Intro Physics I — EXAM 2

2012 February 29, Wednesday — NAME:

Instructions: There are 20 multiple-choice problems each worth 1 mark for a total of 20 marks altogether. Choose the **BEST** answer, completion, etc., and **DARKEN** fully the appropriate circle on the table provided below. Read all responses carefully. **NOTE** long detailed responses won't depend on hidden keywords: keywords in such responses are bold-faced capitalized. Harder/longer multiple-choice problems (which are sometimes based on full-answer problems) are marked by asterisks: $*$ for easy, $**$ for moderate, $***$ for hard: all in the judgment of the instructor.

There are **THREE** full-answer problems worth 10 marks for a total of 30 marks altogether. Answer them all. It is important that you **SHOW** (**SHOW, SHOW, SHOW**) how you got the answer for the full-answer problem. Don't give up on problems where you can't do the first part: sometimes later parts can be done independently. Some full-answer problems may be multiple-pagers: make sure you have answered everything. And **BOX-IN** your final answers.

This is a **CLOSED-BOOK** exam. **NO** cheat sheets allowed. An **EQUATION SHEET** is provided. Calculators are permitted—but **ONLY** for calculations. There are **SCRATCH PAGES** for auxiliary calculations. Remember your name (and write it down on the exam too).

The exam is out of 50 marks altogether and is a 50-minute exam.

Answer Table for the Multiple-Choice Questions

	a	b	c	d	e		a	b	c	d	e
1.	O	O	O	O	O	11.	O	O	O	O	O
2.	O	O	O	O	O	12.	O	O	O	O	O
3.	O	O	O	O	O	13.	O	O	O	O	O
4.	O	O	O	O	O	14.	O	O	O	O	O
5.	O	O	O	O	O	15.	O	O	O	O	O
6.	O	O	O	O	O	16.	O	O	O	O	O
7.	O	O	O	O	O	17.	O	O	O	O	O
8.	O	O	O	O	O	18.	O	O	O	O	O
9.	O	O	O	O	O	19.	O	O	O	O	O
10.	O	O	O	O	O	20.	O	O	O	O	O

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005 qmult 00532 1 1 1 easy memory: Newton's 2nd law: 2

1. Newton's 2nd law is:

   a) $\vec{F} = m\vec{a}$. b) $m\vec{F} = \vec{a}$. c) $E = mc^2$. d) $E = mc^3$. e) $m = Ec^2$.

   SUGGESTED ANSWER: (a)

   Wrong answers:
   c) This is the Einstein equation.

   Redaction: Jeffery, 2008jan01

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005 qmult 00710 2 3 5 moderate math: stopping a bike
Extra keywords: physci KB-60-21

2. A bicycle-rider system has a mass of 80 kg. The bike is traveling on level and has initial velocity 6 m/s north. What is the constant force needed to stop the bike in 4 s?

   a) 80 N south. b) 80 N north. c) 80 N east. d) 100 N south.
   e) 120 N south.

   SUGGESTED ANSWER: (e)

   The acceleration to stop the bike is

$$a = \frac{v - v_0}{t} ,$$

   where $v = 0$ is the final velocity, $v_0 = 6$ m/s is the initial velocity, $t = 4$ s is the stopping time, and I've chosen north at the positive direction. Thus the constant stopping force is thus

$$F = ma = m\left(\frac{v - v_0}{t}\right) = 80 \times (-1.5) = -120\,\text{N} .$$

   South is the negative direction and thus the stopping force is 120 N south.

   Wrong answers:
   c) East. Are you trying to tip the bike.

   Redaction: Jeffery, 2001jan01

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005 qmult 00950 2 5 4 moderate thinking: diving woman and gravity
Extra keywords: physci KB-60-27

3. What is the approximate mass of a woman who weighs 500 N? What is gravitational force that Earth exerts on her. After she jumps **UPWARD** from a diving board, what is her acceleration in the absence of air drag?

   a) About 50 kg, 500 N, and 9.8 m/s^2 downward once she starts moving downward, but **ZERO** before that.
   b) About 50 kg, 50 N, and 9.8 m/s^2 downward once she starts moving downward, but **ZERO** before that.
   c) About 50 kg, 50 N, and 9.8 m/s^2 downward at **ALL** times.
   d) About 50 kg, 500 N, and 9.8 m/s^2 downward at **ALL** times.

e) None of these questions can be answered with the given information.

SUGGESTED ANSWER: (d)

Remember that weight near the Earth's surface is mg where m is mass and $g = 9.8\,\text{m/s}^2$ is the acceleration due to gravity constant. Now 500 N obviously describes the woman's weight: thus her mass is this value divide by about 10. Her weight is the gravitational force that Earth exerts on her. Once she's left the board the only force on her is gravity and she must accelerate downward at $9.8\,\text{m/s}^2$ no matter what direction she is moving in.

Wrong answers:
e) As Lurch would say: "Aaarh."

Redaction: Jeffery, 2001jan01

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005 qmult 01150 1 3 4 easy math: example tension fore of a rope

4. A **MOTIONLESS** mass of 10 kg is suspended from a rope. What is the tension force that the rope exerts on the mass?

   a) 100 N downward. b) 200 N downward. c) 200 N upward.
   d) 100 N upward. e) 200 N horizontally.

SUGGESTED ANSWER: (d)

Well one has to remember about tension. And to be motionless gravity has to be balanced by tension.

Wrong answers:

Redaction: Jeffery, 2001jan01

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005 qmult 11112 2 3 3 moderate math: fuzzy dice at angle 2 fullmult

5.** You are in a car accelerating at a constant $7.5\,\text{m/s}^2$ with constant direction. The car is on level ground. There are fuzzy dice hanging by a cord from the mirror. The dice cord is a (massless) ideal rope. Assuming the dice are a point mass, what is the angle of the dice cord from the **VERTICAL**? **HINT:** Draw a free body diagram for the dice. Remember the class mantra: "$\vec{F}_{\text{net}} = m\vec{a}$ is always true and it's true component by component".

   a) 27°. b) 32°. c) 37°. d) 42°. e) 47°.

SUGGESTED ANSWER: (c)

I omit the diagram.

Note the fuzzy dice, like the car, must be accelerating in the x-direction and at the car's rate of acceleration. They are constrained to do so.

Since the cord is massless ideal rope only two forces act on it. The dice tension force at one end and the mirror holder tension force at the other end. Gravity can't act on the cord since it is massless. Because the only forces on the cord are at the endpoints the cord must be follow a straight line and tension force it exerts on the dice must be aligned with the cord. The tension must be a constant in the cord since no parallel forces act on it except at the endpoints.

We have the two following equations for the dice from $\vec{F} = m\vec{a}$ using a horizontal-vertical set of coordinate axes:

$$ma_x = T\sin\theta \qquad \text{and} \qquad 0 = T\cos\theta - mg \ ,$$

where m is the dice mass, T is the cord tension, and θ is the angle from the vertical.

