Chapter #3 : We are not going to grind on infinite examples this Chapter.

Now that we've mastered 1 dimension now for 2 dimensions

3.1 Just a reminder

$$\Delta \underset{\sim}{r} = \underset{\sim}{r} - \underset{\sim}{r}_0$$

$$\Delta \underset{\sim}{v} = \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t}$$

$$\Delta \underset{\sim}{a} = \lim_{\Delta t \to 0} \frac{\Delta \underset{\sim}{v}}{\Delta t}$$

— are all vectors.

— in particular this means that even if $|\underset{\sim}{v}|$ the magnitude of velocity (i.e., speed is constant) there ~~can~~ still acceleration the direction of velocity changes.

3.2 Kinematic Equations in 2-d — for constant acceleration

— Remarkably

Newton's laws (when we get to them)

tell us that the motion in 2 perpendicular directions is completely independent

⇒ if the ~~force~~ components in those directions are independent.

In this chapter we assume they are and mostly concentrate on projectile motions where horizontal ~~has~~ constant v and vertical ~~has~~ acceleration g down.

We also do some relative motion problem.

So our Kin. Eq. with constant a — Write on board as a table

$x, x_0, v_x, v_{0x}, a_x, t$ / Δx ⇒ ⇐ $y, y_0, v_{y0}, v_y, a_y, t$ / Δy

Mutatis mutandis for y.

	Missing
$v = at + v_0$	$x, x_0, \Delta x$
$x = \frac{1}{2}at^2 + v_0 t + x_0$	v
$x = \frac{1}{2}(v_0 + v)t + x_0$	a
$v^2 = v_0^2 + 2a(x - x_0)$	t
$x = vt - \frac{1}{2}at^2 + x_0$	v_0

$\Delta x = x - x_0$

Now search for the ~~3~~ 4 knowns in ~~only~~ both dimensions

the 2 unknowns

- usually t is the same for both motions (or else the problem is kind of stupid)
- most wanted
- least wanted
- but you must have one of x, x_0 in order to solve for the other.

Ex 1 ~~Rocket~~ Spacecraft

In this case, we don't have the trickery of needing to solve for v, v_0, t
So just the 1st 2-kin. eq. in parallel

$t = 7$

$$v_x = \underset{24}{a_x}\, \overset{7}{t} + \underset{22}{v_{0x}} \doteq 190 \qquad v_y = \underset{12}{a_y}\, t + \underset{14}{v_{0y}} \doteq 100$$

$$x = \tfrac{1}{2} a_x t^2 + v_{0x} t + \overset{0}{x_0} \qquad y = \tfrac{1}{2} a_y t^2 + v_{0y} t + \overset{0}{y_0}$$

$$= 12 \cdot 50 + 150 + 0 \quad t = 7\,s \qquad = 6 \cdot 50 + 100$$

$$\doteq 750\ m\ (\text{Ans } 740) \qquad = 400\,m\ (\text{Ans } 390\,m)$$

Final velocity in component form

$\underline{v} \hat{=} \langle 190, 100 \rangle$ in m/s

This is a full specification actually, but one can put in magnitude, direction form. { which often demanded in question

$$v = \sqrt{190^2 + 100^2}$$

$$\hat{=} \sqrt{2^2 + 1^2} \cdot 10$$

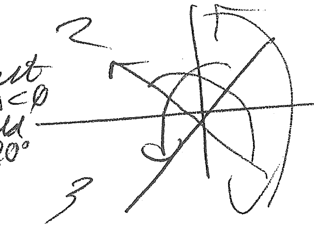
$$= 2\cancel{3}0 \text{ m/s}$$ (real ans. 210 m/s)

$$\theta = \tan^{-1}\left(\frac{100}{200}\right)$$

$$= 27^\circ$$

(ans. 27°)

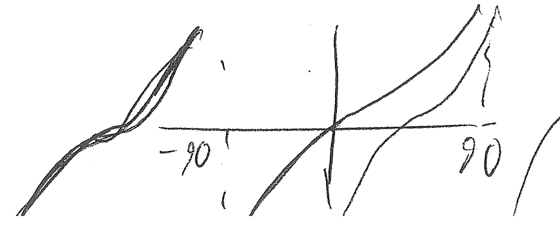
{ note if ~~both~~ $x < 0$, then you have an ambiguity with $\tan^{-1}$



push $x<0$ add 180°

(x, y <0 add 180) $\tan^{-1}$ only knows quadrants 1 & 4

because tan is periodic over 180°



3.3 Example 3

Most of the problems are of this sort

— Projectile motion — neglecting air resistance

a)

