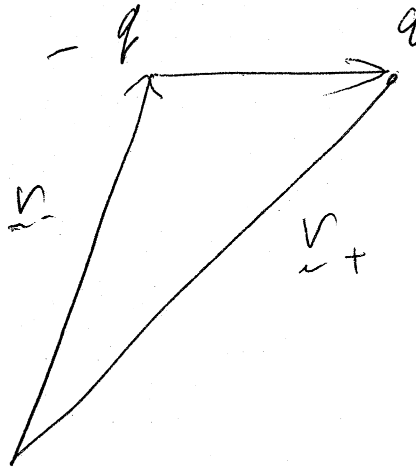
(Gr-EM-149 & 150)

The integral is over all space

$$\underset{\sim}{p} = \sum \underset{\sim}{r}_i\, q_i$$ for a set of point charges.

If one has only q and $-q$

then $\mathbf{p} = q\mathbf{r}_+ - q\mathbf{r}_-$



$= q(\mathbf{r}_+ - \mathbf{r}_-)$

which is the same as on p. 23-70b

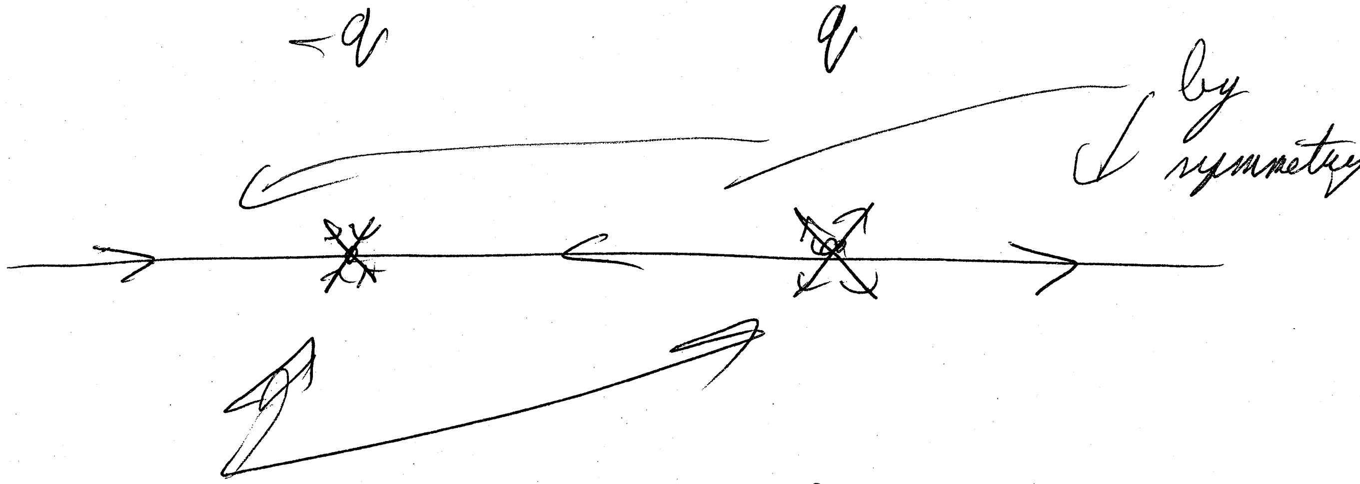
If the distribution is overall neutral, then $\mathbf{p}$ is independent of the origin.

— which we won't prove

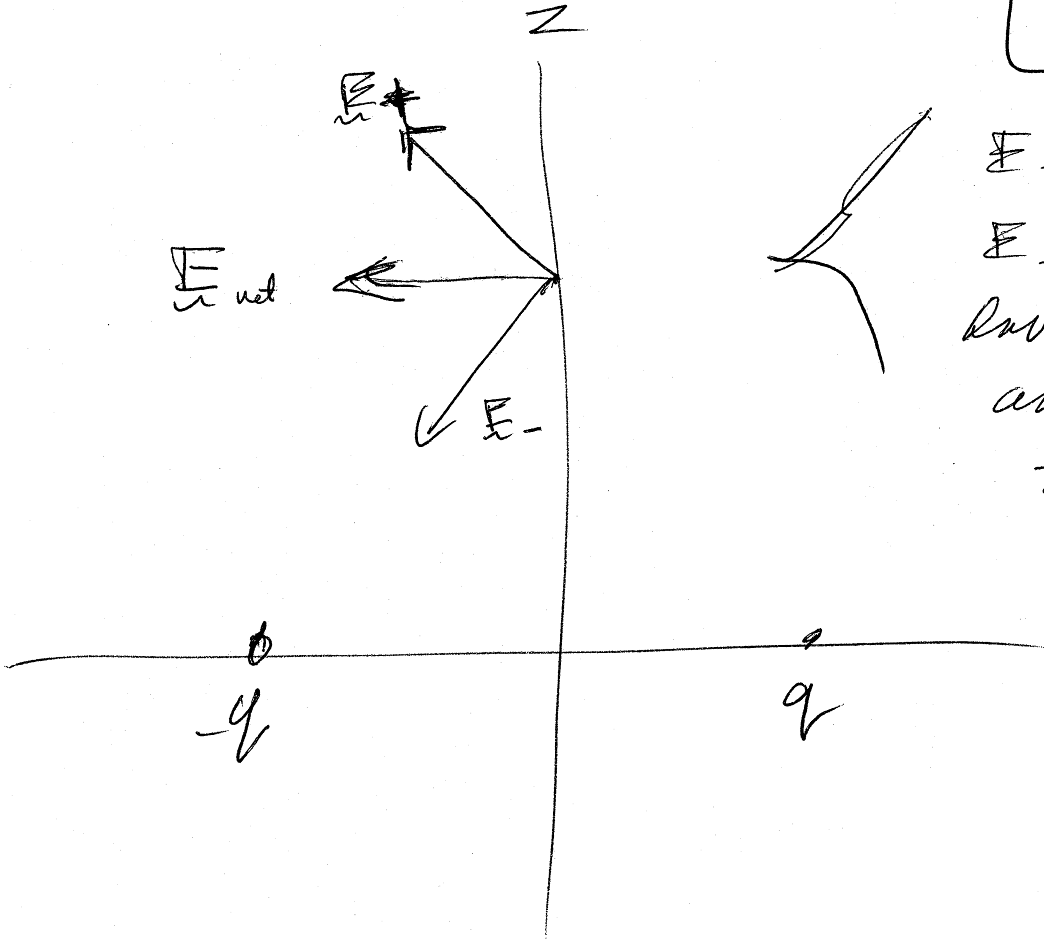
— we primarily only need the physical dipole dipole moment formula.