Although solving a problem symbolically is best, I usually set to zero immediately quantities that are zero: this saves me from tedious generality.

In this case, we don't know m, T, or θ. So we have 3 unknowns in only 2 equations, and so in general can't solve for all the unknowns. But that is in general. Sometimes in particular cases partial solutions can be extracted. In this case, we can divide

$$ma_x = T\sin\theta \qquad \text{by} \qquad mg = T\cos\theta$$

to get

$$\frac{a_x}{g} = \tan\theta \ ,$$

and thus

$$\theta = \tan^{-1}\left(\frac{a_x}{g}\right) = 37.4^\circ \ .$$

Curiously the angle doesn't depend on the mass. Fuzzy dice or an elephant, it's all the same. The cord tension does depend on the mass, of course. In fact, measuring the angle of a hanging object is a way of measuring acceleration:

$$a_x = g\tan\theta \ .$$

We have no way to solve for m and T with the information given.

Wrong answers:

a) Too low.

Fortran-95 Code

```
          print*
          gg=9.8d0
          ax=7.5d0
          theta=atan(ax/gg)*raddeg
          print*,'raddeg,theta'
          print*,raddeg,theta
!         57.2957795130823          37.4270513594566
```

Redaction: Jeffery, 2008jan01

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005 qmult 01230 1 2 5 moderate memory: friction coefficient sizes

6. Which is larger: the coefficient of static or kinetic friction?

   a) They are always equal.
   b) Neither. The larger depends on the materials involved and its about a 50-50 split on which is larger.

c) The kinetic coefficient is always larger.
d) The kinetic coefficient is usually larger.
e) The static coefficient is almost always (always?) larger.

SUGGESTED ANSWER: (e)

The static coefficient is almost always larger. I hesitate to say always because perhaps there is some weird material interface where it isn't. One can sort of understand why static should be larger. In a sliding situation some microscopic bonds may not have a chance to form. Note the fact that the static coefficient is larger is why its recommended that in car slides on ice that you pump your breaks rather than locking them. This is so that the wheels keep rolling just fast enough to maintain a static friction resistance to motion. Actually, it is probably very difficult to do this trick optimally, and I've usually just lock my breaks on the few uncontrolled, but short, ice skids I've been in. And if you are skidding sideways into oncoming traffic (I wasn't at the wheel that time), you can't do much. But computerized systems can reduce skid distance by 30 % or more (reference ???).

Wrong answers:

Redaction: Jeffery, 2001jan01

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006 qmult 00410 1 1 3 easy memory: centripetal acceleration in UCM
Extra keywords: physci

7. In uniform circular motion, the acceleration has:

   a) a constant magnitude and always points **OUTWARD** from the center of motion.
   b) a constant magnitude and always points **ALONG** the circular path (i.e., tangent to the circular path).
   c) a constant magnitude and always points **INWARD** to the center of motion.
   d) a zero value.
   e) a nonconstant magnitude, but a constant direction.

SUGGESTED ANSWER: (c) The magnitude of the acceleration is a constant, but its direction is continually changing so that it always points toward the center.

Wrong answers:
d) I've lived in vain.

Redaction: Jeffery, 2001jan01

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006 qmult 00450 1 4 3 easy deducto-memory: centripetal force defined

8. The centripetal force is:

   a) a mysterious force that **APPEARS** whenever an object goes into uniform circular motion.
   b) a mysterious force that tries to throw you **OFF** playground merry-go-rounds.
   c) in fact $m\vec{a}$ of $\vec{F}_{\rm net} = m\vec{a}$ when this equation is specialized to the case of uniform circular motion. It is **NOT** a mysterious force that appears whenever you have uniform circular motion: it is a force requirement to be satisfied for uniform circular motion. Particular physical forces (e.g., gravity, tension force, and normal

force) must act (sometimes in combination) to give a centripetal force which then causes uniform circular motion.

d) in fact $m\vec{a}$ of $\vec{F}_{\rm net} = m\vec{a}$ when this equation is specialized to the case of uniform circular motion. The force itself is **ALWAYS** a field force emanating from the center of motion that pulls on the circling object atom by atom.

e) a mysterious force that **DISAPPEARS** whenever an object goes into circular motion.

SUGGESTED ANSWER: (c)

At least it is easy with all the easy ways to eliminate wrong answers and the fact that it conforms to the longest-answer-is-right rule.

I the term centripetal force is reserved for circular motion, but that motion doesn't have to be uniform (i.e., constant speed) and it need only be a part of a circle. See Fr-108, 200, 557.

Wrong answers:

d) No it's not always a field force. It can be a tension force (which is a contact force) in the case of a sling for example. It can be friction or a normal force. Gravitational orbits are, of course, important cases where the force is indeed a field force.

e) All things are wrong.

Redaction: Jeffery, 2001jan01

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006 qmult 00474 3 5 3 tough thinking: lifting from a hump

Extra keywords: physci KB-61-39

9. There is a hump on the road with a radius of curvature of 100 m just at the top. In an idealized picture, above about what horizontal speed must a car at the top of the hump lift from the hump?

a) 12 m/s. b) 25 m/s. c) 31 m/s. d) 36 m/s. e) 43 m/s.

SUGGESTED ANSWER: (c)

The car at the top of the hump is executing circular motion about the hump's center of curvature which, of course, is a point beneath the ground. But to execute circular motion there must be a centripetal force. The combination of gravity on the car and the normal force of the ground supplies the centripetal force on the car. But the ground normal force is the reaction force to the car normal force on the ground. As the car moves faster, there is less car normal force because more of the gravity force on the car is needed to keep the car moving in a circle and less is pushing the car into the road. The car normal force goes to zero when all of the gravity force is needed to maintain circular motion. But this means that the ground normal force goes to zero by the 3rd law. Now neither normal forces can be attractive. So when the car's speed exceeds the speed where the entire gravity force is needed to supply the centripetal force then there is not enough force to keep the car on the hump and it must lift.

The explanation is a bit complex, but the phenomenon of cars lifting as you drive to fast over humps is not uncommon.

The maximum car speed without lifting is given when the centripetal force is just the gravitational force alone:

$$mg = m\frac{v^2}{r}$$

which leads to

$$v = \sqrt{rg} = \sqrt{100. \times 9.8} \approx \sqrt{1000} \approx 30\,\mathrm{m/s}\ ,$$

or more exactly, 31.3 m/s If the car speed exceeds 31.3 m/s, the car tends to lift. Now 31.3 m/s is 112 km/h or 70 mi/h. The hump in this case is not very curved, and so a pretty high velocity is needed just to lift from the top.