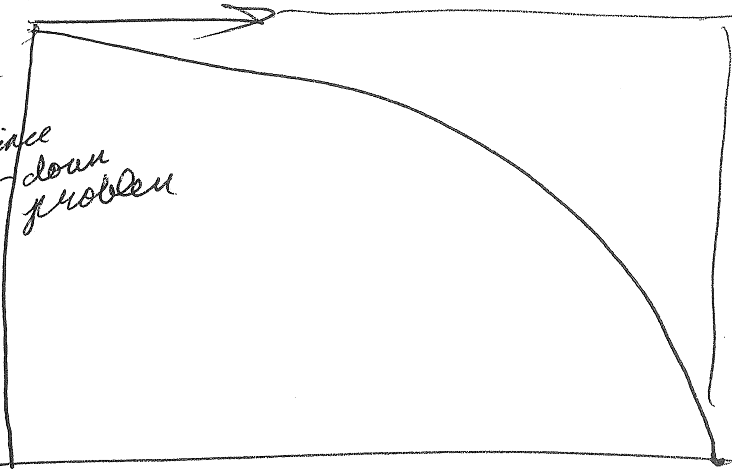
$x=0$ $v_x = 115$ m/s

$y=0$ count down as down as +ve, since an all-down problem

$v_y = 0$

y @ $= 1050$ m

$g = 9.8$ m/s down.



in absence of air resistance
- package hits right under plane.
- air resistance will make it ~~less far along~~ ~~probably~~ back away from plane.

Question

find t for hitting ground.

In this case, remember that x & y motions are independent

$$y = \frac{1}{2} g t^2$$

$$v_y = a_y t + v_{y0}$$

$$y = \frac{1}{2} a_y t^2 + v_{y0}^2 + y_0$$

but in straight falling

$$t = \sqrt{\frac{2y}{g}}$$

$$= \sqrt{\frac{2 \cdot 1050}{9.8}}$$

$$\simeq \sqrt{200}$$

$$= 15 \text{ s}$$

you see how easy it is when you round off in the calculation

(ans. 14.6 s)

b) They now want ~~us~~ to find the final velocity. — CJ make heavey weather of it.

3-6a

No air resistance

No forces to cause acceleration in the x-direction

$\parallel 0$

$v_y = gt$ $10 \cdot 15$

$= 150$

$v_x = a_{0x} t + v_{x0}$

$= 115$ ~~15~~

~~= 1700~~

~~$v = (1700, 150)$ in m/s~~

~~a com specified~~ a complete specification

~~recall we took down as +ve~~

~~$v = \sqrt{1700^2 + 150^2}$~~

~~$= 10 \cdot \sqrt{17^2 + 1.5^2}$~~

~~$= 10 \cdot 17 = 1700$ m/s~~

$v_x = (115, 150)$ m/s and this is a complete specification

$v = \sqrt{1^2 + 1.5^2} * 10$

$= 1.7 \cdot 10 = 170$ m/s

(Ans 184 m/s)

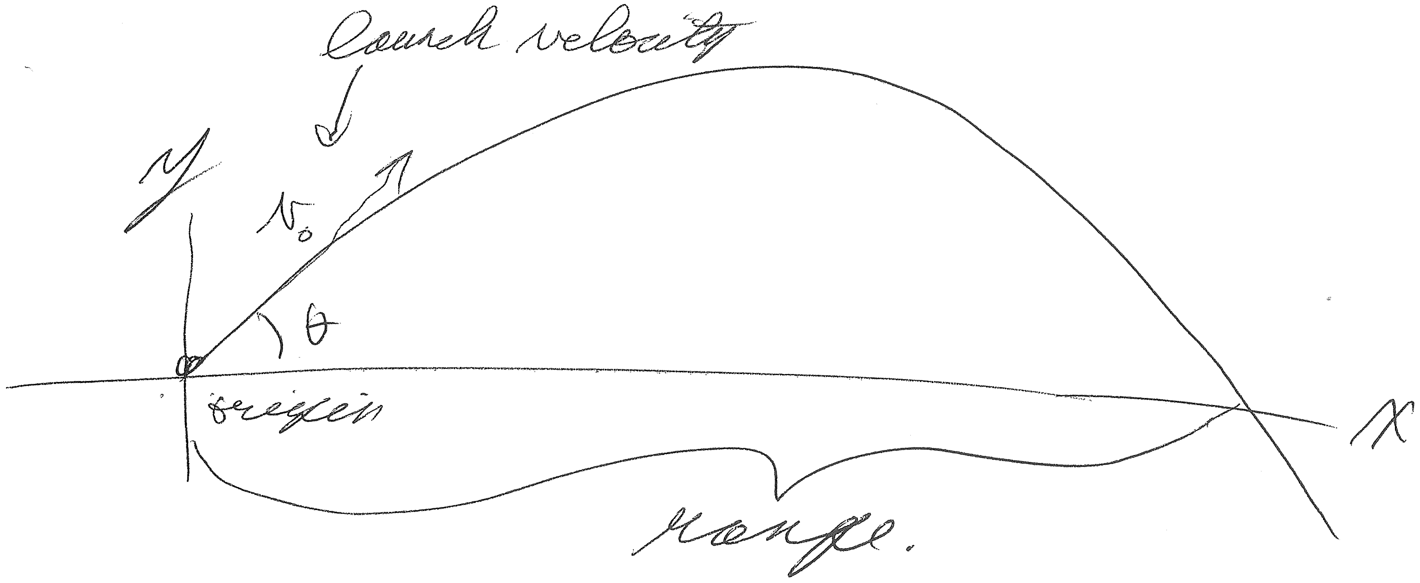
$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$
$$= \tan^{-1}\left(\frac{150}{115}\right)$$
$$\simeq \tan^{-1}(1.3)$$
$$= 53^\circ$$