27-70d

E-field lines of a (physical) dipole

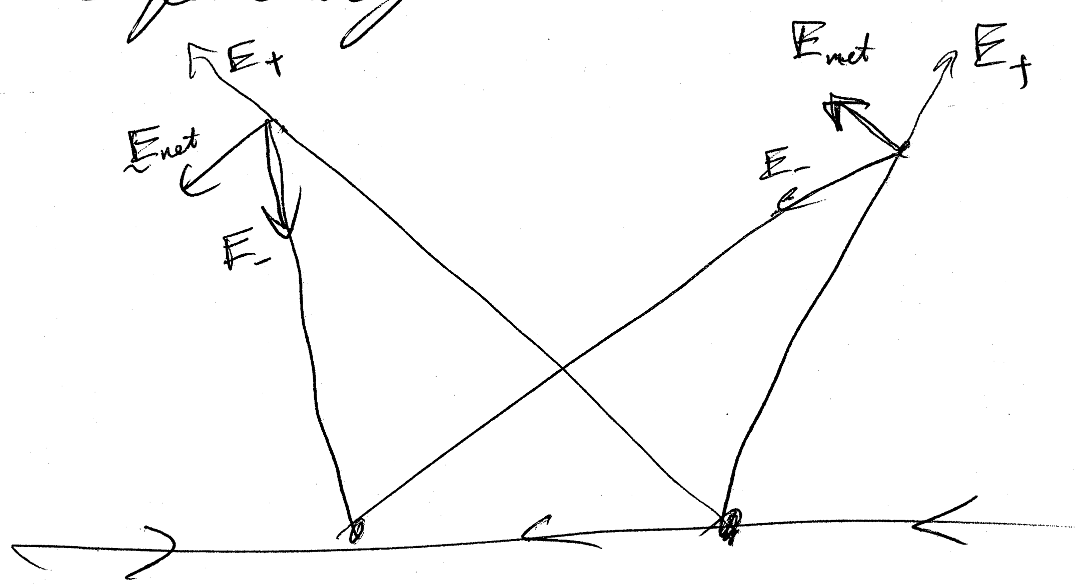


point charge like up close



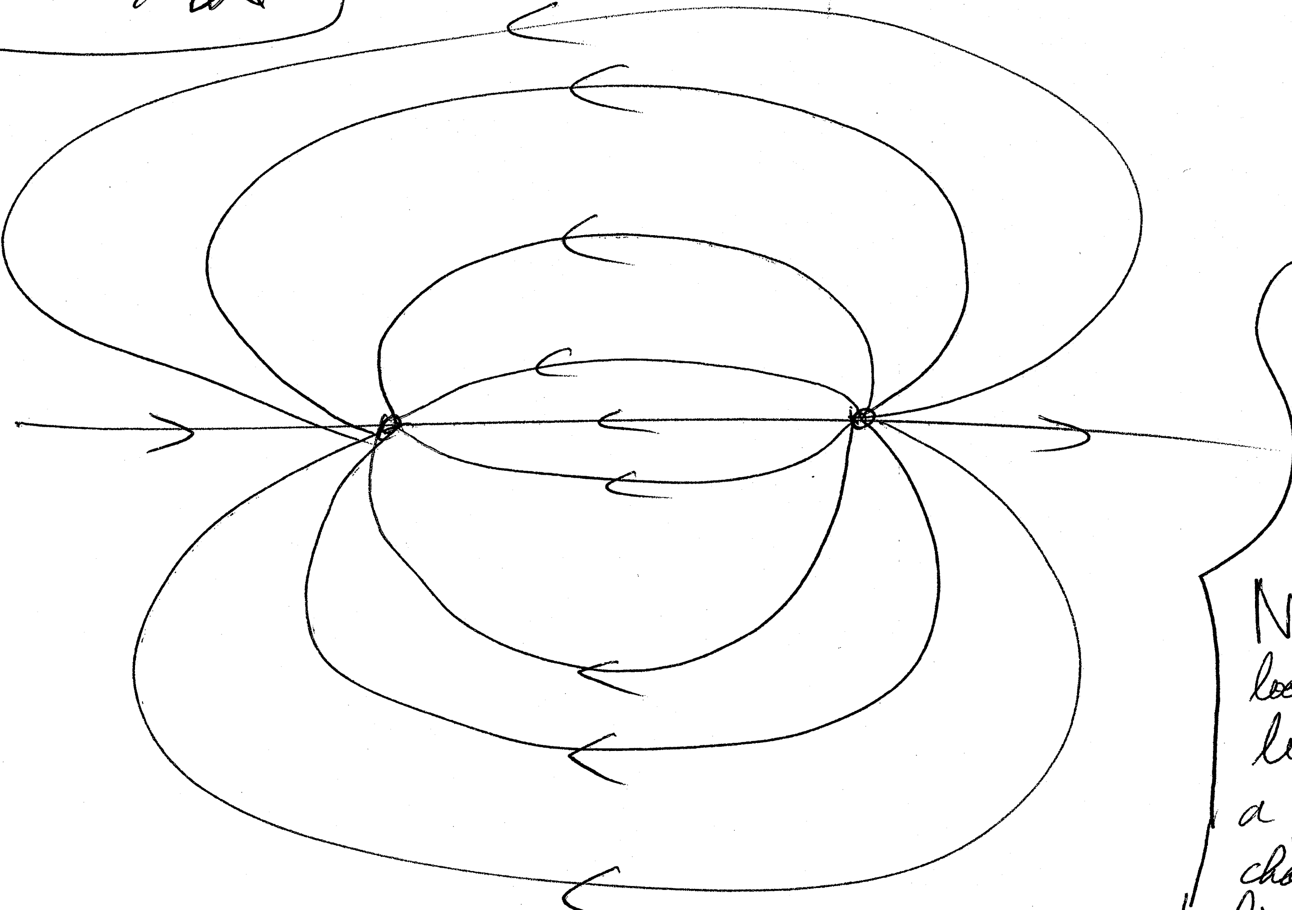
E_+
E_- must have equal and opposite z-components