Fortran-95 Code

```
          print*
          gg=9.8d0
          rr=100.d0
          vv=sqrt(gg*rr)
          vv2=vv*(1.d0/1000.d0)*(3600.d0)
          vv3=vv2*(1.d0/1.609344d0)
          print*,'vv,vv2,vv3'
          print*,vv,vv2,vv3
!    31.3049516849971        112.697826065989
70.0271825451795
```

Wrong answers:

c) Did you forgot to take the square root?

Redaction: Jeffery, 2001jan01

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006 qmult 00770 2 5 2 moderate thinking: circular motion in non-inertial frame

Extra keywords: physci KB-59-21 , but I've corrected it

10. In what situations, if any, can a body move in a circular path at constant speed without a centripetal force?

a) None. b) In certain special non-inertial frames.
c) In all non-inertial frames. d) In all inertial frames. e) Always.

SUGGESTED ANSWER: (b)

Is there an example of such a special inertial frame? Sure. Consider a bug sitting anywhere on an old-fashioned record turntable. From the bug's perspective the whole outside world is going around him/her. But there is no centripetal force causing the whole outside world to do this.

As another example, consider the Earth-Sun system. From the Earth's point of view, the Earth is at rest and the Sun orbits the Earth. Geometrically this is perfectly true. But the Earth's frame is not inertial and the Sun's is—or at least is much more inertial than the Earth's. In the Sun's inertial frame the Earth orbits the Sun. The gravitational force of the Earth on the Sun causes this acceleration. The Earth's gravitational force on the Sun is equally strong in magnitude by the

3rd law, but because of the Sun's much greater mass, it's the Earth that orbits in the Sun's inertial frame. Actually, the Sun is somewhat accelerated around the center of mass of the solar system, somewhat accelerated around the center of mass of the Milky Way, somewhat accelerated around the center of mass of the Local Group (of galaxies), and somewhat accelerated in some other astronomical frames too.

Wrong answers:
a) This answer is wrong unless it is understood that the question is taken as referring to inertial frames, but I am not implying such an "understood." Clearly not since answer (b) is the right general answer.
e) As Lurch would say: "Aaaarh."

Redaction: Jeffery, 2001jan01

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006 qmult 00100 1 1 1 easy memory: five special forces

11. Five special forces in physics are:

a) gravitational force, normal force, tension, friction force, and the linear force (the Hooke's law force).
b) gravitational force, normal force, tension, friction force, and left force (the Captain Hooke's law force).
c) gravitational force, normal force, tension, friction force, and right force.
d) gravitational force, normal force, tension, friction force, and air force.
e) gravitational force, normal force, tension, friction force, and **the Force**.

SUGGESTED ANSWER: (a)

Wrong answers:
b) Avast mateys.
e) May the Force be with you.

Redaction: Jeffery, 2001jan01

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007 qmult 00132 1 1 4 easy memory: conservation of energy 2
Extra keywords: physci

12. In the physics, the conservation of energy means energy:

a) shouldn't be wasted on cars. b) is never destroyed.
c) is never created. d) is never created or destroyed.
e) is perpetually created.

SUGGESTED ANSWER: (d)

Wrong answers:
e) Well no.

Redaction: Jeffery, 2001jan01

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007 qmult 00200 1 4 2 easy deducto-memory: work defined

13. "Let's play *Jeopardy*! For $100, the answer is: In physics, it is a macroscopic process of energy transfer."

What is ____________, Alex?

a) energy b) work c) force d) weight e) sloth

SUGGESTED ANSWER: (b)

Wrong answers:
e) Now is this likely?

Redaction: Jeffery, 2008jan01

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007 qmult 00282 1 3 1 easy math: work lifting ostrich
Extra keywords: physci KB-93-21

14. The work done by the lifting force of a person lifting a 30 kg ostrich to a height of 30 m without acceleration is about:

a) 9000 J. b) 900 J. c) 300 J. d) 3 J. e) 4500 J.

SUGGESTED ANSWER: (a)

For there to be no acceleration, the lifting force must cancel gravity. Therefore

$$\vec{F}_{\text{lift}} = mg\hat{y} \ .$$

The work done by the lifting force is

$$W = \int_{\Delta y} \vec{F}_{\text{lift}} \cdot d\vec{s} = mg \int_{\Delta y} dy = mg\Delta y \approx 30 \times 10 \times 30 \times 1 = 9000\,\text{J} \ ,$$

where Δy is the y displacement of the ostrich center of mass.

One actually barely needs a calculation. The work done by the lifter must create the gravitational potential energy which is $mg\Delta y$.

I think that in practice, lifting an ostrich might take more work than this. But on the other hand, it's a rather small ostrich. They usually weigh between about 60 kg and about 130 kg. Still it's likely to be kicking.

Wrong answers:
b) Maybe you forgot to multiply by g.

Redaction: Jeffery, 2001jan01

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007 qmult 00310 1 1 3 easy memory: work-kinetic-energy theorem

15. The work-kinetic-energy theorem is:

a) $KE = \frac{1}{2}mv^2$. b) $\Delta E = W_{\text{non}}$. c) $\Delta KE = W$. d) $\Delta KE = \frac{1}{2}W$.
e) $\Delta KE = \frac{1}{2}mv^2$.

SUGGESTED ANSWER: (c)

Wrong answers:
a) This is the kinetic energy formula.
b) This is the work-energy theorem.

Redaction: Jeffery, 2008jan01

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008 qmult 00120 1 4 1 easy deducto-memory: general potential energy formula

16. "Let's play *Jeopardy*! For $100, the answer is: $\Delta PE = -W$."

    a) What is the formula relating **POTENTIAL** energy change in a conservative force field to work done by the conservative force (i.e., what is the general potential energy formula), Alex?
    b) What is Faraday's law, Alex?
    c) What are capacitors, Alex?
    d) What is ... no, no wait ... what is unicorn circular motion, Alex?
    e) What is the formula relating **KINETIC** energy change in a conservative force field to work done by the conservative force (i.e., what is the work-kinetic-energy theorem), Alex?

    SUGGESTED ANSWER: (a)

    Wrong answers:

    d) A rhinoceros chasing its tail?
    e) U is pretty much common for potential energy and never used for kinetic energy to my knowledge.

    Redaction: Jeffery, 2001jan01

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008 qmult 00144 1 1 1 easy memory: nonconservative force example

17. An example of nonconservative force is:

    a) kinetic friction. b) gravity. c) the linear force. d) work.
    e) power.

    SUGGESTED ANSWER: (a)

    Wrong answers:

    d) Not a force.
    e) Not a force.

    Redaction: Jeffery, 2001jan01

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008 qmult 00220 1 1 4 easy memory: workless constraint forces

18. Frequently, in conservation-of-mechanical energy problems, one encounters non-conservative forces that guide the motion and cause accelerations. Mechanical energy is conserved because these ____________ do work because they are always ____________ to the direction of motion. Actually, conservative forces can also be ____________ when they are ____________.

    a) work-doing constraint forces; parallel
    b) work-doing constraint forces; perpendicular
    c) workless constraint forces; parallel
    d) workless constraint forces; perpendicular
    e) worthless unconstrained forces; peculiar

    SUGGESTED ANSWER: (d)

    Wrong answers:

a) Exaclty wrong.

