Lecture 7

A Center of Mass and $F = ma$

B Conservation of Momentum

C Impulse - Momentum Theorem

D Collisions in One dimension

F) Energy + Energy Thm for systems of particles

A) Center of Mass and $F = ma$

Hitherto we've regarded objects as particles, which doesn't mean small necessarily

— It just means we can ignore their

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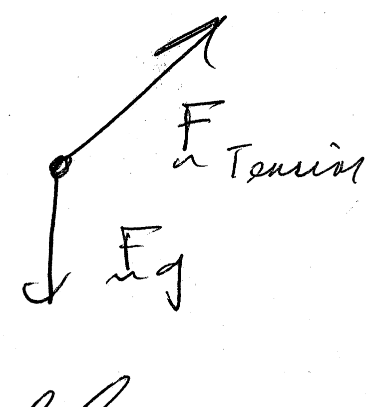
internal structure and motions.

→ and just treat them as being all at one point both for motions of the object and forces on the object as in our free-body diagrams.

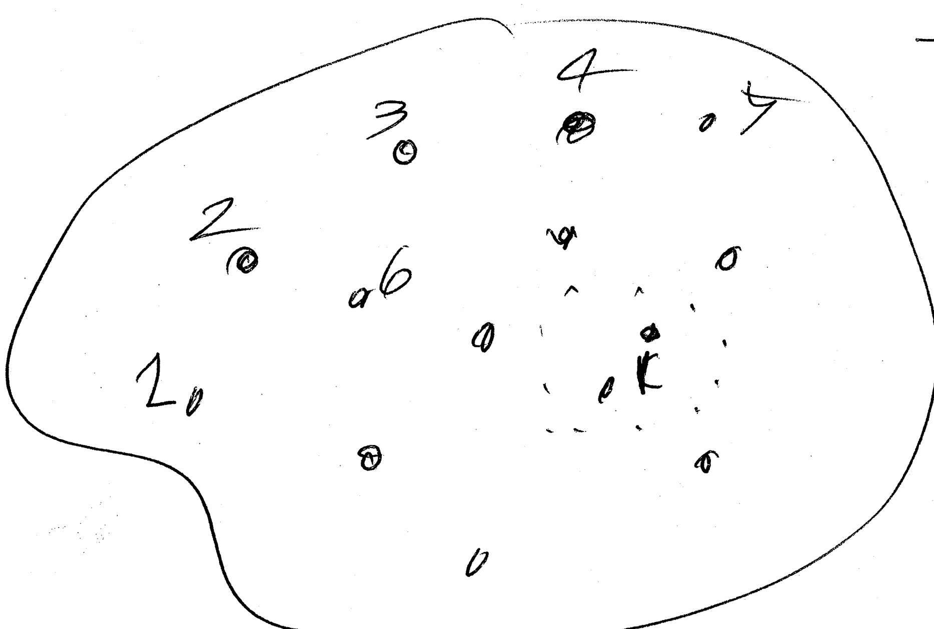


How is this procedure justified.

→ by the concept of center of mass and generalization or special application.

of $\underset{\sim}{F}_{net} = m\underset{\sim}{a}$

Consider a system of particles



— they may make up a rigid object or a squishy object or they may be a lump of ~~l~~ liquid or a gas.

Whatever.
We are being general.

— the particles are as small as we need them to be so that they really have ignorable structure — ~~but~~ the assumption

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is ~~that there~~

is such a small enough.

But the particles aren't so small as being quantum mechanical objects → Newtonian physics still applies to them.

Consider particle i

— a general particle

— it has mass m_i

— and on particle there is a net force

$\underset{\sim}{F}_i$

for simplicity I don't write net but it's implied.

Reasonable. I think one can say that the external ~~motio~~ forces govern the mean motion of the "particles". Now there are many kinds of means but as the particles grow small all means should asymptotically converge to a single value — the true particle limit. We can get as close to that as to be virtually exact while still in the macroscopic realm for macroscopic systems.

F = ma can be applied to particle i

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$\underset{\sim}{F}_i = m_i \underset{\sim}{a}_i$ } acceleration of particle i

Now $\underset{\sim}{F}_i$ can be partitioned or summandized into internal and external contributions

} factorize numbers ~~[illegible]~~ break into factors

summandize is to break into summands or addition terms or addendize

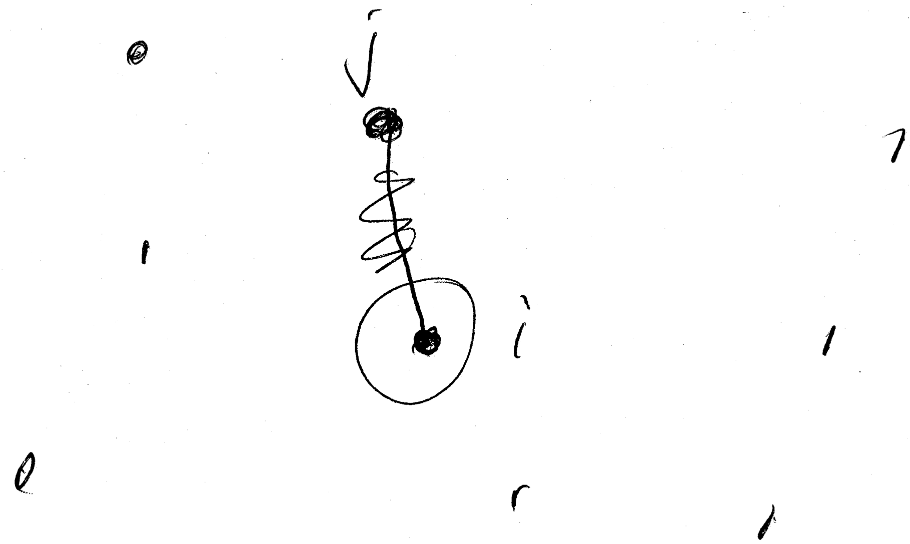
$$\underset{\sim}{F}_i = \underset{\sim}{F}_{i\,ext} + \underset{\sim}{F}_{i\,int}$$

— nothing forbids this

$\underset{\sim}{F}_{i\,ext}$ is forces on i from outside the system

$\underset{\sim}{F}_{i\,int}$ is the forces from inside the system.

$\boxed{7-6}$



Let $\underset{\sim}{F}_{ji}$ be the force particle j exerts on particle i

$$\underset{\sim}{F}_{i\,int} = \sum_{\substack{j \\ j \neq i}} \underset{\sim}{F}_{ji}$$

sum over all particles in the system.

So $\underset{\sim}{F}_i = m_i \underset{\sim}{a}_i$

$$\underset{\sim}{F}_{i\,ext} + \sum_{\substack{j \\ j \neq i}} F_{ji} = m_i a_i$$

$\boxed{7\text{-}8}$

$$\sum_i \underset{\sim}{F}_{i\,ext} = \sum_i m_i \underset{\sim}{a}_i$$

$$\parallel$$

$$\underbrace{\underset{\sim}{F}_{ext}}_{\text{The net external force}}$$

Define $m = \sum_i m_i$ as the total mass.

$$\underset{\sim}{F}_{ext} = m \underbrace{\left(\frac{\sum_i m_i \underset{\sim}{a}_i}{m} \right)}_{\underset{\sim}{a}_{CM}}$$

The acceleration of the center of mass.