<u>Ans</u> 51.3°

Parabolic Trajectory

- at this point, I'm going to deviate a bit from the text.
- It goes on repeating the same kind of calculation but never seems to

that projectile motion is parabolic. — and never gives the range formula.



$$x = v_x t = (v_0 \cos\theta) t$$

$v_x = v_0 \cos\theta$ by trig

— there is no acceleration in the x-direction (neglecting air resistance.

$$y = \underbrace{\tfrac{1}{2}(-g)}_{\text{gravitational acceleration in free-fall}} t^2 + v_y t$$

$$v_y = v_0 \sin\theta$$
by trig

$$\therefore \quad y = -\tfrac{1}{2} g t^2 + v_0 \sin\theta\, t$$

These equations give us position in x and y as a function of time t.

But we can eliminate time and get y as a function of x.

$$x = (v_0 \cos\theta)\, t$$

$$t = \frac{x}{v_0 \cos\theta}$$

$$y = -\frac{1}{2} g \frac{x^2}{v_0^2 \cos^2\theta} + (\tan\theta)\, x$$

y has a quadratic equation — which means the shape the trajectory is parabolic.

The <u>range formula</u> is for the distance the object travels in returning to it's initial height.

So set $y = 0$ and solve for x

$$0 = -\frac{1}{2} g \frac{x^2}{v_0^2 \cos^2\theta} + (\tan\theta)x$$

One solution is $x = 0$. which just means the object is at $y = 0$ when $x = 0$ at launch.

For the other solution, $x \neq 0$, we can divide through by x to get

$$0 = -\frac{1}{2} g \frac{x}{v_0^2 \cos^2\theta} + \tan\theta$$

$$x = \frac{2v_0^2}{g} \underbrace{\cos^2\theta \tan\theta}_{\sin\theta\cos\theta}$$

$$= \frac{v_0^2}{g} \underbrace{(2\sin\theta\cos\theta)}_{\sin 2\theta}$$

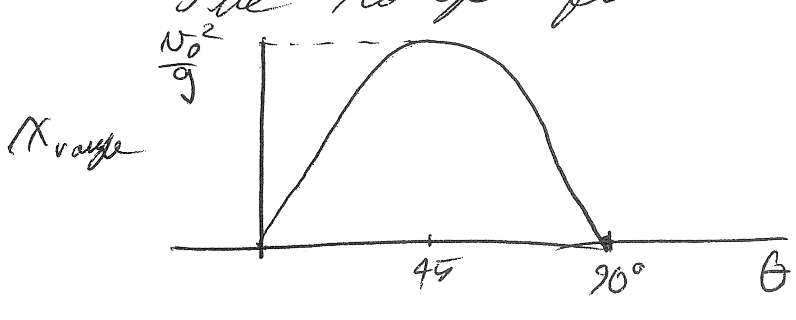
by a trig identity

$\sin 2\theta = \sin\theta\cos\theta + \cos\theta\sin\theta$

$= 2\sin\theta\cos\theta$

$$x_{range} = \frac{v_0^2}{g}\sin 2\theta \qquad \text{(SJ-79)}$$

The range formula.