Redaction: Jeffery, 2008jan01

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008 qmult 00272 1 3 3 easy math: dog drops brick mech. energy conserved 2
Extra keywords: physci

19. A brick has mass 2.0 kg. A dog—from a joke that I'll tell you someday—drops the brick (which it was holding in its mouth or, one might say, with its jowl) 2.0 m. The brick started from rest and air drag is negligible. What is the kinetic energy of the brick just before it hits the ground?

a) 9.8 J. b) 19.6 J. c) 39.2 J. d) about 50 J. e) about 160 J.

SUGGESTED ANSWER: (c)

From the work-energy theorem

$$\Delta E = W_{\rm non} ,$$

one obtains

$$KE = KE_0 - \Delta PE + W_{\rm non} = 0 - mg\Delta y + 0 = 39.2\,{\rm J} ,$$

where $KE_0 = 0$, $\Delta y = -2.0\,{\rm m}$, and $W_{\rm non} = 0$.

Wrong answers:
d) Well no.

Redaction: Jeffery, 2001jan01

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008 qmult 0364 1 1 4 easy memory: restoring force of table and gravity

20. A block sits at rest on table. Gravity pulls it down and a normal force pushes it up. The two forces cancel each other. The block is in static equilibrium—a stable static equilibrium. If a tiny force pushes the block up, the normal force turns off since it is a contact force. If a tiny force pushes the block down, the normal force strengthens to cancel it since it is a force of reaction—as Marx would say. Together, gravity and the normal force constitute a:

a) strong force. b) friction force. c) linear or Hooke's law force.
d) restoring force. e) weak force.

SUGGESTED ANSWER: (d)

Wrong answers:
a) Ambiguous.

Redaction: Jeffery, 2008jan01

005 qfull 00950 2 3 0 moderate math: rocket pod descent on Callisto

Extra keywords: David Bowman and 2001: A Space Odyssey

21. As this is (or was within living memory) 2001, let's say you are David Bowman and you've just arrived at Jupiter. Before going off to investigate that monolith (and go beyond humankind), you decide on a little excursion to Callisto, one of Jupiter's 4 major moons. Assume you are so close to Callisto's surface throughout the maneuvers of this question the gravitational field $g_{\rm Cal}$ can be approximated as a constant.

    a) As your landing pod descends straight down to the Callisto surface and when your are relatively close to touchdown, your rocket thrust is 3260 N and your descent velocity is **CONSTANT**. What is the gravitational force on your pod? Take the upward direction as the positive direction.

    b) Say you reduce thrust to 2200 N and find that the pod has a downward acceleration of $0.39\,{\rm m/s^2}$. What is the mass of your pod including yourself?

    c) What's the free-fall acceleration magnitude due to gravity near the Callisto surface (i.e., $g_{\rm Cal}$, the analog to g for gravity near Earth's surface)? The free-fall acceleration acceleration magnitude is also the gravitational field magnitude.

    d) Say you have a mass of 70 kg. What's your **WEIGHT** on Callisto and what is your Callisto weight divided by your Earth weight (i.e., what is the weight **RATIO**)?

    e) Now the hard part. After finishing your excursion on the icy surface, you launch and go into uniform circular motion, low-Callisto orbit. The gravitational acceleration is approximately the same as the surface gravitational acceleration and the radius of the orbit is approximately just Callisto's radius of 2400 km. Calculate the **ORBITAL SPEED**. Then find the **ORBITAL PERIOD** P (i.e., the time to orbit once) in seconds and in hours. **HINT:** Remember centripetal acceleration and $\vec{F}_{\rm net} = m\vec{a}$.

    SUGGESTED ANSWER:

    a) Well $\vec{F} = ma$ is always true and its always true component by component. If there is no acceleration, then the force of gravity must cancel the rocket thrust. Thus the gravitational force is

    $$F_g = -mg = -F_{\rm th,a} = -3260\,{\rm N}$$

    and the direction is downward to the center of Callisto, of course.

    b) Well $\vec{F} = ma$ is always ..., and so

    $$ma = F_{\rm th,b} - mg$$

    and

    $$m = \frac{F_{\rm th,b} - mg}{a} = \frac{F_{\rm th,b} - F_{\rm th,a}}{a} = \frac{-1060}{-0.39} = 2720\,{\rm kg}\ .$$

    c) Using parts (a) and (b),

    $$g_{\rm Cal} = \frac{|F_g|}{m} = \frac{F_{\rm th,a}}{m} = 1.2\,{\rm m/s^2}\ .$$

The actual equatorial value is $1.235\,\mathrm{m/s^2}$ (Wikipedia: Callisto (moon)).

d) Using part (c),

$$W_{\mathrm{Bow,Cal}} = m_{\mathrm{Bow}} g_{\mathrm{Cal}} = 84\,\mathrm{N}\ .$$

The weight ratio is given by

$$\frac{W_{\mathrm{Bow,Cal}}}{W_{\mathrm{Bow,Earth}}} = \frac{m_{\mathrm{Bow}} g_{\mathrm{Cal}}}{m_{\mathrm{Bow}} g} = \frac{g_{\mathrm{Cal}}}{g} \approx \frac{1.2}{10} = 0.12\ .$$

e) Well the magnitude of centripetal acceleration for uniform circular motion is

$$a_{\mathrm{cen}} = \frac{v^2}{r}\ .$$

The only force causing this acceleration is gravity. Thus

$$F_g = m g_{\mathrm{Cal}} = m a_{\mathrm{cen}} = m\frac{v^2}{r}\ .$$

The orbital speed is then

$$v = \sqrt{g_{\mathrm{Cal}} r} = 1700\ \mathrm{m/s}$$

and the orbital period P is

$$P = \frac{2\pi r}{v} = 2\pi\sqrt{\frac{r}{g_{\mathrm{Cal}}}} = 8900\,\mathrm{s} = 2.5\,\mathrm{h}\ .$$

Fortran Code

```
         print*
         fa=3260.
         fb=2200.
         aa=-0.39
         xmpod=(fb-fa)/aa
         gg=fa/xmpod
         xmbowman=70.*gg
         ratio=gg/9.8
         print*,'xmpod,gg,xmbowman,ratio'
         print*,xmpod,gg,xmbowman,ratio
*              2717.94873  1.19943392  83.9603729  0.122391216
         rcal=2400.e+3
         vv=sqrt(gg*rcal)
         pp=2.*pi*sqrt(rcal/gg)
         print*,'vv,pp,pp/3600.'
         print*,vv,pp,pp/3600.
*              1696.65601  8887.8623  2.46885061
```