Nothing forbids us from summing over all particles.

$$\underbrace{\sum_i \mathbf{F}_{i\,ext} + \sum_{\substack{i\,j \\ i \neq j}} \mathbf{F}_{ji}}_{\text{For each force } \mathbf{F}_{kl} \text{ there is a force } \mathbf{F}_{lk}} = \sum_i m_i a_i$$

~~Now remember~~ by the 3^{rd} law — for every force there is an equal (in magnitude) ~~and opposite)~~ and opposite force.

$\therefore$ ~~for~~ $\mathbf{F}_{lk}$ is the equal and opposite force to $\mathbf{F}_{kl}$

$$\mathbf{F}_{lk} = -\mathbf{F}_{kl}$$

Note ~~these~~ forces do things — they can cause internal acceleration but they cancel out of this sum.

$$\therefore \sum_{\substack{i\,j \\ i \neq j}} \mathbf{F}_{ji} = 0$$

The forces cancel out pairwise.

The center of mass itself is

$$\underset{\sim}{r}_{cm} = \frac{\sum_i m_i \underset{\sim}{r}_i}{M}$$

Now

$$\underset{\sim}{v}_{cm} = \lim_{\Delta t \to 0} \frac{\Delta \underset{\sim}{r}_{cm}}{\Delta t}$$

$$= \frac{\sum_i \lim_{\Delta t \to 0} \frac{\Delta \underset{\sim}{r}_i}{\Delta t}}{M}$$

$$\underset{\sim}{a}_{cm} = \lim_{\Delta t \to 0} \frac{\Delta \underset{\sim}{v}_{cm}}{\Delta t}$$

$$\cancel{\lim} = \frac{\sum_i \overbrace{\left(\frac{\lim \Delta \underset{\sim}{v}_i}{\Delta t} \right)}^{\underset{\sim}{a}_i}}{M}$$

So the $\underset{\sim}{a}_{cm}$ is the acceleration of the center of mass point

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as foreshadowed.

The center of mass is a mass-weighted average position for the system.

So all along when we've used

$$\vec{F}_{net} = m\vec{a}$$

for finite objects we've really been using

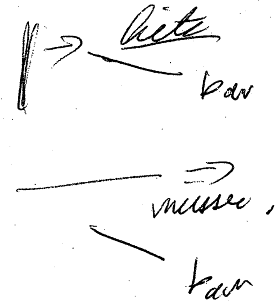
$$\vec{F}_{ext} = m\vec{a}_{cm}.$$

(net external force)

<u>Note</u> - an extended system is not a particle really.

Which external forces act on a system depend on its structure.

Ex Whether a flying ruler hits a bar depends in part on its orientation

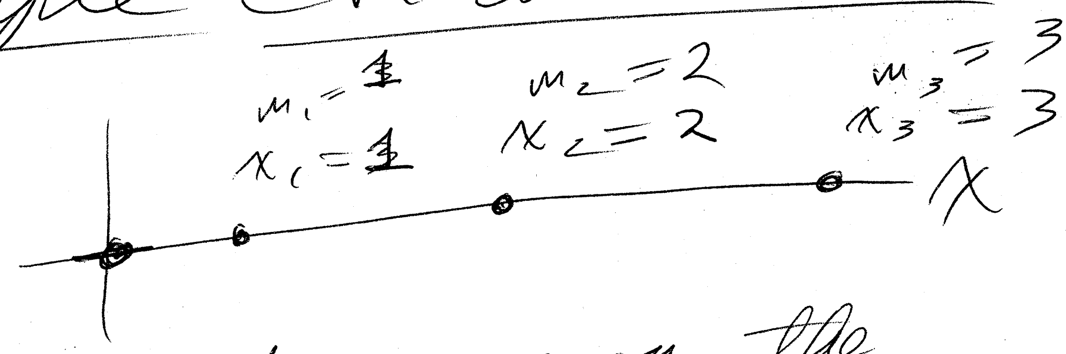


internal forces do things but they don't affect the CM motion

→ and if the CM motion is all you need then one can neglect them as we have hitherto.

Example CM calculation

1)



Three point masses on the x-axis

$$x_{cm} = \frac{\sum_i m_i x_i}{\sum m_i} = \frac{1^2 + 2^2 + 3^2}{1 + 2 + 3} = \frac{14}{6}$$

$$= 2.333\ m.$$

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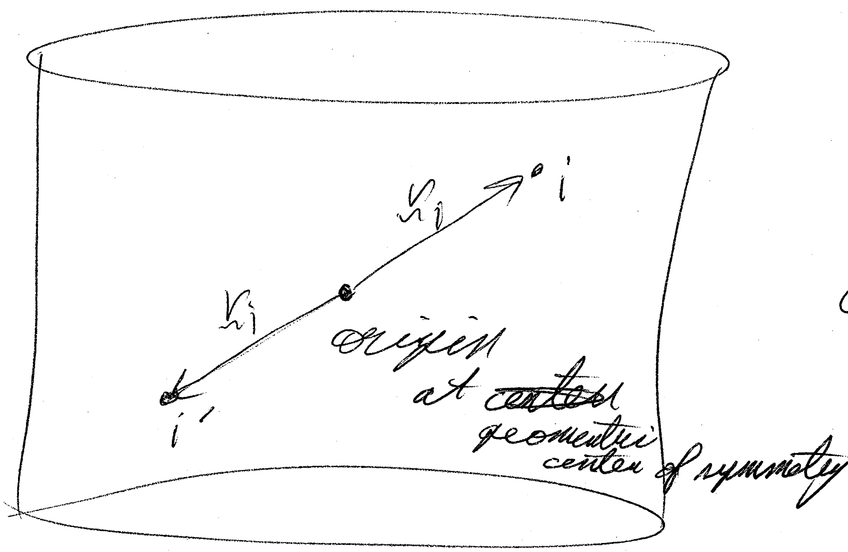
symmetric in 3-dimension

Ex 2

Any symmetric Object of constant density.



Cylinder as an example

Consider the symmetric particles i and i'

in summation $\sum_i m_i \underset{\sim}{r}_i$

one has $m_i \underset{\sim}{r}_i + m_{i'} \underset{\sim}{r}_{i'}$

but $m_i = m_{i'}$

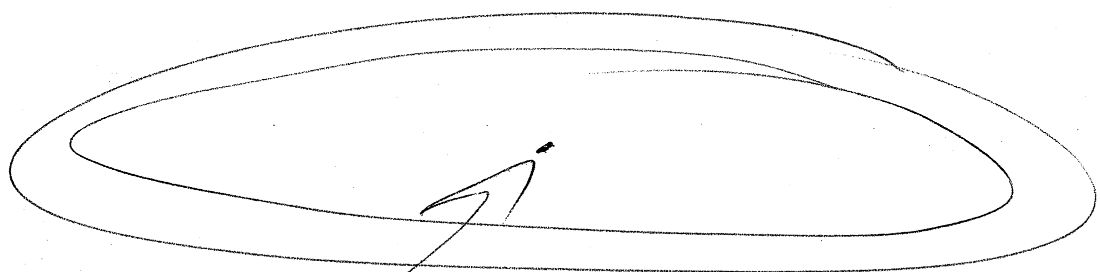
and $\underset{\sim}{r}_{i'} = -\underset{\sim}{r}_i$

$\therefore$ the pair cancels out and this happens for all particles. $\therefore$ $r_{cm} = 0$ for origin at center of symmetry.