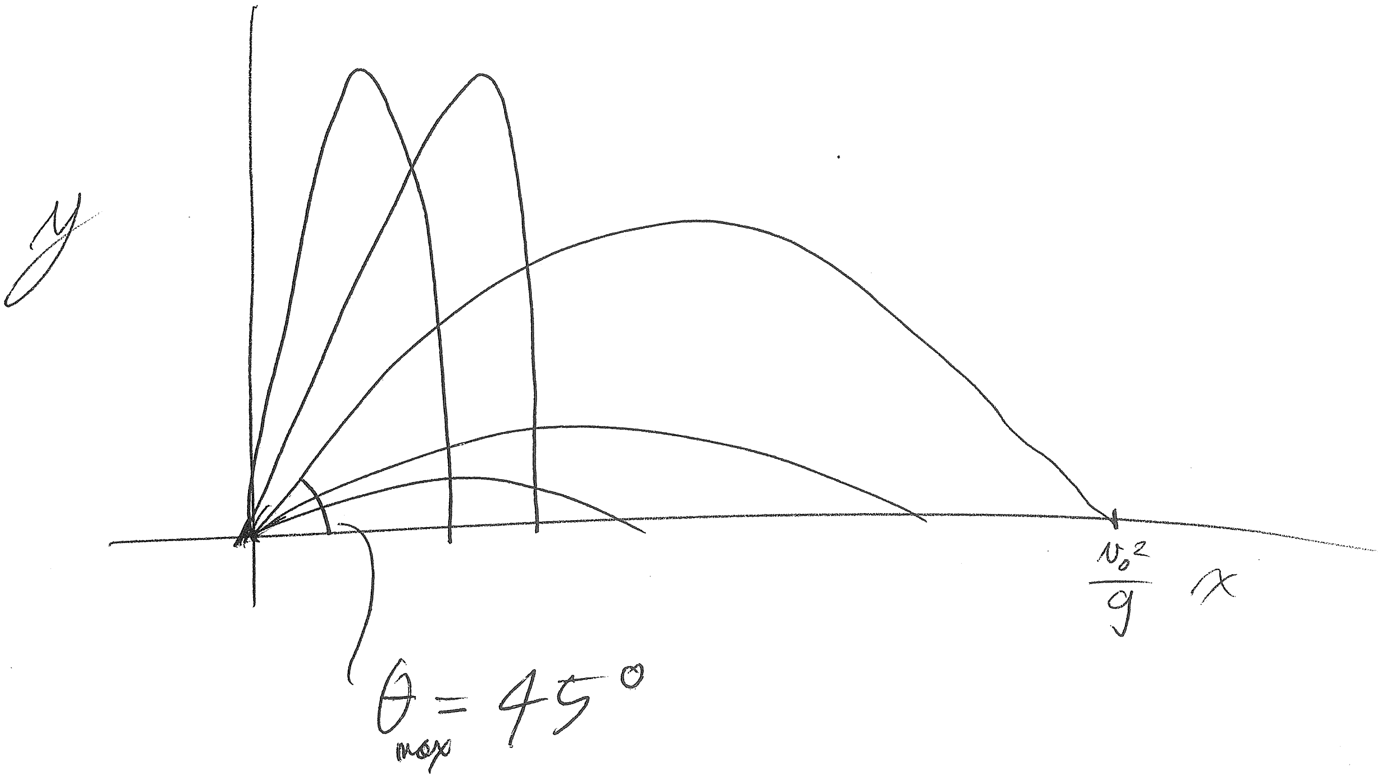


The sine function maximizes at 90°

$\therefore$ $\sin 2\theta$ maximizes at $2\theta = 90°$

or $\theta_{max} = 45°$

So $x_{range\,max} = \frac{v_0^2}{g}$



— air resistance actually causes the maximizing angle to deviate from $\theta_{max} = 45°$.

— I think it gets a bit lower, but there is probably a complex behavior depending on object shape, ~~size~~, density

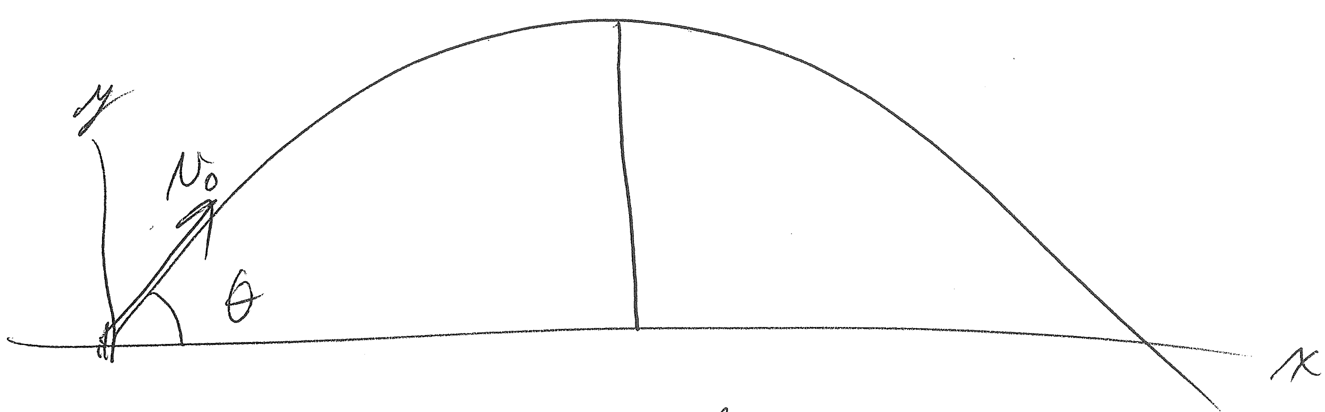
Examples

$v_0 = 10 \text{ m/s}$

$$x_{\text{range max}} = \frac{v_0^2}{g} \simeq \frac{10^2}{10}$$

$$= 10 \text{ m}$$

What about maximum height



Parabolic symmetry tells us that

$$x_{\substack{\text{maximum} \\ \text{for } y}} = \frac{1}{2} x_{\text{range}}$$