More generally



So now we can qualitatively plot the whole field in cross section

23-72



There are no funny little structures — trust me.

Never looks like a point charge field in the far-field limit because it is overall neutral

— The butterfly-like dipole E-field.

— Remember it actually has axial symmetry.

What is the far-field E-field expression?

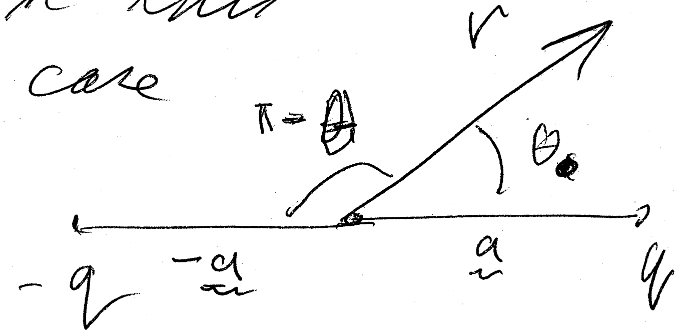
Well for $\sum q_i = 0$

$$\underset{\sim}{E}(\underset{\sim}{r}) \overset{\Delta}{=} \sum_i \frac{k q_i}{r^2}\left(-\frac{\underset{\sim}{a_i}}{r} + 3\frac{r_i}{r}\cos\theta_i\, \hat{r}\right)$$

far-field

(see p. 23-69

In this case



$$\underset{\sim}{E}(\underset{\sim}{r}) = \frac{kq}{r^2}\left(-\frac{a}{r} + 3\frac{a}{r}\cos\theta\,\hat{r}\right)$$

$$+ \frac{k(-q)}{r^2}\left(\frac{a}{r} + 3\frac{a}{r}(-\cos\theta)\,\hat{r}\right)$$

$$= \frac{k(2q)}{r^2}\left(-\frac{\underset{\sim}{a}}{r} + 3\frac{a}{r}\cos\theta\,\hat{r}\right)$$

It's usual at this point to ~~define~~ use the ~~DIPOLE MOMENT~~ dipole moment

$$\underset{\sim}{p} = 2q\underset{\sim}{a}$$

It's a vector with MKS units of Coulomb-meter

recall

$$\therefore \underset{\text{far field}}{E(\underset{\sim}{r})} = \frac{k}{r^2}\left(\frac{3p}{r}\cos\theta\,\hat{r} - \frac{\underset{\sim}{p}}{r}\right)$$

$$= \frac{k}{r^3}\left(3p\cos\theta\,\hat{r} - \underset{\sim}{p}\right)$$

dipole moment field falls off as $\sim \frac{1}{r^3}$ as we get ~~in general (usually)~~ not always) for many cases of $\sum q_i = 0$

(see p. 23-69).

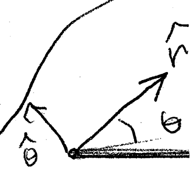
There are other standard ways of writing

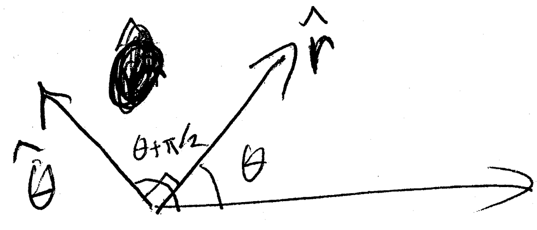
$$\underset{\sim}{E}(\underset{\sim}{r}) = \frac{k}{r^3}\left(3p\cos\theta\,\hat{r} - \underset{\sim}{p}\right)$$

$$\underset{\sim}{p}\cdot\hat{r} = p\cos\theta$$

(GrEM-155)

$$\underset{\sim}{E}(\underset{\sim}{r}) = \frac{k}{r^3}\left(3(\underset{\sim}{p}\cdot\hat{r})\,\hat{r} - \underset{\sim}{p}\right)$$





In spherical polar coordinates

$$\underset{\sim}{p} = p(\cos\theta, \cos(\theta+\pi/2))$$

$$= p(\cos\theta, -\sin\theta)$$

$$\underset{\sim}{E} = \frac{k}{r^3}\left(3p\cos\theta\,\hat{r} - p(\cos\theta\,\hat{r} - \sin\theta\,\hat{\theta})\right)$$

$$= \frac{k}{r^3}\left(2p\cos\theta\,\hat{r} + p\sin\theta\,\hat{\theta}\right)$$