Thus the CM is where you'd guess, at the geometric center of symmetry.

— Any place else would be asymmetric and there is no cause for that.

Where is the CM of a hula-hoop?



at the center of symmetry.

— The CM does NOT have to be at a material point

it doesn't have to be in the object.

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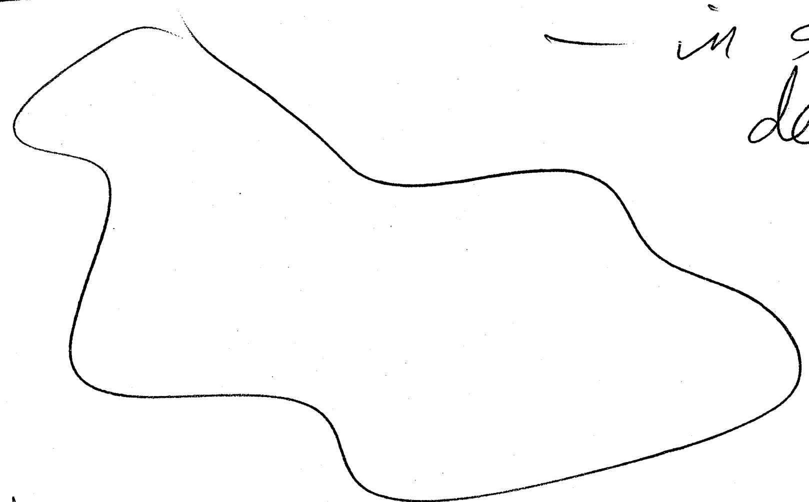
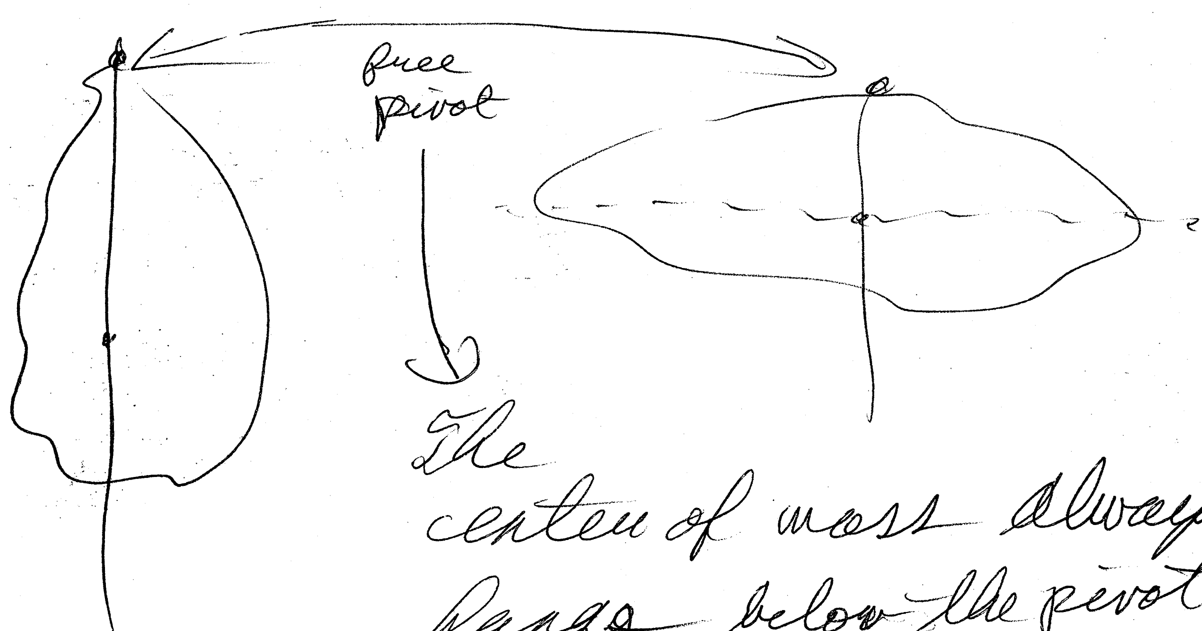
Internal forces hold it rigid.

3 Irregular rigid Object

— in shape, density.



— hw tricky
— calculation is tough
but a simple measurement helps.



The center of mass always hangs below the pivot point for a resting object.

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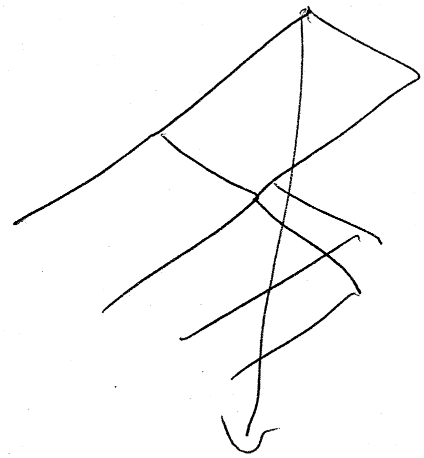
— no proof here — actually takes rotational dynamics which we'll skip probably

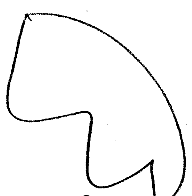
— but you can find it.

—  chair example.

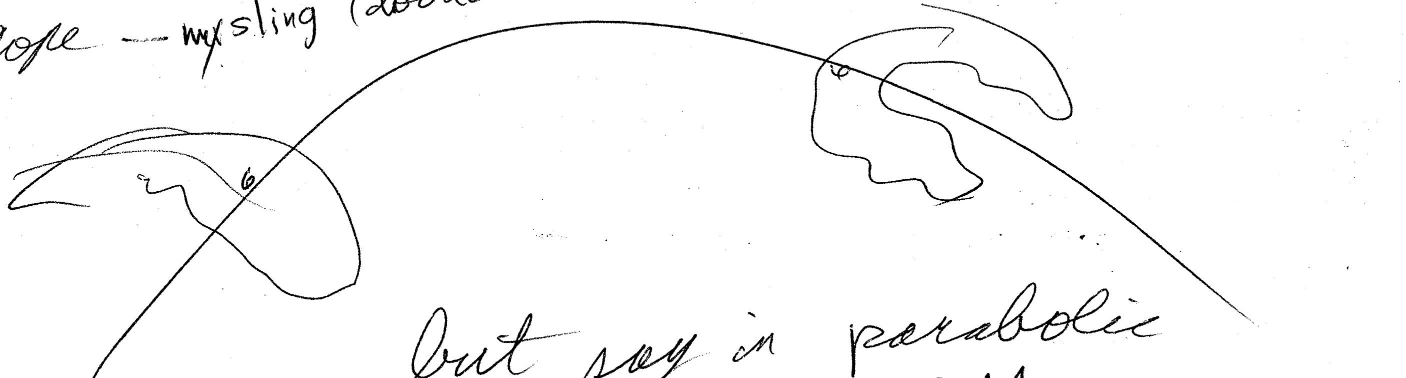
4) Squishy or flexible object.