$$= \frac{v_0^2}{g} \sin\theta \cos\theta$$

Let's shift origins to the point

by transformation $x = x' + \frac{v_0^2}{g}\sin\theta\cos\theta$

$$y = -\frac{\frac{1}{2}g}{v_0^2\cos^2\theta}\left(x'^2 + \left(\frac{v_0^2}{g}\sin\theta\cos\theta\right)^2 + 2x'\frac{v_0^2}{g}\sin\theta\cos\theta\right)$$

$$+ \tan\theta\left(x' + \frac{v_0^2}{g}\sin\theta\cos\theta\right)$$

$$= -\frac{\frac{1}{2}gx'^2}{v_0^2\cos^2\theta} - \frac{1}{2}\frac{v_0^2}{g}\sin^2\theta$$

$$- x'\tan\theta + (\tan\theta)x'$$

$$+ \frac{v_0^2}{g}\sin^2\theta$$

$$= -\frac{\frac{1}{2}gx'^2}{v_0^2\cos^2\theta} + \frac{1}{2}\frac{v_0^2}{g}\sin^2\theta$$

Since we've eliminated the linear ~~term~~ term this was the correct shift.

The function is now clearly a parabola

(one can always eliminate the linear term in a quadratic by the appropriate origin shift).

$x' = 0$ gives y_{max}

$$y_{max} = \frac{1}{2}\frac{v_0^2}{g}\sin^2\theta \qquad \text{(SJ-78)}$$

Example

$v_0 = 10\,m/s$ for $\theta = 45^\circ$

$$y_{max} = \frac{1}{2}\,\frac{100}{10}\left(\underbrace{\sin^2 45}_{\frac{1}{2}}\right)$$

$$= \frac{10}{4} = 2.5\,m.$$

So 10 m/s lauch speed and $\theta = 45^\circ$ gives a range of 10 m and ~~we max hi with~~ max height 2.5 m

Now some examples from the homework.

$y = \frac{1}{2} g t^2$

$\cancel{x} = v_{x_0} t$

$t = x / v_{x_0}$

$y = \left(\frac{1}{2} \frac{g}{v_{x_0}^2}\right) x^2$

So the path is parabolic

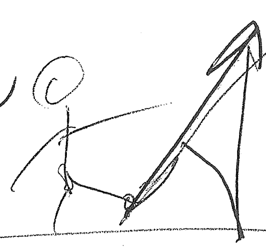
$\theta = \tan^{-1}\left(\frac{150}{115}\right)$

$\simeq \tan^{-1}(1.3)$

$= 53^0$

(Ans. 51.3^0)

Ex 6



← max ~~height~~

↳ that's the clue

$v_y = 0$ at max height

a) need t ↳ ⓪

— no use the timeless equation

$v_y^2 = v_{y0}^2 = 170^0 + 2a(y - y_0)$, $a = -g$

$y = \frac{-v_{y0}^2}{2a}$

$= \frac{-170}{-2 \cdot 10}$

$= +8.5$

(Ans. 10 m) g cooked!

$\theta = 40^0$

$v_{y0} = v \sin\theta = 22 \cdot \sin 40^0$

$= 22 \cdot .6$

$= 13$

① $v_y = \cancel{a}_y t + v_{y0}$, $a_y = -g$

$t = \frac{v_{y0}}{g} = \frac{13}{10} = 1.3\,s$

$y = \frac{1}{2} a_y t^2 + v_y t + y_0$, $a_y = -g$, $v_y = 13$, $y_0 = 0$

$= -8.5 + 17 = 8.5\,m.$

b) time of flight for kickoff

3-8

~~v_y~~

$$\cancel{x = \frac{1}{2}(v + v_0)t + x_0 = 0}$$

$$\cancel{= \frac{1}{2}(22.8)8.8}$$

$$\cancel{= 17.6\,m}$$

Here's a case where the 5 kin eq. for x direction peculiar.

v, v_0, a, t and x

2 unknowns but the equations blah here

all sets of equations have their peculiarities and in this case the 5 eqn. collapse to 1

1. $v = v_0$
2. $x = v_0 t$
3. $x = v_0 t$
4. $v = v_0$
5. $x = v_0 t$.

same as for

not really an eq.

3 eq for 2 unknowns — so it can't be done

3-9

So we need something else.

- We have our y-equations and they'll tell us when the ball is back at the ground.

$$y = \frac{1}{2}at^2 + v_{oy}t$$

$$0 \quad \quad -9 \quad \quad 13$$

$$0 = \frac{1}{2}at + v_0$$ since one t cancels out.

$$t = \frac{-2v}{a}$$

$$= \frac{26}{10}$$

$$= 2.6 \text{ s}$$ (Ans. 2.9 s)

It was the t = 0 solution - since the ball is at the ground at the start.