Redaction: Jeffery, 2001jan01

006 qfull 00700 1 3 0 easy math: general drag force in falling from rest case

22. An object falling from rest is subject to a drag force of magnitude $f(v)$, where v is the object's speed. The function $f(v)$ is monotonically increasing with v and $f(0) = 0$, but is otherwise general. The inverse function to $f(v)$ is $f^{-1}(x)$.

a) Apply Newton's 2nd law to the object taking the downward direction as positive. **HINT:** You just write down the Newton's 2nd law for this particular case.

b) What is terminal velocity, what is the condition necessary for it to hold for a falling object, and why should the falling object reach it? What is the formula for the terminal velocity v_{ter}? Make use of $f^{-1}(x)$.

c) The evolution to terminal velocity can be crudely divided into two phases: a linear growth phase (when the velocity is growing approximately linearly) and an asymptotic phase (when the velocity is asymptotically approaching terminal velocity). The characteristic time t_{ch} boundary between the two phases is obtained by setting $f(v) = 0$ in the equation of motion and solving for the time when $v = v_{\text{ter}}$. Beyond this time, the pure linear growth must be over. **DERIVE** the formula for t_{ch}. Then **DERIVE** the formula for the characteristic length ℓ_{ch} which is the distance fallen in time t_{ch} assuming $f(v) = 0$.

d) The formula

$$v = v_{\text{ter}}(1 - e^{-t/t_{\text{ch}}})$$

is a crude approximate solution for velocity in general. **SKETCH** a v-versus-t plot for the approximate solution.

If we have linear drag $f(v) = bv$ (where b) is a constant, then the approximate solution becomes exact. There are cases where linear drag holds: for very low speeds and no turbulence. Find t_{ch} and ℓ_{ch} for the linear drag force case. Verify by substitution into the equation of motion (i.e., our Newton's 2nd law application from) that the velocity formula is exact for $f(v) = bv$. A good way to do this is to evaluate with the solution the left-hand side (LHS) and right-hand side (RHS) of the equation of motion separately. Then observe that LHS and RHS are equal.

SUGGESTED ANSWER:

a) Behold:

$$mg - f(v) = ma \ .$$

b) Terminal velocity is a constant velocity that an object falling in a fluid medium reaches if it falls sufficiently long. The condition needed for terminal velocity is that the gravitational and the drag forces on the object cancel. The two forces are equal in magnitude, but opposite in direction. When the condition holds, the net force on the object is zero, its acceleration is then zero, and it has a constant velocity which is the terminal velocity itself. An object should reach terminal velocity. Initially $f(v)$ is zero and the object accelerates. As v increases, $mg - f(v)$ decreases, but stays positive and so the acceleration is positive and velocity continues to increase. But when $mg-f(v)$ reaches zero, the acceleration goes to zero, and the object's velocity becomes constant and stays constant. This constant velocity is the terminal velocity.

It takes a mathematical analysis to show this, but for usual drag forces terminal velocity is formally only reached at time equals infinity. The velocity asymptotically approaches terminal velocity. In reality, the fluid medium is subject to velocity fluctuations and once the difference between object velocity and terminal velocity becomes smaller than these, the object has effectively reached terminal velocity. This happens usually at much less than time equals infinity. It usually happens after a few times the characteristic time that we find in the part (c) answer.

The terminal velocity formula is

$$v_{\rm ter} = f^{-1}(mg) \ .$$

c) If

$$mg = ma \ ,$$

then

$$v = gt \ ,$$

and so

$$t_{\rm ch} = \frac{v_{\rm ter}}{g} \ .$$

The characteristic length is

$$\ell_{\rm ch} = \frac{1}{2} g t_{\rm ter}^2 = \frac{v_{\rm ter}^2}{2g} \ .$$

d) You will have to imagine the sketch.

for the linear drag force case, we find

$$v_{\rm ter} = \frac{mg}{b} \qquad \text{and} \qquad t_{\rm ch} = \frac{m}{b} \ .$$

Now for the verification that the approximate solution is exact for the linear drag force case. We have

$$\text{LHS} = mg - bv = mg - bv_{\rm ter}(1 - e^{-t/t_{\rm ch}}) = mge^{-t/t_{\rm ch}}$$

and

$$\text{RHS} = ma = m\frac{v_{\rm ter}}{t_{\rm ch}} e^{-t/t_{\rm ch}} = mge^{-t/t_{\rm ch}} \ .$$

Since $LHS = RHS$, the solution satisfies the equation of motion, and, so it is, in fact, the exact solution—the verification is complete.

Redaction: Jeffery, 2008jan01

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008 qfull 00420 2 3 0 moderate math: Lance Armstrong work and power

Extra keywords: save for exams

23. Lance Armstrong has won the Tour de France bicycle race a record 7 times.

a) What is the main forward **EXTERNAL** force on the Armstrong-bike system. What are at least two of the main reverse **EXTERNAL** forces on the Armstrong-bike system. The forward forces acting alone accelerate and the reverse forces acting alone decelerate. Assume that Lance is riding on a completely horizontal road. **HINT:** We are not concerned with internal forces: e.g., forces that Lance Armstrong exerts on the bike and that the bike exerts on Lance.

b) Say Lance biked 135 km (still on the horizontal road) and started and ended at rest. What is the **NET** work done on the Armstrong-bike system by all external forces? A numerical value is expected for the answer.

c) Say Lance travels at an average speed of 12.0 m/s for the distance of 135 km. How long was the trip in seconds? How long was the trip in hours?

d) Say Lance outputs about 2.0 kW of power on average (which may be a little too high in reality) during his trip of 135 km. This power is total power: some of the energy goes into kinetic energy of his body and bike before becoming waste heat and some goes into heat directly. How much energy has has he output in total in joules?

e) Assuming Lance outputs about 0.15 kW of power for the **REST** of the of 24 hours, what is his daily average output power in **WATTS** (i.e., his sustained metabolic rate)? By "average", it is meant the kind of average that allows one to calculate the energy Lance outputs in any period of time by multiplying by that time.

SUGGESTED ANSWER:

a) Static friction of the road on the Armstrong-bike system is the main force forward external force. It acts directly on the bike tires, of course. If you imagine a case of zero road friction, Lance wouldn't be able to move himself much: the air might blow him along or he could try paddling air—once moving he might go a long way, of course.

   The question of reverse external forces is a bit involved. Air drag is almost certainly the dominant one in terms of removing kinetic energy from the Armstrong-bike system. However, if the wind is blowing at Lance's back and he is moving slower than the wind, air drag will give him a forward external force. But usually he will be moving faster than wind speed. Air drag can be reduced by traveling in the wake of other bikers and I believe this a common strategy.

   When braking the static friction force of the road on the bike is a main revserse external force. But Lance probably tries not to brake at all, except after crossing the finish line. Still he might have to brake at corners if there is no banking and for strategic reasons or safety reasons. Certainly, I would count the static friction force as a main reverse external force for control purposes though probably not for work done.