— the CM can move relative to the constituent particles

internal forces partially control the squishy.



rope — ~~my~~ sling (too dominated by drag)



but say in parabolic flight the CM follows the Parabola.

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B) Momentum and Conservation of Momentum.

We define a new dynamic variable that turns out to be useful.

Linear momentum or momentum for short

— the other kind of momentum is angular momentum (always called ang. mom.)

$$\underset{\sim}{p} = m \underset{\sim}{v}$$

symbol p usually

SI units kg m/s
- no special name or symbol
- almost no other units of momentum are used.

It is a vector.

For a system of particles

$$\underset{\sim}{p}_i = m_i \underset{\sim}{v}_i$$ for each particle.

$$\sum \underset{\sim}{p}_i = \sum_i m_i \underset{\sim}{v}_i$$

$$\parallel$$

$$\underset{\sim}{p}_{total} = m_{total} \left(\frac{\sum m_i \underset{\sim}{v}_i}{m} \right)$$

$$\underset{\sim}{p}_{total} = m_{total} \underset{\sim}{v}_{CM}$$

$$\underset{\sim}{p} = m \underset{\sim}{v}$$ drop subscripts if you know what you mean.

Now consider

either a particle or a system of particles — the equation can be read either way

$$\underset{\sim}{F}_{net} = m \underset{\sim}{a} = m \lim_{\Delta t \to 0} \frac{\Delta \underset{\sim}{v}}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{m \Delta \underset{\sim}{v}}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta \underset{\sim}{p}}{\Delta t}$$

$\boxed{7\text{-}18}$

true for a particle and for a system of particles ~~by the same~~

$$\mathop{F}_{\substack{net\\ext}} = \lim_{\Delta t \to 0} \frac{\Delta P_{total}}{\Delta t}$$

A new version of Newton's 2nd law

— we've derived it assuming constant mass

— but it actually applies for mass-varying objects if one introduces momentum transport or flux forces.

but we won't go there. — beyond our scope.

What if $\underset{\sim}{F}_{\substack{net \\ ext}} = 0$ for a finite time

$a_{cm} = 0$

v_{cm} constant

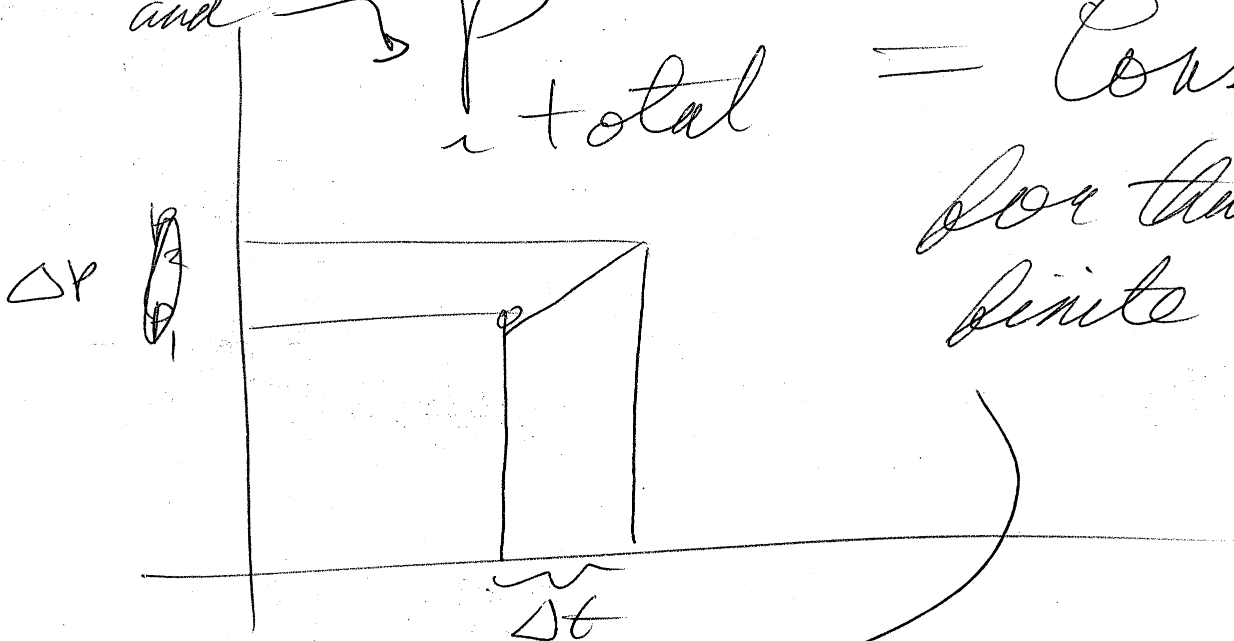
$P_{total} = m \underset{\sim}{v}_0$ constant

Then the instantaneous rate of change of momentum is zero and

$$0 = \lim_{\Delta t \to 0} \frac{\Delta P_{total}}{\Delta t}$$

$$\underset{\sim}{P}_{total} = \text{Constant}$$

for that finite time.



In the absence of external forces total momentum is conserved.

Internal forces can act

Momentum can get redistributed among the particles, but total momentum is conserved.

For some useful numerical examples using this conservation law, we need the Impulse-Momentum Theorem

C) Impulse-Momentum Theorem

Recall $\underset{\sim}{F}_{net} = \lim_{\Delta t \to 0} \frac{\Delta P}{\Delta t}$

Just think of it for a particle in this case.

$$\underset{\sim}{F}^{net}_{ave} = \frac{\Delta P}{\Delta t}$$

time averaged

Really to prove this rigorously from Newton's 2nd law we need integration.

— but it's plausible without a proof.

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So we believe

$$\underset{\sim}{F}_{\text{ave net}} = \frac{\Delta \underset{\sim}{p}}{\Delta t}$$

$$\therefore \quad \Delta \underset{\sim}{p} = \underbrace{\underset{\sim}{F}_{\text{ave net}} \Delta t}$$

We define ~~this~~ to be impulse $\underset{\sim}{J}$

or net impulse to be exact

$$\underset{\sim}{J} \equiv \underset{\sim}{F}_{\text{ave}} \Delta t$$

And Impulse-Momentum Thm is

$$\Delta \underset{\sim}{p} = \underset{\sim}{J}$$

$\underset{\sim}{J}_{\text{force } i} = \underset{\sim}{F}_{i \text{ ave}} \Delta t$
So ~~there~~ each force can give an impulse.

— it's really just Newton's 2nd law in ~~any~~ ~~be~~

another version.