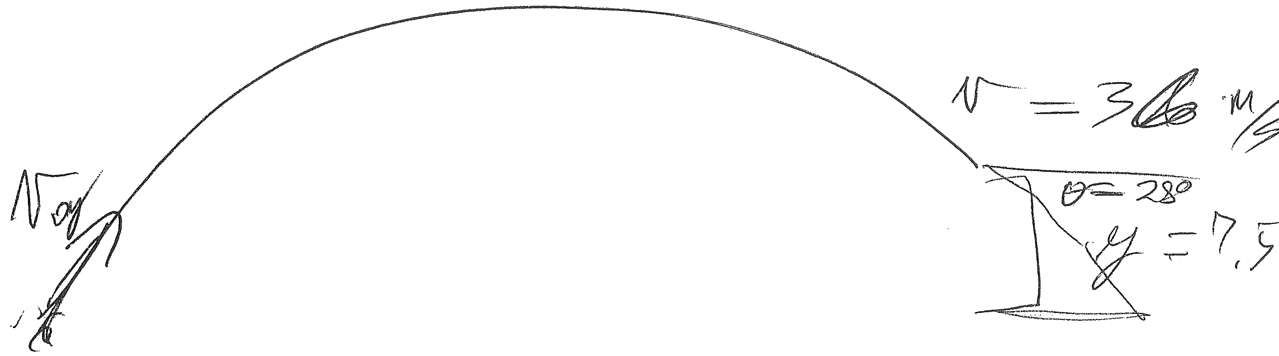
c) :. $v_x = v\cos\theta$

$= 22 \cdot .8$

$= 17 \text{ m/s}$

$$x = v_x t$$
$$= 17 \cdot 2.6$$
$$= 43\text{ m} \quad (\text{Ans. } 49\text{ m})$$

Ex 9 Homerun



Mostly a y problem since no x-acceleration.

$y_0 = 0$	y	v	v_{oy}	a	t
	$= 7.5$	$= v_f \sin(-28°) \to -18$ m/s	$= ?$	$= -g = -10$	$= ?$

2 unknowns we can solve with equation 5 No, the timeless equation works.

$$v_y^2 = v_{oy}^2 + 2ay$$
$$v_{oy} = \pm\sqrt{v_y^2 - 2ay}$$
$$= \pm\sqrt{320 + 150} = \pm 23\text{ m/s}$$

The $v_{yo} = +23$ m/s
is the solution
we want.

The other in this case is
~~the time reversal~~
~~solution.~~

~~we know $v_f < 0$~~
~~but the ~~ equation
~~doesn't.~~

<u>Negative</u> — here I'm caught
in my own trap from
last Friday on what the
other solution means.

and after more minutes than I care to think of.

— Note v_y^2 came into the
equation as a square
and $a\,y$ as a product.

$v_{yo} = -23$ m/s

$v_y = 18$ m/s
and $y = -7.5$
$a = +9.8$ m/s
would lead to same result.

ground as mirror

So it's the mirror image solution which is not physical
since a points down only.

3-12

Still need v_{x0}

but no acceleration in x-direction

$$\therefore v_{x0} = v \cos\theta$$

$$= \cancel{36} \cdot .9$$

$$= 32 \text{ m/s}$$

$$\therefore \underset{\sim}{v_0} = (32, 23)$$

$$\therefore v_0 = \sqrt{1000 + 500}$$

$$= 35 \text{ m/s} \quad (\text{Ans. } 38 \text{ m/s})$$

not bad

$$\theta = \tan^{-1}\left(\frac{\overset{23}{\cancel{500}}}{\underset{32}{1\cancel{00}0}}\right)$$

$$\cancel{\approx 27^\circ} \quad (\text{Ans. } 33^\circ)$$

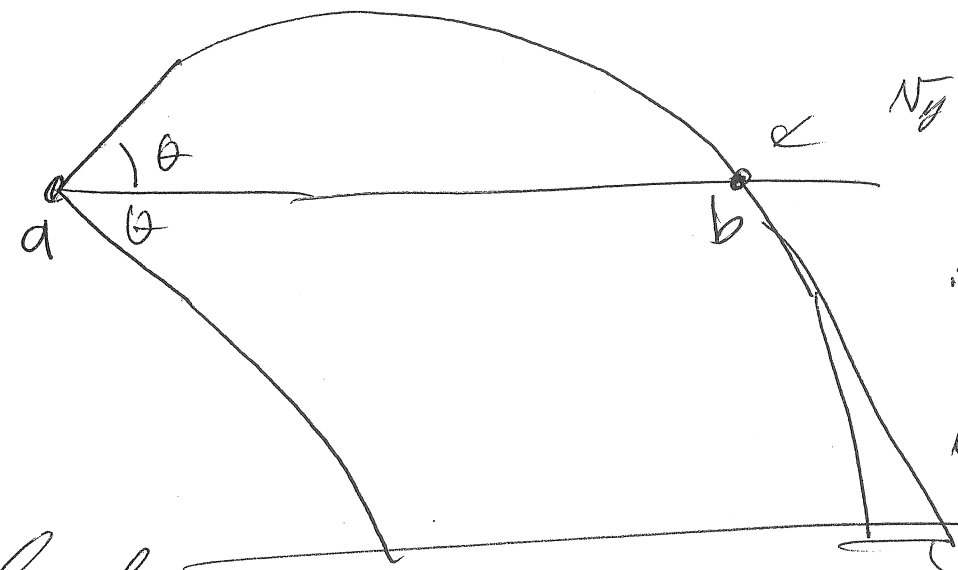
not so good — order of.