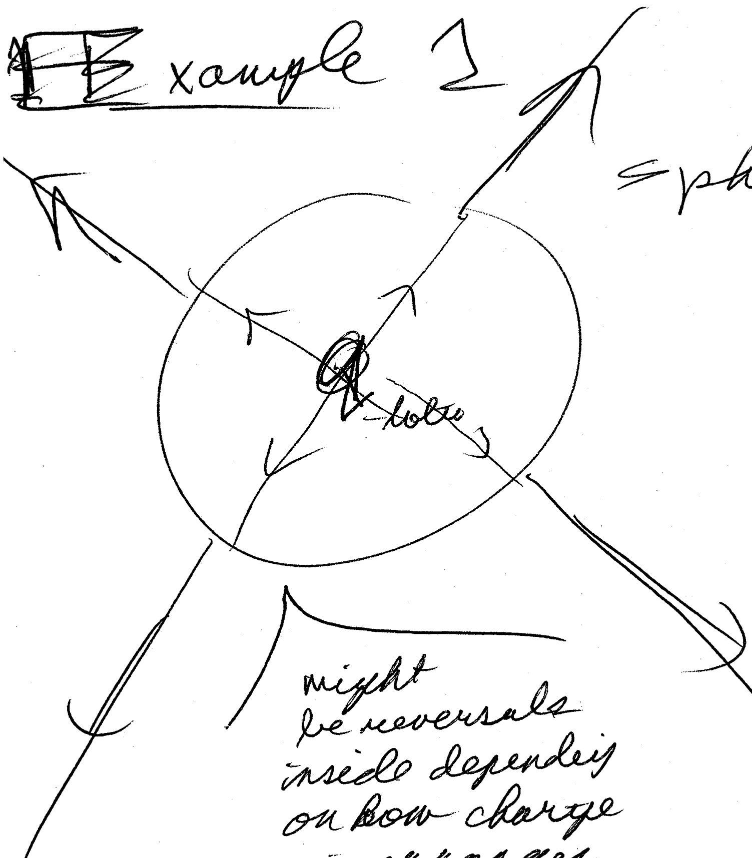
(GrEM-153)

23-76

~~one more example~~

A few more examples of Field lines without math

Example 1



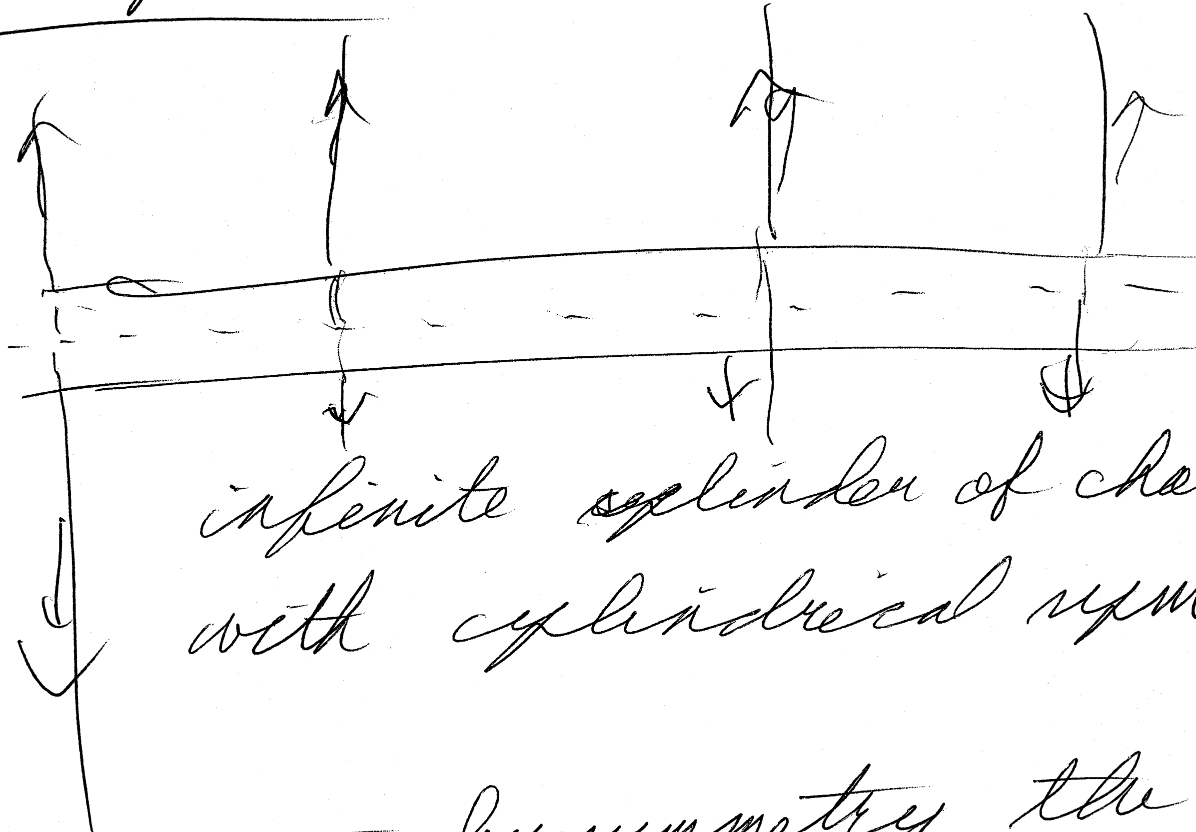
spherically symmetric charge distribution

$\vec{E}$ is radially outward (or inward if $q_{tot} < 0$) by symmetry.

In fact outside the distribution the $\vec{E}$-field is the same as point charge of q_{tot} at the origin

We'll prove this by Gauss' law in Ch 24.