Why have another version?

— For ~~general problems~~ exact work

no need.

— But the Impulse-Momentum Thm

~~allows one to approximate~~

leads to useful approximation

Recall

$$\Delta \underset{\sim}{P} = \underset{\sim}{J} = \underset{\sim}{F}_{ave, net} \Delta t$$

So net average force

to be exact.

But say for a short

time interval Δt

$$\underset{\sim}{F}_{net} = \underset{\sim}{F}_{strong} + \underset{\sim}{F}_{other}$$

where

$$\underset{\sim}{F}_{strong} \gg F_{other}$$

then $\underset{\sim}{J} \cong \underset{\sim}{F}_{strong} \Delta t$

for Δt

and $\Delta p \cong \underbrace{\underset{\sim}{F}_{strong} \Delta t}_{\underset{\sim}{J}_{strong\ force}}$.

Such situations happen in, e.g., collision events where during the collisions only the ~~only the~~ collision forces are non-negligible.

In collision approximation neglect other forces.

Ex Hit Baseball

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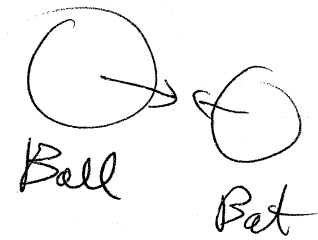
— given $\Delta t = 1.6 \times 10^{-3}\ s$

measured by fast camera for example.

collision time

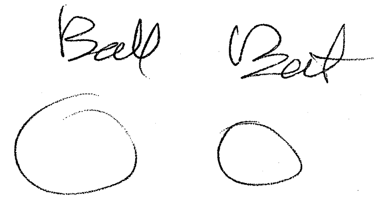
— time of contact between ball & bat



— $v_i = -38\ m/s$

$v_f = 58\ m/s$

$m = .14\ kg$



Solve for $\underset{\sim}{F}_{\text{bat on}}$ of bat on ball

the collision force

$$\Delta p = m(v_f - v_i)$$

$$= .14 * (96)$$

$$\simeq 14\ kg\,m/s \text{ in positive x-direction}$$

Now $\Delta \underset{\sim}{P} = \underset{\sim}{J}_{\cancel{net}}$

$$\underset{\sim}{J} = (\underset{\sim}{F}_{bat} + \underset{\sim}{F}_{grav} + \underset{\sim}{F}_{air\ resistance})\Delta t$$

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$$= \underset{\sim}{J}_{Bat} + \underset{\sim}{J}_{grav} + \underset{\sim}{J}_{air}$$

$$mg(-\hat{y})\Delta t$$

$$= .14 \cdot 10 \cdot 1.6 \times 10^{-3}$$

$$\simeq 2.5 \times 10^{-3}\ kg\ m/s$$

$$|\underset{\sim}{J}_{grav}| << |\underset{\sim}{J}| = |\Delta r|$$

Well we haven't studied quantitatively but in this context

$$|\underset{\sim}{F}_{air}| \sim |\underset{\sim}{F}_{grav}|$$

— they are of the same size scale

Thus gravity and air resistance are negligible and we can make the collision approximation

$$\therefore \quad \Delta \underline{P} = \underset{\sim}{J} \cong J_{bat}$$

$$= \underset{\sim}{F}_{bat} \Delta t$$

$$F_{bat} = \frac{|\Delta \underset{\sim}{P}|}{\Delta t}$$

$$\simeq \frac{14}{1.6 \times 10^{-3}}$$

$$\doteq 9 \times 10^{3}$$

$$\doteq 10^{4}\ N$$

Ans 8400N

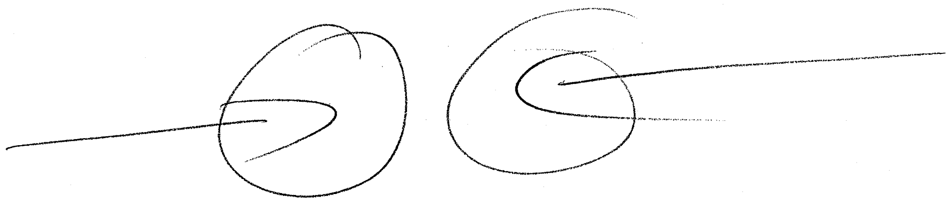
$\simeq$ 2000 lb.

— So if you get hit by a swung bat, ~~the force~~ is really, really big.

7-28)

D) Collisions

— In physics ~~as~~ in life collisions are events in which relatively strong forces act ~~for~~ relatively ~~sh~~ short times.



— at our level we usually think of balls colliding in empty space

— traditionally for intro analysis we break collisions into 3 broad classes

a) inelastic collisions } KE not conserved

b) perfectly or completely inelastic collisions } the objects stick together { subset of (a) really

c) elastic collisions in which KE is conserved

~~the~~ not at every moment of time necessarily ~~but~~ but before and after KE ~~a~~ are the same value

We'll just do two body collisions in 1-d.

That's enough to get the flavor and tricky enough.

We'll make the collision approximation — only collision forces ~~act~~ are non-negligible during the collisions.

~~d) Inelastic collisions with no sticking together.~~

~~d) So~~

and we'll regard those forces as internal to the system of colliders.

$$\therefore \quad \underset{\sim}{P}_{total} = \sum_i \underset{\sim}{p}_i$$

is conserved during the collision

$$\underset{\sim}{F}_{net\ ext} \cong 0$$

approximately.

— So thru the collision

momentum is conserved.

a) Inelastic collisions

— total momentum conserved thru collision

$$m_1 v_{i1} + m_2 v_{i2} = m_1 v_{f1} + m_2 v_{f2}$$

is the equation we have.

→ ordinarily in we think of solving for the future (outcome) given the present / past

→ v_{i1}, v_{i2} are past

v_{f1}, v_{f2} are future

— we assume the masses are known.

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With one equation we can't solve for the future of 2 variables

The complete outcome.

— conservation of momentum is insufficient alone.

— detailed ~~knowledge~~ analysis of the collision interaction would would allows to solve for the future

But we don't want to do that.

— So we must be given ~~one~~ some of the future v_{f1} or v_{2f} ~~and~~ and we can solve for the other.

More generally given any 5 of the 6 variables we can solve for the 6th and ~~the~~ without more machinery, that's the ~~[illegible]~~ all we can do.

Ex 1

$m_1 = 1\ kg$

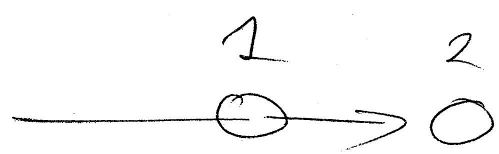
$m_2 = 2\ kg$

$v_{1i} = 2\ m/s$

$v_{2i} = 1\ m/s$

$v_{1f} = ?$

$v_{2f} = 6\ m/s$



1 catches up to 2 and bangs it.