$$= \tan^{-1}\left(\frac{3}{4}\right)$$

$$= 37^\circ$$

Ex 10 — Conceptual

Stone throw problem with v_0 as initial speed



$v_y^2 = v_{y0}^2 + 2a\Delta y$ from the timeless eqn.

$\therefore v_y = \pm v_{y0}$

+ case was the initial $y = 0$

$v_y = -v_{y0}$ is the final.

but then it's traj. is the same stone and so fast out and at a later time

Which hits the ground with greater magnitude of velocity — neglect air resistance

$v_{x_a} = v$ ~~sin~~ θ

$v_{x_b} = v\cos(-\theta) = v_{x_a}$

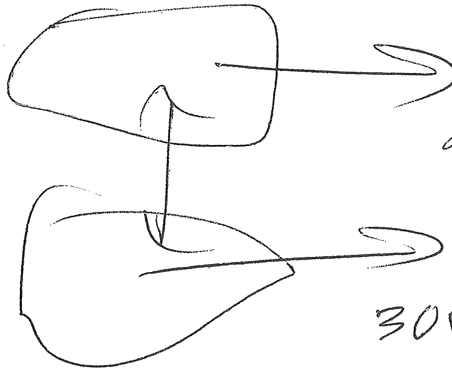
So it's only the y-component that ~~can~~ differs

So $\underset{\sim}{v} = (v_{x0}, v_y)$

it hits with the same velocity, whatever v_y is then

Omit from p. 3-7

3.4 Relative Velocity



the two drivers ~~see~~ are at rest with respect to each other.

$30\,m/s \cong 78\,mi/h$

pedestrians
but pedestrian sees them whizz by at $30\,m/s$

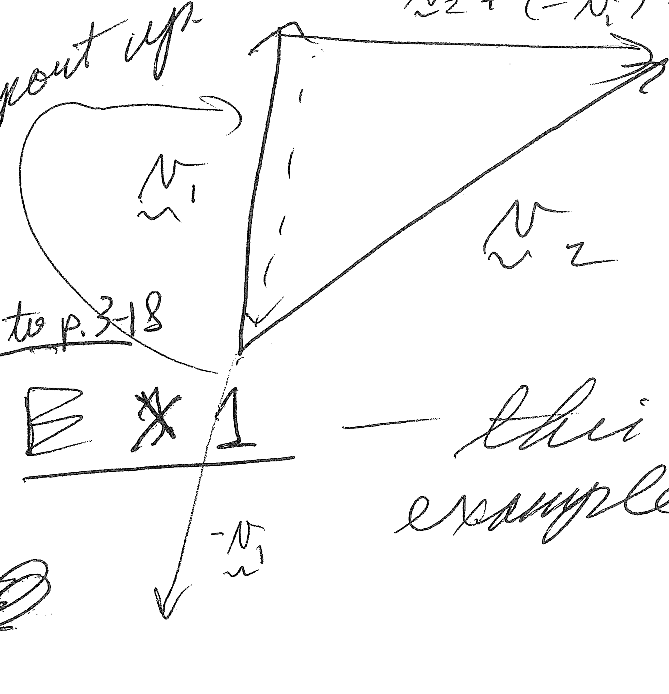
— measured velocity is always relative to some frame of reference.

— if some object defines ~~your~~ frame then it is the standard of rest

$$v_{rel_{12}} = v_2 - v_1$$

velocity of 2 relative to 1.

$v_2 + (-v_1) = v_2$



Omit to p. 3-18

EX 1 — ~~this is a tricky~~ 1-d example from the Hulk

3-15

Train A $v = 60\,m/s$ Train B $v = 50\,m/s$

W → ← E

1000m

They are on a collision course.

- at 1000 m apart the engineers notice each other and slam on the breaks simultaneously.

- assume constant relative acceleration.

What must it be for a zero-rel velocity collision?

Really just a 1-d kinematic ~~eqn.~~ problem.

v_{rel} $\underline{\text{B rel to A}} = v_B - v_A$

$= -50 - (60)$

$= -110\,m/s$

$v_0 = 110\,m/s$

$x_0 = 1000$

$x = 0$

$v = 0$

$t = ?$

$a = ?$

$x_0 = 0$

~~$x_0 = 1000$~~

$\Delta x = -1000 = 0 - (+1000)$

Solve from the timeless equation.