Example 2

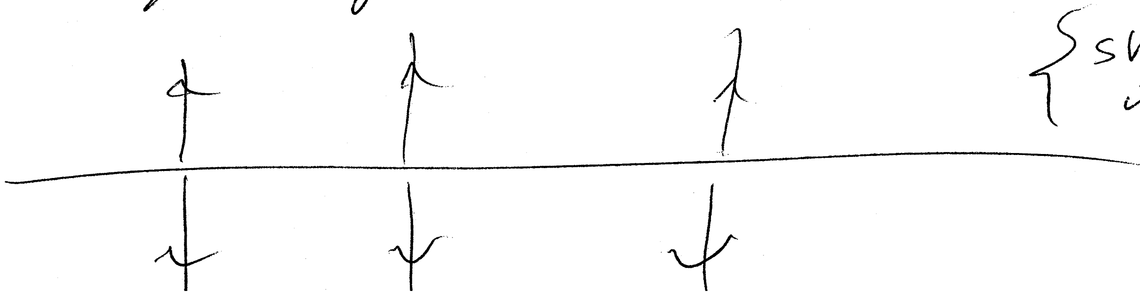


infinite ~~cylinder~~ cylinder of charge
with cylindrical symmetry

— by symmetry the
field lines run radially
outward to infinity
~~of~~ the direction depends
on the cylindrical
charge distribution

Example 3

Infinite plane of ~~uniform~~ charge uniformly spread

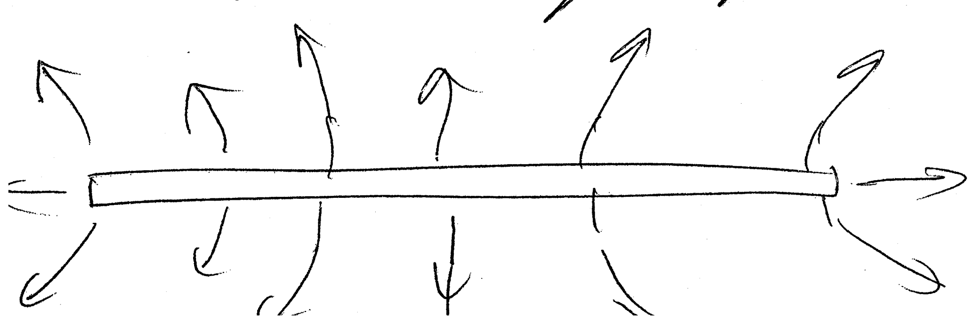

{ shown in cross section.

23-78

By symmetry the field lines must run straight away from the plane on either side.
(or toward if the surface charge density $\sigma < 0$)

There are no infinite cylinders or planes of charge of course.

But close to ~~a~~ finite ones the fields are approximately those of infinite cases.



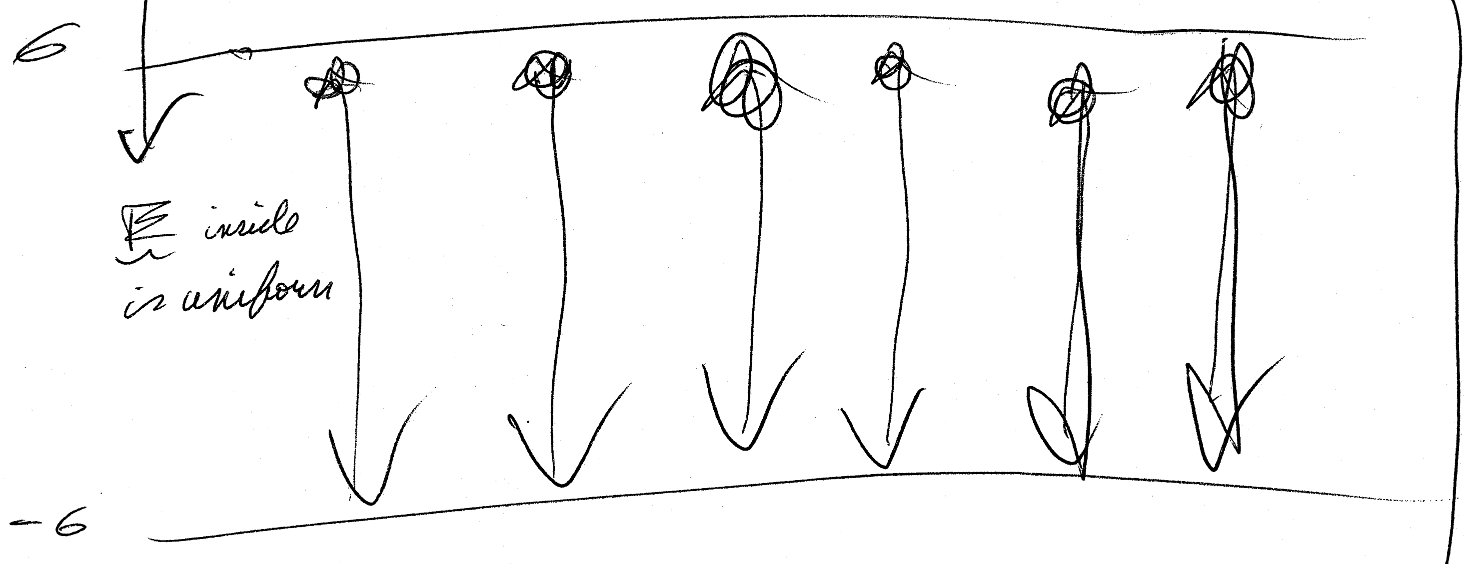
finite cylinder of charge
— far away its like a point charge.

Close to, it's like an infinite cylinder.