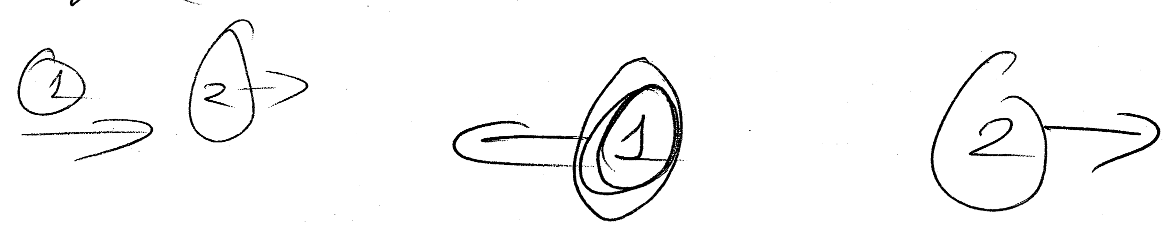
$$\boxed{7-34}$$

$$v_{1f} = \frac{m_1 v_{1i} + m_2 v_{2i} - m_2 v_{2f}}{m_1}$$

$$= \frac{4 - 2 \cdot 6}{1}$$

$$= -8 \text{ m/s}$$

before



Rather interesting thing here

$$KE_i = \tfrac{1}{2} m_1 v_{1i}^2 + \tfrac{1}{2} m_2 v_{2i}^2$$

$$= \tfrac{1}{2} \cdot 4 + \tfrac{1}{2} \cdot 2 \cdot 1$$

$$= 3 \text{ J}$$

$$KE_f = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2$$

$$= \tfrac{1}{2} \cdot 1 \cdot 64 + \tfrac{1}{2} \cdot 2 \cdot 36$$

$$= 32 + 36 = 68 \text{ J}$$

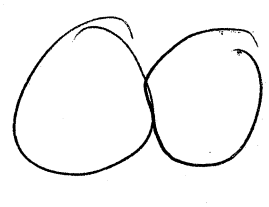
Inelastic

— energy has not been conserved.

— Is this possible?

Yes — some sort of explosion on contact.

In examples one frequently only thinks of loss of KE.

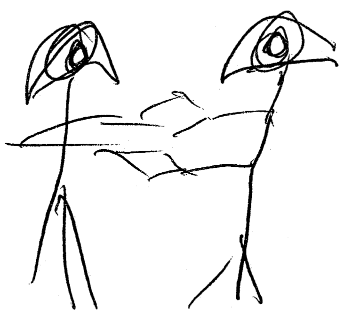


when the balls collide KE goes into ~~Elastic~~ PE but internal friction turns some ~~KE~~ energy into lost waste heat.

but one can imagine a desired reaction — or a spring device which increases KE.

It's still an inelastic collision

Ex 2. Two ice skaters push off each other



Sort of a reverse completely inelastic collision

Initially $v_{1i} = v_{2i} = 0$

$m_1 = 54$ kg

$m_2 = 88$ kg

— we need a bit of outcome information $v_{1f} = 2.5 \frac{m}{s}$

$$0 = m_1 v_{1f} + m_2 v_{2f}$$

$$v_{2f} = -\frac{m_1 v_{1f}}{m_2} = -\frac{54 \cdot 2.5}{88}$$

$$= -\frac{135}{88} \cong -1.5 \quad \text{Ans}$$

In this case human chemical energy turned into KE of the arms turned into KE of translation of the two skaters.

b) <u>Completely Inelastic Collisions</u>

— the two bodies stick together.

$\longrightarrow$ only one final velocity

$$m_1 v_{1f} + m_2 v_{2f} = \underbrace{(m_1 + m_2)}_{M} v_f$$

So because of the extra constraint $v_{1f} = v_{2f} = v_f$, we can solve for the whole

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outcome

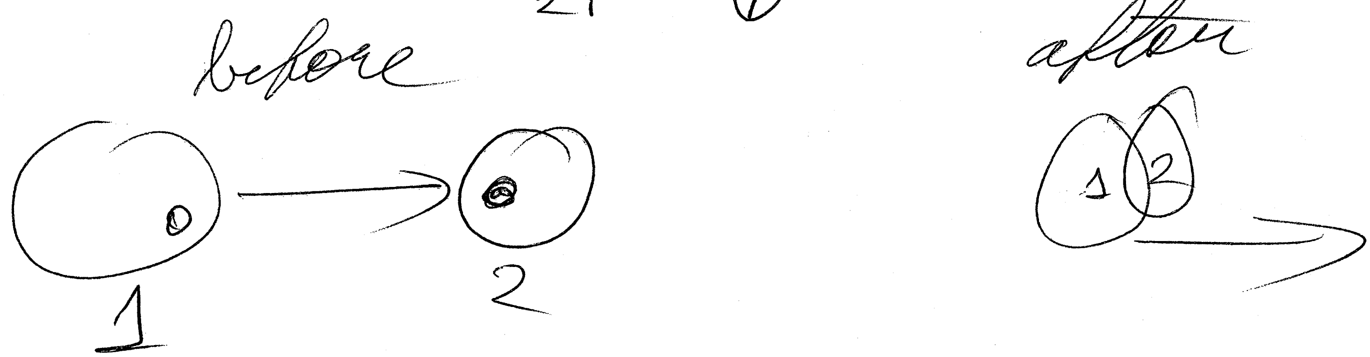
Ex

$m_1 = .25$ kg

$v_{1i} = 5$ m/s

$m_2 = .8$ kg

$v_{2i} = 0$



$$v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$$

$$= \frac{1.25 + 0}{1.05}$$

$$\simeq 1.2 \text{ m/s}$$

Actually it can be shown that

$$KE_i \geq KE_f \quad \text{in general}$$

$\sum_i \frac{1}{2} m_i v_i^2$ ← v_i of CM of particle i

$\frac{1}{2} m v^2$ ← velocity of CM of clump

for any number of objects that collide and stick together in all 3-dimensions

— where does the lost energy go?

Can't really know without detailed specification

- waste heat? some maybe all.
- rotational KE
- vibrational KE & PE
  - usually turned into waste heat by

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internal friction

Omit Proof?

$$\underset{\sim}{v}_i = \underset{\sim}{v}_{cm} + \Delta \underset{\sim}{v}_i$$

$m \underset{\sim}{v}_{cm}$
$\sum m_i \underset{\sim}{v}_i$
$= \sum m_i \underset{\sim}{v}_{cm}$
$+ \sum m_i \Delta \underset{\sim}{v}_i$
$= m v_{cm} + \underbrace{\sum_i m_i \Delta \underset{\sim}{v}_i}_{0}$

$$\underset{\sim}{v}_{cm} = \frac{\underset{\sim}{P}_{total}}{m}$$ is conserved thru the collision.

$$KE_i = \sum_i \frac{1}{2} m_i v_i^2$$

$$= \sum_i \frac{1}{2} m_i (\underset{\sim}{v}_{cm} + \Delta \underset{\sim}{v}_i)^2$$