3-16

$$v^2 = v_0^2 + 2a\Delta x$$

$$a_{rel} = \frac{v^2 - v_0^2}{2\Delta x}$$

$$= \frac{0 - 12100}{-2 \cdot 1000}$$

$$= +\frac{12.1}{2}$$

$$= +6.05 \text{ m/s}^2$$

positive acceleration

it's a ~~deceleration~~.

train B is slowing down moving in the negative direction

Are the trains at rest with respect to the ground?

Not necessarily.
Not likely.
– In fact we can't know how they are moving with respect to the ground without more information.
We'd have to know how to partition a_{rel} between the two trains.

3-17

b) What is the acceleration in g-force

i.e., ~~a~~ in units of g

$$a_{rec} = \frac{+6.05}{9.8}\left(\frac{1g}{9.8\,m/s^2}\right)$$

$$\simeq +.6\,g$$

fraction of unity.

c) Do the passengers experience this g-force?

⇒ a sense of this much acceleration

Tricky at this stage.

But ~~the~~ accelerated frames are different from rest frames.

You know you sense acceleration — it seems like a body force is throwing you around.

there is a real physical difference

the trains are both accelerated frames relative to the ground.

which is relatively unaccelerated

— they only have a sense of being accelerated relative to the ground.

Think about cars in motion.

- You perceive relative ~~motion~~ velocity
- but acceleration as a bodily sensed only relative to the ground.
- We go into this in Ch. 4

d) $v = 60\,\frac{m}{s}\cdot\left(\frac{1\,km}{10^3\,m}\right)\left(\frac{3600\,s}{1\,h}\right)$ 3-18

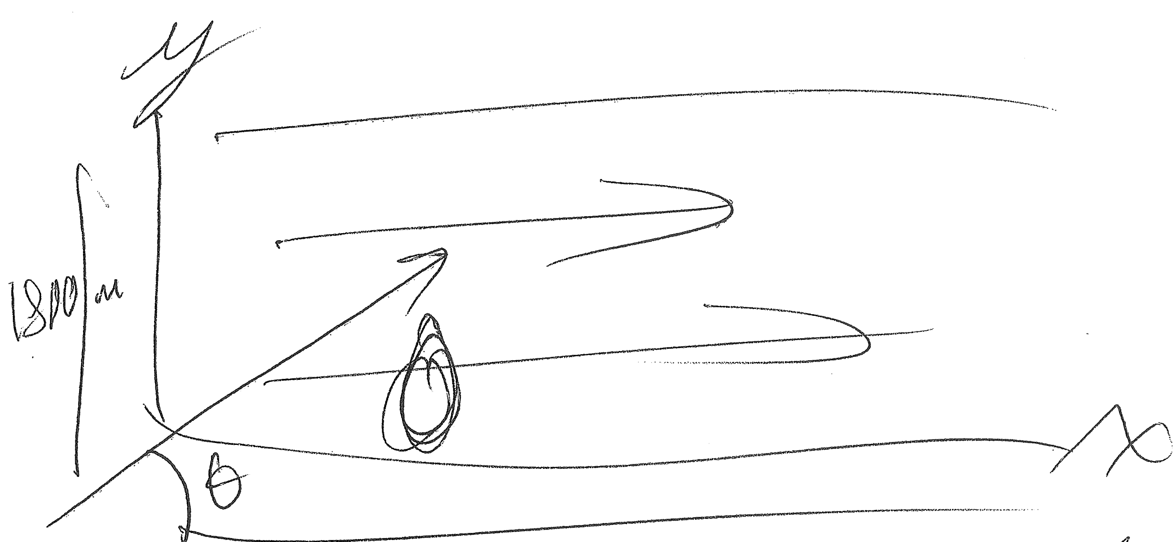
$= 60\cdot 3.6\ km/h$

$= 216\ km/h$

— a pretty fast train.

Omitted from p. 3-1

Ex 11 Boat crossing river



the boat is heading straight across relative to water, but with respect the land.

a) $\underset{\sim}{v}_{BW} = 4.0\ m/s\ \hat{y}$

$\underset{\sim}{v}_{WS} = 2.0\ m/s\ \hat{x}$

water shore

What is $\underset{\sim}{v}_{BS}$?

$$\underset{\sim}{v}_{BS} = (2.0, 4.0)$$

or $v_{BS} = \sqrt{4 + 16}$

$$\cong 4.5 \, m/s \quad (\text{Ans. } 4.5 \, m/s)$$

$$\theta = \tan^{-1}\left(\frac{4}{2}\right)$$

$$= \tan^{-1}(2) = 64^\circ \quad (\text{Ans. } 63^\circ)$$

b) $y = v_y t$

$$t = \frac{y}{v_y} = \frac{180 \, m}{4.0 \, m/s}$$

$$= 450 \, s \left(\frac{1 \, min}{60}\right)$$

$$= 7.5 \, min. \quad (\text{Ans. } \underline{450 \, s})$$