   Rolling friction is probably the reverse external force that does the most work against the motion after air drag. But this is just a guess.

   I will accept any two of air drag, static friciton, and rolling friction as answers. Three answers I will not accept are kinetic friction, gravity, and the normal force.

Kinetic friciton as a reverse external force is probably minor. Probably Lance only skids obviously in emergencies or when showing off, but all fast accelerations and braking may actually involve little skidding. I'm not enough of a bike expert to know.

If Lance were ascending or descending, gravity would also act on the Armstrong-bike system acting. Ascending it is a reverse external force; descending, a forward external force.

The normal force is always perpendicular to the direction of motion. So it is never a forword or reverse external force and it does no work at all on the Armstrong-bike system.

b) Since the system starts and ends at rest, there is zero change in kinetic energy, and thus the external forces have done zero net work on the system. Recall the work-kinetic energy theorem:

$$\Delta KE = W \ ,$$

where ΔKE is the change in kinetic energy and W is the net work done by all external force. If $\Delta KE = 0$, then $W = 0$.

Of course, a lot of internal work was done by Lance on the bike and the Armstrong-bike system did a work on the external world—but the sum would be zero since that work is equal in magnitude, but opposite in sign the work the external world did on him. In fact, all the kinetic energy generated from Lance's internal store of chemical energy went into waste heat. But he has moved far and fast. If the bike ascended during the trip, some gravitational potential energy will have been added to the Armstrong-bike system. If the bike descended during the trip, some gravitational potential energy was removed and ended up as waste heat.

c) Well this is an old exhaustion time question. Clearly,

$$t = \frac{d}{v} = \frac{135 \times 10^3}{12} = 11.25 \times 10^3\,\text{s} = 3.13\,\text{h} \ .$$

d) Behold:

$$E = pt = 2000 \times 11.25 \times 10^3 = 22.5 \times 10^6\,\text{J} = 22.5\,\text{MJ} \ .$$

e) Behold:

$$p_{\text{ave}} = \frac{22.5 \times 10^6 + 150 \times (86400 - t)}{86400} = 390\,\text{W} \ .$$

NOTE: The numbers used in this question are actually only approximate. It's hard to get reliable numbers. However, the sustained metabolic scope of about 5 that one gets from the numbers is at about the actual human upper limit. Sustained metabolic scope is the ratio of sustained metabolic rate (which is average daily power which is about 390 W for "Lance") to basal or resting metabolic rate which is about 80 W (which is typical for Tour-de-France cyclists and anyone else). Only elite endurance athletes like Tour-de-France cyclists and maybe exceptional traditional hunters have sustained metabolic scopes of about 5. Most folks in fairly sedentary lifestyles have sustained metabolic scopes of 1.4 to 1.8.

Fortran-95 Code

```
          print*
          daysec2=86400.d0
          bmr=6870.d0*(1.d3/daysec2)  ! typical value
          smr=33000.d0*(1.d3/daysec2) ! typical value
          print*,'bmr,smr in watts'
          print*,bmr,smr
    !  79.5138888888889 381.9444444444444
          phigh=2.d3
          plow=150.d0
          d=135.d3
          v=12.d0
          t=d/v
          th=t/3600.d0
          en=t*phigh
          trest=daysec2-t
          pave=(en+trest*plow)/daysec2
          print*,'t,th,en,pave'
          print*,t,th,en,pave
    !  11250.0 3.125 22500000.0 390.8854166666667
```

Redaction: Jeffery, 2008jan01

SCRATCH PAGE

SCRATCH PAGE

Equation Sheet for Introductory Physics Calculus-Based

This equation sheet is intended for students writing tests or reviewing material. Therefore it is neither intended to be complete nor completely explicit. There are fewer symbols than variables, and so some symbols must be used for different things: context must distinguish.

The equations are mnemonic. Students are expected to understand how to interpret and use them.

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1 Constants

$$c = 2.99792458 \times 10^8\,\mathrm{m/s} \approx 2.998 \times 10^8\,\mathrm{m/s} \approx 3 \times 10^8\,\mathrm{m/s} \approx 1\,\mathrm{lyr/yr} \approx 1\,\mathrm{ft/ns} \qquad \text{exact by definition}$$

$$e = 1.602176487(40) \times 10^{-19}\,\mathrm{C}$$

$$G = 6.67428(67) \times 10^{-11}\,\mathrm{N\,m^2/kg^2} \qquad \text{(2006, CODATA)}$$

$$g = 9.8\,\mathrm{m/s^2} \qquad \text{fiducial value}$$

$$k = \frac{1}{4\pi\varepsilon_0} = 8.987551787\ldots \times 10^9 \approx 8.99 \times 10^9 \approx 10^{10}\,\mathrm{N\,m^2/C^2}\text{exact by definition}$$

$$k_{\mathrm{Boltzmann}} = 1.3806504(24) \times 10^{-23}\,\mathrm{J/K} = 0.8617343(15) \times 10^{-4}\,\mathrm{eV/K} \approx 10^{-4}\,\mathrm{eV/K}$$

$$m_e = 9.10938215(45) \times 10^{-31}\,\mathrm{kg} = 0.510998910(13)\,\mathrm{MeV}$$

$$m_p = 1.672621637(83) \times 10^{-27}\,\mathrm{kg} = 938.272013(23),\mathrm{MeV}$$

$$\varepsilon_0 = \frac{1}{\mu_0 c^2} = 8.8541878176\ldots \times 10^{-12}\,\mathrm{C^2/(N\,m^2)} \approx 10^{-11} \qquad \text{vacuum permittivity (exact by definition)}$$

$$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N/A^2} \qquad \text{exact by definition}$$

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2 Geometrical Formulae

$$C_{\mathrm{cir}} = 2\pi r \qquad A_{\mathrm{cir}} = \pi r^2 \qquad A_{\mathrm{sph}} = 4\pi r^2 \qquad V_{\mathrm{sph}} = \frac{4}{3}\pi r^3$$

$$\Omega_{\mathrm{sphere}} = 4\pi \qquad d\Omega = \sin\theta\,d\theta\,d\phi$$

---

3 Trigonometry Formulae

$$\frac{x}{r} = \cos\theta \qquad \frac{y}{r} = \sin\theta \qquad \frac{y}{x} = \tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cos^2\theta + \sin^2\theta = 1$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$c^2 = a^2 + b^2 \qquad c = \sqrt{a^2 + b^2 - 2ab\cos\theta_c} \qquad \frac{\sin\theta_a}{a} = \frac{\sin\theta_b}{b} = \frac{\sin\theta_c}{c}$$

$$f(\theta) = f(\theta + 360^\circ)$$

$$\sin(\theta + 180^\circ) = -\sin(\theta) \qquad \cos(\theta + 180^\circ) = -\cos(\theta) \qquad \tan(\theta + 180^\circ) = \tan(\theta)$$