Example 4 Infinite Parallel

Planes of charge of uniform charge density σ and $-\sigma$

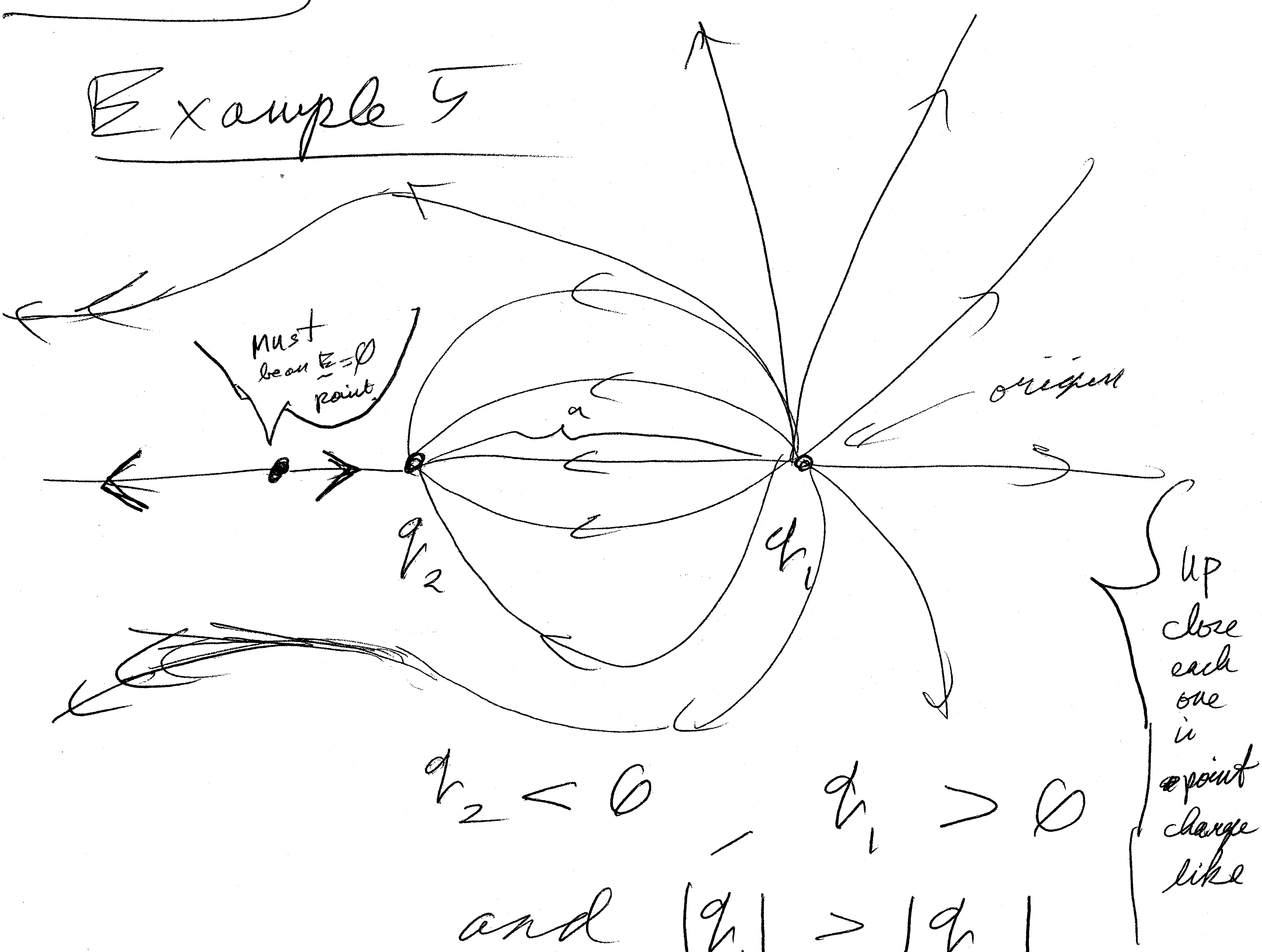
$\underline{E} = 0$ ← prove using Gauss's law in Chapter 24



$\underline{E} = 0$

A parallel plate capacitor approximates this case.

Example 5



$q_2 < 0$, $q_1 > 0$

and $|q_1| > |q_2|$

(Serway - 661)

In far field it must be like a point charge of charge $q_1 + q_2$ to zeroth order in $\frac{a}{r}$

$\underset{\sim}{E} = \frac{k(q_1+q_2)}{r^2}\hat{r}$ (see p. 23-68)

Example 6

— this is just a foreshadowing

(an adumbration even)

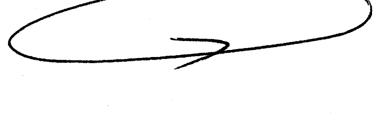
— a ~~changing~~ time-varying magnetic field gives rise to an electric field without charge present.

Faraday's law
Ch. 31

— such fields have field lines with ~~to~~ no ends.

they extend to infinity or they can form closed loop

E



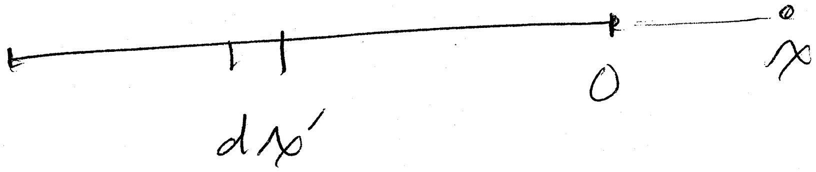
23-82)

There is a reciprocal process

— a time varying electric field causes a magnetic field. (B-field)

In fact, in EMR (electromagnetic radiation), both processes occur where time varying E-fields and B-fields give rise to each other and cause self-propagation without charge around.