$\underset{\sim}{v}_{cm} \cdot \underset{\sim}{v}_{cm} = v_{cm}^2$, $\Delta \underset{\sim}{v}_i \cdot \Delta \underset{\sim}{v}_i = \Delta v_i^2$

$$= \sum_i \frac{1}{2} m_i (v_{cm}^2 + 2 \underset{\sim}{v}_{cm} \cdot \Delta \underset{\sim}{v}_i + \Delta v_i^2)$$

$$= KE_f + \underset{\sim}{v}_{cm} \sum_i m_i \Delta \underset{\sim}{v}_i + \sum_i \frac{1}{2} m_i \Delta v_i^2$$

$$= KE_f + \underbrace{\sum_i \frac{1}{2} m_i \Delta v_i^2}_{\geq 0 \text{ always}}$$

or all bits already together

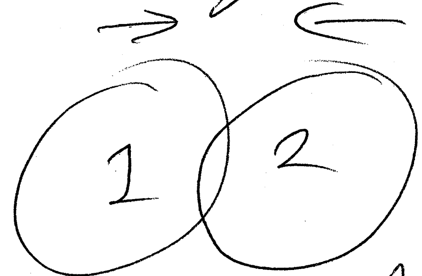
So $KE_i \geq KE_f$ equality only for all $\Delta \underset{\sim}{v}_i = 0$

c) Elastic Collision

Here we assume

KE is conserved.

but only in that $KE_i = KE_f$



— during the collision event, KE is converted into elastic PE of objects.

→ But there is no loss to waste heat by internal friction.

— So all the PE gets converted back to KE.

In this case for 2 particles in 1-d the

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Full outcome can be predicted since we have 2 equations

① $m_1 v_{1i} + m_2 v_{2i} = m_1 v_{f1} + m_2 v_{2f}$

② $\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{f1}^2 + \frac{1}{2} m_2 v_{2f}^2$

①, ② two equations in 2 unknowns.

They can be solved for v_{f1} and v_{2f}

but one equation is non-linear which makes things tricky

v_{f1} and v_{2f} can be found given all other variables

But we have tricks.

③ $m_1 (v_{1i} - v_{1f}) = -m_2 (v_{2i} - v_{2f})$

④ $m_1 (v_{1i}^2 - v_{1f}^2) = -m_2 (v_{2i}^2 - v_{2f}^2)$

Recall difference of squares

$$\boxed{7-43}$$

$$a^2 - b^2 = (a-b)(a+b)$$

$\therefore$ We can divide ④/③ { Assuming ③ not equal to zero.

⑤ $$v_{1i} + v_{1f} = v_{2i} + v_{2f}$$

$$v_{2f} - v_{1f} = -(v_{2i} - v_{1i})$$

$$v_{relf} = -v_{rel\,i}$$

an interesting result in itself.

⑥ $$v_{2f} = v_{1i} + v_{1f} - v_{2i}$$

substitute into ①

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{f1} + m_2 (v_{1i} + v_{1f} - v_{2i})$$

$$(m_1 - m_2) v_{1i} + 2 m_2 v_{2i} = m v_{f1} \qquad m \equiv m_1 + m_2$$

$\boxed{7\text{-}44}$

Not in text but should be

$$v_{1f} = \frac{(m_1 - m_2)\,v_{i1} + 2m_2 v_{2i}}{m_1 + m_2} \quad \checkmark$$

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$$v_{2f} = ?$$

$$= \frac{(m_2 - m_1)\,v_{2i} + 2m_1 v_{1i}}{m_1 + m_2}$$

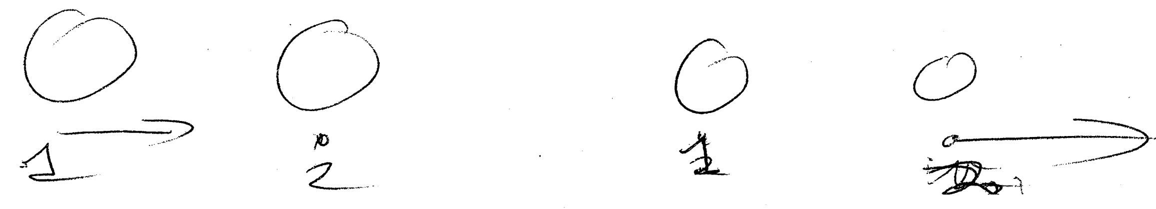
by symmetry
Just interchange indices.

<u>Special case</u> — not exhaustive

1.) $m_1 = m_2 = m$

$$\left.\begin{aligned} v_{1f} &= v_{2i} \\ v_{2f} &= v_{1i} \end{aligned}\right\}$$ they interchange their values

Say $v_{2i} = 0$, $v_{1f} = 0$,

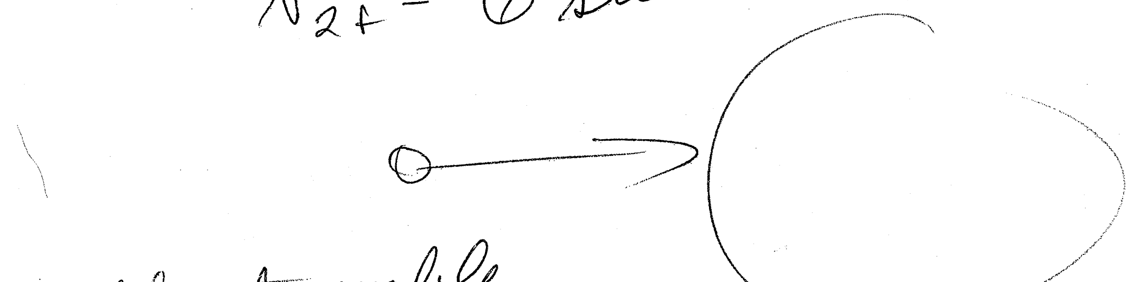


<u>Demo</u> with
— multiball pendulum in box.

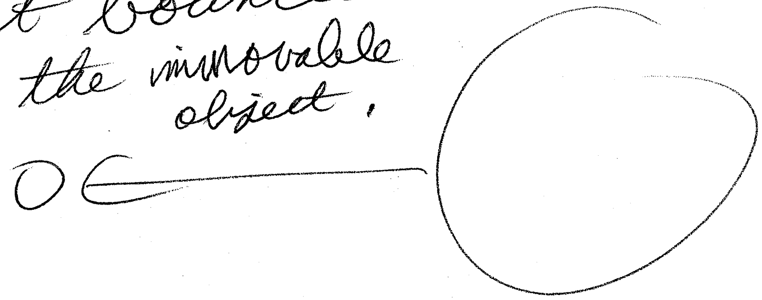
2) $M_2 \to \infty$, $v_{2i} = 0$

$v_{1f} = -v_{i1}$

$v_{2f} = 0$ still.



— the stoppable object bounces off the immovable object.



Actually another solution to the ~~Cons of momentum~~ & elastic collision problem.