$$\sin(-\theta) = -\sin(\theta) \qquad \cos(-\theta) = \cos(\theta) \qquad \tan(-\theta) = -\tan(\theta)$$

$$\sin(\theta + 90^\circ) = \cos(\theta) \qquad \cos(\theta + 90^\circ) = -\sin(\theta) \qquad \tan(\theta + 90^\circ) = -\tan(\theta)$$

$$\sin(180^\circ - \theta) = \sin(\theta) \qquad \cos(180^\circ - \theta) = -\cos(\theta) \qquad \tan(180^\circ - \theta) = -\tan(\theta)$$

$$\sin(90^\circ - \theta) = \cos(\theta) \qquad \cos(90^\circ - \theta) = \sin(\theta) \qquad \tan(90^\circ - \theta) = \frac{1}{\tan(\theta)} = \cot(\theta)$$

$$\sin(a + b) = \sin(a)\cos(b) + \cos(a)\sin(b) \qquad \cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$\sin(2a) = 2\sin(a)\cos(a) \qquad \cos(2a) = \cos^2(a) - \sin^2(a)$$

$$\sin(a)\sin(b) = \frac{1}{2}\left[\cos(a - b) - \cos(a + b)\right] \qquad \cos(a)\cos(b) = \frac{1}{2}\left[\cos(a - b) + \cos(a + b)\right]$$

$$\sin(a)\cos(b) = \frac{1}{2}\left[\sin(a - b) + \sin(a + b)\right]$$

$$\sin^2\theta = \frac{1}{2}[1-\cos(2\theta)] \qquad \cos^2\theta = \frac{1}{2}[1+\cos(2\theta)] \qquad \sin(a)\cos(a) = \frac{1}{2}\sin(2a)$$

$$\cos(x) - \cos(y) = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos(x) + \cos(y) = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin(x) + \sin(y) = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

4 Approximation Formulae

$$\frac{\Delta f}{\Delta x} \approx \frac{df}{dx} \qquad \frac{1}{1-x} \approx 1 + x \ : \ (x << 1)$$

$$\sin\theta \approx \theta \qquad \tan\theta \approx \theta \qquad \cos\theta \approx 1 - \frac{1}{2}\theta^2 \qquad \text{all for } \theta << 1$$

5 Quadratic Formula

$$\text{If} \quad 0 = ax^2 + bx + c\ , \quad \text{then} \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = -\frac{b}{2a} \pm \sqrt{\left(\frac{b}{2a}\right)^2 - \frac{c}{a}}$$

6 Vector Formulae

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2} \qquad \theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) + \pi? \qquad \vec{a} + \vec{b} = (a_x + b_x, a_y + b_y)$$

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} \qquad \phi = \tan^{-1}\left(\frac{a_y}{a_x}\right) + \pi? \qquad \theta = \cos^{-1}\left(\frac{a_z}{a}\right)$$

$$\vec{a} + \vec{b} = (a_x + b_x, a_y + b_y, a_z + b_z)$$

$$\vec{a} \cdot \vec{b} = ab\cos\theta = a_x b_x + a_y b_y + a_z b_z$$

$$\vec{c} = \vec{a} \times \vec{b} = ab\sin(\theta)\hat{c} = (a_y b_z - b_y a_z, a_z b_x - b_z a_x, a_x b_y - b_x a_y)$$

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7 Differentiation and Integration Formulae

$$\frac{d(x^p)}{dx} = px^{p-1} \quad \text{except for } p = 0; \qquad \frac{d(x^0)}{dx} = 0 \qquad \frac{d(\ln|x|)}{dx} = \frac{1}{x}$$

Taylor's series
$$f(x) = \sum_{n=0}^{\infty} \frac{(x-x_0)^n}{n!} f^{(n)}(x_0)$$
$$= f(x_0) + (x-x_0)f^{(1)}(x_0) + \frac{(x-x_0)^2}{2!} f^{(2)}(x_0) + \frac{(x-x_0)^3}{3!} f^{(3)}(x_0) + \ldots$$

$$\int_a^b f(x)\,dx = F(x)|_a^b = F(b) - F(a) \quad \text{where} \quad \frac{dF(x)}{dx} = f(x)$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad \text{except for } n = -1; \qquad \int \frac{1}{x}\,dx = \ln|x|$$

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8 One-Dimensional Kinematics

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} \qquad v = \frac{dx}{dt} \qquad a_{\text{avg}} = \frac{\Delta v}{\Delta t} \qquad a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

$$v = at + v_0 \qquad x = \frac{1}{2}at^2 + v_0 t + x_0 \qquad v^2 = v_0^2 + 2a(x - x_0)$$

$$x = \frac{1}{2}(v_0 + v)t + x_0 \qquad x = -\frac{1}{2}at^2 + vt + x_0 \qquad g = 9.8\,\text{m/s}^2$$

$$x_{\text{rel}} = x_2 - x_1 \qquad v_{\text{rel}} = v_2 - v_1 \qquad a_{\text{rel}} = a_2 - a_1$$

$$x' = x - v_{\text{frame}}t \qquad v' = v - v_{\text{frame}} \qquad a' = a$$

9 Two- and Three-Dimensional Kinematics: General

$$\vec{v}_{\text{avg}} = \frac{\Delta\vec{r}}{\Delta t} \qquad \vec{v} = \frac{d\vec{r}}{dt} \qquad \vec{a}_{\text{avg}} = \frac{\Delta\vec{v}}{\Delta t} \qquad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

10 Projectile Motion

$$x = v_{x,0}t \qquad y = -\frac{1}{2}gt^2 + v_{y,0}t + y_0 \qquad v_{x,0} = v_0\cos\theta \qquad v_{y,0} = v_0\sin\theta$$

$$t = \frac{x}{v_{x,0}} = \frac{x}{v_0\cos\theta} \qquad y = y_0 + x\tan\theta - \frac{x^2 g}{2v_0^2\cos^2\theta}$$

$$x_{\text{for } y \text{ max}} = \frac{v_0^2\sin\theta\cos\theta}{g} \qquad y_{\text{max}} = y_0 + \frac{v_0^2\sin^2\theta}{2g}$$

$$x(y = y_0) = \frac{2v_0^2\sin\theta\cos\theta}{g} = \frac{v_0^2\sin(2\theta)}{g} \qquad \theta_{\text{for max}} = \frac{\pi}{4} \qquad x_{\text{max}}(y = y_0) = \frac{v_0^2}{g}$$

$$x(\theta = 0) = \pm v_0\sqrt{\frac{2(y_0 - y)}{g}} \qquad t(\theta = 0) = \sqrt{\frac{2(y_0 - y)}{g}}$$