23-84) divide and conquer



$$d\underset{\sim}{E} = \frac{k\lambda\, dx'}{(x-x')^2}\,\hat{x}$$

Just using Coulomb's law where $dq = \lambda\, dx'$

$$\underset{\sim}{E} = \int_{-\ell}^{0} \frac{k\lambda\, dx'}{(x-x')^2}\,\hat{x}$$

x' is the dummy variable not x.

$$= \frac{+k\lambda}{(x-x')}\Bigg|_{-\ell}^{0}\,\hat{x}$$

$$= +k\lambda\hat{x}\left[\frac{1}{x} - \frac{1}{(x+\ell)}\right]$$

$$= \frac{k\lambda\ell\,\hat{x}}{x(x+\ell)} = \frac{kQ\,\hat{x}}{x(x+\ell)}$$

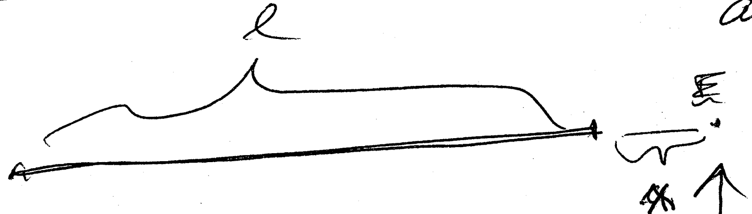
(Serway-657 mutatis mutandis)

§ 23.5 $\underline{E}$-field of ~~continuous charge~~ distributions

We need to do some integrations.

– We'll just do a couple not so tedious examples.

Ex 23.6 Thin rod of length l and total charge Q.



$\lambda = \frac{Q}{l}$ is the linear charge density.

What is $\underline{E}$ at this point on the axis.

We'll assume $Q > 0$ and so $\underline{E}$ points in the $\hat{x}$ direction.

(If Q ~~equals zero~~ < 0, it just points the other way.

Note if $x \to 0$,

$\underset{\sim}{E}$ diverges — sort of like going to a point charge.

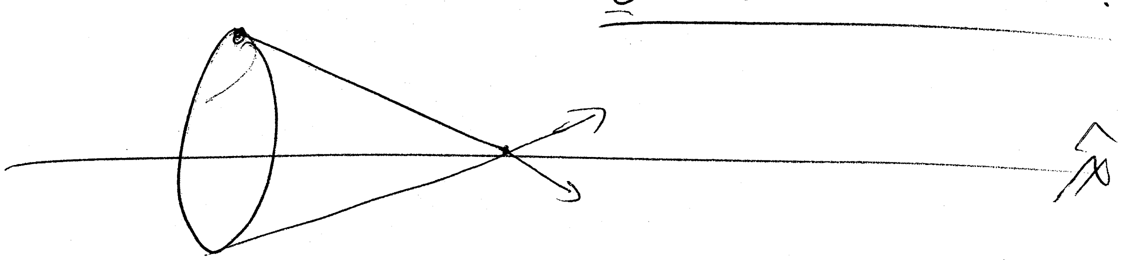
If $x \gg l$,

$$\underset{\sim}{E} = \frac{kQ\hat{x}}{x^2}$$ just

as we'd expect from our general analysis on p. 23-66.

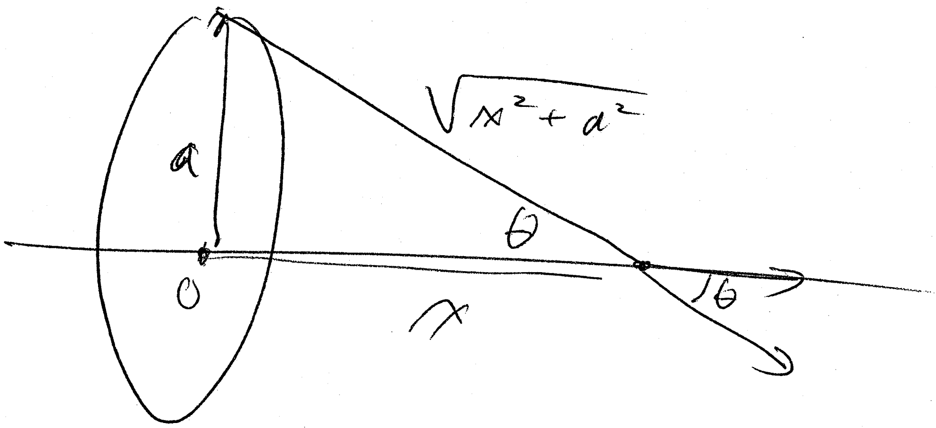
Ex 23.7 E-field of a uniform thin ring

total charge $Q > 0$ on the axis.



23-8b

By symmetry all non-x components cancel out.



We can integrate around the ring instantly
$d\underline{E} = \dfrac{k \frac{Q}{2\pi a}\, a\, d\phi}{x^2+a^2} \cos\theta$;
integrate ϕ from 0 to 2π.