Which is? $v_{1f} = v_{if}$

$v_{2f} = v_{2i}$

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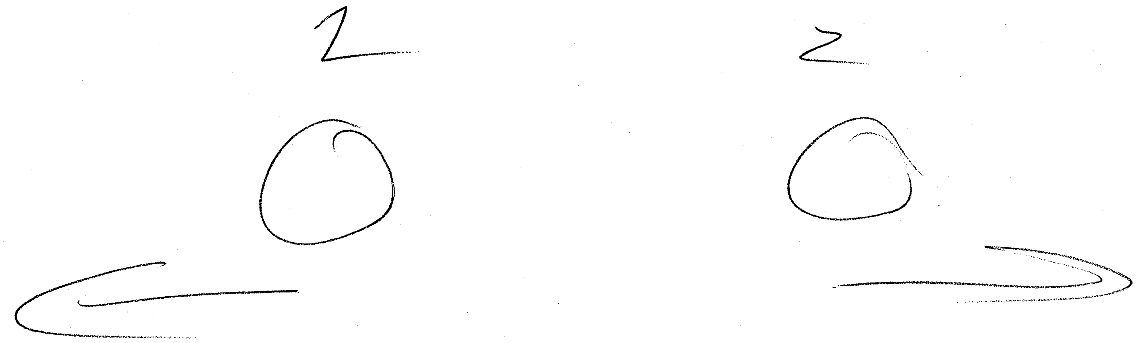
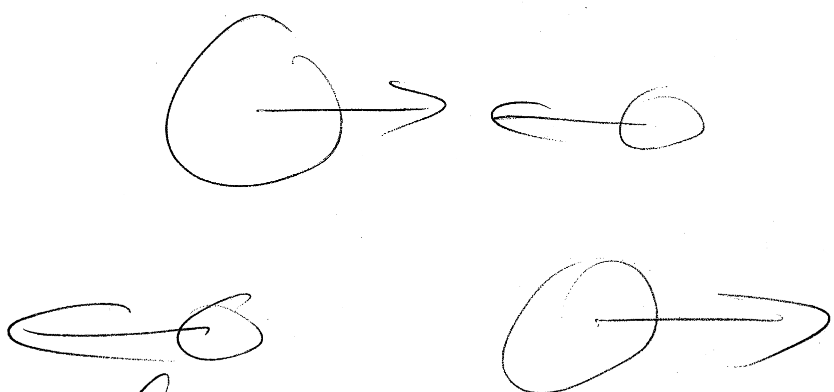


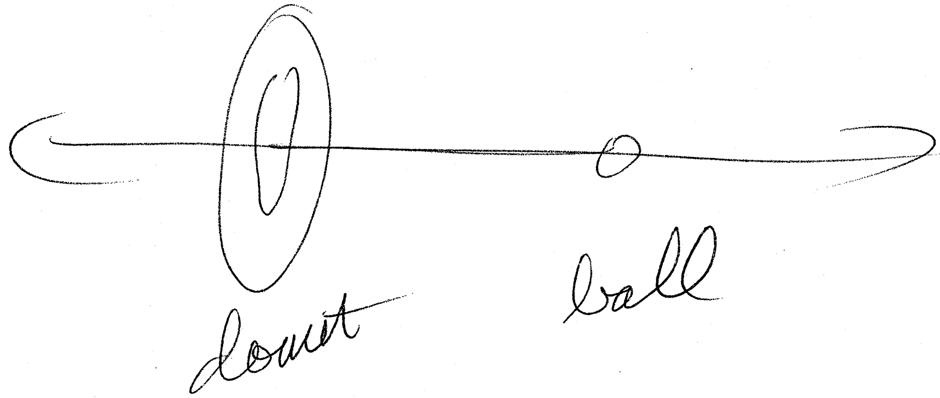
this solution objects moving away from each other

— but there is the ghost solution



where they just pass thru without interaction.

I have thought of semi-real case.



F) Energy, Work, Energy Thm

— we derived or defined for particles

$$W = \underset{\sim}{F} \cdot \Delta \underset{\sim}{s}$$

$$\Delta KE = W$$

$$KE = \frac{1}{2} m v^2$$

$$\Delta E = \Delta KE + \Delta PE = W_{non}$$

What happens when the object is a system of particles or an extended object.

7-48

Well not if you are only concerned w/ the center of mass motion,

~~everything~~ is the same

Definition for system $W_{CM} = \underbrace{F_{ext}}_{\text{external force}} \underbrace{\Delta s_{CM}}_{\text{displacement of center of mass}}$

Work-Energy Thm for center of mass

$\underbrace{\Delta KE_{CM}}_{\frac{1}{2} \underbrace{M}_{\text{total mass}} v_{cm}^2} = \underbrace{W_{net}}_{\text{net work on CM}}$

Defined by. $\frac{1}{2} M v_{cm}^2$

net work on CM. derived using

$F_{ext, net} = M a_{cm}$

Potential energy is potentially trickier.

Can one define a PE for center of mass.

— Maybe not absolutely in general.

But one can for gravity.

Recall $PE = mgy$

with respect to an arbitrary zero-point.

For a system of ~~total~~ mass m, $PE_i = m_i g y_i$

$$PE = \sum PE_i = \sum m_i g y_i$$

$$= mg\left(\frac{\sum m_i y_i}{\sum_i m_i}\right) = mg\,y_{cm}$$

7-50

The grav potential energy of all the particles sums to the PE of the CM

$$PE_{total} = mg\,y_{cm}$$

So for gravity at least the work energy thm is recovered and we can for some purposes treat the mass as concentrated at the CM.

$$\Delta E_{CM} = \Delta KE_{CM} + \Delta PE_{CM} = W_{\substack{\text{non}\\ \text{net}\\ \text{ext.}}}$$

Other forms of potential energy can be dealt with similarly sometimes ~~usually~~, but not ~~quite~~ always —

6

For example in Newton's 2nd law applied to systems

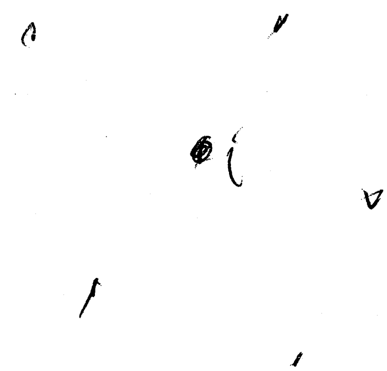
$$\underset{\sim}{F}_{\substack{net\\ext}} = m\,\underset{\sim}{a}_{cm}$$

the motion is as if all mass were concentrated at the CM.

This is true for gravitational force law near the Earth's surface as we've just proven.

<u>But</u> in generally for systems

$$\underset{\sim}{F}_{grav} = \sum_i m_i\,\underset{\sim}{g}_i$$



mass of particle i

gravitational field at particle i

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$$\underset{\sim}{F}_{grav} = m \left(\frac{\sum m_i \underset{\sim}{g}_i}{\sum m_i} \right)$$

This will not be the CM gravitational field $\underset{\sim}{g}(\underset{\sim}{r}_{cm})$ in general.

It is in the special case ~~that~~ $\underset{\sim}{g} = g(-\hat{y})$ because $\underset{\sim}{g}$ is a constant and so must be $\underset{\sim}{g}(\underset{\sim}{r}_{cm})$.

— So one <u>cannot</u> always ~~treat~~ use the CM a valid mean position for all calculations.