11 Relative Motion

$$\vec{r} = \vec{r}_2 - \vec{r}_1 \qquad \vec{v} = \vec{v}_2 - \vec{v}_1 \qquad \vec{a} = \vec{a}_2 - \vec{a}_1$$

12 Polar Coordinate Motion and Uniform Circular Motion

$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\vec{r} = r\hat{r} \qquad \vec{v} = \frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{r} + r\omega\hat{\theta} \qquad \vec{a} = \frac{d^2\vec{r}}{dt^2} = \left(\frac{d^2r}{dt^2} - r\omega^2\right)\hat{r} + \left(r\alpha + 2\frac{dr}{dt}\omega\right)\hat{\theta}$$

$$\vec{v} = r\omega\hat{\theta} \qquad v = r\omega \qquad a_{\tan} = r\alpha$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} = -r\omega^2\hat{r} \qquad a_{\text{centripetal}} = \frac{v^2}{r} = r\omega^2 = v\omega$$

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13 Very Basic Newtonian Physics

$$\vec{r}_{\text{cm}} = \frac{\sum_i m_i\vec{r}_i}{m_{\text{total}}} = \frac{\sum_{\text{sub}} m_{\text{sub}}\vec{r}_{\text{cm sub}}}{m_{\text{total}}} \qquad \vec{v}_{\text{cm}} = \frac{\sum_i m_i\vec{v}_i}{m_{\text{total}}} \qquad \vec{a}_{\text{cm}} = \frac{\sum_i m_i\vec{a}_i}{m_{\text{total}}}$$

$$\vec{r}_{\text{cm}} = \frac{\int_V \rho(\vec{r})\vec{r}\,dV}{m_{\text{total}}}$$

$$\vec{F}_{\text{net}} = m\vec{a} \qquad \vec{F}_{21} = -\vec{F}_{12} \qquad F_g = mg \qquad g = 9.8\,\text{m/s}^2$$

$$\vec{F}_{\text{normal}} = -\vec{F}_{\text{applied}} \qquad F_{\text{linear}} = -kx$$

$$f_{\text{normal}} = \frac{T}{r} \qquad T = T_0 - F_{\text{parallel}}(s) \qquad T = T_0$$

$$F_{\text{f static}} = \min(F_{\text{applied}}, F_{\text{f static max}}) \qquad F_{\text{f static max}} = \mu_{\text{static}}F_{\text{N}} \qquad F_{\text{f kinetic}} = \mu_{\text{kinetic}}F_{\text{N}}$$

$$v_{\text{tangential}} = r\omega = r\frac{d\theta}{dt} \qquad a_{\text{tangential}} = r\alpha = r\frac{d\omega}{dt} = r\frac{d^2\theta}{dt^2}$$

$$\vec{a}_{\text{centripetal}} = -\frac{v^2}{r}\hat{r} \qquad \vec{F}_{\text{centripetal}} = -m\frac{v^2}{r}\hat{r}$$

$$F_{\text{drag,lin}} = bv \qquad v_{\text{T}} = \frac{mg}{b} \qquad \tau = \frac{v_{\text{T}}}{g} = \frac{m}{b} \qquad v = v_{\text{T}}(1 - e^{-t/\tau})$$

$$F_{\text{drag,quad}} = bv^2 = \frac{1}{2}C\rho A v^2 \qquad v_{\text{T}} = \sqrt{\frac{mg}{b}}$$

14 Energy and Work

$$dW = \vec{F} \cdot d\vec{s} \qquad W = \int \vec{F} \cdot d\vec{s} \qquad KE = \frac{1}{2}mv^2 \qquad E_{\text{mechanical}} = KE + PE$$

$$P_{\text{avg}} = \frac{\Delta W}{\Delta t} \qquad P = \frac{dW}{dt} \qquad P = \vec{F} \cdot \vec{v}$$

$$\Delta KE = W_{\text{net}} \quad \Delta PE_{\text{of a conservative force}} = -W_{\text{by a conservative force}} \quad \Delta E = W_{\text{nonconservative}}$$

$$F = -\frac{dPE}{dx} \qquad \vec{F} = -\nabla PE \qquad PE = \frac{1}{2}kx^2 \qquad PE = mgy$$

15 Momentum

$$\vec{F}_{\text{net}} = m\vec{a}_{\text{cm}} \qquad \Delta KE_{\text{cm}} = W_{\text{net,external}} \qquad \Delta E_{\text{cm}} = W_{\text{not}}$$

$$\vec{p} = m\vec{v} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}_{\text{total}}}{dt}$$

$$m\vec{a}_{\text{cm}} = \vec{F}_{\text{net non-flux}} + (\vec{v}_{\text{flux}} - \vec{v}_{\text{cm}})\frac{dm}{dt} = \vec{F}_{\text{net non-flux}} + \vec{v}_{\text{rel}}\frac{dm}{dt}$$

$$v = v_0 + v_{\text{ex}} \ln\left(\frac{m_0}{m}\right) \qquad \text{rocket in free space}$$

16 Collisions

$$\vec{I} = \int_{\Delta t} \vec{F}(t)\, dt \qquad \vec{F}_{\text{avg}} = \frac{\vec{I}}{\Delta t} \qquad \Delta p = \vec{I}_{\text{net}}$$

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f} \qquad \vec{v}_{\rm cm} = \frac{\vec{p}_1 + \vec{p}_2}{m_{\rm total}}$$

$$KE_{{\rm total}\ f} = KE_{{\rm total}\ i} \qquad \text{1-d Elastic Collision Expression}$$

$$v_{1'} = \frac{(m_1 - m_2)v_1 + 2m_2v_2}{m_1 + m_2} \qquad \text{1-d Elastic Collision Expression}$$

$$v_{2'} - v_{1'} = -(v_2 - v_1) \qquad v_{\rm rel'} = -v_{\rm rel} \qquad \text{1-d Elastic Collision Expressions}$$

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17 **Rotational Kinematics**

$$2\pi = 6.2831853\ldots \qquad \frac{1}{2\pi} = 0.15915494\ldots$$

$$\frac{180^\circ}{\pi} = 57.295779\ldots \approx 60^\circ \qquad \frac{\pi}{180^\circ} = 0.017453292\ldots \approx \frac{1}{60^\circ}$$

$$\theta = \frac{s}{r} \qquad \omega = \frac{d\theta}{dt} = \frac{v}{r} \qquad \alpha = \frac{d^2\theta}{dt^2} = \frac{d\omega}{dt} = \frac{a}{r} \qquad f = \frac{\omega}{2\pi} \qquad P = \frac{1}{f} = \frac{2\pi}{\omega}$$

$$\omega = \alpha t + \omega_0 \qquad \Delta\theta = \frac{1}{2}\alpha t^2 + \omega_0 t \qquad \omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

$$\Delta\theta = \frac{1}{2}(\omega_0 + \omega)t \qquad \Delta\theta = -\frac{1}{2}\alpha t^2 + \omega t$$