So $$\underline{E}(x) = \frac{kQ}{x^2+a^2}\cos\theta\,\hat{x}$$

where $$\cos\theta = \frac{x}{\sqrt{x^2+a^2}}$$

$$\underline{E}(x) = \frac{kQx}{(x^2+a^2)^{3/2}}\,\hat{x}$$

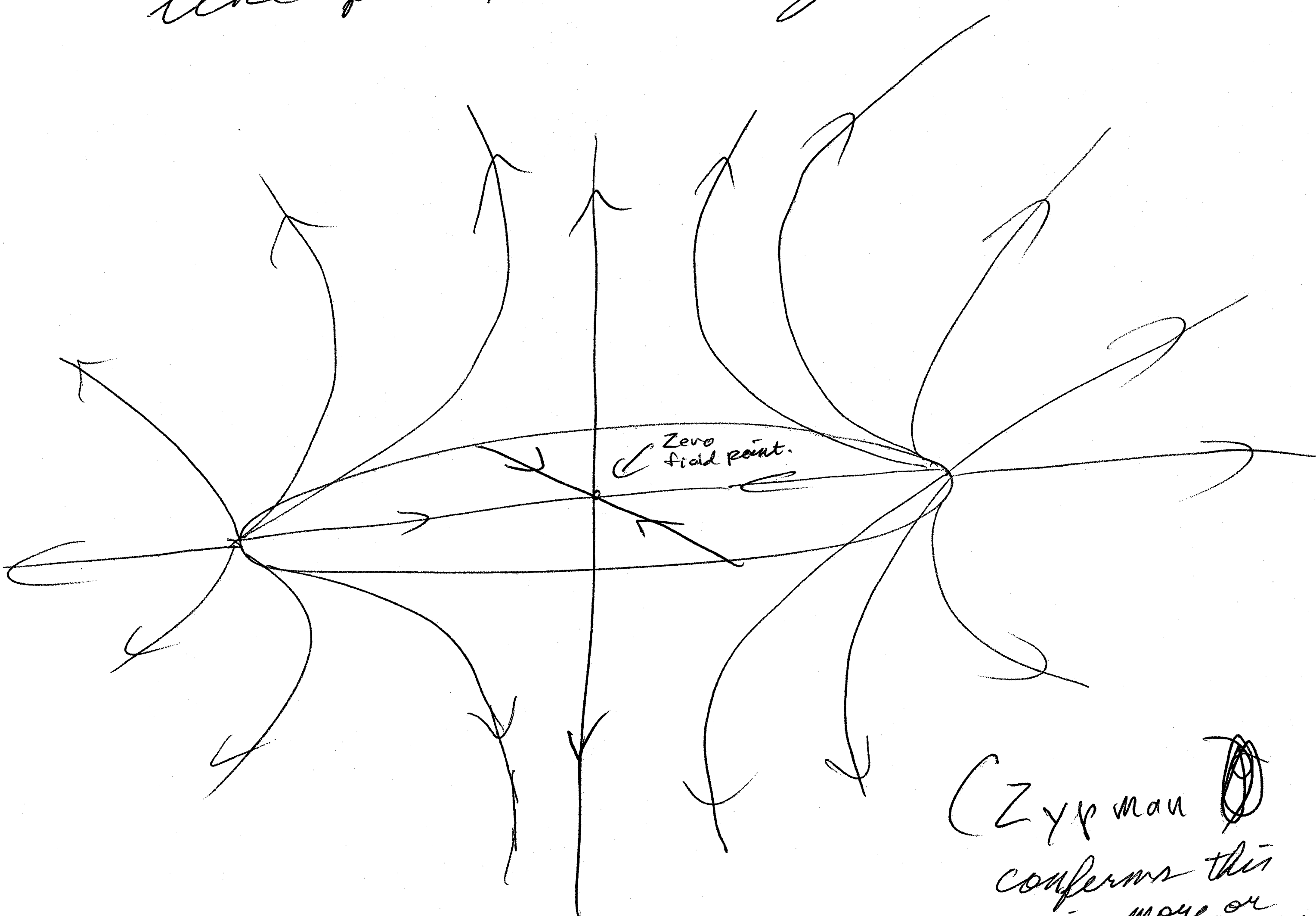
$$\underline{E}(0) = 0$$
$$\underline{E}(x \gg a) = \frac{kQ}{x^2}\hat{x}$$ as expected.

~~a charge q put there is in equilibrium but unstable equilibrium if $q>0$~~

~~a slight perturbation gives a positive~~

What is the E-field like for the ring?

23-87



(Zypman confirms this is more or less right)

Something like this.
In the far field, it must be like a point charge of Q as we proved on p. 23-66

A ~~positive~~ charge q ~~>0~~ is at equilibrium at the ring center.

Since $\vec{E}=0$ there.

23-88

~~But it's an unstable equilibrium~~

~~— any perturbation out of the plane and it will be accelerated off to infinity.~~

~~What of $q < 0$ at the ring center.~~

~~— It's also an equilibrium.~~

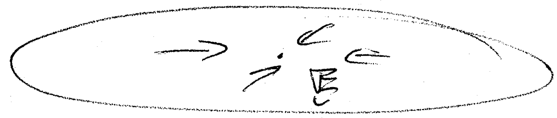
~~— Is it stable?~~

~~— for any perturbation out of ring plane, there is a restoring force.~~

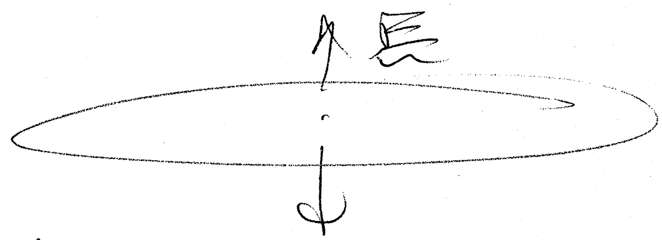
~~— But in the plane?~~

But it's an unstable equilibrium.

A positive charge is stable to perturbation in plane of ring since field points inward



but unstable to perturbations out of plane since ~~pla~~ the field points outward.



any perturbation and the +ve charge will accelerate off to infinity

Actually that this is so is plausible, but to prove it takes a lot of work see Zypman.

A -ve charge is in the reverse situation.
- ~~any~~ perturbation out of plane is restored, but in the plane and the charge accelerates to the ring.